Aperture Averaging of Scintillation for Space-to-Ground Optical Communication Applications

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15 August 1983

APPROVED FOR PUBLIC RELEASE;
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Prepared for
SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
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This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-82-C-0083 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by D. H. Phillips, Director, Electronics Research Laboratory. Lieutenant L.J. Zappone, SD/YKXL, was the Air Force project officer.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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## Aperture Averaging of Scintillation for Space-to-Ground Communication Applications

### Aperture Averaging Optical Communications

### Atmospheric Turbulence Scintillation

We derive a useful engineering formula for the aperture averaging factor of optical scintillation that exhibits the explicit parametric dependence on wavelength, collector diameter, and path weighted integrals of the index structure constant profile. Numerical results presented here indicate that care should be exercised in the application of available index structure constant profile models to optical communication system performance studies.
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1. Aperture Averaging Scale Height and Reduction Factor for Various $c_n^2$ Profiles
Communication from space-to-ground stations which utilize coherent short wavelength radiation (~1μm) as the carrier must contend with atmospherically induced scintillation. The amount of scintillation with which the communication system must deal is a function of the diameter of the collecting entrance pupil in the ground station optical system. As is well known, increasing the size of the aperture diameter reduces the amount of scintillation measured by the detector.\textsuperscript{1,2,3}

The aperture averaging factor is defined as the ratio of the variance of irradiance obtained from a finite size collecting aperture to the corresponding quantity obtained from a "point aperture." In this letter the aperture averaging factor is calculated for both spherical and plane waves. The Kolmogorov spectrum with an arbitrary index structure constant $C_n^2$ profile, as would be encountered in a space-to-ground propagation geometry, is used here. A useful engineering formula for the aperture averaging factor that exhibits the explicit parametric dependence on wavelength, collector diameter and path weighted integrals of $C_n^2$ is presented below. In Ref. 3 it is shown that the aperture averaging factor, $A$, can be written as

\begin{equation}
A = \frac{1}{8} \int_0^1 \frac{C_L(Dx)}{C_L(0)} M_L(x) \text{d}x
\end{equation}

where $C_L(\phi)$ is the covariance of irradiance and

\[ M_L(x) = \frac{1}{2\pi} \left[ \cos^{-1} x - x (1-x^2) \right]^{1/2} \]
is the optical transfer function of an unaberrated circular entrance aperture. Here we consider weak scintillation conditions only where $C_i(0) \ll 1$.

Following the development given in Ref. 3, it can be shown for spherical waves that the aperture averaging factor can be written as

$$A = \frac{B}{C_i(0)}$$

where

$$B = 4\pi^2 \int_0^z dz' (z-z')^2 \left(\frac{z'}{z}\right)^2 \int_0^\infty f s f R K^5 dK$$

$$C_i(0) = 4\pi^2 \int_0^z dz' (z-z')^2 \left(\frac{z'}{z}\right)^2 \int_0^\infty f s f R K^5 dK$$

$$f_s = \text{sinc}^2[K^2 z'(z-z')/2kz]$$

$$f_R = \left[\frac{2J_1(KDz'/2z)}{(KDz'/2z)}\right]^2$$

and, for the Kolmogorov spectrum,

$$\phi(K,z') = 0.033 C_n^2(z')K^{-11/3}. $$
In Eqs. (3)-(6), \( z' = 0 \) corresponds to the location of the source, \( z \) is the distance between the source and the receiver, \( k \) is the optical wavenumber, \( \text{sinc}(u) = \frac{\sin(u)}{u} \), \( D \) is the diameter of the circular collector, and \( J_1 \) is the Bessel function of the first kind of order one. The quantities \( f_s \) and \( f_R \) can be regarded as the spherical wave turbulent diffraction and receiver filter functions, respectively.\(^3\) To modify Eqs. (3) - (6) for plane wave propagation the factor \( z'/z \) is replaced by unity.

We are primarily concerned with downward propagation from an exoatmospheric laser source to a receiver located within the atmosphere or on the ground. For the corresponding case of upward propagation, the transverse irradiance correlation scale length is typically much greater than the diameter of collection apertures of practical concern, and hence no appreciable aperture averaging effects will result (i.e., \( A = 1 \)).

For spherical-wave propagation, \( f_R \) possesses spatial frequencies \( K < K_R = \frac{2z}{Dz'} = \frac{2}{D} \). The relative importance of turbulent diffraction effects and aperture averaging can be seen by comparing \( K_R \) and \( K_s \), where \( K_s = \left(\frac{2kz}{z'(z-z')}\right)^{1/2} = \left(\frac{2k}{z_0}\right)^{1/2} \). For

\[
D^2 \gg \frac{z_0}{k},
\]

\( K_R \ll K_s \) and all the turbulence lies in the near field of the aperture. In this case the geometrical optics formulation (i.e., \( f_s = 1 \)) can be used in Eq. (3) to compute the collected scintillation. On the other hand, for \( D^2 \ll \frac{z_0}{k} \) it follows that \( K_R \gg K_s \), and one may use the approximation \( f_R = 1 \) in Eq. (3) with the result that the aperture averaging factor approaches unity. The quantity \( z_0 \) is of the order \( 10 \) km and thus, for example, apertures of diameters larger than a few inches will exhibit averaging effects for visible light propagation.
In the downward propagation direction, the atmospheric turbulence is limited to the latter portion of the propagation path where \( z' = z \). The function \( C_n^2(z') \) is nonzero only in the range \( z - z_o < z' < z \), where for an exoatmospheric source \( z_o \ll z \). A change of variables from \( z' \) to \( \eta = z - z' \) is indicated with \( C_n^2(z') \) replaced by \( C_n^2(\eta) \), where \( \eta = 0 \) corresponds to the location of the receiver. If inequality (8) is satisfied, then \( f_s = 1 \) in Eq. (3) and the aperture averaging factor is given by

\[
A = A_0 \left( \frac{\lambda h_o}{b^2} \right)^{7/6}
\]

where

\[
A_0 = 4 \frac{\int_{-2/3}^{\infty} J_1^2(x) \, dx}{\int_{-11/16}^{\infty} \sin^2 x \, dx} = \frac{\Gamma^5(1/3)}{\pi^{14/3} [3^{1/2} - 1]} \approx 0.90, \tag{10}
\]

\[
\frac{\int \frac{dnC_n^2(\eta)n^2(1-\eta)z^{-1/3}}{path}}{\int \frac{dnC_n^2(\eta)[n(1-\eta)^{-1}]}{path}} {6/7}
\]

\[
\frac{\int \frac{dnC_n^2(\eta)n^2}{path}}{\int \frac{dnC_n^2(\eta)n^{5/6}}{path}} {6/7}
\]

\( \lambda \) is the optical wavelength, and \( \Gamma \) is the gamma function. The quantity \( h_o \) can be regarded as an atmospheric turbulence aperture averaging scale height. For plane waves or for spherical waves where \( z \gg z_o \) (i.e., \( \eta/z \ll 1 \)) we have that

\[
h_o = \left[ \frac{\int \frac{dnC_n^2(\eta)n^2}{path}}{\int \frac{dnC_n^2(\eta)n^{5/6}}{path}} \right]^{6/7} \tag{12}
\]
An important attribute of Eq. (9) is that it reveals the explicit parametric dependence of the aperture averaging factor on optical wavelength, collector diameter, and path weighted integrals of $C_n^2$. For propagation at zenith angle $\theta$ it is easy to show that $h_o$ in Eq. (9) is replaced by $\h_o \sec \theta$, and $C_n^2(\eta)$ in Eqs. (11) and (12) is now given by the index structure constant profile for the situation of interest.

An engineering formula can be constructed from Eq. (9) which can be useful even when inequality (8) is violated (i.e., for small apertures). Indeed, for $D^2 \ll z_o/k$, $A = 1$, and thus without requiring detailed knowledge of $C_n(\rho)$, the aperture averaging factor can be approximated by the engineering formula:

$$A \approx \frac{1}{1 + A_o^{-1}[D^2/\lambda h_o \sec \theta]}^{7/6}$$

(13)

where $A_o^{-1} = 1.1$. This expression gives the aperture averaging factor for space-to-ground propagation conditions in closed form and is thus suitable for system studies of a broad class of atmospheric turbulence models.

For illustrative purposes we present, in Table 1, numerical values of $h_o$ and $A$ for zenith propagation, $\lambda = 1 \, \mu m$, $D = 1 \, m$, and various $C_n^2$ profiles quoted in the literature. In addition, we give the corresponding weak scintillation values for the "point aperture" variance of irradiance:

$$C_l(0) = 2.24 \, k^{7/6} \int C_n^2(\eta)^{5/6} d\eta.$$  

Examination of Table 1 reveals for the turbulence models considered here that both $h_o$ and $A$ based on the NAVY/DARPA model are about a factor of two to
four less than the corresponding quantities based on Barletti's and Hufnagel's models. For example, the NAVY/DARPA daytime model and the Hufnagel model (for $V = 27$ m/sec) both indicate that the variance of irradiance at a "point" is about 0.1. On the other hand, the ratio of the corresponding aperture averaging factors is about 3.6. Hence, for $\lambda = 1$ $\mu$m and a 1 m entrance aperture diameter, the variance of irradiance based on the Hufnagel model is 3.6 times larger than that obtained from the NAVY/DARPA daytime model. As this example illustrates, care should be exercised in the application of available $C_p^2$ models to optical communication system performance studies.
Table 1. Aperture averaging scale height and reduction factor for various $C_n^2$ profiles. The quantity $V$ is the rms wind speed between 5 and 20 km altitude above mean sea level.  

<table>
<thead>
<tr>
<th>Turbulence Model</th>
<th>Scale Height $h_0$ (km)</th>
<th>Aperture Averaging Factor: &lt;br&gt; Zenith Propagation, $\lambda = 1\mu m$, $D = 1m$</th>
<th>Point Irradiance Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAVY/DARPA (4) (DAYTIME)</td>
<td>3.4</td>
<td>$1.2 \times 10^{-3}$</td>
<td>0.095</td>
</tr>
<tr>
<td>NAVY/DARPA (NIGHTIME)</td>
<td>5.3</td>
<td>$2.0 \times 10^{-3}$</td>
<td>0.053</td>
</tr>
<tr>
<td>Barleti et al. (5)</td>
<td>9.8</td>
<td>$4.1 \times 10^{-3}$</td>
<td>0.18</td>
</tr>
<tr>
<td>HUFNAGEL (6) (V = 18 m/sec)</td>
<td>9.1</td>
<td>$3.7 \times 10^{-3}$</td>
<td>0.063</td>
</tr>
<tr>
<td>HUFNAGEL (V = 27 m/sec)</td>
<td>10.3</td>
<td>$4.3 \times 10^{-3}$</td>
<td>0.11</td>
</tr>
<tr>
<td>HUFNAGEL (V = 36 m/sec)</td>
<td>10.8</td>
<td>$4.6 \times 10^{-3}$</td>
<td>0.19</td>
</tr>
</tbody>
</table>
REFERENCES


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