SIMPLY INSTRUMENTABLE AND OPTIMAL DIGITIZATION OF
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INST TROY NY DEPT OF ELECTRICAL COMPUT. W A PEARL
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Annual Report for the Period
30 June 1982 to 29 June 1983
on Grant No. AFOSR-81-0188
entitled
Simply Instrumentable and Optimal
Digitization of Analog Information Sources
by
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ABSTRACT

The following is a progress report for the period 30 June 1982 to 29 June 1983 on Grant Number AFOSR-81-0188, "Simply Instrumentable and Optimal Digitization of Analog Information Sources", William A. Pearlman, principal investigator. New techniques for encoding sources with and without memory are described in Section 1 and the papers, presentations, and theses supported by the research are listed in Section 2. Three new optimal coding methods, generalizing previous ones, have been discovered for stationary Gaussian sources and the squared-error distortion measure. These methods, which store codewords on trees and trellises and utilize systematic search algorithms, are being applied to actual speech and image data. Also, the code storage requirements are not large, as this research has shown that small code letter alphabets give nearly optimal results.
0. **INTRODUCTION**

The following is a report on the activity and progress for the period 30 June 1982 to 29 June 1983 on the grant entitled, "Simply Instrumentable and Optimal Digitization of Analog Information Sources", Grant Number AFOSR-81-0188, by William A. Pearlman, principal investigator. For the sake of completeness and establishing the framework, we describe some work which was begun in the previous year and carried over to the current period. In Section 1, we present past and current work on techniques for encoding sources with and without memory; and in Section 2 we list the papers, presentations and theses supported by the research.
1. SUMMARY OF PROGRESS

A. Memoryless Sources

A.1 $R_{LM}(D)$ Calculations

We undertook the calculation of the $R_{LM}(D)$, the constrained-size reproduction alphabet rate-distortion function, for the Laplacian source with squared-error distortion measure to complement the previous Gaussian results. The calculation of this function gives necessary parameters for accomplishing coding simulations and furnishes absolute bounds on coding performance with a size $M$ reproduction alphabet. As with the Gaussian case, we confirm the theory in that performance close to $R(D)$, the rate-distortion bound, can be obtained with reasonably small values of $L$, the size of amplitude range partition, and $M$. The value of $M$ for Laplacian needs to be larger than that for Gaussian in order to obtain the same closeness to $R(D)$ at the same rate. These calculations were used in two other tasks in memoryless source coding (to be described) and will be important for tasks yet to be undertaken in coding sources with memory.

A.2 Sliding-Block and Random Source Coding

With the source coding parameters furnished by the $R_{LM}(D)$ calculations, we were able to simulate at the rate of one bit per source symbol new sliding-block codes and random trellis codes for Gaussian and Laplacian sources with the squared-error distortion measure. These codes used only a four letter reproduction alphabet and achieved performance either comparable or superior to previous simulations with continuous reproduction alphabets. The method and results are published in the IEEE Transactions on Communications (Paper A.1 in Section II).
A.3 Value and Probability Constraints

The $R_{LM}(D)$ theory, where only the size of the reproduction alphabet is fixed, leads naturally toward imposing further constraints on the reproduction alphabet. These constraints are specifically fixing the values and probabilities of the letters in this alphabet. The rate-distortion functions defined with these reproduction alphabet constraints are absolute bounds in performance when encoding under these same constraints on choice of the reproduction alphabet. Furthermore, fixing either the values or probabilities or both obviates a calculation of a rate-distortion function to find the missing coding parameters. Reasonable choices of reproduction values and probabilities which come to mind are those of the optimum $M$-level quantizer. These parameters are easily found through existing published tables or calculated by straightforward numerical algorithms. The ultimate question is whether these additional constraints on the reproduction alphabet impose intolerable losses in coding performance. Through calculations of the appropriately constrained alphabet rate-distortion functions for Gaussian and Laplacian sources with squared-error distortion, we have found that the loss in performance is quite small under these additional constraints on the reproduction alphabet. Various aspects of the theory and results have been presented at the 1982 IEEE International Conference on Communications, whose Proceedings contains the full paper (see B.2 in Section 2), and at the 1982 IEEE International Symposium on Information Theory (see C.3 in Section 2). A more detailed exposition has been submitted for publication review to the IEEE Transactions on Information Theory (see A.2, Section 2).

A.4 Source Mismatch, Adaptive Coding, and Channel Errors

The source coding techniques in this research are designed to be instrumentable. In this spirit we have investigated aspects of constrained
alphabet codes for memoryless sources which often arise in practice or implementa-
tion. The first is the effect of source mismatch, which arises when one assumes a source probability distribution different from the actual one. Mismatch in distributional shape, variance, or a combination of both are possible. First, both constrained and unconstrained codes seem to be relatively insensitive to shape mismatch, as shown through calculations on a variety of shapes in the generalized Gaussian family. Moreover, as long as \( L \) and \( M \), the input range partition and alphabet sizes, are set to give theoretical performance close to the rate-distortion bound, the alphabet constrained codes are no better or worse than the unconstrained ones in their sensitivity to source mismatch. These results appear in J. World's thesis (see D.2, Section 2).

In the case of channel errors, the conclusion is the same. If \( L \) and \( M \) are chosen appropriately as above, the alphabet constrained codes have sensitivity to channel errors about equal to that of unconstrained codes. Both analysis and simulation confirm this conclusion. The results are reported in H. Aschmann's thesis (see D.1, Section 2).

We just completed a study of coding of memoryless sources adaptively to overcome the deleterious effects of variance mismatch. The model paradigm is the periodically quasi-stationary driving noise of the auto-regressive source used to model speech. We have found the constrained alphabet code to be slightly less robust and a little more difficult to adapt, but giving nearly the same performance except for small mismatch, in which case the unconstrained code performs far worse. The complete results appear in A. Chaddha's thesis (see D.3, Section 2).

We plan to write one or more papers to report these results on mismatch, adaptive coding, and effects of channel errors.
B. Sources with Memory

Sources with memory have the greater potential payoff with efficient data compression than sources without memory. Until now there has been only one coding theorem proved for a source with memory that yields an implementable encoding scheme. This coding theorem applies only to a Gaussian autoregressive source with squared-error distortion above a so-called critical rate. Under this grant we have proved three new theorems which yield implementable coding schemes for Gaussian sources (auto-regressive or not), with squared-error distortion, applicable to all nonzero rates. In chronological order of discovery, our first theorem proves that an optimal tree code can be constructed to operate on the Karhunen-Loeve (KL) transform of the source. Our second theorem proves that there exists an optimal tree code operating on the actual time values of the source. The last one proves the existence of an optimal trellis code for the KL transform of the source. The proofs of all three theorems provide a methodology for building a code with performance approaching the rate-distortion bound. The two tree codes are theoretically not decodable because of the requirement at the decoder of storing a tree, the number of branches of which grows exponentially with the length of the source sequence. In practice, however, one can use knowledge of the seed of a pseudo-random number generator to generate the reproduction values on the tree branches at the decoder without storing the tree. The trellis code, however, can be decoded theoretically and practically, because the storage requirement increases only linearly and not exponentially with the source sequence length. Encoding simulations have been undertaken using all three methods given by these theorems. They all give performance close to the rate-distortion bound. As yet we see no advantage of one method over the other in terms of performance versus search intensity for the various
Gaussian sources we have used. The results of this work on ideally stationary Gaussian sources with memory are now published in two conference papers (Section 2, B.1 and B.3). The transform trellis code construction and the coding theorem will be published in the IEEE Transactions on Information Theory (Section 2, B.3), while the simulation results and method of code construction will be presented and appear in full in the conference record of ICC '83 (Section 2, D.4).

A great deal of effort has been undertaken to apply these new coding methods to actual image and speech sources. In his Ph.D. dissertation (Sec. 2, D.4), P. Jakatdar has applied the transform tree technique toward encoding images. He has compared the performance with that of transform quantization and has investigated theoretically and experimentally, the effects of channel errors on both schemes. The technique shows promise in increased SNR over quantization, but needs to be adapted more effectively to the changing statistics over the image area. A paper describing the method and results has been submitted for presentation and inclusion in the record of ICC '84 (sec. 2, B.5).

The transform trellis code technique has been used to encode actual speech. Here some crude adaptation of the code has been attempted to account for the quasi-stationary nature of speech. The SNR figures are markedly superior to comparable ones in the literature. A paper describing the method and results has been submitted for presentation at the forthcoming IEEE International Symposium on Information Theory (Sec. 2, C.6). We can report no subjective results with this speech coding technique, as we have no facilities for listening to the coded speech.

Much material now written only in P. Jakatdar's and B. Mazor's Ph.D. dissertations (Sec. 2, D.4 and D.5) is suitable for journal publication. We plan to write these articles as soon as possible.
2. **PAPERS**

Listed below are the articles and conference presentation describing the results of the research.

**A. Journal Articles**


**B. Conference Proceedings Articles**


C. Conference Presentations
1. See B.1 above
2. See B.2 above
4. See B.3 above
5. See B.4 above
7. See B.5 above.

D. Theses and Reports


This report summarizes progress during the period 30 Jun 82-29 Jun 83. New techniques for encoding sources with and without memory are described in Section 1 and the papers, presentations, and theses supported by the research are listed in Section 2. Three new optimal coding methods, generalizing previous ones, have been discovered for stationary Gaussian sources and the squared-error distortion measure. These methods, which store codewords on trees and trellises and utilize systematic search algorithms, are being applied to actual speech and image data. Also, the code storage requirements (CONT.)
ITEM #20, CONTINUED: are not large, as this research has shown that small code letter alphabets give nearly optimal results.