ESTIMATING THE PROBABILITY THAT ACADEMY PERSONNEL WILL CHOOSE MILITARY HOUSING

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DEPT OF ECONOMICS

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This research report is presented as a competent treatment of the subject, worthy of publication. The United States Air Force Academy vouches for the quality of the research, without necessarily endorsing the opinions and conclusions of the author.

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This research report has been reviewed and is approved for publication.

THOMAS E. McCANN, Lt Col, USAF
Director of Research and Continuing Education
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UNITED STATES AIR FORCE ACADEMY
DEPARTMENT OF ECONOMICS
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Abstract

This report estimates the probability that Academy personnel will live on or off base. The report uses a Logit model with stratification by income; it relates certain socio-economic characteristics of the income groups to the probability that they will choose to live on base. Results indicate that it would be very difficult to impossible to fill Academy housing with a free choice option. The addition of Peterson to the population would probably, given local economic conditions, suffice to fill Academy housing. Some tentative recommendations are made, based on an analysis of the data, to upgrade the amount of usable living space.
Introduction

This study addresses the probability of a military member with dependents choosing to live on or off base at the U.S. Air Force Academy. Housing policy decisions are made by the Academy Chief of Staff in compliance with command dictates, DOD Regulation 4165.44 and Air Force Regulation 90-1. We attempt to give numerical estimates, insofar as the data will allow, of the probability that a given class of military families will live on or off base--given that the policy decision is made to adopt a free choice approach to housing. In general, for morale, recruiting, and personal financial reasons, the authors feel that a free choice approach is the best alternative; however, it is recognized that other constraints, such as housing occupancy rates or command considerations may dictate a different policy. As in most statistical reports, the results are the "best available data" and are not a substitute for informed judgement or experience in a job. The figures are compiled in a scientifically "valid" way, but there is clearly a margin for error; therefore these figures should be one input to the total policy making process. Statistical methods used are contained in an appendix with appropriate references. To facilitate the presentation of the material, the body of the report will use general explanations of the techniques used with the applicable results.
The Model

The thrust of the model is straightforward—military families are faced with a choice as to whether they will live on or off base. They make this choice based on a number of socio-economic characteristics; e.g., income level, availability of alternative housing, family size, and expected period of occupancy. The model assumes that socio-economic characteristics are the dominant choice variables. It does not rule out personal taste, but relegates this to a random influence. It is assumed that taste will balance out on average—to the extent that there is a systematic preference not based on the above characteristics, there will be some bias in the model. The model uses a statistically valid sample of Air Force families who made the housing choice decision at four representative bases in 1978. We observe the choice that is actually made, and then relate this choice to the socio-economic characteristics of each Air Force family in the sample. The estimating technique is a Logit model—the Logit model estimates the parameters of an assumed function which are "most likely" to have generated the observed outcome. Consider Figure 1.

1 See Appendix 1 for details.
The probability of any one family choosing to live off or on base is zero or one; however, the probability associated with a large number of families can range anywhere from zero to one, depending on the X values (socio-economic characteristics). It should be noted that this functional form implies that the maximum impact of the X values will occur at the middle of the probability distribution (.5), or, more concretely, at the indifference point between living on or off base--this is in accord with our intuition as to how choices are actually made! More specifically, the model considers (1) rent differential; that is, the difference between BAQ plus VHA plus tax advantage (the cost of living on base) and the cost of owning a home (inclusive of tax
advantage and assumed capital gain)\(^2\) or the cost of renting; (2) difference in distance to work center; (3) difference in number of bedrooms (surrogate for size); (4) income; (5) number of family members; and (6) expected period of occupancy. These factors are used to figure a probability of living on or off base. An example will help make this clear: consider an E-4 with dependents-- what is the probability that he or she will live on base? Take the data; that is, on average we observe that E-4s who live off base on the average are located 12 miles from work, have a four bedroom house, pay $350 in rent, and remain 48 months at their duty station. Put these figures into the estimated formula for the Air Force sample, and we can calculate the probability that this type of individual will live off base. In this case the off-base probability comes out to approximately .3, or a .7 probability of living on base.

Some cautions are necessary. We are not estimating the probability that any particular family will live off base--only the probability that, given a large group of families with the same characteristics, a predictable percentage will choose to live off base. To the extent that local conditions differ from those in the original sample, we will observe differences in the estimated probabilities.

\(^2\)See Appendix 2 for details.
As Table 1 makes clear, due to a set of largely fortuitous circumstances, the local conditions in the Colorado Springs area are extremely close to the original values; therefore, we can be reasonably confident in using the original coefficients.

3 Rents and imputed rents increased with inflation, but Air Force members began receiving VHA, so the net difference is about the same.
### Table 1
Statistics for Logit Model Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imputed rent difference (homeowner vs. military rental)</td>
<td>-107.90</td>
<td>128.00</td>
<td>-102.00</td>
<td>127.00</td>
</tr>
<tr>
<td>Imputed rent difference (private rental vs. military rental)</td>
<td>-70.91</td>
<td>96.95</td>
<td>-70.02</td>
<td>54.45</td>
</tr>
<tr>
<td>Distance to work difference (private housing vs. military housing miles)</td>
<td>6.54</td>
<td>5.10</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>Number of bedrooms difference (homeowner vs. military rental)</td>
<td>0.32</td>
<td>0.66</td>
<td>0.28</td>
<td>1.0</td>
</tr>
<tr>
<td>Number of bedrooms difference (private rental vs. military rental)</td>
<td>0.20</td>
<td>0.86</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Expected period of assignment (months)</td>
<td>47.42</td>
<td>3.67</td>
<td>42.49</td>
<td>5.96</td>
</tr>
</tbody>
</table>

4 Income and family comparison are not necessary, since families will be categorized by income class and family size. Furthermore, we assume that income increases since 1978 have kept pace with inflation, and real income has remained relatively constant.
We assume, as mentioned earlier, that the military population at the Academy does not differ appreciably from the same ranks in the Air Force with regard to housing preferences. We believe this is a reasonable assumption. Finally, a statistical note: in estimating the model, income was a dominant variable; that is, with income included, the effect on choice was so large that the other variables were relegate to an extremely minor role. This is a problem of estimating technique. It does not mean that the other variables are unimportant, only that the estimating procedure attributes all variation to income when it is present in the model. Fortunately, the difficulty can easily be overcome by stratifying the model according to income (rank) and running the regression for each income (rank) class. This was done, and these are the results presented. The other variables take on their average values.

The Results

Consider Table 2 which presents the results. The first probability to consider is the probability that an E-1, 2, 3 will live on base (.55). This probability needs further explanation. The sample assumes that Air Force members had free choice of housing. For E-4 and above this is a good assumption; however, during the time period of the study, this was not a good assumption for E-3 and below. The bases in question reserved housing for E-4 and above. E-3s and below were allowed to live on base,
but only when the occupancy rate fell below requirements. E-3s and below could sometimes move on base immediately, but more often had long waits and uncertainty involved in getting a house on base. For these reasons the probability for E-3s is an extremely conservative figure. As an example of what might happen, the Academy now has 65 E-3s and below who are eligible for housing; of this group 61 now live on base for a probability .93. The other probabilities are reasonable.

Using these probabilities some calculations can be made. The Academy has 1242 adequate housing units of all types; these are divided exactly in half with respect to officers and enlisted. Considering the eligible population we can estimate the number of houses which should be allotted to the various ranks and the probability that they will be occupied. Again, an example will make this clear. There are 652 eligible E-4s, 5s, and 6s at the Academy. This group has a .69 probability of living on base; therefore, our best guess as to the number of houses to allot to this group, with the expectation that they will be occupied is 450, or 652 times .69.
Table 2
Probability of Living on Base by Rank

<table>
<thead>
<tr>
<th>Pay Grade</th>
<th>Probability of Living On Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1,2,3</td>
<td>.550</td>
</tr>
<tr>
<td>E-4,5,6</td>
<td>.686</td>
</tr>
<tr>
<td>E-7,8,9</td>
<td>.339</td>
</tr>
<tr>
<td>0-1</td>
<td>.686</td>
</tr>
<tr>
<td>0-2</td>
<td>.339</td>
</tr>
<tr>
<td>0-3</td>
<td>.284</td>
</tr>
<tr>
<td>0-4,5,6</td>
<td>.367</td>
</tr>
</tbody>
</table>

Consider Table 3. We have done the same calculation for each group. Note that we are using the conservative figure for E-1s, 2s, and 3s.

Table 3
Expected Occupancy by Rank

<table>
<thead>
<tr>
<th>Rank</th>
<th>Eligible</th>
<th>Probability</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1-3</td>
<td>65</td>
<td>(.55)</td>
<td>= 36</td>
</tr>
<tr>
<td>E4-6</td>
<td>535</td>
<td>(.69)</td>
<td>= 369</td>
</tr>
<tr>
<td>E7-9</td>
<td>186</td>
<td>(.34)</td>
<td>= 63</td>
</tr>
<tr>
<td>0-1</td>
<td>7</td>
<td>(.69)</td>
<td>= 5</td>
</tr>
<tr>
<td>0-2</td>
<td>46</td>
<td>(.34)</td>
<td>= 16</td>
</tr>
<tr>
<td>0-3</td>
<td>511</td>
<td>(.28)</td>
<td>= 143</td>
</tr>
<tr>
<td>0-4,5,6</td>
<td>408</td>
<td>(.38)</td>
<td>= 155</td>
</tr>
</tbody>
</table>

|               |               |             | 787         |

9
It is clear that the Academy will not be able to voluntarily fill its housing under any allocation scheme as long as only Academy people are eligible to live on base. Adding Peterson to the eligible population increases occupancy. Peterson presently has a waiting list of approximately 200 families and E-3 and below are not allowed to live on base. There are approximately 160 eligibles in the E-3 and below category— at the lowest rate (.55) they could be expected to occupy 88 more houses. The 200 on the waiting list have indicated a desire for on-base housing. Based on past experience approximately 67 (1/3) might occupy Academy housing. These assumptions bring the expected occupancy to 942. Taking the .98 occupancy rate, the Academy must fill 1217 houses. If we add in the actual Academy figure for E-3s and below (61 vs. our projected 36) and add approximately 40 houses for other service personnel and DVPs we get a figure of 1007. This still leaves a shortfall of 210. There is no doubt that these figures are on the conservative side! Preliminary indications indicate that continued inflation and high interest rates, together with a noticeable drop in capital gains for housing sales, will increase the probability of living on base. Our best guess is that the military population of the Academy and Peterson will fill Academy housing to the .98 occupancy rate. This policy
might require a reallocation of approximately 50 houses⁵ to the lower grade NCO housing. If Academy housing were opened to the entire military population of Colorado Springs, it is an absolute certainty that the desired occupancy rate would be achieved.

Summary

This report uses the "best available" figures to give rough estimates of the probability that on-base housing will be occupied. Principle conclusions are: (1) The Academy will not achieve .98 occupancy rate with a free choice policy, (2) The Academy and Peterson would most likely achieve occupancy rates of .98 given our expectations of the state of the local economy and, (3) Opening the Academy to the entire military population would definitely achieve desired occupancy rates.

The report has not addressed certain "quality of living" considerations. It is apparent from the figures in the original study and from our analysis of local home sales, that one of the chief considerations, apart from income, is usable space within a house. At the present time Academy housing is not as competitive in this area as it might be. The typical split-level house in Colorado Springs provides up to 2,000 square feet of

⁵This number depends on the distribution of new individuals choosing to live on base. If this is about equal across ranks then 50 would be a reasonable number. This suggests that the present allocation of housing is about right.
finished living spaces--on two or more levels! Preliminary indica-
tions indicate that improvements in this area as, for example,
a standard finishing of Academy basements (one or two bedroom,
bath and exit) would substantially increase the attractiveness of
Academy housing. Cost considerations might, of course, be
prohibitive; however, over the years we have spent a great deal
on improving the engineering and energy efficiency of housing--
some of this money might profitably be diverted to increasing
living space, but this topic would need further study.

Housing presents complex management problems with strong
"emotional" content. Freedom of choice tends to substantially
lower the "emotional" content of the issue but, as this report
points out, leaves difficult policy/management choices (espe-
cially when viewed in light of the 98% occupancy requirement).
Appendix 1

Mathematical Presentation of Model

In this appendix, we provide a model which explains the probability of choice of home ownership, private rental, or military rental as a function of the independent variables listed in the following table. The variables identified by X symbols are housing attributes and the variables identified by Y symbols are fairly socio-economic characteristics.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁</td>
<td>Imputed Rent</td>
</tr>
<tr>
<td>X₂</td>
<td>Distance to Work</td>
</tr>
<tr>
<td>X₃</td>
<td>Number of Bedrooms</td>
</tr>
<tr>
<td>Y₁</td>
<td>Military Income (RMC)₁</td>
</tr>
<tr>
<td>Y₂</td>
<td>Number of Persons in Family</td>
</tr>
<tr>
<td>Y₃</td>
<td>Expected Period of Assignment</td>
</tr>
</tbody>
</table>

To obtain probabilities of choice based upon the variables presented in Table 1, we adapted a qualitative choice multinomial

₁Regular Military Compensation (RMC) is defined as the sum of basic pay, quarters, and subsistence allowances and the tax advantage to the tax-exempt status of the allowances. The tax advantage is calculated by determining the amount of additional taxable earnings required to pay the tax and still be left with the same take-home pay.
Logit model originally developed by McFadden (1973) and refined by others, including Li (1977).²

Consistent with the Logit analysis methodology, we note that under freedom of choice conditions, each military family must choose a dwelling unit in one of three housing sectors: home ownership, private rental, or military rental housing. The choice between sectors is a function of the difference in housing attributes between sectors and a given family's socio-economic characteristics. Thus, the conditional probability of choice of sector k compared to alternative sector m can be expressed as follows:

\[ P(k/k,m) = f\left( (X_{l1} - X_{l1m}), (X_{l2} - X_{l2m}), (X_{l3} - X_{l3m}), Y_1, Y_2, Y_3 \right) \]

where \( k = 1, 2, 3, k \neq m \). \hspace{1cm} (1)

or, in functional form,

\[ P(k/k,m) = \exp\left( b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4Y_1 + b_5Y_2 + b_6Y_3 \right) \]

\hspace{1cm} (la)

Where the \( b \)'s are the coefficients and the \( X \)'s and \( Y \)'s are the variables.

Following McFadden, it can be shown that the natural logarithms of the ratio of the conditional probabilities $P_k$ and $P_m$ can be expressed as:

$$\log \frac{P_k}{P_m} = a_{km} + b_1 (X_{1k} - X_{1m}) + b_2 (X_{2k} - X_{2m}) + b_3 (X_{3k} - X_{3m}) + b_4 Y_1 + b_5 Y_2 + b_6 Y_3 + u. \tag{2}$$

where $P_k + P_m = 1$ and $u$ is a stochastic error term.$^3$

In Gertcher (1981), 1,822 observations of housing sector choices by military families were used to estimate the coefficients. The 1,822 observations were taken in Calendar Year 1975 from Ellsworth, MacDill, Travis, and Tinker Air Force bases. These installations were selected randomly from all installations that were complying with the freedom of choice condition. Thus, these observations represent a random sample of the military population in the continental United States that experienced freedom of choice with regard to housing sectors.

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The coefficients and resulting probabilities of choice estimated in the 1981 study can be applied to an analysis of housing choice at the Air Force Academy.\footnote{As explained earlier, freedom of housing sector choice has not existed at the Air Force Academy for military families in certain categories since at least Calendar Year 1978.} From the 1981 study, we obtained estimates of \( P_1 \), the probability of home ownership given freedom of choice between our three housing sectors. We also obtained estimates of \( P(2/2,3) \), the conditional probability of private rental given that a family chooses either private rental or military rental. Estimates of these two probabilities were obtained for different categories of families.

Consider the following decision tree, where \( P(\emptyset) \) equals the probability of renting and \( P_1 \) is the probability of home ownership, given the freedom of choice condition.
Obviously, we can state that:

\[ P_1 = P(1/1,2,3) \]  \hspace{1cm} (3)

which is the probability of choice of home ownership given freedom of choice between home ownership, private rental, and military rental. However, we must calculate \( P_2 \) and \( P_3 \) from the conditional probabilities \( P(2/2,3) \) and \( P(3/2,3) \) and the probability of renting \( P(\theta) \). To do this, we note that the probabilities at nodes 1 and 2 must sum to one. Thus, we have

\[ P(\theta) = 1 - P_1 \]  \hspace{1cm} (4)

\[ P(3/2,3) = 1 - P(2/2,3) \]  \hspace{1cm} (5)

and for the case of \( P_1 + P_2 + P_3 = 1 \),
\[ P_2 = P(\theta)P(2/2, 3) \]
\[ P_3 = P(\theta)P(3/2, 1) \]

Referring to our Logit model, we can easily show that \( P_1 \) can be derived from equation (1a), given that observations of home ownership, private rental, and military rental are considered simultaneously. In a similar fashion, \( P(2/2, 3) \) can be derived from equation (2), where \( k = 2, m = 3 \), and \( P_2 + P_3 = 1 \).

We are now ready to consider categories of families to facilitate our policy analysis. In the 1981 study, families were first grouped according to annual income. Within each income class, they were further grouped by family size. Income classes were: below $7,000; $7,000 to $9,999; $10,000 to $14,999; and by $5,000 increments, up to $29,999. The final income class was for families $30,000 to $37,000. The family size categories were: 2, 3-4, 5, and 6 and above. Thus, 48 computer runs were necessary to obtain \( P_1 \) and \( P(2/2, 3) \) for all categories. With these numbers, we can estimate the probability that a class of individuals will live on base—these probabilities are presented in Table 2 of the main text.
Appendix 2

Computation of Imputed Rent
With Allowance for Capital Gain

Consider the imputed rent associated with owning a home. In addition to the monthly mortgage payment (principle and interest), home owners pay property taxes, insurance, utilities, and maintenance expenses. They also experience closing costs and a down payment cost at the time of purchase and additional closing costs at the time of sale. However, there are benefits due to the capital gain over the period of ownership and the income tax deduction. All cash flows, both negative (expenses) and positive (benefits) must be included in the calculation of an imputed rent.

Essentially, we converted the cash flows over the expected period of home ownership to a uniform monthly imputed rent using standard time value of money amortization techniques.\(^1\) We used the mortgage interest rate as the discount rate, which reflects the opportunity cost associated with the home ownership investment.

\(^1\)A number of references provide the necessary techniques for converting uneven cash flows to a uniform monthly amount (imputed rent). For example, See E.J. Mishan, Cost Benefit Analysis, (New York: Praeger Publishers, 1976)
Except for the capital gain component, the calculation of the imputed rent is rather straightforward. However, the estimate of capital gain involves both an estimate of the period of ownership and an estimate of the future selling price of the home.

The estimate of the period of ownership was assumed to be the expected period of assignment to the Academy for each military pay grade. For example, a Captain assigned to the faculty has an expected period of assignment of four years. A Staff Sergeant assigned to base supply has an expected period of assignment of three years.

Estimates of expected selling price may reasonably vary. We expressly recognize the possibility for bias. Forecasts were made of annual appreciation rates of median selling prices of single family homes. To account for possible bias, we made optimistic, realistic, and conservative forecasts. Rates of 15, 12, and 6 percent were obtained, respectively, for Colorado Springs. The 12 percent appreciation rate over the expected period of ownership was used to estimate selling price. In turn, the expected selling price was used in the calculation of capital gain for given home owners.

In general, the amortized capital gain and income tax benefits serve to reduce the imputed rent considerably below the sum of monthly principle, interest, taxes, insurance, and utilities.
payments. Thus, a home owner with an $80,000 mortgage at a 12 percent interest rate experiences monthly payments of about $640. However, his monthly imputed rent is $1,00, assuming a 12 percent appreciation rate over four years and a 20 percent marginal tax bracket. If the appreciation rate were halved, holding everything else constant, his monthly imputed rent would be $500, still considerably below his monthly outlays on principal, interest, taxes, etc.

As is apparent from the above example, the imputed rent of home ownership is very sensitive to the appreciation rate. However, we found that home buyers are relatively optimistic about the appreciation of their home investment. It is important to note that expectations are the important element in the decision between home ownership private rental, and military rental housing. Based on this and on the findings of previous studies, we feel confident that, at the present time, an appreciation rate of 12 percent for Colorado Springs is reasonably accurate with regard to home buyer expectations.²

²See Gertcher, op. cit.
BIBLIOGRAPHY


