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MULTIACCESS PROTOCOLS FOR VARIABLE LENGTH PACKETS

by

Jacek Jachner

This report is based on the unaltered thesis of Jacek Jachner, submitted in partial fulfillment of the requirements for the degree of Master of Science at the Massachusetts Institute of Technology in May, 1983. The research was conducted at the M.I.T. Laboratory for Information and Decision Systems, with support provided by the Advanced Research Project Agency under Contract No. N00014-75-C-1183.
MULTIACCESS PROTOCOLS FOR
VARIABLE LENGTH PACKETS

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MULTIACCESS PROTOCOLS FOR
VARIABLE LENGTH PACKETS

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Submitted to the Department of
Electrical Engineering and Computer Science
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for the Degree of Master of Science in
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ABSTRACT

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free observations of the channel. High channel utilization
is achieved without breaking up variable length messages
into fixed length packets; the new protocols offer an
alternative to existing Pure and Slotted Aloha protocols.

The Variable Length Slotted Aloha (VLSA) protocol
exploits any apriori knowledge about packet lengths to
schedule transmissions so as to minimize the risk of
collision; the traffic throughput of VLSA is consistently
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(RHVLSA) protocol, the packet header is used to place a
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bution. The message throughput of the RHVLSA protocol
is larger than that of Slotted Aloha for certain message
length distributions.

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1. INTRODUCTION

Multiaccess protocols govern the sharing of a common resource by a number of independent users; the issue arises for scarce and expensive resources or when a high degree of connectivity between the users is desired. In communication systems, protocols address the problem of controlling access to a common channel so as to efficiently allocate the available communication bandwidth amongst the contending transmitting stations.

The earliest protocols, frequency or time division multiple access, allocated channel bandwidth to the stations in a static fashion, independently of their activity(1). These fixed assignment techniques have proven to be very successful for stream type traffic. Computer traffic however, can often be characterized as bursty, as in the case of interactive terminal traffic. This burstiness is a result of a large degree of randomness in both the message generation times and message sizes, and the relatively small transmission delay tolerated by interactive terminals. A transmitting station that carries traffic from such bursty sources demands the channel rather infrequently, but when it does, it requires a rapid response. In such environments, fixed subchannel allocation schemes would need to assign to each station sufficient capacity to meet its peak transmission rates, which would result in low overall channel utilization. Instead, dynamic allocation of the channel resources is preferred; such is the case in packet communication systems, where the entire channel bandwidth is allocated to a single station, based on need, but only for a short period of time. Transmissions are
formatted into packets, which along with the message data contain overhead information such as source and destination addresses, error control bits, and a synchronizing sequence required for the correct reception of the packet.

The difficulty, then, resides in scheduling these transmissions on a channel which must carry its own control information. A class of protocols, called random access (1), resolves this issue by granting the transmitting stations full access to the channel according to their randomly varying needs; since collisions may result, channel performance depends on proper collision avoidance and retransmission scheduling. Another approach is adopted in demand assignment techniques, or reservation protocols, which require that explicit control information regarding the stations' need for the channel be exchanged before message transmissions are allowed. This avoids packet collisions but the conflict resolution problem now arises with the reservations themselves. The channel utilization can be superior to that achievable with random access, but the reservation placement requirement imposes a minimum delay on packet transmissions. A number of adaptive strategies which integrate the two approaches have been proposed (2), (3), which seek to maintain near-optimum performance at all times.

The environment in which a multiaccess protocol is to function is important in its formulation. Protocols have been designed for satellite channels, ground radio networks and local area communication. This thesis focuses on satellite channels, which are characterized by star connectivity, assuming a satellite can receive signals from any earth station in its coverage pattern, and can
transmit to all such stations. Another important feature is the inherently long roundtrip propagation delay of 0.25 sec., which is usually large compared to a packet transmission time; hence a station is not aware of another station's transmission while that transmission is in progress.

The original multiaccess protocol proposed for this environment was Pure Aloha(4) which was soon modified to Slotted Aloha(5) and later enhanced by tree conflict resolution(6),(7),(8). These protocols deal with the randomness in the message generation times of bursty sources. The variability in message length is, however, handled rather poorly; both Slotted Aloha and tree protocols impose a uniform length for all packets. This seriously degrades their performance when the message lengths are non-constant. After a brief overview of these protocols in the next section, this thesis will introduce a protocol designed to enhance Pure Aloha performance for packets of variable length. This Variable Length Slotted Aloha will be further improved in the last section, where we propose the use of packet headers to make reservations.
2. RANDOM ACCESS PROTOCOLS

Historically, the first random multiaccess protocol was Pure Aloha, proposed by Abramson(4). It is an inherently simple technique, which permits a station to transmit anytime it desires. Feedback from the channel determines whether a transmission has been successful, or whether a collision has occurred. In the latter case, a retransmission is required, which, to avoid continuing conflicts, is attempted only after a random retransmission delay. If the delays are sufficiently randomized, and the number of stations is sufficiently large, the attempted packet transmissions can be considered to be independent events, with memoryless inter-arrival times. The transmission attempts then form a Poisson process, which leads to a simple analysis of channel performance.

There are several criteria for evaluating multiaccess protocols. High channel bandwidth utilization, or information throughput, is desirable, as is low message delay. For Pure Aloha, the throughput rate of successful packets $S$, is related to the rate of packet transmission attempts $G$, which includes both new and retransmitted packets, by the probability of transmission success. For uniform length packets and assuming a Poisson process, the probability that all the stations remain idle for the two packet lengths required for conflict-free transmission is $e^{-2G}$. Hence,

$$S = G e^{-2G}$$  \(1\)

The highest throughput achievable is only 18.4% at $G = \frac{1}{2}$. 
due to conflicts and idle time. The throughput decreases for rates \( G \) above the optimal, which makes Pure Aloha unstable as the channel may enter a clogged state where many stations attempt transmission but few are actually successful. The message delay, which is the elapsed time between message generation and successful reception, can be characterized by the expected number of transmissions required for conflict-free reception, which in the steady state, is just \( G/S \).

The throughput of the Aloha technique may be improved by slotting the channel time, as first proposed by Roberts(5). Slotted Aloha requires that any packets wishing to begin transmitting do so only at slot boundaries, and terminate their transmission within one slot; thus packet lengths are restricted to fit within one slot. This technique prevents mid-packet collisions and doubles the packet per slot throughput to a maximum of 36.8% with

\[
S = G e^{-G}
\]

Slotted Aloha is still unstable, but various stabilizing control algorithms have been proposed(9).

Subsequent improvements in conflict resolution have led to the tree algorithms(6)-(8). Information about collisions is used to reschedule collided packets as if traversing a tree, decreasing the probability of collision at each retransmission. Such approaches yield the twin benefits of throughput values of 43-48%, and of stable performance at these levels. These techniques, however, are designed to resolve conflicts only between uniform
length packets, and, as we point out in the next section, their performance suffers if the message generating sources serviced by the transmitting stations produce messages of variable length.
3. VARIABLE LENGTH MESSAGES

The random access protocols discussed so far deal with the randomness in the message generation times of bursty sources. The other random aspect of such sources is the variation in message size. Variable length packets can be used in a Pure Aloha protocol, but as shown by Ferguson(10), the system performance deteriorates from the optimum when packets are of equal length. For Slotted Aloha and tree conflict resolution algorithms, uniform packet lengths are a design requirement. For these protocols, variable length messages generated by the sources must be formatted into fixed length packets. This however, reduces the actual message part of the throughput $S$ for both the short messages, which leave part of the packet unfilled, and for the long ones which must be broken up into several packets and incur the overhead costs for each. Even with the optimal selection of the packet length for a given message length distribution, the throughput advantage of these protocols over Pure Aloha is seriously degraded for variable length messages(10). The breaking up of long messages also increases the complexity of the system, requiring packet numbering and message buffering at the receiver.

A multiaccess protocol specifically designed for variable length messages would avoid these drawbacks; we shall now propose a protocol which achieves significant throughput gains over Pure Aloha without restricting packets to be uniform in length.
4. VARIABLE LENGTH SLOTTED ALOHA

In this section, we introduce an effective protocol which extends the advantages of channel time slotting to non-uniform length packets. Any apriori knowledge of the lengths of packets is used to schedule transmissions so as to minimize the risk of collision.

The packet structure in our formulation of the data communication channel constrains the length of the packet to be at least the length of the overhead information in the header. We slot the channel time on the minimum packet length, with packet transmissions beginning exclusively at slot boundaries, but allowed to continue through several slots. Transmission starting times are further restricted to coincide with what we shall call starting slots, whose selection may vary for different packet lengths. Packet transmissions which last i slots are assigned starting slots every i slots, to create a Slotted Aloha environment amongst packets of a given length.

Interference between different length packets is minimized, for any packet length distribution, if at least one slot on the channel is chosen as a starting slot for packets of all lengths. Overlap between packets is further reduced if the longer packet lengths are restricted to be integer multiples of all shorter ones, that is, if the packet lengths are mutually divisible. In such a case, both the starting and ending slots of a packet coincide with the corresponding slots for any shorter packets. The number of starting slots for packets
whose transmission would overlap the transmission of a packet of length \( i \) is thus restricted to \( \frac{i}{j} \) starting slots for packets of length \( j \), for \( 0 < j \leq i \), and exactly one for any packets longer than \( i \).

An implementation of such a channel is shown in figure 1, with packets of length \( 2^k \) ranging from one slot long header-only packets, which may be used to send very short messages, up to the longest \( 2^m \) slot packets. This length distribution yields the largest number of different length packets for any upper limit on packet length, but it is not unique in satisfying the mutually divisible lengths condition.

The assumptions used in the analysis of such a protocol are similar to the ones for Slotted Aloha. The stations are assumed to be time synchronized with the slots on the channel. All the stations listen to the channel traffic and make error-free observations of the outcome of transmissions. There is a delay in making such observations which exceeds the maximum packet length; hence the stations are unaware of the outcome while a transmission is still in progress. Messages are either correctly received or are completely destroyed by a collision and must be retransmitted. If the stations are numerous enough and the retransmission times are sufficiently randomized, the transmission attempts can be considered to be independent with memoryless inter-arrival times, thus forming a Poisson process.
FIGURE 1. VLSA Channel Structure

packet lengths 1 to $2^m$ slots
(illustrated $m = 3$)
The attempted traffic transmission rate, $G$, is measured in slots per slot of channel time. Packets of length $1_k = 2^k$ account for a fraction $r_k$ of this rate, that is for $G_k = r_k G$ slots of traffic per slot of channel time, or $G_k$ packets every $2^k$ starting slot. The packet transmission rate for packets of length $1_k$ is $1 / 1_k$ of the traffic transmission rate for such packets. The traffic throughput $S$ is the rate of successfully transmitted traffic slots per channel slot, of which a fraction $r_k$ is in $2^k$ long packets. Such packets successfully carry traffic at a rate of $S_k = r_k S$, which is just the rate of attempted traffic transmissions which do not suffer collision:

$$S_k = G_k \times \Pr(\text{channel idle at start}) \times \Pr(\text{idle for } 2^k \text{ slots}| \text{idle at start})$$

Due to the Poisson nature of the transmission process, a packet will survive if there is no other transmission during its starting slot, an event of probability $e^{-G_k}$, and no other transmissions start until the end of the $2^k$ long packet. The channel structure limits transmissions after the starting slot to be shorter packets, and these occur at a combined rate of $\beta_k G$, where $\beta_k$ is defined for packets of length $1_k$ as:

$$\beta_k = \sum_{j=0}^{k-1} \left( \frac{1_k}{1_j} - 1 \right) r_j$$

$$\beta_k = \sum_{j=0}^{k-1} \left( 2^{k-j} - 1 \right) r_j$$
The probability that no such transmissions occur is then \( \exp(-G/\beta_k) \). The traffic throughputs for individual packet lengths as well as for the entire channel can be expressed as

\[
S_k = G_k e^{-G(1 + \beta_k)} \quad /5/ 
\]

\[
S = G e^{-G} \sum_{k=0}^{m} \gamma_k e^{-G/\beta_k} \quad /6/ 
\]

The distribution of traffic throughput amongst the packet lengths is related to the attempted transmission rate distribution by the transformation

\[
\gamma_k = \frac{S_k}{S} = \frac{\gamma_k e^{-G/\beta_k}}{\sum_{i=0}^{m} \gamma_i e^{-G/\beta_i}} \quad /7/ 
\]

Since the \( \gamma_k \)'s are fractions that sum to unity, the following relation results from equation /7/.

\[
1 = \left[ \sum_{k=0}^{m} \gamma_k e^{G/\beta_k} \right] \left[ \sum_{i=0}^{m} \gamma_i e^{-G/\beta_i} \right] \quad /8/ 
\]

The reverse transformation to that in equation /7/ is now obtained using /8/.
The packet length distribution is most usefully specified for newly generated packets, which, in the steady state, is the same as the distribution of successfully transmitted packets. The throughput equation /6/ can be restated, using /8/, in terms of the traffic throughput fractions $r^*_k$:

$$S_k = r^*_k S$$  \hspace{1cm} /10/$$

$$S = \frac{G e^{-G}}{\sum_{k=0}^{m} r^*_k e^{G/3_k}}$$  \hspace{1cm} /11/$$

Equations /4/ and /9/ can be combined to express $\beta_k$ in terms of the $r^*$'s:

$$\beta_k = \frac{\sum_{j=0}^{k-1} (2^{k-j} - 1) r^*_j e^{G/3_j}}{\sum_{i=0}^{m} r^*_i e^{G/3_i}}$$  \hspace{1cm} /12/$$
We can determine the traffic throughput $S$, given any distribution $\nu_k$ and attempted traffic transmission rate $G$, by numerically solving the implicit equations for the $\beta_k$'s. The throughput varies with $G$ as in the Aloha protocols, increasing to reach a maximum and then decreasing. The throughput value at the maximum varies with packet distribution, as shown in figure 2 for the case of two packet lengths, one twice as long as the other; it is upperbounded by $e^{-1}$, which is attained only if all traffic consists of one packet type. The maximum possible throughput for any distribution $\nu$ decreases as the number of allowed packet lengths increases, as shown in figure 3 for traffic equally distributed over allowed packet types.

The analysis of the Variable Length Slotted Aloha, or VLSA, channel can easily be extended to evaluate Pure Aloha performance for variable length packets. All the equations /5/ through /11/ still apply, only the form of the $A_k$ equations /4/ and /12/ is changed. For Pure Aloha, transmission starting times are not restricted, hence the rate $A_{PAk} G$ of transmissions which may interfere with a transmission in progress is larger than that for VLSA. In fact, for Pure Aloha, with packets of length $l_k$ comprising a fraction $\nu_k$ of the attempted transmission traffic and $p_k$ of the successful transmission traffic, the rate $A_{PAk} G$ is just the sum of the attempted packet transmission rates, $\nu_j / l_j$, over all packet lengths and for the duration of transmission $l_k$.

$$A_{PAk} G = \sum_{j=0}^{m} \frac{l_k}{l_j} \nu_j$$  \[13/\]
FIGURE 2. Maximum Throughput v.s. Small Packet Fraction

2 packet lengths
length ratio 2:1

\[ \frac{S_{\text{small}}}{S_{\text{total}}} \]

\[
\begin{align*}
S_{\text{max}} & \\
0 & \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \\
0 & \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4
\end{align*}
\]
FIGURE 3. Maximum Throughput v.s. Number of Packet Types

packets are $2^k$ slots long
traffic throughput is equally distributed
A comparison of equations /12/ and /14/ shows that $\beta_{PAk}$ always exceeds $\beta_k$ for VLSA, hence the traffic throughput is always larger for VLSA. The throughput performance of the two protocols is compared in figure 3, for traffic equally distributed over variable length packets. VLSA achieves a significant improvement over Pure Aloha for any packet length distribution, but the throughput of both protocols tends to zero as the number of packet types and the ratio of allowed packet lengths becomes large.

It should be noted that in the VLSA protocol, as in Pure Aloha, the attempted traffic transmission rate and traffic throughput rate distributions differ. In fact, from equation /9/,

$$\gamma_k = \frac{\sum_{j=0}^{m} \gamma_j e^{G/\beta_j}}{\gamma_k e^{G/\beta_k}}$$

This ratio increases monotonically with k, since $\beta_k$, as defined in /12/, is also a monotonically increasing function of k. Hence, the protocol is not fair in the sense that the transmission delay varies for different length packets; the expected number of transmissions.
required for a $2^k$ long packet is:

$$\frac{G_k}{S_k} = \frac{Y_k G}{\gamma_k S}$$

The long packets are more likely to collide on the channel, hence incur longer delays since they require more retransmissions than short packets.

This fairness problem can be remedied by a modification to VLSA which we shall introduce in the following section; in addition, the channel throughput will thereby be improved.
5. RESERVATION HEADERS

Appropriate scheduling of packet transmissions in the VLSA protocol described above restricts the collision time from exceeding the length of the longest of the collided packets. Packets are assumed to be completely destroyed by a collision, even though the variable length packets may not overlap on their entire lengths. We will now show that relaxing this assumption allows useful information to be obtained from collisions, which, if used to resolve the conflicts, leads to significant improvements in the protocol's performance.

In a collision between packets of different lengths, parts of the longer packet are not interfered with by the shorter one, and could be useful if they were correctly received. Correct reception occurs only if the receivers can lock-in on the transmission by recognizing the synchronizing sequence in the packet header. Should this sequence be corrupted by a collision, no further part of the packet can be decoded. For a collision which occurs anytime after the header, the receiver decodes the packet but the error check bits at the end show the reception is incorrect. The extent of the damage cannot be determined unless there are additional error checks on the packet parts. In particular, the inclusion of an error detecting capability in the overhead at the beginning of the packet allows the header to be decoded independently. An attempted transmission now leads to one of three outcomes:

1. Entire packet successful
2. Header successful, message part suffered collision
3. Entire packet is destroyed
The headers, being shorter than entire packets, have a better chance of survival in the contention environment, and can be used to resolve conflicts. A header decoded correctly not followed by a correctly decoded message, can reserve future channel time for the packet retransmission, providing the header contains packet length information. The station which placed the reservation retransmits the entire packet after a fixed delay sufficient for all the other stations to have heard the reservation. The other stations, aware of the request and the length of the packet, refrain from transmitting during that time.

The channel can be in one of two modes. In the contention mode, stations compete to transmit packets or at least reservation headers, which are then recorded during the conflict-free reserved mode. The VLSA protocol is used while in the contention mode, with the first slot of each packet acting as a reservation header. As illustrated in figure 4, packets which have placed reservations are delayed until the end of the VLSA frame in progress, each frame being as long as the longest allowed packet. At that time all are serviced, in the order of reservation header transmission, including any reservations that arrive while the channel is in the reserved mode; only then does contention resume.

In the analysis of this Reservation Header Variable Length Slotted Aloha, or RHVLSA, channel, we denote the attempted traffic transmission rate during contention as $S_C$, with a fraction $\gamma_k$ in $2^k$ long packets, and the traffic throughput rate in contention as $S_C$. In addition, we
FIGURE 4. RHVLSA Channel

- Transmission delay
- Reservation
- Collision contention period
- Reserved period
define $R_{ck}$ as the traffic slots per contention slot, which are in packets $2^k$ long whose headers have not incurred collision. Again using the Poisson nature of the transmission process,

$$R_{ck} = G_{ck} e^{-G_o} \quad /17/$$

The entire packet will succeed if the header is successful, and no interfering packets transmit while the transmission is in progress. The VLSA channel structure limits the rate of such transmissions to $\beta_k G$; hence,

$$S_{ck} = R_{ck} e^{-G_o \beta_k} \quad /18/$$

where

$$\beta_k = \sum_{j=0}^{k-1} (2^{k-j} - 1) \gamma_j \quad /19/$$

Packets with successful headers but corrupted messages generate reservations at a rate of $G_{rk}$ (slots of traffic in $2^k$ long packets per contention slot).

$$G_{rk} = R_{ck} - S_{ck} \quad /20/$$

$$G_T = \sum_{k=0}^{m} G_{rk} \quad /21/$$
The total throughput $S_T$, for both channel modes, is easily obtained by observing that any packet whose header succeeds by contention is assured of successful transmission, either immediately, or in the reserved mode. The total throughput is then the rate of successful headers scaled by the fraction of channel time spent in the contention mode.

$$S_{Tk} = \frac{R_{ck}}{1 + G_r} = \frac{G_{ck} e^{-G_c}}{1 + G_r}$$ \hspace{1cm} /22/

$$S_T = \sum_{k=0}^{m} S_{Tk} = \frac{G_c e^{-G_c}}{1 + G_r}$$ \hspace{1cm} /23/

Using equations /17/, /18/, /20/, /21/, we express $G_r$ as

$$G_r = G_c e^{-G_c} - G_c e^{-G_c} \sum_{k=0}^{m} \gamma_k e^{-G_c/3k}$$ \hspace{1cm} /24/

Since the probability of header success is the same for all packets, the traffic throughput distribution is equal to the contention channel attempted traffic transmission distribution; from equations /22/ and /23/ we have:

$$\gamma_k = \frac{G_{ck}}{G} = \frac{S_{Tk}}{S} = \gamma_k$$ \hspace{1cm} /25/

This important feature of RHLVLSA makes the protocol fair in the sense that the expected number of transmissions
in the contention mode is the same for all packet classes.

\[
\frac{G_{ck}}{S_{Tk}} = \frac{G_c}{S_T} \quad \forall k
\]

The packets that use the reserved mode do however, incur one additional transmission delay.

To illustrate the protocol, consider a channel with only two packet types, one twice as long as the other, which is slotted on the short packet length. Further let the traffic throughput rate be the same for both packet types \( \nu_C = \nu_1 = \frac{1}{2} \); the traffic throughput is plotted in figure 5 with respect to \( G_c \). All short packets are transmitted on the contention channel; in contrast, only some of the long packets succeed by contention, while the rest make reservations. The greater vulnerability of long packets in the contention mode is exactly compensated for by transmissions in the reserved mode.

The maximum traffic throughput \( S_T \) is achieved for the value of \( G_c \) which satisfies the equation \( \frac{dS_T}{dG_c} = 0 \), which reduces to

\[
1 = G_c + G_c \sum_{j=0}^{m} \beta_j e^{-G_c \beta j}
\]
FIGURE 5. Two Packet RHVLSA Channel

2:1 packet length ratio
equal traffic throughput
This maximum throughput depends on the distribution of traffic amongst the packet types. For the two packet type channel, with a 2:1 packet length ratio, figure 2 shows how $S_{T_{max}}$ varies with $p_0$, decreasing from $e^{-1}$ for a single packet type channel. The reservation header protocol is clearly less sensitive to the packet distribution than VLSA alone.

As the number of allowed packet lengths increases, the maximum throughput decreases as shown in figure 3 for the case of equally distributed throughputs. The RHVLSA protocol is more robust than VLSA, maintaining throughput levels of around 30% even for packets with length ratios of 64:1. In fact, since $G_c$, as defined in equations /20/ and /24/, is upper bounded by $G_c \exp(-G_c)$ even if no packets succeed in the contention channel, the reservation header protocol throughput $S_{T_{max}}$ is always lower bounded by:

\[
S_{T_{max}} \geq \frac{G_c e^{-G_c}}{1 + G_c e^{-G_c}} \quad G_{c_{opt}} = 1
\]

\[
S_{T_{max}} \geq \frac{1}{e + 1}
\]

Thus regardless of the traffic distribution and the number of allowed packet lengths, the RHVLSA traffic throughput will always exceed 26.9%. In fact, since the $\lambda$'s have been eliminated from equation /28/, 26.9% is a lower bound even for a reservation header protocol without the VLSA restriction of mutually divisible packet lengths.
Due to packet overhead, the throughput of actual messages $S_m$ of any protocol is only a fraction $\delta$, called the transmission efficiency, of the traffic throughput.

$$S_m = \delta S_T$$  \hspace{1cm} (29)$$

For Slotted Aloha, Ferguson (10) defined this efficiency in terms of the mean message length $\overline{m}$, the mean number of packets per message $\overline{n}$ and the packet length $L_p$.

$$\delta_{SA} = E \left[ \frac{\text{length of message}}{\text{length of transmission}} \right] = \frac{\overline{m}}{\overline{n} L_p}$$  \hspace{1cm} (30)$$

For variable length protocols, with messages of length $l_m$ transmitted using $n_k$ packets of length $l_k$, the transmission efficiency is

$$\delta_{VL} = E \left[ \frac{l_m}{\sum_k n_k l_k} \right]$$  \hspace{1cm} (31)$$

A comparison can now be made between the message throughputs of Slotted Aloha and variable length protocols. For message lengths which closely match the allowed packet lengths, the superior transmission efficiency of RHVLSA gives it a message throughput advantage over Slotted Aloha, since less overhead is required when entire messages are sent in single packets. For instance, consider a two packet type RHVLSA channel, with a 2:1 packet length ratio and equally distributed traffic throughput; the short packets are therefore twice as numerous as the long ones. Furthermore, we assume that the overhead comprises half of the short packet length, and a quarter of the long
packet length. This channel would be best utilized if it were servicing a source which produced short messages, equal in length to the overhead, at twice the rate of long messages, which were three times as long as the overhead. The traffic throughput of RHVLSA for this packet length distribution is shown in figure 3 to be 34.4%, and the transmission efficiency, given the message length distribution, can be determined from equation /31/ to be .625. Slotted Aloha traffic throughput, from equation /2/, is 36.3%, but the transmission efficiency calculated using equation /30/ is at most .5, even for the optimal choice of packet length $L_p$. The message throughput of RHVLSA and Slotted Aloha are then, using equation /29/, 21.5% and 18.4% respectively; the variable length protocol is better suited for traffic from sources which produce messages with the postulated length distributions.
6. CONCLUSIONS

The multiaccess protocols we have introduced provide an alternative to the breaking up of variable length messages into fixed length packets. In the Variable Length Slotted Aloha protocol, apriori knowledge about packet lengths is used to schedule transmissions to minimize the risk of collision. The performance of VLSA bridges the gap between Pure and Slotted Aloha, with channel traffic throughput always better than that of Pure Aloha. The protocol discriminates against long packets, which incur a longer transmission delay. Furthermore, traffic throughput is sensitive to the distribution of packet lengths, and falls to zero as the number of allowed packet lengths becomes large.

Reservation headers have been proposed as a method of compensating for the deleterious effects of long packets in the VLSA protocol, since reservations are most advantageous when the ratio of packet to reservation header length is large. The reservation header protocol is fair in the sense that the expected number of transmissions in the contention channel is the same for packets of any length. In addition, the traffic throughput always exceeds 26.9%, regardless of the number of packet types or the distribution of traffic between them. The performance of the Reservation Header Variable Length Slotted Aloha protocol has been evaluated for packets $2^k$ long, but the analysis is general enough to accommodate any choice of packet lengths which satisfy the mutually divisible lengths condition. If the allowed packet lengths are selected to match the message lengths, the message throughput of RHVLSA
will exceed that of Slotted Aloha, due to the reduced overhead when a message is transmitted in a single packet, and to the gain obtained by scheduling long packet transmissions using reservations.

The RHVLSA protocol should be considered as an effective alternative in applications which currently employ Pure or Slotted Aloha for variable length messages, including mixed-mode protocols which integrate pure reservation and Aloha techniques.
REFERENCES


