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TECHNICAL REPORT NO. 81

AUGUST 1982

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A globally convergent, modified Newton method is developed for the least squares estimation of target from a DME type range measuring system. The resulting algorithm is used to evaluate the effects of truncation error on the accuracy of the position estimates for the RMS/MTTS system located at MacGregor Range.

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INTRODUCTION

In DME type instrumentation systems, the ranges from several distinct locations to a common target are measured and recorded. If $x(t_i)$, $y(t_i)$, and $z(t_i)$ are the cartesian components of the target position at time t_i in some reference coordinate system, and x_α, y_α, z_α, $\alpha=1, M$ are the Cartesian components of the measurement sites in the same reference coordinate system, the range measurements from each site to the target can be modelled as:

$$R_\alpha(t_i) = \{(x(t_i)-x_\alpha)^2 + (y(t_i)-y_\alpha)^2 + (z(t_i)-z_\alpha)^2\}\ 1/2 + e_\alpha(t_i)\ \alpha=1,M \tag{1}$$

where $e_\alpha(t_i)$ is an error associated with the measurement of range from the αth site to the target. The process of estimating the Cartesian positions, $x(t_i)$, $y(t_i)$, $z(t_i)$ from the measured ranges has often been termed multilateration. The term, trilateration, is a more common term used to denote the process of obtaining cartesian positions from three measured ranges. Several methods have been proposed for multilateration, [1].

These methods include nonlinear least squares and if $M \geq 4$, a linear least squares method.

The nonlinear least squares method minimizes the sum of squares

$$\sum_{\alpha=1}^{M} (R_\alpha(t_i) - \sqrt{(x(t_i)-x_\alpha)^2 + (y(t_i)-y_\alpha)^2 + (z(t_i)-z_\alpha)^2}\ )^2 \tag{2}$$

If $M \geq 4$, a linear least squares method can be applied to the multilateration problem. M-1 linear equations are obtained by differencing the squares of the range measurements. Thus,

$$\begin{aligned} R^2_{\alpha+1}(t_i)-R^2_\alpha(t_i) = {} & 2x(t_i)(x_\alpha-x_{\alpha+1}) + 2y(t_i)(y_\alpha-y_{\alpha+1}) + 2z(t_i)(z_\alpha-z_{\alpha+1}) \\ & + (x_\alpha-x_{\alpha+1})^2 + (y_\alpha-y_{\alpha+1})^2 + (z_\alpha-z_{\alpha+1})^2 \end{aligned} \tag{3}$$

is linear in the Cartesian positions. Rearranging (3) results in the M-1 linear equations.

$$d_{x_\alpha} x(t_i) + d_{y_\alpha} y(t_i) + d_{z_\alpha} z(t_i) = P_\alpha(t_i) \quad i=1, M-1 \tag{4}$$

where d_{x_α}, d_{y_α}, d_{z_α} are the direction cosines of the vector between measuring sites,

$$d_{x_\alpha} = \frac{x_\alpha - x_{\alpha+1}}{R_{\alpha,\alpha+1}}, \quad d_{y_\alpha} = \frac{y_\alpha - y_{\alpha+1}}{R_{\alpha,\alpha+1}}, \quad d_{z_\alpha} = \frac{z_\alpha - z_{\alpha+1}}{R_{\alpha,\alpha+1}} \tag{5}$$

and $R_{\alpha,\alpha+1}$ is the distance between measuring sites

$$R_{\alpha,\alpha+1} = \sqrt{(x_\alpha - x_{\alpha+1})^2 + (y_\alpha - y_{\alpha+1})^2 + (z_\alpha - z_{\alpha+1})^2} \tag{6}$$

The right hand side of (4) is defined by

$$P_\alpha(t_i) = \frac{R^2_{\alpha+1}(t_i) - R^2_\alpha(t_i) - R^2_{\alpha,\alpha+1}}{2R_{\alpha,\alpha+1}} + (x_\alpha d_{x_\alpha} + y_\alpha d_{y_\alpha} + z_\alpha d_{z_\alpha}) \tag{7}$$

The linear least squares method estimates $x(t_i)$, $y(t_i)$, $z(t_i)$ by minimizing

$$\sum_{\alpha=1}^{M-1} [P_\alpha(t_i) - (d_{x_\alpha} x(t_i) + d_{y_\alpha} y(t_i) + d_{z_\alpha} z(t_i))]^2 \tag{8}$$

For applications where the measuring sites are all ground based and the altitude of the vehicle being tracked is relatively low, both the linear and nonlinear least squares methods experience great difficulty in obtaining reliable solutions. In the nonlinear least squares case this ill-conditioning sometimes results in the failure of the algorithm to converge or the convergence of the solution to something other than a strong local minimum. These convergence problems exist even under the most ideal conditions of no error in the measured ranges. This report describes a nonlinear least squares algorithm for multilateration which will always converge to a strong local minimum. In the ideal case of no measurement error, this algorithm will converge to the true solution provided the

starting solution is within certain prescribed limits. The application of this algorithm to the estimation of a vehicle trajectory from the measured ranges of the General Dynamics RMM/MTTS system at MacGregor Range is described. The limits on the starting solution are presented for the case of perfect measurements and the effect of measurement errors on the vehicle position error is presented for a wide variety of geometries.

LEAST SQUARES ALGORITHM FOR MULTILATERATION

The nonlinear least squares algorithm for multilateration must minimize

$$f(\bar{x}) = 1/2 \sum_{\alpha=1}^{M} (R_\alpha - f_\alpha(\bar{x}))^2 \tag{9}$$

where $\bar{x}$ is the position vector with coordinates (x, y, z) in some reference coordinate system. R_α is the range measured by the αth measurement site and

$$f_\alpha(\bar{x}) = \sqrt{(x-x_\alpha)^2 + (y-y_\alpha)^2 + (z-z_\alpha)^2} \tag{10}$$

where $(x_\alpha, y_\alpha, z_\alpha)$ are the coordinates of the measuring site in the reference coordinate system.

Suppose $f(\bar{x})$ is approximated in the neighborhood of $\bar{x}_0$ by a quadratic,

$$f(\bar{x}) \approx f(\bar{x}_0) + F^T(\bar{x}_0)(\bar{x}-\bar{x}_0) + 1/2\ (\bar{x}-\bar{x}_0)^T H(\bar{x}_0)(\bar{x}-\bar{x}_0) \tag{11}$$

where $F(\cdot)$ is the gradient vector of $f(\cdot)$, and $H(\cdot)$ is the Hessian matrix.

Setting the derivative of $f(\cdot)$ in (11) to zero,

$$F(\bar{x}) = F(\bar{x}_0) + H(\bar{x}_0)(\bar{x}-\bar{x}_0) = 0 \tag{12}$$

solving for x gives

$$\bar{x} = \bar{x}_0 + H^{-1}(\bar{x}_0)F(\bar{x}_0) \tag{13}$$

provided $H^{-1}(\cdot)$ exists. The resulting $f(\bar{x})$ is

$$f(\bar{x}) = f(\bar{x}_0) - 1/2 F^T(\bar{x}_0)H^{-1}(\bar{x}_0)F(\bar{x}_0) \tag{14}$$

From (14) it is apparent that if $H(\bar{x}_0)$ is positive definite, $f(\bar{x}) < f(\bar{x}_0)$. This is the standard result for minimization by Newton iteration. If the Hessian

of the function being minimized is positive definite, a sufficiently small step in the Newton direction will always result in a decrease of the objective function. In terms of the measurement functions the gradient vector is

$$F(\bar{x}) = -\sum_{\alpha=1}^{M} F_\alpha(\bar{x})(R_\alpha - f_\alpha(\bar{x})), \tag{15}$$

where $F_\alpha(\bar{x})$ is the gradient vector of $f_\alpha(\bar{x})$

$$F_\alpha(\bar{x}) = \frac{1}{\sqrt{(x-x_\alpha)^2 + (y-y_\alpha)^2 + (z-z_\alpha)^2}} \begin{bmatrix} x-x_\alpha \\ y-y_\alpha \\ z-z_\alpha \end{bmatrix} \tag{16}$$

The Hessian matrix $H(\bar{x})$ is given by

$$H(\bar{x}) = \sum_{\alpha=1}^{M} (F_i(\bar{x})F_i^T(\bar{x}) - H_\alpha(\bar{x})(R_\alpha - f_\alpha(\bar{x}))), \tag{17}$$

where

$$H_\alpha(\bar{x}) = 1/f_\alpha(\bar{x})\ I_3 + (1/f_\alpha(\bar{x}))^3\ (\bar{x}-\bar{x}_\alpha)(\bar{x}-\bar{x}_\alpha)^T \tag{18}$$

If a Newton iteration is used to solve the multilateration problem, the Hessian may often not be positive definite so that the objective function may increase at some steps and the direction of the iteration may go astray. How then should we proceed at a point where the Hessian is not positive definite? Even if the Hessian is positive definite so that the Newton direction is a descent direction, the least squares Newton step may not result in a decrease of the sum of squares. Thus, we have two problems to solve in applying the Newton method to the multilateration problem:

(1) Replace the Hessian matrix with a matrix $H(\bar{x})$ which is guaranteed positive definite and a reasonable approximation to the Hessian.

(2) After obtaining an appropriate H matrix, choose a step length α in the Newton direction which results in a decrease in the sum of squares.

THE GILL-MURRAY HESSIAN MODIFICATION

In order to insure that the H matrix used in the Newton iteration is positive definite, Gill and Murray [2] suggested a method for modification of the Hessian based on an LDT^T factorization of the Hessian. The following is a summary of the Gill-Murray modification.

If the Hessian H is positive definite, then

$$H = LDL^T \tag{19}$$

where L is unit lower triangular and D is diagonal with $D_i > o$. When in computing the L-D factors of the Hessian we encounter a $D_i < o$, modify H so that

$$\bar{H} = H + E \tag{20}$$

so that $\bar{H}$ will be positive definite and $\bar{H} = \bar{L}\bar{D}\bar{L}^T$, $\bar{D}_i > o$. E is diagonal. The elements of E are chosen to satisfy three requirements;

(1) All elements of $\bar{L}\bar{D}^{1/2}$ are bounded above by a chosen value β;

(2) All elements of $\bar{D}$ are bounded below by a chosen value ξ, and;

(3) $\bar{H} = H$ when H is positive definite and sufficiently well conditioned. Thus, we have to specify the parameters β and ξ. ξ is chosen to be

$$\xi = \max \{m_\varepsilon ||H||, m_\varepsilon\} \tag{21}$$

where m_ε is near the smallest floating point number representable by the computer being used and

$$||H|| = \{\sum_{i,j} H_{ij}^2\}^{1/2} \tag{22}$$

A choice of β which will satisfy the requirements is

$$\beta^2 = \max \{\tau_{1/3}, \tau_2, m_\varepsilon\}, \tag{23}$$

where

$$\tau_2 = \max_{j=1,3}\{|H_{jj}|\}, \tag{24}$$

and

$$\tau_1 = \max_{j=1,3} \{n_j\}, \tag{25}$$

with

$$n_j = \max_{k=j+1,3} \{|H_{kj}|\}, \tag{26}$$

let

$$\psi = H_{jj} - \sum_{p=1}^{j-1} L_{jp}^2 D_p \tag{27}$$

and

$$c_{k_j} = L_{kj} \bar{D}_j \tag{28}$$

Then define

$$t_j = \max_{k>j} \{|c_{kj}|\} \tag{29}$$

The elements of $\bar{D}$ are defined as

$$\bar{D}_j = \begin{Bmatrix} \xi \text{ if } \xi \geq \max \{|\psi_j|, t_j^2/\beta^2\} \\ |\psi_j| \text{ if } |\psi_j| \geq \max \{\xi, t_j^2/\beta^2\} \\ t_j^2/\beta^2 \text{ if } t_j^2/\beta^2 \geq \max \{\xi, |\psi_j|\} \end{Bmatrix} \tag{30}$$

thus, the elements of E are

$$E_j = \begin{Bmatrix} \xi - \psi_j \text{ it } \xi \geq \max \{|\psi_j|, t_j^2/\beta^2\} \\ |\psi_j| - \psi_j \text{ it } |\psi_j| \geq \max \{\xi, t_j^2/\beta^2\} \\ t_j^2/\beta^2 - \psi_j \text{ it } t_j^2/\psi^2 \geq \max \{\xi, |\psi_j|\} \end{Bmatrix} \tag{31}$$

Thus, we have the mechanism to specify a modified Newton direction which is a descent direction. We must now specify the length of the step to take in this direction.

CHOICE OF STEP LENGTH

Having determined the direction $\bar{d}$,

$$\bar{H}(\bar{x}_0)\bar{d} = F(x_0) \tag{32}$$

along which we will move to obtain a decrease in the sum of squares, we must determine the length of the step α to take in this direction

$$x = x_0 + \alpha\bar{d} \tag{33}$$

A value of $\alpha=1$ corresponds to taking the full least squares step along $\bar{d}$. We employ the following strategy. Let the sum of squares f along the direction $\bar{d}$ be parameterized by the scalar variable α. Thus, we denote

$$f(\alpha) = f(\bar{x}_0 + \alpha\bar{d}) \tag{34}$$

and the derivative $f'(\alpha)$ by

$$f'(\alpha) = F^T(\bar{x}_0 + \alpha\bar{d})\bar{d} \tag{35}$$

Since $\bar{d}$ is a descent direction, $f'(o) = F^T(x_0)\bar{d}<o$. If $f(1)<f(o)$ and $f'(1)<0$, we move the whole least squares step, $\alpha=1$, so that

$$\bar{x} = \bar{x}_0 + d \tag{36}$$

If $f(1)<f(o)$ and $f'(1)>o$, we choose α in the interval [0,1] by cubic interpolation. In this case

$$\alpha=1 - \frac{f'(1) + a-b}{f'(1) - f'(o) + 2a} \tag{37}$$

where

$$a = (b^2-f'(1)f'(0))^{1/2} \tag{38}$$

$$b = 3(f(0)-f(1)) + f'(0) + f'(1) \tag{39}$$

This value for α minimizes $f(\alpha)$ on the interval [0,1] assuming that $f(\alpha)$ is a cubic on this interval. If $f(1)>f(0)$, we halve the interval considered and repeat the above strategy including the cubic interpolation on the interval [0,1/2].

CONVERGENCE CRITERIA

The Newton iteration is considered to have converged when

$$|x_i - x_{o_i}| \le \varepsilon_1(|x_{o_i}| + \varepsilon_2), \quad i = 1,2,3 \tag{40}$$

for three consecutive iterations. The constants used are $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-3}$. It is possible that the iteration described above will converge to a point $\bar{x}_o$ for which $F(\bar{x}_o)=0$ but $H(\bar{x}_o)$ is not positive definite. This situation is detected when $\| E \| > \varepsilon$. Gill and Murray [2] suggest that if this situation is encountered the iteration be continued in the following sequence. Solve,

$$\bar{L}^T y = e_j \tag{41}$$

where e_j is a unit vector such that

$$\psi_j - E_j = \min_{k=1,3} \psi_k - E_k \tag{42}$$

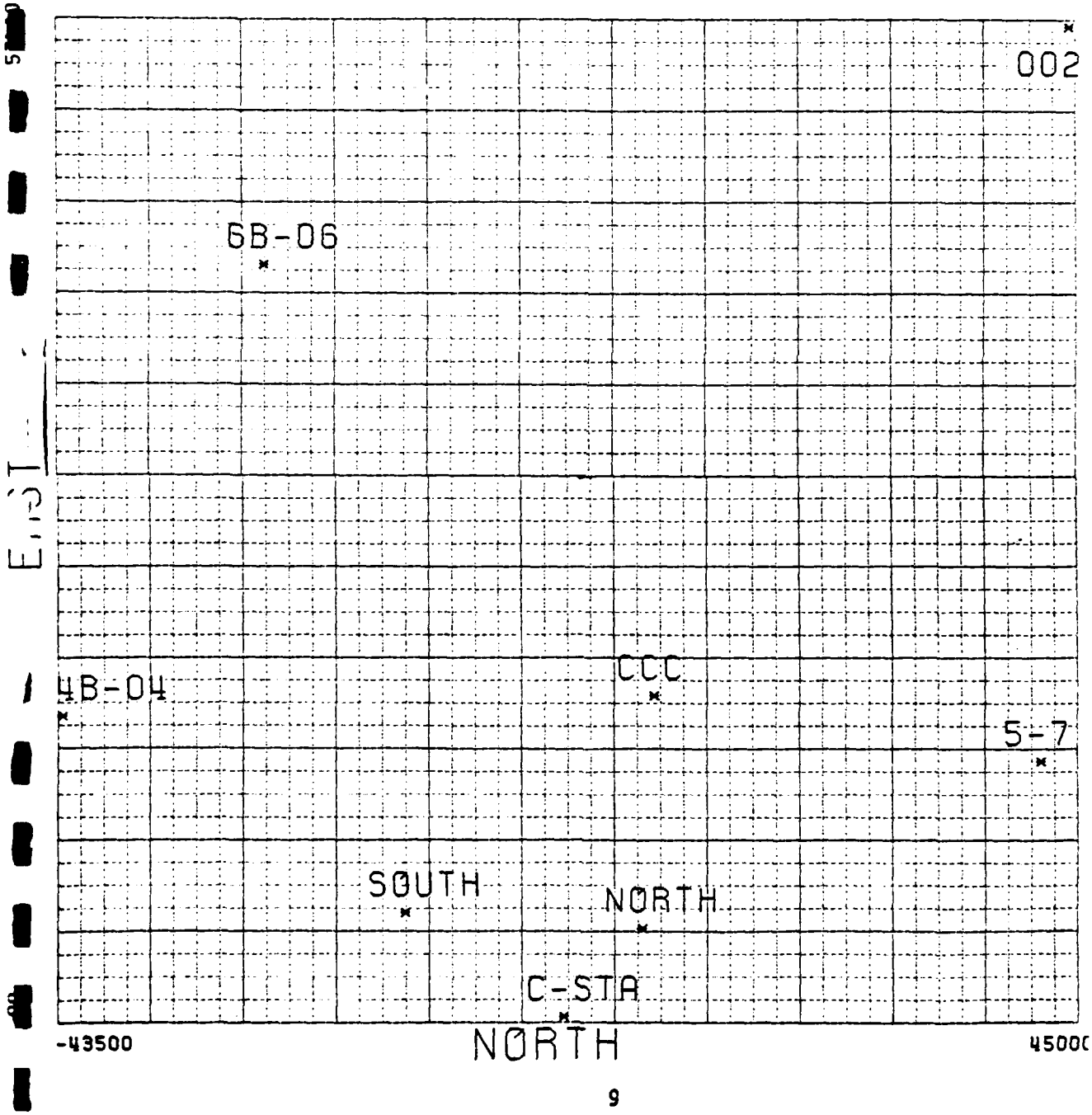
Then choose the direction for search as

$$\bar{d} = \begin{cases} -\mathrm{sign}(y^T F(x_o))y, & \text{if } o < \|F(x_o)\| < \varepsilon \\ y, & \text{if } \|F(x_o)\| = 0 \end{cases} \tag{43}$$

APPLICATION TO THE RMS/MTTS SYSTEM

The current configuration of the General Dynamics RMS/MTTS system located at MacGregor Range consists of eight range measuring receivers. The geometry of the receiver locations is given in the Fig 1. We will consider the convergence limits and position errors of the nonlinear least squares algorithm for target x,y positions within the field specified by the limits displayed in Fig. 1 and for target z values of z = 100, 200, 300, 500, 700, 1000, 1500, 2000, 3000, 5000 ft. The reference coordinate system for this description has the x, y plane tangent to the 1866 Clarke spheroid at C-station with x is positive East, y is positive North, and z is perpendicular to the x, y plane and positive upward. For purpose of evaluation we divide the field of Fig. 1 into cells by dividing both the x and y limits into ten equal intervals. Thus, we will evaluate the convergence limits and errors at 121 x, y grid points.

FIG. 1



We first consider the limits on the starting value of the z-coordinate of target position. Suppose we have exact measurements of the range from each of the eight range measuring sites to the target. We obtain a starting solution x_0, y_0, z_0, for the nonlinear least squares algorithm by assuming a z_0 close to the true z value and estimating x_0, y_0, using the linear least squares algorithm described in the introduction with $z = z_0$. Thus, we are using the linear least squares method in two dimensions. At each grid point on the field the nonlinear least squares algorithm will converge to a strong local minimum and if z_0 is close enough to the true z value this strong local minimum will be the true value of the target position. The limits on the starting z value for the algorithm to converge to the true target position when presented with exact range measurements are summarized in the following table. δz_u denotes the limit on how far above the true solution that z_0 can be and δz_L denotes how far below the true solution that z_0 can be. The notation (a, b) in the table indicates that the limit is somewhere between a and b and the notation >a indicates that the limit is greater than a.

z (ft)	δz_L (ft)	δz_u (ft)
100	(100,150)	(> 200)
200	(150,200)	(250,300)
300	(200,250)	(150,200)
500	(300,350)	(150,200)
700	(200,250)	(150,200)
1000	(0, 50)	(200,250)
1500	(150,200)	> 800
2000	(400,500)	> 3000
3000	> 1000	> 3000
5000	> 3000	> 5000

x

y

$z = 100 \quad z_0 = -100$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-200.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-100.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-200.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 100 \quad z_0 = -50$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-166.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 100 \quad z_0 = 0,50,100,---,300$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 200 \quad z_0 = 100$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-300.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-300.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-300.	-221.	0.	0.	0.	0.	0.	0.	0.

$z = 200 \quad z_0 = -50$

x

y

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-3	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-221.	0.	0.	0.	0.	0.	0.	0.

$z = 200 \quad z_0 = 0$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-221.	0.	0.	0.	0.	0.	0.	0.

$z = 200 \quad z_0 = 50 - 450$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-0.	0.	0.	0.	0.	0.	0.

$z = 200 \quad z_0 = 500$

0.	0.	531.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	435.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x

y

$z = 200 \quad z_0 = 550$

0.	0.	531.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	435.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	455.	0.	0.	0.	0.

$z = 200 \quad z_0 = 600$

0.	0.	531.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	435.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	567.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	455.	0.	0.	0.	0.

$z = 300 \quad z_0 = -100$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-537.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-365.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-400.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-400.	-400.	-400.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-395.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-400.	-362.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-400.	-400.	0.	0.	0.	0.	0.	0.	0.

$z = 300 \quad z_0 = -50$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-537.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-365.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-400.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	-400.	-400.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-400.	-362.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-400.	0.	0.	0.	0.	0.	0.	0.

$z = 300 \quad z_0 = 0$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-337.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-385.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-400.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-400.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-302.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-400.	0.	0.	0.	0.	0.	0.	0.

$z = 300 \quad z_0 = 50$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-337.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-400.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-302.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 300 \quad z_0 = 100 - 450$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 300 \quad z_0 = 500 - 600$

0.	0.	331.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

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x

y

z = 300 z_0 = 650

0.	0.	331.	619.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	534.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	547.	0.	0.	0.

z = 300 z_0 = 700

0.	0.	331.	619.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	534.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	674.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	547.	0.	0.	0.

z = 50 z_0 = 100

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	-477.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	-558.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-532.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-600.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-585.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-600.	-428.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-579.	0.	0.	0.	0.	0.	0.

z = 500 z_0 = 150

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	-477.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-428.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-579.	0.	0.	0.	0.	0.	0.

x

y

z = 500 z_0 = 200 - 650

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

z = 500 z_0 = 700

0.	0.	0.	192.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

z = 500 z_0 = 750

0.	0.	0.	192.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	423.	0.	0.

z = 500 z_0 = 800

0.	0.	0.	192.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	423.	0.	0.

x

y

z = 500 z_0 = 850

0.	0.	0.	192.	497.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	408.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	549.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	423.	0.	0.

z = 500 z_0 = 900

0.	0.	0.	192.	497.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	408.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	549.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	423.	708.	0.

z = 700 z_0 = 300

0.	-706.	-465.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-484.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-441.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-565.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	-572.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-415.	0.	0.	0.	0.

z = 700 z_0 = 350

0.	0.	-465.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-484.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-415.	0.	0.	0.	0.

x

y

$z = 700 \quad z_0 = 400$

0.	0.	-465.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-484.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 700 \quad z_0 = 450$

0.	0.	-465.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 700 \quad z_0 = 500 - 850$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 700 \quad z_0 = 900$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	303.	0.

$z = 700 \quad z_0 = 950$

x

y

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	421.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	303.	0.

$z = 700 \quad z_0 = 1000$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	421.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	303.	577.

$z = 700 \quad z_0 = 1050$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	519.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	421.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	303.	577.

$z = 700 \quad z_0 = 1100$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	707.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	519.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	493.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	421.	696.
0.	0.	0.	0.	0.	0.	0.	0.	0.	303.	577.

x

y

$z = 1000 \quad z_0 = 600$

0.	0.	0.	-763.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-390.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-660.	-387.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-535.	-278.	-23.

$z = 1000 \quad z_0 = 650$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-387.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-535.	-278.	-23.

$z = 1000 \quad z_0 = 700$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-387.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-535.	-278.	-23.

$z = 1000 \quad z_0 = 750$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-278.	-23.

$z = 1000 \quad z_0 = 800$

x →

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-778.	-73.

y ↓

$z = 1000 \quad z_0 = 850 - 950$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-73.

$z = 1000 \quad z_c = 1000 - 1200$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1000 \quad z_0 = 1250 - 1300$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	440.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x

y

$z = 1000 \quad z_0 = 1350$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	483.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	440.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1000 \quad z_0 = 1400$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	483.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	440.	733.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	508.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1500 \quad z_0 = 1100$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-393.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-557.	-233.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-684.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1500 \quad z_0 = 1150$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-393.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-557.	-233.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1500 \quad z_0 = 1200$

x →

y ↓

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-557.	-233.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1500 \quad z_0 = 1250 - 1350$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-233.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 1500 \quad z_0 = 1400 - 2300$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 2000 \quad z_0 = 1100$

0.	0.	0.	0.	0.	0.	-1711.	-1353.	-991.	0.	0.
0.	0.	0.	0.	0.	0.	-1780.	-1447.	-1124.	-789.	0.
0.	0.	0.	0.	0.	0.	-1869.	-1557.	-1253.	-944.	0.
0.	0.	0.	0.	0.	0.	0.	-1703.	-1403.	-1095.	-725.
0.	0.	0.	0.	0.	0.	0.	-1884.	-1578.	-1267.	-914.
0.	0.	0.	0.	0.	0.	0.	0.	-1769.	-1460.	-1132.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1658.	-1352.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1843.	-1554.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1730.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1884.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-2022.

$z = 2000 \quad z_0 = 1300$

x →

y ↓

0.	0.	0.	0.	0.	0.	0.	-1353.	-991.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-1447.	-1124.	-789.	0.
0.	0.	0.	0.	0.	0.	0.	-1557.	-1253.	-944.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-1403.	-1095.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1267.	-914.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1132.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 2000 \quad z_0 = 1500$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-1175.	-789.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-944.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-775.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$z = 2000 \quad z_0 = 1700 - 5000$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

The limits on the starting z solution given in the table are based on the convergence at all 121 (x, y) grid points of the field. The limits on z_0 for some (x, y) grid points may be much greater than the limits in the table. More detailed information on the convergence at each individual grid point is given in the preceeding tables. Each table entry gives the error between the converged z solution and the true z solution. The heading for each table is the true z solution and the starting solution z_0. The errors in the table which are non-zero are due to the convergence of the algorithm to a local minimum which is not the true z-value.

ESTIMATION ERROR DUE TO TRUNCATION

After correcting the measured MTTS ranges for lags and refraction, one of the most important errors remaining in the data is the two meter truncation error. This truncation error may be magnified many times by geometric factors as will be seen. This truncation error is a basic limitation of the MTTS instrumentation system. We evaluate the position error caused by the truncation at each of the 121 (x, y) grid points previously selected for each of the z-coordinates, z = 100, 200, 300, 500, 700, 1000, 1500, 2000, 3000, 5000 ft. The results are presented in the following tables. For each altitude a pair of error tables is given. The first error table is the error in the estimated z-coordinate for each of the 121 grid points in the field. The second error table gives the error in the estimated ground range, i.e., the square root of the sum of squared errors in the estimated x and y coordinates. The errors in both tables are given in feet. The z-coordinate estimation errors are summarized in Fig. 2 which plots the z estimation error averaged over the 121 (x, y) grid points, the minimum estimation error for these grid points, and the maximum estimation error for these grid points as a function of z. It is possible to improve the estimation errors due to the truncation by making a simple correction to the measured ranges.

z = 100 2m truncation

-99.	-55.	-4.	51.	41.	44.	46.	-44.	-17.	49.	-50.
84.	217.	142.	157.	55.	36.	32.	-13.	-16.	27.	-6.
78.	139.	258.	198.	49.	56.	57.	12.	-7.	14.	6.
98.	158.	206.	268.	171.	99.	68.	52.	72.	21.	59.
-24.	107.	168.	257.	402.	190.	81.	65.	11.	42.	-18.
-26.	88.	114.	214.	562.	335.	173.	129.	145.	125.	83.
-50.	-7.	16.	103.	362.	272.	218.	145.	129.	220.	31.
-151.	-57.	22.	199.	236.	380.	234.	206.	276.	54.	80.
-131.	-152.	7.	78.	201.	337.	457.	274.	167.	184.	56.
-63.	-90.	-93.	5.	132.	232.	280.	119.	199.	58.	47.
40.	109.	-43.	-49.	49.	149.	79.	160.	56.	58.	4.

x →

y ↓

7.4	5.3	3.3	1.6	3.2	2.8	4.8	3.4	2.5	3.0	4.3
3.2	5.5	3.3	6.2	3.4	4.5	2.3	4.4	4.8	3.2	4.0
3.7	3.4	5.5	7.7	3.5	3.3	4.5	3.1	3.6	3.0	6.0
2.6	4.3	6.4	7.6	3.6	5.5	3.6	4.4	5.2	4.3	5.3
4.2	.7	5.0	6.8	9.4	6.1	4.7	4.8	3.8	4.5	3.8
5.0	1.9	5.0	7.1	15.5	8.2	3.2	4.9	4.9	4.6	3.2
5.4	2.4	2.7	4.0	8.6	7.6	5.9	4.4	3.5	5.0	2.9
7.6	6.1	2.3	3.4	3.8	8.2	6.8	4.8	4.8	5.3	3.2
7.3	6.0	4.2	1.4	3.1	8.3	8.7	8.1	5.5	4.2	4.7
5.3	7.5	4.1	3.0	3.9	5.0	6.9	6.7	7.0	2.8	3.8
5.0	5.0	5.4	4.9	1.7	5.2	2.3	4.5	4.5	4.8	2.5

z = 200 2m truncation

-247.	-70.	10.	74.	116.	44.	43.	-5.	-20.	64.	-27.
-46.	95.	108.	181.	136.	93.	57.	12.	-6.	1.	-11.
-5.	39.	206.	140.	117.	95.	37.	11.	16.	23.	5.
-2.	58.	184.	301.	236.	97.	111.	62.	14.	-8.	53.
-115.	7.	118.	226.	302.	272.	95.	66.	76.	45.	75.
-109.	-15.	56.	160.	263.	294.	209.	147.	47.	158.	116.
1.	-21.	-45.	83.	262.	267.	343.	213.	175.	131.	202.
17.	-158.	-78.	99.	160.	235.	439.	265.	176.	233.	117.
-68.	-171.	-93.	-22.	101.	283.	307.	243.	251.	84.	74.
-66.	-21.	-159.	-79.	32.	173.	348.	365.	64.	36.	77.
-57.	9.	-15.	-9.	18.	139.	267.	256.	158.	50.	-0.

x →

y ↓

11.8	7.0	3.0	3.0	4.1	2.2	4.3	4.3	2.3	3.0	4.2
5.2	3.4	3.9	8.1	5.4	4.4	1.9	4.9	4.7	3.2	3.9
3.0	3.4	6.6	3.2	5.3	3.6	4.6	3.2	3.6	3.0	5.0
2.6	4.3	6.4	7.1	5.5	6.1	4.1	4.6	5.3	4.3	5.4
6.2	.7	3.9	6.9	9.4	8.5	6.1	5.1	5.0	4.4	3.9
5.7	.7	2.6	7.2	7.3	5.9	5.0	5.4	3.4	5.2	3.7
4.5	3.2	3.4	4.0	8.6	6.9	8.8	7.0	4.5	3.6	4.4
5.4	6.1	3.0	3.4	2.8	5.3	11.7	5.9	4.8	5.4	2.9
7.1	10.6	4.2	1.4	3.1	6.4	8.6	5.2	8.4	4.2	4.6
6.4	4.2	7.2	3.9	3.9	4.0	9.0	11.5	3.4	3.3	4.2
4.6	5.0	3.4	3.8	3.8	4.3	5.4	6.2	5.6	5.8	2.8

z = 300 2m truncation

-173.	22.	-12.	53.	142.	155.	34.	-2.	-28.	-36.	33.
-12.	42.	125.	185.	174.	157.	10.	24.	30.	-21.	-9.
38.	-2.	130.	235.	153.	81.	74.	3.	-5.	41.	24.
-54.	-103.	67.	201.	136.	58.	95.	77.	38.	68.	-30.
-108.	-93.	35.	126.	195.	148.	133.	166.	89.	128.	103.
-176.	-97.	-69.	92.	184.	194.	359.	235.	227.	58.	16.
-12.	-33.	-27.	-65.	148.	167.	243.	286.	273.	194.	124.
-88.	-155.	-13.	-1.	60.	135.	191.	165.	76.	125.	141.
-168.	-129.	-193.	-122.	1.	173.	258.	380.	160.	114.	63.
-32.	-96.	-126.	-182.	-79.	73.	248.	361.	121.	127.	101.
55.	-5.	-79.	-239.	-82.	89.	169.	156.	238.	4.	4.

x

y

9.2	4.3	5.3	2.5	4.2	3.4	4.1	4.3	4.2	3.1	3.9
5.7	4.3	2.1	5.9	6.3	5.4	2.7	5.1	3.8	3.3	3.9
4.0	4.9	3.9	5.7	6.6	4.1	5.3	2.4	3.8	3.3	5.0
7.8	4.1	2.6	7.1	5.5	3.1	4.8	5.9	5.8	3.7	4.1
6.7	.7	1.5	6.9	4.1	6.9	7.5	7.3	5.5	6.4	4.4
3.6	2.6	.4	5.4	7.4	5.9	9.2	7.5	6.3	5.2	3.7
3.8	3.4	3.2	1.5	5.4	6.9	8.8	7.0	7.1	4.6	3.1
5.4	1.7	3.2	3.4	2.8	5.3	5.7	5.9	4.8	4.0	4.0
7.1	7.6	4.2	3.2	3.1	4.6	8.8	9.5	6.2	4.0	3.8
4.7	2.4	5.6	6.6	3.1	4.0	9.0	9.1	5.0	5.9	3.9
5.7	4.0	3.1	8.8	3.8	3.8	5.4	6.2	8.1	5.0	3.4

z = 500 2m truncation

-50.	20.	-26.	106.	220.	121.	-16.	62.	16.	30.	27.
-30.	-106.	-33.	29.	348.	97.	66.	40.	-21.	-46.	44.
-174.	-41.	-98.	95.	142.	138.	85.	10.	7.	-27.	67.
11.	-231.	-128.	-9.	160.	237.	61.	91.	78.	58.	75.
-52.	-120.	-81.	-30.	120.	191.	112.	262.	187.	66.	49.
-110.	-110.	-108.	-82.	49.	175.	283.	303.	203.	211.	136.
5.	-62.	[illegible]	-37.	-44.	54.	262.	258.	429.	182.	346.
-91.	-124.	-163.	-1.	-120.	87.	131.	246.	392.	330.	122.
-105.	-96.	-268.	-148.	-166.	-116.	99.	215.	318.	265.	102.
-48.	-71.	-100.	-150.	-200.	-121.	20.	123.	278.	129.	95.
-46.	-10.	-13.	-5[illegible].	-128.	-154.	-17.	87.	88.	70.	2.

x

y

5.2	4.9	6.7	6.1	4.9	4.8	3.6	4.6	4.8	4.1	2.8
7.1	6.3	2.5	3.1	12.4	5.8	4.2	5.2	3.6	4.7	4.2
5.6	4.1	6.1	5.1	5.0	6.5	4.4	2.6	5.0	3.9	3.9
3.3	8.3	3.3	1.7	6.3	8.4	3.9	4.0	3.7	3.8	4.0
2.7	6.2	2.3	.6	5.5	6.4	3.8	7.5	8.6	3.5	5.6
4.1	3.0	2.8	1.8	4.4	6.1	10.5	9.7	5.8	6.6	5.4
3.9	2.2	1.3	.6	.9	4.4	7.4	8.4	13.8	6.8	9.0
3.7	5.7	3.7	2.1	2.3	5.6	5.2	9.1	11.7	10.0	5.8
2.4	3.3	11.2	5.4	6.4	3.8	3.7	7.5	9.5	7.2	3.7
1.8	5.7	7.0	3.8	4.8	4.2	3.7	5.4	7.9	4.7	5.6
3.6	3.1	5.4	4.4	7.2	5.2	4.7	5.9	2.9	3.9	4.8

z = 700 2m truncation

-132.	-95.	5.	-42.	20.	172.	-42.	71.	10.	84.	56.
-65.	-197.	-188.	-90.	24.	172.	-16.	40.	-22.	34.	17.
-13.	-86.	-248.	-157.	24.	142.	123.	9.	31.	80.	13.
-22.	-59.	-128.	-110.	-6.	107.	181.	103.	61.	-15.	68.
-28.	-100.	-201.	-116.	-110.	77.	96.	271.	186.	43.	106.
-41.	-79.	-67.	-70.	-145.	8.	8.	230.	388.	251.	183.
-8.	-45.	-30.	-3.	-80.	-103.	-29.	193.	114.	351.	164.
-58.	-71.	-75.	-123.	-157.	-204.	10.	139.	100.	285.	305.
-55.	-114.	-116.	-204.	-232.	-151.	-118.	25.	145.	311.	339.
-36.	-9.	-57.	-102.	-144.	-281.	-127.	-25.	84.	160.	82.
51.	32.	14.	-82.	-173.	-41.	-366.	-83.	23.	115.	-1.

x

y

4.3	3.0	2.9	3.9	4.9	9.1	2.9	4.4	4.5	5.1	4.5
4.0	4.7	8.2	3.0	4.1	8.3	5.4	5.1	2.9	3.3	3.5
4.2	4.6	10.4	7.0	3.1	5.0	5.8	5.0	1.3	4.0	4.0
4.3	5.4	6.2	1.5	2.3	4.8	7.0	5.9	4.3	3.8	3.6
4.8	5.2	5.0	2.3	1.8	5.2	7.3	9.2	5.4	4.5	4.7
3.5	2.7	.6	1.0	3.2	2.0	3.5	8.3	12.8	6.8	6.1
4.4	2.5	2.9	1.7	3.4	1.4	3.6	9.6	3.9	10.3	7.3
1.9	4.2	4.5	2.2	3.6	3.5	4.1	7.8	5.4	9.0	8.8
7.5	4.7	4.5	6.5	6.7	6.3	4.3	1.7	7.6	9.7	9.4
3.5	4.7	5.6	6.0	5.6	8.4	5.7	3.6	3.1	7.6	3.7
5.0	6.4	4.7	6.7	5.8	2.9	12.6	5.7	4.7	6.0	2.3

z = 1000 2m truncation

14.	12.	-9.	-48.	-43.	48.	-98.	17.	78.	124.	132.
-11.	-75.	-93.	-132.	-268.	-122.	63.	25.	-75.	-71.	50.
-120.	-61.	-139.	-166.	-179.	-128.	33.	51.	-33.	78.	19.
-18.	-36.	-153.	-174.	-163.	-156.	-46.	149.	224.	192.	-1.
-91.	-56.	-98.	-60.	-261.	-147.	-19.	-0.	229.	286.	284.
-42.	-57.	-72.	-78.	-74.	-145.	-198.	-52.	177.	246.	-3.
-9.	-26.	-14.	-6.	-39.	-76.	-232.	-106.	22.	29.	234.
-23.	-37.	-44.	-62.	-148.	-186.	-270.	-173.	-87.	73.	248.
-22.	-53.	-68.	-71.	-79.	-227.	-425.	-142.	-64.	-70.	227.
-7.	-25.	-27.	-61.	-94.	-66.	-137.	-290.	-144.	-70.	43.
4.	25.	-20.	-19.	-20.	-136.	-173.	-349.	-177.	-121.	-11.

x →

y ↓

3.6	3.6	4.1	3.8	1.5	3.6	3.3	4.7	4.9	4.3	5.4
4.4	6.4	3.2	6.8	7.0	3.0	6.4	3.6	3.5	3.2	3.0
2.3	5.8	4.2	7.9	3.0	3.0	3.6	5.3	3.7	6.0	2.9
2.3	3.9	7.3	5.1	6.0	3.6	1.0	8.3	9.1	8.2	4.4
3.2	3.6	3.9	.8	6.6	1.1	3.4	3.4	9.9	10.1	11.9
3.9	4.2	3.3	3.5	.1	3.0	2.6	1.6	8.0	11.0	5.5
4.0	1.6	1.2	1.7	1.6	1.8	5.4	2.8	2.4	2.7	8.3
3.4	3.1	2.8	3.4	6.9	6.7	7.2	2.1	2.0	6.0	10.6
4.9	6.2	5.0	2.8	1.7	5.5	10.5	.6	2.7	2.3	8.4
5.6	3.8	4.2	4.2	3.7	4.2	5.6	7.2	2.0	2.5	4.1
2.7	6.0	2.2	4.6	4.9	7.1	4.1	8.6	4.5	8.7	3.9

z = 1500 2m truncation

15.	-26.	-3.	-38.	-40.	-53.	41.	-276.	-31.	11.	197.
7.	-9.	-57.	-40.	-35.	-63.	-82.	-96.	186.	-176.	-61.
-46.	-30.	-28.	-79.	-70.	-134.	-96.	-32.	39.	52.	-157.
7.	-36.	-85.	-95.	-101.	-156.	-112.	-176.	-149.	1.	-139.
-17.	-25.	-58.	-58.	-51.	-106.	-305.	-250.	-263.	-138.	-6.
-24.	-47.	-20.	-51.	-49.	-30.	-87.	-42.	-77.	-223.	-259.
-8.	-19.	-2[illegible].	-14.	-52.	-1[illegible]8.	-141.	-149.	-284.	-271.	-221.
-16.	-47.	-3[illegible].	-4[illegible].	-41.	-8[illegible].	-184.	-166.	-277.	-225.	-142.
-20.	-3[illegible].	-51.	-57.	-6[illegible].	-57.	-110.	-120.	-258.	-125.	-259.
-26.	-9.	-2[illegible].	-25.	-75.	-117.	-100.	-142.	-53.	-122.	-222.
5.	-1.	2.	-22.	-40.	-51.	-44.	-36.	-38.	-14.	0.

x

y

5.0	4.9	5.0	4.8	2.5	4.2	4.3	5.5	3.2	5.7	10.1
5.2	4.4	1.5	3.2	2.2	2.3	1.9	1.8	9.4	3.4	2.4
1.0	3.7	3.5	2.6	2.7	4.3	2.1	4.0	5.0	4.8	2.9
4.3	2.8	2.9	4.4	4.8	4.1	1.5	1.0	.9	3.5	1.6
3.7	3.3	2.4	1.9	1.0	2.4	9.6	5.5	4.0	.8	3.5
2.7	2.4	.9	2.6	1.2	2.3	.9	1.1	.6	2.4	1.7
1.4	1.9	.7	1.2	1.0	3.3	4.9	4.1	8.5	4.0	3.2
1.8	5.3	.9	.8	3.2	3.7	7.4	5.9	6.9	2.9	1.1
2.7	3.7	6.7	2.9	2.9	2.4	4.5	4.1	6.9	.7	7.4
3.7	5.6	5.5	3.3	4.3	4.0	2.7	5.9	2.1	4.5	6.5
5.2	3.9	5.2	2.8	5.4	3.6	4.8	3.2	4.4	3.0	3.8

z = 2000 2m truncation

-7.	-2.	-6.	-2.	-45.	-26.	-7.	-62.	-111.	-192.	-448.
-5.	-32.	-27.	-48.	-39.	-36.	-35.	-24.	-65.	112.	-60.
2.	-20.	-32.	-38.	-46.	-38.	-49.	-5.	-4.	-87.	-241.
-1.	-20.	-20.	-61.	-70.	-65.	-79.	-62.	-77.	-68.	-218.
-11.	-9.	-45.	-56.	-46.	-110.	-127.	-118.	-156.	-161.	-237.
-15.	-23.	-35.	-38.	-27.	-44.	-108.	-78.	-191.	-63.	-176.
0.	-10.	-12.	-2.	-47.	-64.	-55.	-147.	-71.	-75.	-316.
-4.	-12.	-34.	-37.	-55.	-41.	-92.	-132.	-103.	-179.	-99.
-25.	-34.	-24.	-40.	-48.	-88.	-95.	-95.	-77.	-147.	-81.
-29.	-16.	-13.	-32.	-39.	-37.	-45.	-44.	-59.	-39.	-41.
3.	0.	1.	-13.	-45.	-49.	-32.	-49.	-17.	-25.	-2.

x

3.	3.	5.	2.	2.	3.	6.	2.	0.	4.	12.
6.	5.	3.	3.	2.	3.	3.	2.	4.	8.	2.
4.	5.	3.	3.	2.	3.	1.	3.	3.	4.	6.
4.	3.	2.	1.	3.	1.	2.	1.	2.	1.	5.
4.	3.	3.	3.	1.	3.	5.	4.	3.	4.	6.
4.	2.	2.	2.	1.	3.	5.	3.	6.	1.	3.
5.	2.	1.	2.	3.	3.	1.	4.	2.	2.	8.
5.	5.	1.	1.	3.	2.	3.	5.	2.	5.	1.
2.	3.	4.	2.	3.	3.	2.	5.	1.	7.	1.
1.	3.	3.	4.	4.	3.	2.	2.	4.	3.	2.
5.	3.	5.	7.	4.	3.	3.	5.	2.	2.	4.

y

z = 3000 2m truncation

-15.	-7.	-3.	-6.	-20.	-25.	-5.	-4.	40.	23.	-65.
-7.	-1.	-28.	-21.	-15.	-43.	1.	-17.	4.	-25.	-53.
-3.	-25.	-20.	-39.	-32.	-30.	-23.	-3.	1.	-4.	-86.
-4.	-16.	-36.	-21.	-29.	-13.	-21.	-31.	-54.	-33.	-85.
-5.	-11.	-30.	-24.	-23.	-41.	-34.	-59.	-46.	-47.	-129.
-7.	-13.	-18.	-17.	-14.	-47.	-58.	-58.	-93.	-37.	-56.
-5.	-13.	-4.	-11.	-23.	-42.	-71.	-44.	-19.	-99.	-38.
-17.	-15.	-21.	-32.	-31.	-41.	-45.	-27.	-76.	-34.	-43.
-22.	-9.	-18.	-26.	-26.	-41.	-28.	-52.	-52.	-33.	-71.
-16.	-10.	-9.	-23.	-22.	-40.	-31.	-29.	-36.	-41.	-23.
2.	-6.	-2.	-1.	-5.	-42.	-14.	-40.	-15.	-23.	-4.

x

3.	4.	4.	5.	2.	1.	3.	5.	6.	4.	2.
5.	4.	3.	1.	3.	2.	3.	2.	5.	2.	1.
4.	3.	3.	5.	2.	2.	4.	5.	5.	3.	4.
3.	6.	2.	2.	0.	2.	2.	1.	2.	3.	4.
3.	2.	4.	1.	1.	3.	2.	3.	2.	1.	6.
3.	4.	1.	0.	1.	4.	3.	2.	5.	2.	1.
3.	1.	2.	0.	1.	2.	2.	1.	1.	2.	2.
1.	1.	4.	2.	1.	4.	4.	3.	4.	1.	1.
2.	2.	3.	3.	2.	3.	2.	2.	1.	2.	4.
4.	2.	3.	6.	2.	4.	1.	3.	3.	3.	3.
4.	4.	4.	4.	5.	2.	1.	2.	4.	3.	5.

y

z = 5000 2m truncation

-18.	-1.	-6.	-6.	-7.	1.	7.	-16.	-11.	-9.	13.
-11.	-8.	-12.	-10.	-13.	-14.	1.	-5.	-5.	-20.	-17.
-24.	-2.	-11.	-14.	-17.	-5.	-9.	-7.	-8.	8.	-24.
-5.	-6.	-20.	-15.	-5.	-11.	-23.	-12.	-14.	-8.	-18.
-6.	-11.	-13.	-22.	-14.	-20.	-34.	-34.	-30.	-3.	4.
-7.	-16.	-6.	-14.	-13.	-26.	-17.	-3.	-16.	-24.	-42.
-3.	-9.	-4.	-9.	-8.	-16.	-14.	-33.	-24.	-23.	-13.
-8.	-14.	-13.	-17.	-17.	-7.	-31.	-42.	-14.	-22.	-44.
-6.	-7.	-23.	-20.	-12.	-28.	-28.	-21.	-29.	-33.	-43.
-5.	-4.	-5.	-6.	-7.	-16.	-14.	-18.	-10.	-22.	-8.
3.	-3.	-6.	-12.	-14.	-16.	-15.	-11.	-10.	-5.	-7.

x →

y ↓

1.	5.	4.	4.	4.	4.	5.	3.	3.	4.	5.
3.	1.	1.	3.	3.	3.	3.	4.	4.	1.	4.
1.	4.	1.	2.	2.	2.	3.	3.	2.	4.	3.
5.	4.	2.	2.	2.	1.	0.	3.	3.	4.	2.
6.	2.	1.	2.	2.	2.	1.	0.	2.	5.	5.
3.	5.	1.	0.	2.	3.	1.	2.	2.	1.	2.
4.	3.	1.	2.	1.	1.	3.	2.	2.	2.	2.
4.	2.	1.	4.	2.	1.	2.	4.	2.	1.	2.
5.	4.	3.	4.	2.	1.	3.	1.	3.	2.	3.
3.	3.	4.	3.	2.	1.	2.	2.	2.	2.	4.
7.	2.	4.	4.	5.	2.	3.	3.	6.	4.	3.

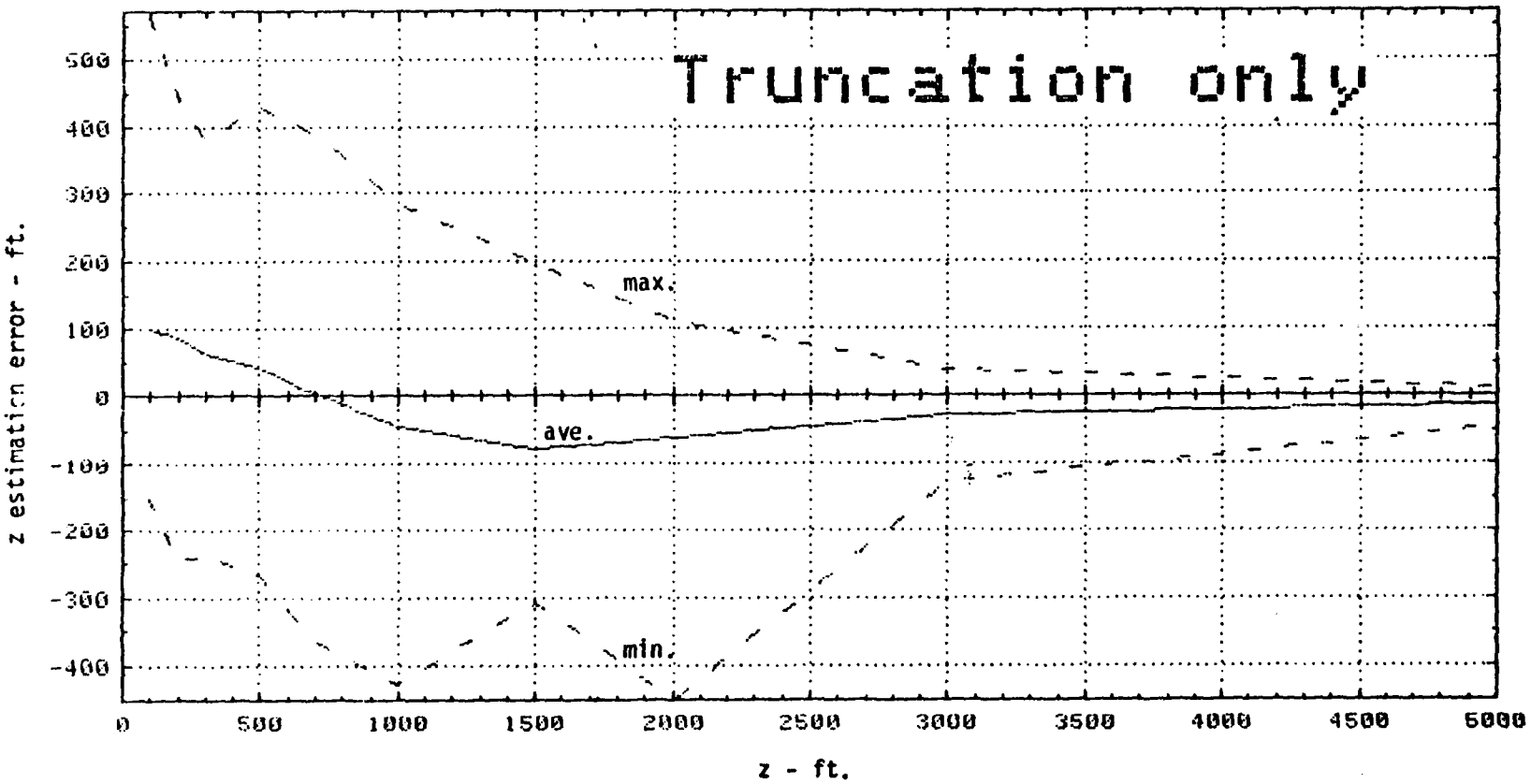
Truncation only
max.
ave.
min.
500
400
300
200
100
0
-100
-200
-300
-400
z estimation error - ft.
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
z - ft.

FIG. 2

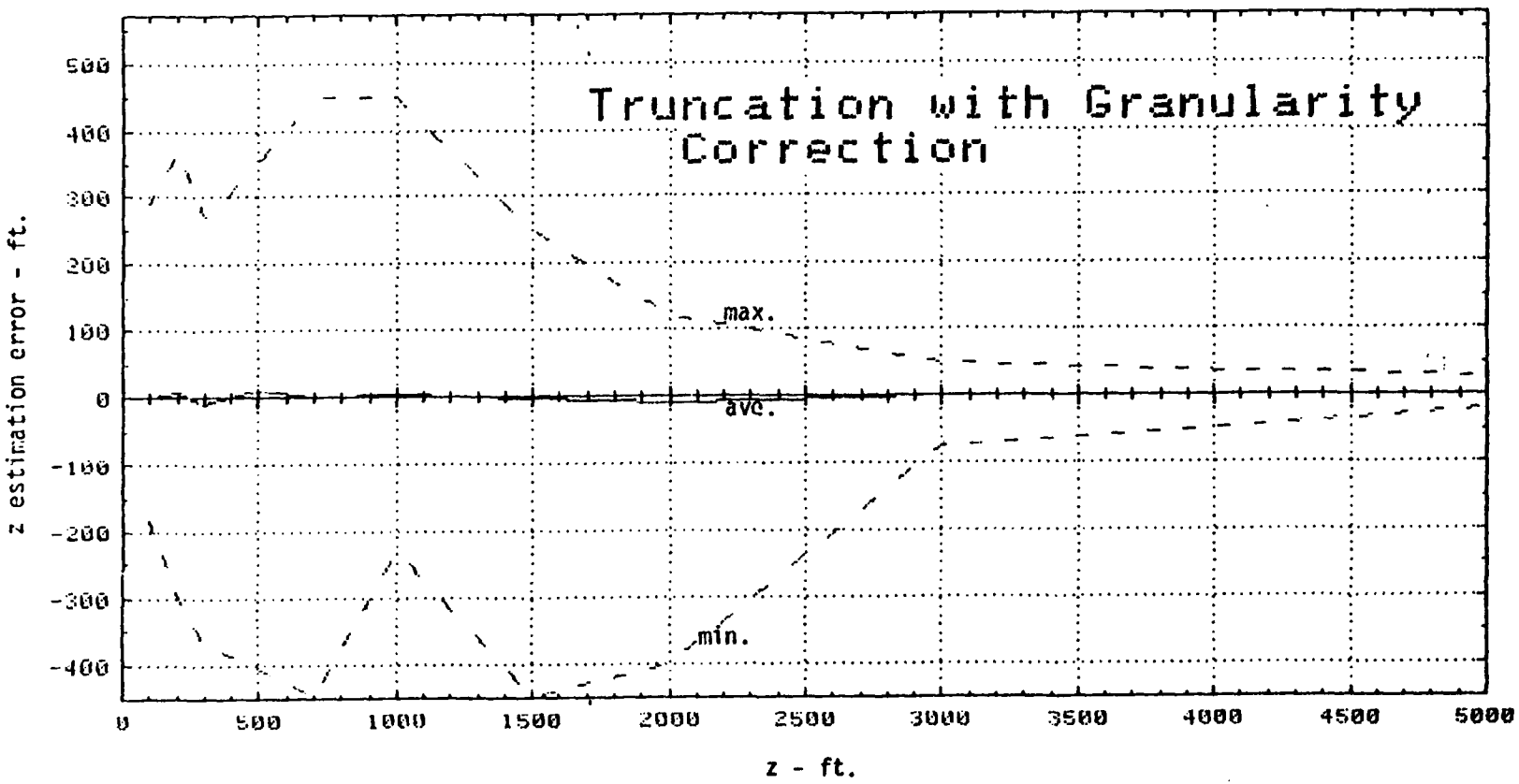
Truncation with Granularity Correction
max.
ave.
min.
z estimation error - ft.
500
400
300
200
100
0
-100
-200
-300
-400
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
z - ft.

FIG. 3

If half of the truncation error is added to each of the measured ranges, the expected value of the measurement errors will be reduced to zero under reasonable assumptions. We call this correction a granularity correction. The granularity correction for the RMS/MTTS system is one meter. The z-coordinate estimation error in the presence of a two meter truncation error and a one meter granularity correction is summarized in fig. 3 where the average, minimum, and maximum errors over the (x, y) grid points are plotted as a function of z. Comparison of figs. 2 and 3 show a significant improvement in the average estimation error when the granularity correction is applied to the measured ranges.

CONCLUSIONS AND FUTURE STUDIES

A globally convergent modified Newton method for multilateration has been presented. This algorithm was applied to the evaluation of the RMS/MTTS measurement system at MacGregor Range.

The preceeding tables and plots show some severe limitations to the use of the RMS/MTTS system at MacGregor Range. At low altitudes, even in the absence of any range measurement errors, a good nominal value of the z-coordinate of the trajectory must be available in order that the trajectory solution converge to the true z-solution. The MTTS range measurements have a two meter resolution. The effect of this two meter resolution on the MTTS Cartesian position solution was evaluated for a variety of z-coordinates and ground positions. The results of this evaluation show that the error in the ground plane position estimates caused by the truncation error to be acceptably small for all altitudes. However, at low altitudes the error in estimated z-coordinate is often unacceptably large, even when a perfect nominal z-value was used to start the solution. It was demonstrated how a correction could be applied to the range measurements to reduce the average error in the z-coordinate caused by the truncation error.

At higher altitudes, say above 3000 ft., the error in the estimate of the z-component of position is of a more acceptable magnitude and the modified Newton algorithm converges to the true z-solution in the absence of measurement error from a wide range of starting z-values.

The tables presented above support the conclusion that the error in the estimated ground plane position is acceptable at any z-value. Thus, the MTTS system offers a highly desirable measuring system for estimating the trajectory of ground targets, especially if a terrain map table look up is used to obtain the z-position from the estimated x, y position estimates. The conclusion that the MTTS is attractive for estimating trajectories for ground targets has been supported by comparing MTTS estimated position with positions of ground targets obtained from the ALTS laser tracker.

It has been suggested that the use of one or more airborne A-station receivers enhances the estimated z-component of position, especially at low altitudes. We have found this conclusion to be correct when using the modified Newton algorithm. The enhancement of the position estimates from the RMS/MTTS at MacGregor Range provided by using airborne A-stations is the subject of a forthcoming report.

It is also reasonable to suggest that the position estimate errors might be reduced by using a position estimation algorithm which in a sense averages over several x, y coordinates while estimating a constant z-position. Although the computation time using such an algorithm would be significantly greater, this increase might be justified by the reduced estimation errors. The use of such an algorithm will be investigated.

REFERENCES

(1) Jackson, H.F., Lechner, James A., and Sookne, David J., A System for Position-Location Based on Ranges, Proceedings of the Eighteenth Conference on the Design of Experiments in Army Research Development and Testing, (1973), pp549-577

(2) Gill, Phillip E., and Murray, Walter, Newton-Type Methods for Unconstrained and Linearly Constrained Optimization, Mathematical Programming, V7, (1974), pp 331-350