**INTAHILS: An Interactive Analytical Hill Shading System Using Entropy as an Information Statistic**

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**Abstract:**
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INTAHILS: AN INTERACTIVE ANALYTICAL HILL SHADEING SYSTEM USING ENTROPY AS AN INFORMATION STATISTIC

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BIOGRAPHICAL SKETCH

Royal Webster is a cartographer in the Techniques Office, Aeronautical Information Department, at the Defense Mapping Agency Aerospace Center. He received his B.S. degree in Geography from Michigan State University, studied cartography at the University of Kansas, and received his M.A. in Geography from Ohio State University, where his area of specialization was computer assisted cartography. His cartographic interests include interactive cartography, map information content, data collection and statistical mapping.

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ABSTRACT

The interactive computer mapping system, INTAHILS, allows the user to generate analytical shaded relief maps. Selected algorithms have been chosen to utilize Digital Terrain Matrix data and display the shaded relief image in a virtual map or softcopy format. Various commands are used to interactively edit or change the image by altering the illumination source angles in selected local areas of the map image. Specific topographic features may then be enhanced, resulting in a more informative visual impression of the terrain. The information statistic is measured numerically by calculating the entropy of the map information presented. The
process thus allows for a maximization of information presented, while minimizing the subjective bias of the cartographer. Good man-machine communication principles were utilized, resulting in a convenient and user-friendly system.

INTRODUCTION

The cartographer's task is to analyze and organize real world data and provide enhancements such that map users will receive an accurate picture of reality from his map. Two map categories exist, depending upon the type of data displayed. Thematic maps show the distribution of some phenomenon, e.g. income. General maps show the location of physical features, e.g. cities, lakes, and relief information. This paper discusses a method used to enhance the displaying of relief information.

BACKGROUND

Methods used to show terrain information include contours, block diagrams, hachures and shaded relief (hill shading). Contours include the capacity to yield quantitative relief information. However, some map users have a perceptual difficulty in visualizing the terrain as shown by contours (Yoeli, 1965). Block diagrams eliminate the perceptual disadvantage but are not planimetrically correct (Peucker, 1980). Shaded relief, although not yielding quantitative terrain data, aids the map user in visualizing the terrain because the map is created with various gray tone intensities. The intensities are analogous to the reflectance properties exhibited when light strikes an object, an event seen daily. Hence, the human eye is trained in perceiving shapes from observing shadow patterns (Tanaka, 1950).

Early attempts to provide an automated solution for hill shading came from Tanaka (1950). His approach was to vary the width of the contours. Using a neutral gray base, Tanaka's contours were drawn in black for surfaces sloping away from the light source and in white on slopes facing the light source. The contour interval was selected so that on an average slope, the contours just coalesced at their widest point, creating both darkened and lightened slope areas. A problem was that when the grade was slight, the map appeared terraced, whereas on steep grades, contour overlap occurred.

The theory for analytical hill shading was developed by Wiechel in 1878. The intensity of light reflected from any point is proportional to the light incident on that point. Mathematically, the light intensity of a plane is propor-
tional to the cosine of the incidence angles of the light rays on that plane. Equation 1 gives the mathematical relationships and figure 1 shows the variables graphically.

\[ \cos e = \cos a \cos b + \sin a \sin b \cos c \]  

(1)

where 
\( e \) = the angle between the light ray and the normal to the slope
\( a \) = the angle between the vertical and the light ray
\( b \) = the angle between the normal to the slope and the perpendicular to the horizontal plane, and
\( c \) = the angle between the azimuth of the light ray and the azimuth of the normal to the slope.

[Diagram of light ray, normal to slope, and variables labeled as described in the equation.]

Figure 1.

This approach was impractical in Wiechel's day because of the number of computations required. However, the equation was developed into a computer algorithm by Yoeli (1965). Using gridded data, Yoeli computed the slope of each surface patch and subjected the slope coefficient to the algorithm. By controlling the amount of overprint with the computed cosine \( e \) value, a shaded relief map was generated via the line printer. This application, although significant in the
development of analytical shaded relief maps, did not allow for any change in the light source position so that selected terrain features may be enhanced.

Brassel (1974) provided a mechanism for changing the light source position by incorporating ridge and valley line information into Yoelit's technique. This information was used to determine, within limits, the azimuth and vertical angles at which the light source should be placed to increase contrast between illuminated and shaded slopes. Brassel's modification allowed for adjusting the vertical angle of illumination, the azimuth of illumination, and correcting for atmospheric perspective. The light source position for a given cell was determined by the degree and direction of slope of that cell's surface. The degree of contrast enhancement was a function of the cell's elevation.

The question may be asked "Why should the light source be changed?" The answer is so that potentially more terrain information may be displayed. Using the conventional northwest light source, both sides of a northwest-southeast ridge will be equally illuminated. However, move the light source to a more westerly position and one side will be lighter than the other. If the light source is moved on toward the southwest, the sides of the ridge will exhibit maximum contrast. However, unlimited azimuthal changes are not allowed lest confusion reign.

A related question is "How does one know if he has shown more terrain information on a given map?" Cartographers are concerned with the amount of information communicated to map users. It is therefore necessary to use some method to measure that information. One tool is information theory, or entropy. This technique was developed by Hartley in 1928 to measure the bits of information transmitted in telecommunication circuits. It was further expanded by Wiener in 1948 and by Shannon and Weaver in 1949 (Robinson and Petchenik, 1976). The equation for entropy (from Shannon and Weaver) is:

\[
H = - \sum_{k=1}^{n} p_k \log p_k
\]

where \( H \) = entropy,
\( p_k \) = the probability of the \( k \)th event, and
\( n \) = the total number of events in the system.

Entropy has been used by cartographers in forming class determination models for choropleth maps (Wasilenko and Moellering, 1977). It has also been used to compare experi-
mental maps with topographic maps produced using standard
topographic map specifications (Taylor, 1975).

Entropy yields information about uncertainty. The more un-
certain an event, the higher the entropy value, i.e. maximum
entropy is complete uncertainty. If all events in a system
are equally likely, an observer is uncertain as to which
event will occur. Conversely, when maximum certainty is
reached, entropy is zero. Since some uncertainty exists in
any system (Khinchin, 1957), minimizing the entropy value
reduces the uncertainty. Applying this concept to shaded
relief maps, it follows that if one is completely uncertain
regarding the terrain’s appearance, the entropy value will
likely be higher than if one is relatively sure of the
terrain.

THE MAPPING SYSTEM

With this background, the goal was to bring together the
concepts of analytical hill shading and information measure-
ment in an interactive mapping environment. Explicitly
stated, the goals were: 1) to develop an interactive com-
puter-assisted cartographic system capable of producing analy-
tical shaded relief maps on a CRT terminal where the user
may change the light source position to alter the shading
pattern such that a more detailed map is generated, and
2) to include a numerical measuring tool as a guide to the
user in adding map detail.

The interactive capability stated in the first goal allows
the system user to intervene during the mapping process.
The user may edit his map in a softcopy state while his map-
ing module is an active task, reducing the time required
relative to batch systems. Interactivity optimizes both
segments of the man-machine system; it allows each to con-
centrate on those tasks it is more suited to perform. The
machine is more efficient at numerical operations, while the
man performs logical operations more easily (Moellering,
1975).

The second goal is stated such that a more effective map may
be created. In this case, effective means the relative
amount of information available to the user is maximized.
Information theory is a useful tool for measuring the effec-
tiveness of a map (Taylor, 1975). Some, however, question
its usefulness. Robinson and Petchenik (1976) state:

"although something conceptually similar to the
technique of information theory may well have some
limited use in special cartographic analyses, the
fundamental concepts upon which information theory
rests make its direct application in cartographic communication impossible." (p. 37).

Having recognized this, it must be stated that the present research is not directed toward examining how the map user reads and interprets map information. The aim here is to ensure that the maximum terrain information possible is placed on the map. In this system, entropy is used as a measure of the relative amount of terrain information on the map. The entropy values are displayed to the user via the CRT screen and serve as an indication to whether any user selected changes in the light source position are meaningful.

The system is designed to utilize terrain elevation matrix data in a regular gridded format. Gridded data is used because the Yoeli algorithm is easily incorporated and because some authors believe gridded data to be the best type for analytical shaded relief map generation (Marsik, 1971; Yoeli, 1975).

Consideration was given to the earth area covered by one map display. Due to the size of a DMA standard dataset, it is not practical to utilize each point in generating the map. Hence, the system was built so that a large area, using the entire dataset, may be displayed at reduced resolution, or so that smaller areas may be examined at greater resolution. The resolution is user selectable and may be changed depending on the terrain examined and the amount of detail the user wishes to display.

Because the system was developed using a vector display terminal (Tektronix 4014), it was necessary to emulate gray shading. This was done by creating a continuous triangular dot pattern and varying the distance between the dots based on the value calculated for cosine e (eq. 1). This technique is based on the principle of varying the gray level as used in creating choropleth maps without class intervals (Tobler, 1973).

Defining local areas in the display which are to have the light source position changed is performed by enclosing the local area in a polygon. The polygon is outlined with the aid of the graphic cursor, and all cells lying within the polygon are flagged by a point-in-polygon routine. New illumination coefficients are then computed for these cells using the changed light source position angles.

The entropy of the total system and of each local area is computed both before and after the light source position is changed. The values are displayed to the user to indicate whether the changes were meaningful. A meaningful change
occurs when the entropy value is reduced. Since maximum entropy is maximum uncertainty, reducing the entropy value from a given value implies a relative reduction in the level of uncertainty, or increasing the relative amount of terrain information on the map.

When the user desires a hardcopy of any map display, he may save the current illumination coefficients which are then used to generate a map on the Versatec plotter. Figure 2 is a generalized flow diagram of the system.

Figure 2.
Flow within the system is controlled by user issued commands. Studies have shown that users will attempt to abbreviate longer commands (Benbasat, et al., 1981), therefore one word commands were employed to maintain an easy to use environment. To minimize user confusion and reduce the possibility of errors, prompts are issued by the system when data or commands are required. The commands, with a brief explanation of each, are listed in Table 1.

Table 1. List of commands utilized by the system.

<table>
<thead>
<tr>
<th>First level</th>
<th>Second level</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>YES</td>
<td></td>
<td>Affirmative answer</td>
</tr>
<tr>
<td>NO</td>
<td></td>
<td>Negative answer</td>
</tr>
<tr>
<td>SUBDivide</td>
<td></td>
<td>Divides dataset into segments for increased resolution</td>
</tr>
<tr>
<td>CHANGE</td>
<td></td>
<td>Indicates changes desired, invokes second level commands</td>
</tr>
<tr>
<td>NONE</td>
<td></td>
<td>No (further) changes, error recovery if CHAN erroneously entered</td>
</tr>
<tr>
<td>BOTH</td>
<td></td>
<td>Change both azimuth and elevation angles</td>
</tr>
<tr>
<td>DIRECTION</td>
<td></td>
<td>Change azimuth angle only</td>
</tr>
<tr>
<td>ELEVATION</td>
<td></td>
<td>Change elevation angle only</td>
</tr>
<tr>
<td>OLD</td>
<td></td>
<td>Indicates further changes in previously defined area</td>
</tr>
<tr>
<td>NEW</td>
<td></td>
<td>Define new area for changes</td>
</tr>
<tr>
<td>SHADE</td>
<td></td>
<td>Performs shading</td>
</tr>
<tr>
<td>PAPER</td>
<td></td>
<td>Generates hardcopy</td>
</tr>
<tr>
<td>NEXT</td>
<td></td>
<td>Loops back for new map region or to change resolution</td>
</tr>
<tr>
<td>QUIT</td>
<td></td>
<td>Terminate interactive session</td>
</tr>
<tr>
<td>INSTRUCTIONS</td>
<td></td>
<td>Displays instructions</td>
</tr>
</tbody>
</table>

Only the first four letters (or less) of each command are required to effectuate the command.

DISCUSSION OF RESULTS

Figure 3 is the first map generated. Little terrain information is shown on the right half of the map image. By defining local areas and entering new azimuth and/or eleva-
tion angles for the light source, it can be seen that more terrain information is shown in the map in figure 4.

Figure 5 shows the steps taken to get from the first map (fig. 3) to the map in fig. 4. The left side of the screen shows the man-machine communication that has occurred. When the user requests a change in the light source angles, the system directs him to define the area. Three different local areas were defined. The graphic cursor was used to pick the nodes of the polygons, each node is denoted by an "X" tick. To avoid confusing individual areas, the ticks of each polygon are joined, explicitly outlining the polygon. The system then computes and displays the entropy value for the local area, denoted "L ENT". The next prompt requests the angles to be changed. If "BOTH" is entered, the system prompts for each angle separately, and echoes the entered values for confirmation. When the user confirms the values correct, the new local area entropy value is computed and displayed. The global entropy value is also displayed, denoted "G ENT". This value may be compared with the global entropy value from the original display, shown in the legend under the map (line 2, "G: 10.933"). In the first area selected for light source angles to be changed, it is noted that the changes resulted in more terrain detail being presented. This may be seen by comparing the maps in figs. 3 and 4. It is also noted that, by comparing the pre- and post-change entropy values, the post-change values are lower. Because a lower entropy value indicates a greater amount of certainty within a system, and because it can be seen (by visual inspection) that the map in fig. 4 shows more terrain detail than the map in fig. 3, one can establish that a lower entropy value indicates a greater amount of terrain detail is presented. Stated another way, as the entropy values decrease and the terrain information increases, the certainty of the landform becomes more apparent to the map user.

However, not all light source angle changes produce the desired effect. In some instances, the entropy values show post-change increases. When this occurs, the user may specify that the same area be examined again. This is done by entering the "OLD" command. Upon exercising this option, the user enters the OLD area number, and the system redraws the polygon from the points stored earlier. In this manner, the user is assured that he will be comparing entropy values from the exact same area, maintaining consistency in his comparisons. He may then select different light source angle changes as often as desired until he is satisfied with the map solution.
CHANGES? paper
CHANGE/NONE change
DEFINE AREA
AREA NMBR 3
L ENT: 6.594
ANGLES CHGD both
ENTER DIR: -15.
ENTER ELE: 39.
D=-15. E=39.?yes
L ENT: 5.755
G ENT: 10.903
MORE CHGS? yes
OLD/NEW AREA new
NEW? yes
DEFINE AREA
L ENT: 6.482
ANGLES CHGD both
ENTER DIR: +75.
ENTER ELE: 39.
D=75. E=39.?yes
L ENT: 6.288
G ENT: 10.894
MORE CHGS? yes
OLD/NEW AREA new
NEW? yes
DEFINE AREA
L ENT: 8.928
ANGLES CHGD both
ENTER DIR: +20.
ENTER ELE: 35.
D=20. E=35.?yes
L ENT: 8.934
G ENT: 10.898
MORE CHGS? yes
OLD/NEW AREA old
OLD? yes

Figure 5.
GEOGRAPHIC LOCATION OF SOUTHWEST CORNER:
LAT: 35 DEG 40.00 MIN  LONG: 105 DEG 50.00 MIN
AREAS: ILLUMINATION SOURCE ANGLES:  ENTROPIES:
GLOBAL:  DIRECTION: 45  ELEVATION: 45  SFC: 11.121  SHADES: 10.886
LOCAL 3:  DIRECTION: 10  ELEVATION: 40  SFC: 8.970  SHADES: 8.685
QUADRANGLE SIZE: 10.00  MIN  SCALE: 1:104351

Figure 6.
After several changes were made, a hardcopy of the map solution shown in figure 4 was generated on a Versatec plotter. It is seen in figure 6.

SUMMARY AND CONCLUSION

The goals in developing this mapping system were to build an interactive analytical hill shading system and to include in that system some statistical tool to indicate to the cartographer when a more informative map had been created. These goals were accomplished. By comparing figures 3 and 4, one may see that a reasonable degree of success was achieved. By observation, it may be seen that the map image in figure 4 yields more terrain information than does the map in figure 3. The reduced entropy value (from map in figure 4) indicates that the uncertainty of the terrain appearance has been reduced. Therefore, the conclusion is reached that a worthwhile mapping system was developed, one which may aid the cartographer in creating analytical shaded relief maps while maximizing the visual impression of the terrain shown on the map.

REFERENCES


Wasilenko, M., and H. Moellering, 1977, "An Information Theoretic Model for the Derivation of Choropleth Classes", Discussion Paper No. 56, Department of Geography, Ohio State University, Columbus.
