The investigator focused on and produced papers with the following titles: "On a damped nonlinear evolution equation," "Nonexistence of smooth electromagnetic fields in nonlinear dielectrics, I infinite cylindrical dielectrics," and "Nonexistence of smooth electromagnetic fields in nonlinear dielectrics, II shock development in a half space." Details on this research are contained in the report.
May 14, 1984

FROM: Professor Frederick Bloom

TO: AFOSR
Bolling, AFB
DC 20332

RE: Interim Scientific Report
AFOSR-81-0171
15 May 1981 - 14 May 1982

During the period 15 May 1981 - 14 May 1982 the principal investigator was on leave from the University of South Carolina. The Fall semester 1981 was spent at the University of Connecticut while the Spring and Summer of 1982 were spent at the University of Maryland. The focus of our work in the Fall of 1981 was the damped nonlinear evolution equation \( w_{tt} = \sigma(w)_{xx} - \gamma w_t \), an equation which arises in several areas of applied mathematics and, in particular, in studies of shearing flows in a nonlinear viscoelastic fluid. A summary of results obtained on existence and nonexistence of solutions for initial-boundary value problems associated with this equation may be found in the attached abstract #1. In the spring and summer of 1982 our work centered on a study of shock formulation for intense plane electromagnetic waves propagating into infinite cylindrical dielectrics and nonlinear dielectric half-spaces; summaries of this work may be found in the attached abstracts #2 and #3. While at the University of Maryland we also initiated joint work with Professor Stuart Antman on the equilibrium states of nonlinearly-elastic current bearing wires which are placed in an ambient magnetic field and are subjected to the type of self-interactive force that results from an application of the Biot-Savart law. Particular interest is centered on the branching that may occur, from one equilibrium configuration to another, when the parameter \( \lambda = IB \) is varied through a countable set of discrete values, where \( I \) is the constant current in the wire and \( B \) is the strength of the ambient magnetic field. This work is ongoing and the existence of straight, plane circular, and wiggly states in the plane which oscillate about a torus, has already been established.

[Signature]

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On the Damped Nonlinear Evolution Equation

\[ w_{tt} = \sigma(w)_{xx} - \gamma w_t \]

by

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ABSTRACT

Initial boundary value problems for the damped nonlinear wave equation \( w_{tt} = \sigma(w)_{xx} - \gamma w_t \) arise in several areas of applied mathematics and, in particular, in studies of shearing flow in a nonlinear viscoelastic fluid; the problems of global existence and nonexistence of smooth solutions have been extensively studied in the strictly hyperbolic case \( \sigma'(0) > \xi > 0, \forall \xi \geq 0 \) as well as in the case where \( \sigma'(0) > 0 \) and the initial data are chosen so small that \( \sigma'(w) > 0 \) for as long as a smooth solution \( w(x,t) \) exists. In this paper we study the global nonexistence problem for the cases \( \sigma'(0) = 0 \) and \( \sigma'(0) > 0 \) but \( \sigma'(\xi) < 0 \) for \( |\xi| \) sufficiently large and derive growth estimates which are valid on the maximal interval of existence of a sufficiently smooth solution.
Nonexistence of Smooth Electromagnetic Fields in Nonlinear Dielectrics

I. Infinite Cylindrical Dielectrics

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Abstract

Coupled nonlinear wave equations are derived for the evolution of the components of the electric induction field $D$ in a class of rigid nonlinear dielectrics governed by the nonlinear constitutive relation $\mathbf{E} = \lambda(D)\mathbf{D}$, where $\mathbf{E}$ is the electric field and $\lambda > 0$ is a scalar-valued vector function. For the special case of an infinite one-dimensional dielectric rod, it is shown that, under relatively mild conditions on $\lambda$, solutions of the corresponding initial value problem for the electric induction field cannot exist globally in time in the $L^2$ sense if it is assumed that the electric field in the rod is perpendicular to the axis of the rod and varies with the coordinate along that axis. The results hold when the electromagnetic field in the rod has compact support. Growth estimates for solutions, which are valid on the maximal time-interval of existence are also derived; these are valid in the simple but physically important case where $\lambda(D) = \lambda_1 + \lambda_0 \| \mathbf{D} \|^2$. We also discuss relations with recent work on the phenomena of self-focusing and self-trapping for high intensity laser beams in a dielectric medium.
Nonexistence of Smooth Electromagnetic Fields in Nonlinear Dielectrics
II. Shock Development in a Half-Space

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Abstract

Implications of the Rankine-Hugoniot and Lax k-shock conditions are explored for a system of hyperbolic conservation laws associated with the propagation of an intense plane wave into an isotropic nonlinear dielectric half-space. Asymptotic estimates are obtained for both $t_{\text{max}}$ and $s_{\text{max}}$, respectively, the time elapsed and distance travelled by the wave into the half-space before shock development occurs.