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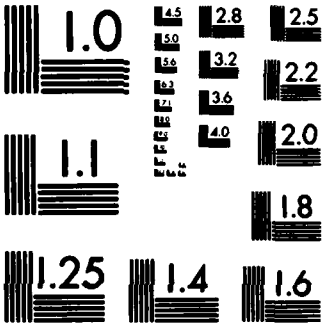

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RESEARCH AND DEVELOPMENT TECHNICAL REPORT

DELEW-TR-83-2

CURVE FIT TECHNIQUE FOR A SMOOTH CURVE USING GAUSSIAN SECTIONS

RYAN MOATS

ELECTRONIC WARFARE LABORATORY

AUGUST 1983

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This report describes the development of a general algorithm that is easily programmable on either a small desk-top computer or calculator and can curve-fit any number of points into smooth Gaussian sections for statistical analysis.

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Table of Contents

Page

INTRODUCTION..... 1

DISCUSSION..... 1

Development of First Gaussian Section..... 1
Development of Subsequent Sections..... 3
Computer Testing..... 5

CONCLUSIONS AND RECOMMENDATIONS..... 8

Figures

1. Output from Sample Curve-fit Program..... 5

2. Listing of Sample Curve-fit Program..... 6

3. Graph of Percent Deviation as a Function of x..... 8

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CURVE FIT TECHNIQUE FOR A SMOOTH CURVE USING GAUSSIAN SECTIONS

INTRODUCTION

In optics, it is often necessary to determine the total luminous flux of a beam by use of sampling beam receivers. If the beam is circular in cross section and it is desirable to approximate this with a Gaussian energy distribution along any diameter, then it is referred to as Gaussian distribution. Receivers placed on a diameter of this beam provide discrete samples so that the total amount of flux may be calculated. Thus, a technique that forms a smooth Gaussian patch between any number of discrete sample points would be useful for a total incoming flux approximation. This report deals with the development of this technique.

DISCUSSION

Development of First Gaussian Section

To simplify the fitting technique, two sampling cases are assumed. One is that one sample point has an intensity greater than all other sample points, and the other is that two adjacent sample points have the same luminous intensity. These sampling cases lead to two different beginnings for the Gaussian sections surrounding the peak point and will be dealt with separately.

If there is a single peak point, represented by (x_0, y_0), the Gaussian section that spans to the next point on either side must be found. A Gaussian section (G_n) has the form

$$G_n = y_k = c_n e^{-\frac{1}{2}\left(\frac{x_k - u_n}{s_n}\right)^2} \qquad \text{Equation (1)}$$

where

(x_k, y_k) are the coordinates of any point which the section passes through

u_n is the mean point of the Gaussian section

s_n is the standard deviation of the Gaussian section

c_n is the normalizing constant for the Gaussian section.

The peak point of any Gaussian has a slope of 0, therefore, the first section must have its peak at (x_0, y_0). Here, the exponential is at its maximum, 1, therefore the exponent is 0, leading us to realize that

$$u_1 = x_0 \qquad \text{Equation (2)}$$

Now, since the exponential has a value of 1, the Gaussian simplifies to

$$c_1 = y_0 \qquad \text{Equation (3)}$$

Rearranging (1) for the standard deviation and letting (x_1, y_1) be the other boundary the Gaussian passes through, we have

$$s_1 = \left(\frac{(x_1 - u_1)^2}{2(\ln c_1 - \ln y_1)}\right)^{\frac{1}{2}} \qquad \text{Equation (4)}$$

and equations (2) through (4) describe the first Gaussian section.

If two points have the same intensity, they can be assumed to be boundaries of the same Gaussian. Then the mean of the section is the midpoint between the two sample points. Here (x_1, y_1) represents a second point with the same intensity as the first, and it is realized that

$$u_1 = (x_1 + x_0)/2 \qquad \text{Equation (5)}$$

To simplify matters, an assumption will be made that the measured intensities at these points are 90% of the peak intensity. This means that

$$c_1 = y_1/0.9 = y_0/0.9 \qquad \text{Equation (6)}$$

By using equation (4), s_1 can now be found. Thus, equations (4) through (6) decribe the first Gaussian section in the second case.

Development of Subsequent Sections

After the first Gaussian section is fitted, subsequent Gaussian sections must be fitted to assure a smooth curve. A smooth curve is one where both functions have equal values and equal slopes at the point of junction. This condition on the second section leads to

$$G'_n(x_k) = G'_{n+1}(x_k) \qquad \text{Equation (7)}$$

G_n is the Gaussian section ending at (x_k, y_k) and G_{n+1} is the Gaussian section extending from (x_k, y_k) to (x_{k+1}, y_{k+1}). Taking the derivative of G_n from (1) at the common boundary point (x_k, y_k) we have

$$G_n'(x_k) = -y_k \frac{(x_k - u_n)}{s_n^2} \qquad \text{Equation (8)}$$

Performing the same derivative for the Gaussian G_{n+1} at (x_k, y_k) we have

$$G_{n+1}'(x_k) = -y_k \frac{(x_k - u_{n+1})}{(s_{n+1})^2} \qquad \text{Equation (9)}$$

Using equation (7), equations (8) and (9) may be set equal to each other. From this, the $-y_k$ factors may be cancelled out and the equation may then be rearranged to solve for s_{n+1}

$$(s_{n+1})^2 = \frac{(x_k - u_{n+1})}{(x_k - u_n)} s_n^2 \qquad \text{Equation (10)}$$

which relates s_{n+1} to s_n. However, this equation has the term u_{n+1} which has yet to be found. To do so, we take the Gaussian G_{n+1} previously defined as being bounded by the points (x_k, y_k) and (x_{k+1}, y_{k+1}). The above Gaussian must then satisfy the following condition:

$$y_{k+1} e^{-\frac{1}{2}\left(\frac{x_k - u_{n+1}}{2s_{n+1}}\right)^2} = y_k e^{-\frac{1}{2}\left(\frac{x_{k+1} - u_{n+1}}{2s_{n+1}}\right)^2} \qquad \text{Equation (11)}$$

From this we take the natural logarithm of both sides of the equation and rearrange it to isolate u_{n+1}

$$2(s_{n+1})^2(\ln y_{k+1} - \ln y_k) = (x_k - u_{n+1})^2 - (x_{k+1} - u_{n+1})^2 \qquad \text{Equation (12)}$$

Substituting the general form for s_{n+1} (from equation (10)), and rearranging terms we have

$$u_{n+1} = \frac{2x_k s_n^2(\ln y_{k+1} - \ln y_k) - (x_k^2 - x_{k+1}^2)(x_k - u_n)}{2s_n^2(\ln y_{k+1} - \ln y_k) - 2(x_k - x_{k+1})(x_k - u_n)} \qquad \text{Equation (13)}$$

This allows u_{n+1} to be found solely in terms of known quantities. Then u_{n+1} may be substituted into equation (10) to find s_{n+1}. These, in turn, may be used in equation (1), along with the point (x_k, y_k), to solve for c_{n+1}. Thus we have a method of finding a Gaussian solely from the preceding Gaussian and its boundary points. This method will work for any case and is therefore general.

Computer Testing (Sample run, figures 1 and 2)

To facilitate validation of the curve-fit technique developed, a computer program was written to perform the task of finding the Gaussian sections of a curve-fit problem by the methods given here. (The program, run on a Tektronix GS-4052 System, is listed in figure 2.)

The data used for the sample run consisted of the peak, 90%, 50%, and 10% points of a Gaussian curve with u = 0, s = 1, c = 0.398942. The printout from a sample run with first one side's curve fit and then the other side's curve fit is shown in figure 1.

Section	Mean	Stan. Dev	Norm Cons
1	0	0.999834481528	0.398942
2	0	0.999668990452	0.398962878929
3	-4.415707426E-4	0.999712877896	0.399308591275

Section	Mean	Stan. Dev	Norm Cons
1	0	0.999834481528	0.398942
2	0	0.999668990452	0.398962878929
3	4.415707426E-4	0.999712877896	0.399308591275

Figure 1. Output from sample curve-fit program.

```
40 T=0
50 DEF FNA(Z)=(X(K)-U(J))*S(I)↑2/(X(K)-U(I))
60 DEF FNT(Z)=2*X(I)*S(M)↑2*LOG(Y(J)/Y(I))-(X(I)↑2-X(J)↑2)*(X(I)-U(M))
70 DEF FNV(Z)=2*S(M)↑2*LOG(Y(J)/Y(I))-(X(I)-X(J))*(X(I)-U(M))
80 DEF FNB(Z)=FNT(Z)/FNV(Z)
90 DEF FND(Z)=Y(K)*EXP((X(K)-U(I))↑2/(2*S(I)↑2))
100 READ X(1),Y(1)
110 READ N9
120 FOR L=2 TO N9+1
130 READ X(L),Y(L)
140 NEXT L
150 C(1)=Y(1)
160 U(1)=X(1)
170 S(1)=(X(2)-U(1))↑2/(2*LOG(C(1)/Y(2)))
180 FOR N=2 TO N9
190 I=N-1
200 J=N
210 M=I
220 K=N
230 U(N)=FNB(1)
240 S(N)=FNA(1)
250 I=N
260 C(N)=FND(1)
270 NEXT N
280 IF T=1 THEN 310
290 PRINT "L"
300 GO TO 340
310 PRINT
320 PRINT
330 PRINT
340 PRINT "section|mean|stan. dev|norm cons"
350 FOR Q=1 TO 70
360 PRINT "-";
370 NEXT Q
```

Figure 2. Listing of sample curve-fit program.

```
380 PRINT
390 FOR Z=1 TO N9
400 PRINT Z;"I";U(Z);"I";S(Z);"I";C(Z)
410 NEXT Z
420 T=T+1
430 IF T=2 THEN 530
440 GO TO 120
450 DATA 0,0.398942
460 DATA 3
470 DATA 0.4590044,0.359048
480 DATA 1.17741,0.199471
490 DATA 2.14597,0.0398942
500 DATA -0.4590044,0.359048
510 DATA -1.17741,0.199471
520 DATA -2.14597,0.0398942
```

Figure 2. Listing of sample curve-fit program (contd).

The sample run output seems quite accurate. To determine the accuracy, a graph was prepared calculating the percent deviation of the curve-fitted curve with the original curve, and the results are shown in figure 3.

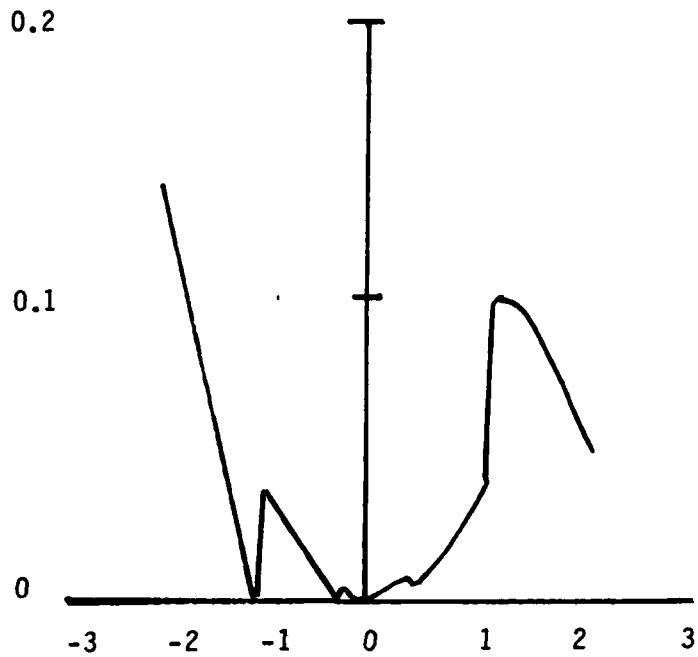

Figure 3. Graph of percent deviation as a function of x.

The greatest deviation is under .15%, an error small enough to be attributed to computer rounding and sample error.

CONCLUSIONS AND RECOMMENDATIONS

The above described technique has led to a deviation from a norm of no greater than .15%. This devation is of a small enough magnitude to validate the developed method of curve-fitting. Furthermore, the algorithm that does the fitting is simple enough to be used on a programmable calculator.

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