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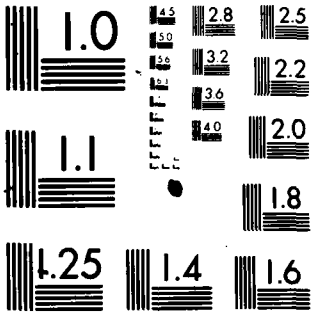

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WOLFQP, WOLFE'S METHOD FOR QUADRATIC PROGRAMMING

BY JOHN W. WINGATE,
JEFFREY S. YOUNGS

RESEARCH AND TECHNOLOGY DEPARTMENT

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This report documents a FORTRAN routine WOLFQP for solving quadratic programming problems. Theory, usage notes and examples are included in the report.

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FOREWORD

This report documents a FORTRAN routine WOLFQP for solving quadratic programming problems. Theory, usage notes and examples are included in the report.

Ira M Blatstein

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CHAPTER 1

WOLFE'S METHOD FOR QUADRATIC PROGRAMMING

Wolfe's quadratic programmming method[1] is based on solving a system of linear relations subject to complementarity conditions. The linear system (without the complementarity conditions) can be solved by the simplex method of linear programming. The pivoting rules of the simplex method can be restricted so that the complementarity conditions are met. When the quadratic form in the objective function is positive definite (for a minimization problem) the pivoting restrictions do not prevent the simplex method from solving the linear system.

The linear system is based on the Karush-Kuhn-Tucker conditions (or Lagrange multiplier rule) for the problem to be solved. Let this problem be:

$$\text{minimize } 1/2 \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} p_1^T & p_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + C$$

subject to the constraints

$$x_1 \geq 0 \qquad (n_1 \text{ variables}),$$

$$x_2 \text{ unconstrained } (n_2 \text{ variables}),$$

$$\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq b_1 \ (m_1 \text{ constraints}),$$

$$\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b_2 \ (m_2 \text{ constraints}).$$

The constant term C is included for evaluation purposes only, it does not affect the minimizing point. If there are no unconstrained variables ($n_2 = 0$)

the matrix $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$ reduces to Q_{11} and $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ reduces to $\begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix}$

Similar interpretations apply when $n_1 = 0$, $m_1 = 0$ or $m_2 = 0$. Inequalities apply componentwise; i.e., $x_1 \geq 0$ is equivalent to $x_{1j} \geq 0$, $j = 1,\ldots,n_1$. Q is positive definite.

Introduce Lagrange multiplier vectors κ_1 for the constraint $x_1 \geq 0$ and λ_1 and λ_2 for the other constraints, and form the Lagrangian

$$
\begin{aligned}
L = 1/2 \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} p_1^T & p_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + C \\
+ \kappa_1^T \quad x_1 \\
+ \lambda_1^T \left(\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_1 \right) \\
+ \lambda_2^T \left(\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_2 \right)
\end{aligned}
$$

from which, by the Lagrange multiplier rule for inequality constrained problems, we get the linear conditions

$$
\kappa_1 + \begin{bmatrix} Q_{11} & Q_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} A_{11}^T & A_{21}^T \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + p_1 = 0,
$$

$$
\begin{bmatrix} Q_{12} & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} A_{12}^T & A_{22}^T \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + p_2 = 0,
$$

along with the original constraints

$$x_1 \geq 0,$$

$$\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq b_1,$$

$$\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b_1,$$

the sign constraints on the multipliers for the inequalities

$$\kappa_1 \leq 0,$$

$$\lambda_1 \geq 0,$$

and the complementary slackness condition

$$\kappa_1^T \; x_1 + \lambda_1^T \left(\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_1 \right) + \lambda_2^T \left(\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_2 \right) = 0.$$

Since each of the $n_1 + m_1 + m_2$ terms in this sum is nonpositive by the constraints, each must individually vanish:

$$\kappa_{1j} = 0 \text{ or } x_{1j} = 0, \; j = 1, \ldots, n_1,$$

etc.

An alternate phrasing of this system of linear inequalities and equations can be made if we introduce auxiliary vector variables κ_2, S_1 and S_2.

Find a solution to:

$$\begin{bmatrix} I & 0 & 0 & 0 & Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ 0 & I & 0 & 0 & Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ 0 & 0 & I & 0 & A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & 0 & I & A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ S_1 \\ S_2 \\ x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -p_1 \\ -p_2 \\ b_1 \\ b_2 \end{bmatrix} \tag{1}$$

$$\kappa_1 \leq 0,\ \kappa_2 = 0,\ S_1 \geq 0,\ S_2 = 0,\ x_1 \geq 0,\ \lambda_1 \geq 0 \ . \tag{2}$$

and

$$\kappa_1^T\ x_1 = 0,\ S_1^T\ \lambda_1 = 0 \ . \tag{3}$$

(The complementary slackness conditions (3) again require each term in the scalar products $\kappa_1^T\ x_1$ and $S_1^T\ \lambda_1$ to vanish.) If we ignore the complementary slackness conditions, we can solve (1) and (2) by minimizing a penalty function

$$P = \sum_{j=1}^{n_1} \max\{0,\kappa_{1j}\} + \sum_{j=1}^{n_2} |\kappa_{2j}| + \sum_{j=1}^{m_1} \max\{-S_{1j},0\} + \sum_{j=1}^{m_2} |S_{2j}|$$

$$+ \sum_{j=1}^{n_1} \max\{-x_{1j},0\} \qquad + \qquad \sum_{j=1}^{m_1} \max\{-\lambda_{1j},0\}$$

Subject to (1) (and no sign constraints). The minimum value of P is 0 if and only if (1) and (2) are consistent. Of course, it is not necessary to weight all the sign constraint violations equally. For example, we could replace $|S_{2j}|$ by $\max\{-a\ S_{2j},\ b\ S_{2j}\}$ where a and b are positive. Changing the weights in this manner does not affect the equivalence of the consistency of (1) and (2) to the minimum being zero. A slightly different procedure is normally used (in a simplex method Phase I). The identity matrix in the first $(n_1 + n_2 + m_1 + m_2)$ columns is used as an initial basis for the primal simplex method, any constraint violations resulting therefrom are penalized, and sign constraints are retained

in such a way that the penalty function is linear. For example, if $b_{2j} > 0$, $S_{2j} = b_{2j}$ in the initial basic solution. The term $\cdots + S_{2j} + \cdots$ appears in the penalty and the restriction $S_{2j} \geq 0$ is imposed; S_{2j} is termed an "artificial variable" in this case. (If b_{2j} were negative, the penalty would be $-S_{2j}$ and S_{2j} kept nonpositive.) For S_1 the treatment is slightly different. If $b_{1j} \geq 0$, set $S_{1j} = b_{1j}$, with no penalty. If $b_{1j} < 0$, divide S_{1j} into the sum of a slack variable S_{1j} (unpenalized, nonnegative) and an artificial variable $S_{1j}{}^a$ (penalized, nonpositive), set to b_{1j} initially. This procedure is, however, equivalent to changing the weights in P. Using $b_{2j} > 0$ again as an example, we could change the weighting so that the penalty on S_{2j} changes from $|S_{2j}|$ to $\max\{(-\infty)S_{2j}, S_{2j}\}$ and in practice we could replace ∞ by a suitable large positive quantity M.

It is not actually necessary to use a linear cost functional. With a little care in programming, the usual simplex method can handle cost functionals like P. The linear programming code LINOPT[2] uses the dual simplex method to solve problems of maximizing a linear functional subject to upper and lower bounds on all variables and constrained quantities. The problem of minimizing P subject to (1) is the dual of such a problem, which could be solved using LINOPT, with the effect of minimizing P subject to (1) by the primal simplex method.

Let us now consider the complementary slackness conditions (3). In terms of the primal simplex method, they can be enforced by ensuring that S_{1j} and its complementary variable λ_{1j} (κ_{1j} and x_{1j}) are not simultaneously basic. This is an easily enforced pivoting restriction. Unfortunately it is too strong to be applied to minimizing P subject to (1) directly: the algorithm may terminate with $P > 0$ even though (1), (2) and (3) are consistent. We get around this difficulty by using a phase I/phase II procedure. Suppose we split κ_1 into a multiplier κ_1^m, subject to complementary pivoting restrictions, and an artificial variable κ_1^a, not so restricted: $\kappa_1 = \kappa_1^m + \kappa_1^a$. Since κ_{1j}^m and κ_{1j}^a cannot both be basic, one of them is always zero and nonbasic.

We can let κ_{1j} be whichever of them is basic (if one is) and keep track of whether this is κ_{1j}^a or κ_{1j}^m.

Suppose now that we have a basic feasible solution to the constraints on x. Then

$$\begin{bmatrix} I & 0 & A_{11} & A_{12} \\ 0 & I & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$S_1 \geq 0,\ S_2 = 0,\ x_1 \geq 0$$

Extend this basis to the constraint matrix for (1) by making κ_1^a and κ_2 basic. If we now minimize $P' = \sum_{j=1}^{n_1} |\kappa_{1j}^a| + \sum_{j=1}^{n_2} |\kappa_{2j}|$ subject to (1) and

$\kappa_1 = \kappa_1^a + \kappa_1^m$, $\kappa_1^m \leq 0$, $x_1 \geq 0$, $S_1 \geq 0$, $S_2 = 0$, $\lambda_1 \geq 0$ and the complementary pivoting restriction, the minimum will be zero, and we will have solved (1), (2) and (3). (The proof of this fact requires the positive definiteness of Q.)

If we wish to enforce the sign constraints by penalty terms added to P' these terms must be so heavily weighted that no change of basis that reduces P' can introduce sign constraint violations.

The phase I is to find the initial basic feasible solution. It is not necessary to work on a smaller problem using only the [I A] part of the full coefficient matrix $\begin{bmatrix} I & 0 & Q & A^T \\ 0 & I & A & 0 \end{bmatrix}$. If we allow κ_1 and κ_2 to be basic and unconstrained in phase I, they will be basic throughout phase I and the initial basis inverse for Phase II will be the final basis inverse for phase I.

Phase I problem:

$$\text{minimize } P_I = M \left\{ \sum_{j=1}^{m_1} \max\{-S_{1j},0\} + \sum_{j=1}^{m_2} |S_{2j}| + \sum_{j=1}^{n_1} \max\{-x_{1j},0\} + \sum_{j=1}^{m_1} \max\{-\lambda_{1j},0\} \right\}$$

subject to:

$$\begin{bmatrix} I & 0 & 0 & 0 & Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ 0 & I & 0 & 0 & Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ 0 & 0 & I & 0 & A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & 0 & I & A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ S_1 \\ S_2 \\ x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -p_1 \\ -p_2 \\ b_1 \\ b_2 \end{bmatrix}$$

where $M >> 1$. Since M scales out of the problem it is not actually needed here. It is included as part of the setup for phase II. A similar comment applied to the penalty on λ_1 which remains nonbasic and zero throughout phase I (and could therefore be ignored). Starting basic variables are κ_1, κ_2, S_1 and S_2. The program actually sets up and solves the dual to this problem. Let k_1 be a vector of variables dual to κ_1, k_2 dual to κ_2, σ_1 dual to S_1, σ_2 dual to S_2, ξ_1 dual to x_1, ξ_2 dual to x_2, ℓ_1 dual to λ_1 and ℓ_2 dual to λ_2. For both phases the dual variables are related by the transpose of the coefficient matrix:

$$\begin{bmatrix} k_1 \\ k_2 \\ \sigma_1 \\ \sigma_2 \\ \xi_1 \\ \xi_2 \\ \ell_1 \\ \ell_2 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \sigma_1 \\ \sigma_2 \end{bmatrix} \tag{4}$$

The dual phase I problem is to maximize $z = [-p_1^T \; -p_2^T \; b_1^T \; b_2]\begin{bmatrix} k_1 \\ k_2 \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$ subject to the bounds:

$$k_1 = 0,\ k_2 = 0,\ -M \le \sigma_1 \le 0,\ -M \le \sigma_2 \le M$$

$$-M \le \xi_1 \le 0,\ \xi_2 = 0,\ -M \le \ell_1 \le 0,\ \ell_2 = 0$$

(The bounds on ℓ_1 and ℓ_2 are obviously always satisfied in phase I.)
The objective function is defined by the right-hand side of (1). The bounds are defined by the penalties in P_I: the bounds for a dual variable deriving from the penalties on the corresponding variable. Violation of nonzero bounds in phases I and II are ignored: they can be made to go away by increasing the penalty on the appropriate variable and we could have set up the penalty function in this way in the first place. Furthermore, ignoring violation of nonzero bounds yields a more efficient procedure.

Assuming that phase I ends with feasibility shown, we start phase II by changing the penalty from P_I to

$$P_{II} = P_I + \sum_{j=1}^{n_1} |\kappa_{1j}^a| + \sum_{j=1}^{n_2} |\kappa_{2j}| + \sum_{j=1}^{n_1} \max\{0, \kappa_{1j}^m\},$$

or equivalently, changing the bounds on k_1 and k_2. For k_2 this is easy: $-1 \le k_2 \le 1$, but for k_1, setting the bounds is complicated by the implicit handling of the split of κ_1 into $\kappa_1^m + \kappa_1^a$. Let us start by making each artificial variable κ_{1j}^a basic, and treat k_{1j} as its dual, with bounds $-1 \le k_{1j} \le 1$. Then k_{1j} will initially be either +1 or -1 (depending on the sign of κ_{1j}^a). Ordinarily, without the pivoting restriction, if $k_{1j} = k_{1j}^a = -1$, the lower bound (zero) on $k_{1j}^m = k_{1j}^a = k_{1j}$ would be violated, and we could pivot k_{1j}^m into the basis with value zero, with no change in the basis inverse matrix. With the pivoting restriction, we can still do this if ξ_{1j} is not basic (and zero--but it will be zero if basic because we ignore violation of nonzero bounds). Since we want to reduce artificial variables to zero, we might as well do so at the start. Thus the initial setup of bounds on k_1 are: $r_j \le k_{1j} \le 1$

where $r_j = -1$ if ξ_{1j} is basic at the end of phase I,

$r_j = 0$ if ξ_{1j} is nonbasic at the end of phase I.

In proceeding through phaseII, we should interpret k_{1j} as dual to k_{1j}^m as soon as possible. Thus r_j should be set to zero whenever ξ_{1j} leaves the basis, not only the lower bound, but k_{1j} itself should be reset.

Since each variable in ℓ_1 is nonbasic at the start of phase II, the complementarity restrictions on ℓ_1 and ℓ_1 cause no problems in setting up for phase II.

Comments in the modified LINOPT code referring to pivoting restrictions are given in terms appropriate to the dual problem (4) with phase II bounds. A further thing to note is that the negation of p_1 and p_2 in the objective row

$$[-p_1^T \quad -p_2^T \quad b_1^T \quad b_2^T]$$

is handled internally, so that this vector should be passed as

$$[\ p_1^T \quad p_2^T \quad b_1^T \quad b_2^T\ ].$$

CHAPTER 2
USER'S NOTES

WOLFQP is a FORTRAN Language computer code written to solve problem (1) with the method described in the previous section. The code is a modified version of LINOPT which takes advantage of the structure in the Quadratic Programming problem to reduce the storage required. Also the user is only required to input the QP problem, it is translated by WOLFQP into the Linear Programming Problem.

Communication with the calling program is accomplished through the calling statement. The formal parameters are described thoroughly in the internal documentation (See Appendix A). This section therefore will only attempt to clarify some of the murkier details.

The scratch array SCR must be dimensioned at least $N^2 + 13N + 6$ ($N = N_1 + N_2 + M_1 + M_2$; where N_1 = Number of sign constrained variables, N_2 = Number of sign unconstrained variables, M_1 = Number of unequality constraints, M_2 = Number of equality constraints). Failure to do this will cause memory to be overwritten and result in indeterminate output. The scratch array is divided into the internal arrays required of LINOPT. The correlation between these internal variables and positions in SCR are detailed in the internal documentation (See Appendix A). For detailed descriptions of the internal variables see reference [1].

It is noted that the unknown vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in problem (1) has sign constrained x_1 and sign unconstrained x_2 variables. For ease of notation the constrained and free variables are grouped separately. WOLFQP does not require the user to so order the variables. The integer vector IND allows the user to permute the variables without permuting the Q, A1 and A2 matrices. This allows the user to form the matrices in a convenient manner.

The First N_1 locations of IND contain the indices for which there are sign constraints. Locations $N_1 + 1 \ldots, N_1 + N_2$ contain the indices for the free variables.

Also it may be inconvenient to reform the constraint matrices for a call to WOLFQP. As an example the user may wish to call WOLFQP with only some of the constraint rows active. Locations $N_1 + N_2 + 1 \ldots N_1 + N_2 + M_1$ of IND contain the indices of the active rows, if any, in the inequality constraint matrix A1. In the same manner locations $N_1 + N_2 + M_1 + 1 \ldots N_1 + N_2 + M_1 + M_2$ of IND contain the indices of the active rows, if any, in A2; the equality constraint matrix.

If the value of the objective function is desired the user should set the formal parameter VALUE to a non-zero value. This results in the calculation of the objectives function value at the optimal solution. The Scalar C is ignored except in this calculation.

Two inputs which control the algorithm are ITMAX and NINVT, corresponding to IPASS (2) and IPASS(8) respectively. ITMAX is the iteration limit. Upon completion of ITMAX iterations control is returned to the calling program with IERR (IPASS (4)) set to 2. All internal storage has been saved. If desired WOLFQP may be recalled and computation continued by calling WOLFQP with FLAG set to true. No other change is necessary or advised.

On output the optimal distribution is placed in the array Y. The value is calculated, if desired. The iteration number and error indicator are stored in the appropriate positions of IPASS (See Appendix A). Also if the user desires to look at the internal variables SCR contains these values.

Other error conditions (IERR = 1,3,4+) must be corrected before re-calling WOLFQP. For these conditions the tableau must be re-initialized. (Flag set to False).

An upper limit on ITMAX should be between 5*N and 10*N. Some problems may take more; most should take fewer iterations. Practically ITMAX should be set so that an inordinate amount of CP time is not consumed before the problem formulation has been thoroughly checked.

NINVT controls the re-inversion of the tableau. Since succeeding inverses are formed as perturbations of previous inverses truncation error can accumulate in the inverse. To remedy this situation the inverse must be re-formed. Every NINVT iterations the inverse will be reformed. To avoid unnecessary calculations this should not occur too frequently. The recommended value is between 2 x N and 3 x N.

The final input controlling calculations is EPS1. This is used to control zero tolerance. Zero tolerance is important in two parts of the algorithm. Tableau entries which are less than EPS1 in magnitude are treated as zero.

Their values are considered to be truncation error. Thus for this purpose EPS1 is set to the truncation error of the machine.

The other spot where zero tolerance is involved is when constraint violations are checked. Constraint violations less than EPS1 are treated as zero. This is also to avoid truncation error but the user must be careful. If the problem is scaled such that entries in the tableau are less than EPS1 their entries will be ignored. For this purpose EPS1 must be on the order of the smallest entries in the tableau (Q, A_1 and A_2). The input value for EPS1 should be the minimum of these two values.

The major modification to LINOPT occurs in PIVROW. This subroutine chooses the incoming basic variable which is required to meet the complementary slackness condition. Flags contained in the internal array BASIC are used to facilitate this checking.

Each word in the BASIC array contains seven bits used as flags and if the complementary slackness condition applied to the variable, (a complementary index). Bits are used to reduce the storage requirement. The layout of each basic word is described in the internal documentation of SETQP. (See Appendix A).

The Bit operations OR, AND, SHIFT and COMPLEMENT are required to access and define the flag bits. These operations are described more thoroughly in machine dependencies (Appendix C).

The Flag Bits are also used in DSIMP. In DSIMP the basic or non-basic bit is set as appropriate and bounds are reset, if applicable, on variables leaving the basis.

CHAPTER 3

EXAMPLES

A short test program appears at the end of the source listing. (Appendix A). The problem is defined by the NAMELIST inputs. Mnemonic names are used in the NAMELIST which are then loaded into the IPASS array. The following examples list the NAMELIST inputs and the output resulting from a subsequent call to QPTAB.

Example 1.

This problem illustrates the basic use of the algorithm.

$$\min \; 1/2 \; [x_1, x_2] \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [6 \; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

s.t.

$$x_1, x_2 \geq 0$$
$$x_1 \leq 2$$
$$x_2 \leq 1$$
$$x_1 + x_2 = 2$$

This problem requires the following inputs

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  IND = 1,2,1,2,1,
  N = 5,
  NCON = 2,
  NUNC = 0,
   NINEQ = 2,
  NINV = 3000,
  ITMAX = 100,
  Q = 4., -2.,
  Q(1,2) = 2.,4.,
  A1 = 1., 0.,
  A1(1,2) = 0.,1.,
  A2(1,1) = 1.,
  A2(1,2) = 1.,
  OBJ = 6.,0.,2.,1.,2.,
  C = 0.,
  VALUE = 1.,
$END
```

output of QPTAB after call to WOLFQP

```
$OUT
N      =  5,
NCON   =  2,
NUNC   =  0,
NINEQ  =  2,
NEQ    =  1,
NINV   =  3000,
ITMAX  =  100,
ITER   =  2,
IERR   =  0,
C      =  0.0,
VALUE  =  .8E+01,
$END
        Q
  .4000E+01   -.2000E+01
 -.2000E+01    .4000E+01


        P
  .6000E+01   0.

   A1*X          B1                A1
  .1000E+01    .2000E+01    .1000E+01  0.
  .1000E+01    .1000E+01   0.           .1000E+01


   A2*X          B2                A2
  .2000E+01    .2000E+01    .1000E+01   .1000E+01


        X
  .1000E+01    .1000E+01
```

Basically the output reprints the inputs with the addition of the solution vector X = [1.,1.], the value of the function VALUE = 8., (VALUE on input was non-zero). The number of iterations required ITER = 2, the error indicator IERR = 0 indicating solution and each constraint is evaluated as a check. (See A1*X and A2*X columns.

Example 2.

This example illustrates how sign constraints can be written implicitly or explicitly. The inputs differ slightly. The explicit version requires more iterations.

$$\min 1/2\ [x_1, x_2, x_3, x_4] \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 4 & 6 & 4 \\ 6 & 6 & 61 & 12 \\ 1 & 4 & 12 & 100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0, & 0, & 0, & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Inputs for Problem 1

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  OBJ = 10*0.,
     N = 4.,
  NCON = 4,
  NUNC = 0,
  NINEQ = 0,
  NINV = 2000,
  ITMAX = 100,
  Q(1,1) = 1.,0.,6.,1.,
  Q(1,2) = 0.,4.,6.,4.,
  Q(1,3) = 6.,6.,61.,12.,
  Q(1,4) = 1.,4.,12.,100.,
  OBJ = 0.,0.,0.,-1.,
  C = 0.,
  VALUE = 1.,
  IND = 1,2,3,4,
$END
```

One notes that the sign constraints are entered implicitly and are the only constraints.

Outputs:

```
$OUT
N      =  4,
NCON   =  4,
NUNC   =  0,
NINEQ  =  0,
NEQ    =  0,
NINV   =  2000,
ITMAX  =  100,
ITER   =  1,
IERR   =  0,
C      =  0.0,
VALUE  =  -.5E+02,
$END
```

```
        Q
  .1000E+01   0.          .6000E+01   .1000E+01
 0.           .4000E+01   .6000E+01   .4000E+01
  .6000E+01   .6000E+01   .6100E+02   .1200E+02
  .1000E+01   .4000E+01   .1200E+02   .1000E+03

        P
 0.           0.          0.          -.1000E+01

        X
 0.           0.          0.           .1000E-01
```

Note only 1 iteration is required.

The following inputs solve the same problem, but x_1 and x_2 are implicitly constrained. x_3 and x_4 have explicit constraints in the A1 matrix.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  A1 = 300*0., A2 = 300*0., Q=900*0., OBJ = 30*0.,
  OBJ = 10*0,
     N = 6,
  NCON = 2,
  NUNC = 2,
  NINEQ = 2,
  NINV = 2000,
  ITMAX = 100,
  Q(1,1) = 1.,0.,6.,1.,
```

```
Q(1,2) = 0.,4.,6.,4.,
Q(1,3) = 6.,6.,61.,12.,
Q(1,4) = 1.,4.,12.,100.,
A1(1,1) = -1.,0.,0.,0.,
A1(1,2) = 0.,-1.,0.,0.,
A1(1,3) = 0.,0.,-1.,0.,
A1(1,4) = 0.,0.,0.,-1.,
OBJ = 0.,0.,0.,-1.,
C = 0.,
VALUE = 1.,
IND = 1,2,3,4,3,4,
$END
```

Output:

```
$OUT
N      =  6,
NCON   =  2,
NUNC   =  2,
NINEQ  =  2,
NEQ    =  0,
NINV   =  2000,
ITMAX  =  100,
ITER   =  3,
IERR   =  0,
C      =  0.0,
VALUE  =  -.5E-02,
$END

        Q
  .1000E+01    0.           .6000E+01    .1000E+01
 0.            .4000E+01    .6000E+01    .4000E+01
  .6000E+01    .6000E+01    .6100E+02    .1200E+02
  .1000E+01    .4000E+01    .1200E+02    .1000E+03

        P
 0.            0.           0.           -.1000E+01

   A1*X            B1              A1
 0.            0.           0.           0.           -.1000E+01   0.
 -.1000E-01    0.           0.           0.           0.           -.1000E+01

        X
 0.            0.           0.           .1000E-01
```

Note the solution is the same as the implicitly constrained example but this solution required 3 interations instead of 1 the problem has increased in size

from N = 4 to N = 6 thus requiring additional storage.

If one wishes to continue in this vein, all sign constraints can be made explicit. The dimension of the problem becomes N = 8. The same solution now requires 7 iterations. The reader may verify this if desired.

Example 3

This is an example displaying the restart capabilities of the code. Since the Q matrix is the identity this is a constrained least squares problem of a particularly simple type. By following the directions in Appendix B the user can eliminate the need to store the identity matrix in memory.

This problem arises as a sub-problem of the feasible direction method. What is required is to find the direction of steepest descent subject to the binding constraints. The feasible direction method can at times require many constrained gradients to be calculated. The constraint matrix is usually full throughout the process but which constraints are binding depend on the given position where the constrained gradient is to be calculated.

This motivates the use of the linked list IND. First to indicate in the first N_1 positions the indices of the sign constrained variables. In the following N_2 positions the indices of sign unconstrained variables. Thus the variables need not be ordered constrained followed by unconstrained. The next M_1 positions of IND are filled with the row indices of the active inequality constraints and finally the last M_2 positions are filled by the indices of the active equality constraints.

Thus for the feasible direction method the full constraint matrix is formed once. Afterward only the row indices of the matrices must be manipulated to pass the current constraint set. Note the objective row containing the right hand side of the constraints must be reformed each time it changes.

The gradient projection problem can be stated:

$$\min_{\hat{g}} 1/2 < \hat{g} - g(x), \hat{g} - g(x) >$$

where g(x) is the gradient at x and $\hat{g}$ is the constrained gradient.
Subject to

$$\hat{g}_i \geq 0 \text{ for all } x_i = 0$$

Maintaining x_i as non-negative

$$A_1 \hat{g} \leq 0$$

$$A_2 \hat{g} = 0$$

where for the gradient project problem A_1 and A_2 are the constraints matrices on K.

Inputs

```
$IN
  EPS = 1.E-10
  FLAG = .FALSE.,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 100,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 1.,
  A1(1,4) = 0.,1.,
  A1(1,5) = 0.,1.,
  A1(1,6) = 0.,1.,
  A1(1,11) = 2*0.,1.,
  A1(1,12) = 2*0.,1.,
  A1(1,13) = 2*0.,1.,
  A1(1,14) = 3*0.,1.,
  A1(1,15) = 3*0.,1.,
```

```
  A1(1,16) = 3*0.,1.,
  A2(1,1) = 300*0.,
  A2(1,1) = 1.,
  A2(1,2) = 1.,
  A2(1,3) = 1.,
  A2(1,4) = 1.,
  A2(1,5) = 1.,
  A2(1,6) = 1.,
     A2(1,7) = 1.,
  A2(1,8) = 1.,
  A2(1,9) = 1.,
  A2(1,10) = 1.,
  A2(1,11) = 0.,1.,
  A2(1,12) = 0.,1.,
  A2(1,13) = 0.,1.,
  A2(1,14) = 0.,1.,
  A2(1,15) = 0.,1.,
  A2(1,16) = 0.,1.,
  A2(1,17) = 0.,1.,
  A2(1,18) = 0.,1.,
  A2(1,19) = 0.,1.,
  A2(1,20) = 0.,1.,
  OBJ = -1.,-.566,-.8187,-.9927,-.8187,-.8187,-.8047,-.7800,-.9643,
     -.7701,-.0659,-.0221,-.0130,-.0667,-.0158,-.0043,14*0.,
  IND = 2,3,5,6,8,10,11,12,13,18,19,20,1,4,7,9,14,15,16,17,1,2,3,4,1,2,
$END
```

Outputs:

```
$OUT
N      = 26,
NCON   = 12,
NUNC   = 8,
NINEQ  = 4,
NEQ    = 2,
NINV   = 3000,
ITMAX  = 100,
ITER   = 26,
IERR   = 0,
C      = 0.0,
VALUE  = -.74708433333331E-02,
$END
.1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. .1000E+01
```

Outputs Continued:

P

-.1000E+01	-.8566E+00	-.8187E+00	-.9927E+00	-.8187E+00	-.8187E+00	-.8047E+00	-.7800E+00	-.9643E+00	-.7701E+00
-.6590E-01	-.2210E-01	-.1300E-01	-.6670E-01	-.1580E-01	-.4300E-02	0.	0.	0.	0.

A1*X B1 A1

0.	0.	.1000E+01	.1000E+01	.1000E+01	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	.1000E+01	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-.1110E-15	0.	0.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	.1000E+01	.1000E+01	.1000E+01	0.	0.	0.
		0.	0.	0.	0.	0.	.1000E+01	.1000E+01	.1000E+01
		0.	0.	0.	0.				
		0.	0.	0.	0.				
		0.	0.	0.	0.				
		0.	0.	0.	0.				

A2*X B2 A2

0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01
-.1110E-15	0.	0.	0.	0.	0.	0.	0.	0.	0.
		.1000E+01	.1000E+01	0.	0.	0.	0.	0.	0.
		0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01
		0.	0.	0.	0.				
		.1000E+01	.1000E+01	[illegible]E+01	.1000E+01				

X

0.	0.	0.	0.	0.	0.	-.7980E-01	0.	.7980E-01	0.
0.	0.	0.	.3777E-01	-.131[illegible]-01	-.2463E-01	0.	0.	0.	0.

Note the constraint matrices. The unequality constraints are groups of three consecutive variables. The equality constraints are groups of ten consecutive variables. Also 26 iterations were required to determine the solution.

This next problem is identical to the previous but instead of using the permutation indice the matrices have been reformed. Sign constrained variables have been ordered first followed by sign unconstrained variables.

If this must be done many times for a given constraint tableau one can see a considerable work would be required. Also ITMAX has been set to 5.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  IND = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,1,2,3,4,1,2,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 5,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 0.,1.,
```

```
  A1(1,4) = 0.,1.,
  A1(1,7) = 2*0.,1.,
  A1(1,8) = 2*0.,1.,
  A1(1,9) = 2*0.,1.,
  A1(1,13) = 1.,
  A1(1,14) = 0.,1.,
  A1(1,17) = 3*0.,1.,
  A1(1,18) = 3*0.,1.,
  A1(1,19) = 3*0.,0.,
  A2(1,1) = 300*0.,
  A2(1,1) = 1.,
  A2(1,2) = 1.,
  A2(1,3) = 1.,
  A2(1,4) = 1.,
  A2(1,5) = 1.,
  A2(1,6) = 1.,
     A2(1,7) = 0.,1.,
  A2(1,8) = 0.,1.,
  A2(1,9) = 0.,1.,
  A2(1,10) = 0.,1.,
  A2(1,11) = 0.,1.,
  A2(1,12) = 0.,1.,
  A2(1,13) = 1.,
  A2(1,14) = 1.,
  A2(1,15) = 1.,
  A2(1,16) = 1.,
  A2(1,17) = 0.,1.,
  A2(1,18) = 0.,1.,
  A2(1,19) = 0.,1.,
  A2(1,20) = 0.,1.,
  OBJ = -.8566,-.8187,-.8187,-.8187,-.78,-.7701,-.0659,-.0221,-.0130,
     3*0.,-1.,-.9927,-.8047,-.9643,-.0667,-.0158,-.0043,11*0.,
$END
```

Outputs:

```
$OUT
N       = 26,
NCON    = 12,
NUNC    = 8,
NINEQ   = 4,
NEQ     = 2,
NINV    = 3000,
ITMAX   = 5,
ITER    = 5,
IERR    = 2,
C       = 0.0,
VALUE   = .1E+01,
$END
```

Q

.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01

Output Continued:

P

-.8566E+00	-.8187E+00	-.8187E+00	-.8187E+00	-.7800E+00	-.7701E+00	-.6590E-01	-.2210E-01	-.1300E-01	0.
0.	0.	-.1000E+01	-.9927E+00	-.8047E+00	-.9643E+00	-.6670E-01	-.1580E-01	-.4300E-02	0.

A1*X B1 A1

0.	0.	.1000E+01	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	.1000E+01
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

0.	0.	0.	0.	.1000E+01	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	0.	0.
.1000E+01	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
.1000E+01	.1000E+01	.1000E+01	0.

A2*X B2 A2

0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	.1000E+01

0.	0.	0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01
.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.	0.	0.	0.

0.	0.	0.	0.
.1000E+01	.1000E+01	.1000E+01	.1000E+01

X

-R	-R	-R	-R	-R	-R	-R	-R	-R	-R
-R	-R	-R	-R	-R	-R	-R	-R	-R	-R

Note IERR returns equal to 2 indicating the iteration limit has been reached. To continue we call WOLFQP changing MAXIT to 100. No other change is necessary.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .TRUE.,
  IND = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,1,2,3,4,1,2,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 100,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 0.,1.,
  A1(1,4) = 0.,1.,
  A1(1,7) = 2*0.,1.,
  A1(1,8) = 2*0.,1.,
  A1(1,9) = 2*0.,1.,
  A1(1,13) = 1.,
  A1(1,14) = 0.,1.,
  A1(1,17) = 3*0.,1.,
  A1(1,18) = 3*0.,1.,
  A1(1,19) = 3*0.,1.,
  A2(1,1) = 300*0.,
  A2(1,1) = 1.,
```

```
  A2(1,2) = 1.,
  A2(1,3) = 1.,
  A2(1,4) = 1.,
  A2(1,5) = 1.,
  A2(1,6) = 1.,
     A2(1,7) = 0.,1.,
  A2(1,8) = 0.,1.,
  A2(1,9) = 0.,1.,
  A2(1,10) = 0.,1.,
  A2(1,11) = 0.,1.,
  A2(1,12) = 0.,1.,
  A2(1,13) = 1.,
  A2(1,14) = 1.,
  A2(1,15) = 1.,
  A2(1,16) = 1.,
  A2(1,17) = 0.,1.,
  A2(1,18) = 0.,1.,
  A2(1,19) = 0.,1.,
  A2(1,20) = 0.,1.,
  OBJ = -.8566,-.8187,-.8187,-8187,-.78,-.7701,-.0659,-.0221,-.0130,
     3*0.,-1.,-.9927,-.8047,-.9643,-.0667,-.0158,-.0043,11*0.,
END
```

```
$OUT
N       = 26.
NCON    = 12.
NUNC    = 6.
NINEQ   = 4.
NEQ     = 2.
NINV    = 3000.
ITMAX   = 100.
ITER    = 16.
IERR    = 0.
C       = 0.0.
VALUE   = -.74708433333331E-02.
$END
```

Q

.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01

P

-.8566E+00	-.8187E+00	-.8187E+00	-.8187E+00	-.7800E+00	-.7701E+00	-.6590E-01	-.2210E-01	-.1300E-01	0.
0.	0.	-.1000E+01	-.9927E+00	-.8047E+00	-.9643E+00	-.6670E-01	-.1580E-01	-.4300E-02	0.

A1*X	B1	A1							
0.	0.	.1000E+01	.1000E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	.1000E+01	.1000E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	.1000E+01	.1000E+01
-.1110E-15	0.	0.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	.1000E+01	0.	0.	0.
		0.	0.	0.	0.	0.	.1000E+01	0.	0.
		.1000E+01	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.				
		0.	0.	0.	0.				
		0.	0.	0.	0.				
		.1000E+01	.1000E+01	.1000E+01	0.				

A2*X	B2	A2							
0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.	0.
-.1110E-15	0.	0.	0.	0.	0.	0.	0.	.1000E+01	.1000E+01
		0.	0.	0.	0.	.1000E+01	.1000E+01	.1000E+01	.1000E+01
		.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.	0.	0.	0.
		0.	0.	0.	0.				
		.1000E+01	.1000E+01	.1000E+01	.1000E+01				

X

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-.7980E-01	.7980E-01	.3777E-01	-.1313E-01	-.2463E-01	0.

After unscrambling the output so that it corresponds to the first problem of this example set one can see the answers are the same.

Example 4

Example 3 illustrated a constrained least-squares problem which consisted of finding a Euclidean projection onto a cone. More general least-squares approximation problems lead, by the same algebraic manipulations which yield the normal equations, to quadratic minimization problems with the Q - matrix not equal to the identity.

Given k values $H_1, \ldots, H_k$, we wish to find n values $x_1, \ldots, x_n$ which minimize $1/2 \sum_{i=1}^{k} h_i^2$ where $h_i = \sum_{i=1}^{n} C_{ij} x_j - H_i$, the i-th residual, subject to upper and lower bounds on the variables x_j, $j=1,\ldots,n$. In matrix terms we can define a vector residual $h = CX - H$, where C is $k \times n$. Expanding the quadratic gives us the problem:

$$\text{minimize } 1/2\ x^T(C^TC)X \ -(H^TC)x + 1/2\ H^TH$$

$$\text{subject to} \quad \begin{bmatrix} I \\ -I \end{bmatrix} X \leq \begin{bmatrix} u \\ -\ell \end{bmatrix}$$

where u and ℓ are vectors of upper and lower bounds. Here C^TC corresponds to Q, $-H^TC$ to p and $1/2\ H^TH$ to C. (More general linear constraints could also be used.)

The following example, for which we are indebted to George Gray of E22, NSWC, treats the approximation of a given magnetic field. The original data, unscaled, make all the Q-matrix entries very small ($\sim 10^{-12}$). Since values less than EPS1 in magnitude are set to zero in the tableau calculations, we must ensure that EPS1 is small enough to ensure that no significant Q - matrix entries are zeroed. We present three cases, one with EPS1 = 10^{-10} (which bombs), one with EPS1 = 0, and one with EPS1 = 10^{-10} but with the data rescaled (x divided by 10^6, C multiplied by 10^6) so that this difficulty does not arise.

Example 4, Case 1

Inputs:

```
$IN
 EPS = 1.E-10,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E-5,2.765742302E-6,-2.179388535E-5,-3.785483192E-5,
    -3.739012681E-5,-4.532612771E-5,-4.154215575E-5,-1.86391216E-5,
    1.286575818E-5,3.16621E+6,8.48499E+6,13.06003E+6,1.62557+7,
    1.91037+7,1.77811+7,1.65675+7,12.75075E+6,5.80300E+6,
    -1.05051E+6,-2.82833E+6,-4.35334E+6,-.54186+7,-.63678+7,-.59270+7,
    -.55225+7,-4.23525E+6,-1.93433E+6,
 Q = 8.706679319E-12,3.293967558E-13,-2.362473067E-12,-1.182179011E-12,
    -4.737248319E-13,-1.97169935E-13,-8.830662389E-14,-4.324526662E-14,-2.291560193E-14,
 Q(1,2) = 3.293967558E-13,7.49676956E-12,3.640488738E-13,-2.343175479E-12,
     -1.17408002E-12,-4.738453526E-13,-1.956924747E-13,-8.802831499E-14,-4.324526662E-14,
 Q(1,3) = -2.362473067E-12,3.640488738E-13,8.384986211E-12,2.517128094E-13,-2.339273365E-12,
 -1.180883408E-12,-4.725728139E-13,-1.956924747E-13,-8.830662389E-14,
 Q(1,4) = -1.182179011E-12,-2.343175479E-12,2.51712809E-13,8.563588812E-12,3.484107527E-13,
   -2.362765444E-12,-1.180883408E-12,-4.738453526E-13,-1.97169935E-13,
 Q(1,5) = -4.737248319E-13,-1.174080021E-12,-2.339273365E-12,3.484107527E-13,7.467745672E-12,
    3.484107527E-13,-2.339273365E-12,-1.174080021E-12,-4.737248319E-13,
 Q(1,6) = -1.97169935E-13,-4.738453526E-13,-1.180883408E-12,-2.362765444E-12,3.484107527E-13,
    8.563588812E-12,2.51712809E-13,-2.343175479E-12,-1.182179011E-12,
 Q(1,7) = -8.830662389E-14,-1.956924747E-13,-4.725728139E-13,-1.180883408E-12,-2.339273365E-12,
    2.517128094E-13,8.384986211E-12,3.640488738E-13,-2.362473067E-12,
 Q(1,8) = -4.324526662E-14,-8.802831499E-14,-1.956924747E-13,-4.738453526E-13,-1.174080021E-12,
    -2.343175479E-12,3.640488738E-13,7.49676956E-12,3.293967558E-13,
 Q(1,9) = -2.291560193E-14,-4.324526662E-14,-8.830662389E-14,-1.97169935E-13,-4.737248319E-13,
    -1.182179011E-12,-2.362473067E-12,3.293967558E-13,8.706679319E-12,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 1

Outputs:

```
$OUT
N       = 27,
NCON    = 0,
NUNC    = 9,
NINEQ   = 18,
NEQ     = 0,
NINV    = 200,
ITMAX   = 200,
ITER    = 3,
IERR    = 4,
C       = .10657777E+04,
VALUE   = .27208508981644E+03,
$END
```

Q

.8707E-11	.3294E-12	-.2362E-11	-.1182E-11	-.4737E-12	-.1972E-12	-.8831E-13	-.4325E-13	-.2292E-13
.3294E-12	.7497E-11	.3640E-12	-.2343E-11	-.1174E-11	-.4738E-12	-.1957E-12	-.8803E-13	-.4325E-13
-.2362E-11	.3640E-12	.8385E-11	.2517E-12	-.2339E-11	-.1181E-11	-.4726E-12	-.1957E-12	-.8831E-13
-.1182E-11	-.2343E-11	.2517E-12	.8564E-11	.3484E-12	-.2363E-11	-.1181E-11	-.4738E-12	-.1972E-12
-.4737E-12	-.1174E-11	-.2339E-11	.3484E-12	.7468E-11	.3484E-12	-.2339E-11	-.1174E-11	-.4737E-12
-.1972E-12	-.4738E-12	-.1181E-11	-.2363E-11	.3484E-12	.8564E-11	.2517E-12	-.2343E-11	-.1182E-11
-.8831E-13	-.1957E-12	-.4726E-12	-.1181E-11	-.2339E-11	.2517E-12	.8385E-11	.3640E-12	-.2362E-11
-.4325E-13	-.8803E-13	-.1957E-12	-.4738E-12	-.1174E-11	-.2343E-11	.3640E-12	.7497E-11	.3294E-12
-.2292E-13	-.4325E-13	-.8831E-13	-.1972E-12	-.4737E-12	-.1182E-11	-.2362E-11	.3294E-12	.8707E-11

P

.2293E-04	.2766E-05	-.2179E-04	-.3785E-04	-.3739E-04	-.4533E-04	-.4154E-04	-.1864E-04	.1287E-04

A1*X	B1	A1							
.1051E+07	.3166E+07	.1000E+01	0.	0.	0.	0.	0.	0.	0.
.2828E+07	.8485E+07	0.	.1000E+01	0.	0.	0.	0.	0.	0.
.4353E+07	.1306E+08	0.	0.	.1000E+01	0.	0.	0.	0.	0.
.5419E+07	.1626E+08	0.	0.	0.	.1000E+01	0.	0.	0.	0.
.6368E+07	.1910E+08	0.	0.	0.	0.	.1000E+01	0.	0.	0.
.5927E+07	.1778E+08	0.	0.	0.	0.	0.	.1000E+01	0.	0.
.5523E+07	.1657E+08	0.	0.	0.	0.	0.	0.	.1000E+01	0.
.4235E+07	.1275E+08	0.	0.	0.	0.	0.	0.	0.	.1000E+01
.1934E+07	.5803E+07	0.	0.	0.	0.	0.	0.	0.	0.
-.1051E+07	-.1051E+07	-.1000E+01	0.	0.	0.	0.	0.	0.	0.
-.2828E+07	-.2828E+07	0.	-.1000E+01	0.	0.	0.	0.	0.	0.
-.4353E+07	-.4353E+07	0.	0.	-.1000E+01	0.	0.	0.	0.	0.
-.5419E+07	-.5419E+07	0.	0.	0.	-.1000E+01	0.	0.	0.	0.
-.6368E+07	-.6368E+07	0.	0.	0.	0.	-.1000E+01	0.	0.	0.
-.5927E+07	-.5927E+07	0.	0.	0.	0.	0.	-.1000E+01	0.	0.
-.5523E+07	-.5523E+07	0.	0.	0.	0.	0.	0.	-.1000E+01	0.
-.4235E+07	-.4235E+07	0.	0.	0.	0.	0.	0.	0.	-.1000E+01
-.1934E+07	-.1934E+07	0.	0.	0.	0.	0.	0.	0.	0.

Example 4, Case 1
Outputs Cont.:

```
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                     .1000E+01
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    -.1000E+01

      X
.1051E+07   .2828E+07   .4353E+07   .5419E+07   .6368E+07   .5927E+07   .5523E+07   .4235E+07   .1934E+07
```

Example 4, Case 2

Inputs:

```
$IN
 EPS = 0.,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E-5,2.765742302E-6,-2.179388535E-5,-3.785483192E-5,
    -3.739012681E-5,-4.532612771E-5,-4.154215575E-5,-1.86391216E-5,
    1.286575818E-5,3.16621E+6,8.48499E+6,13.06003E+6,1.62557+7,
    1.91037+7,1.77811+7,1.65675+7,12.75075E+6,5.80300E+6,
    -1.05051E+6,-2.82833E+6,-4.35334E+6,-.54186+7,-.63678+7,-.59270+7,
    -.55225+7,-4.23525E+6,-1.93433E+6,
 Q = 8.706679319E-12,3.2939667558E-13,-2.362473067E-12,-1.182179011E-12,
    -4.737248319E-13,-1.97169935E-13,-8.830662389E-14,-4.324526662E-14,-2.291560193E-14,
 Q(1,2) = 3.293967558E-13,7.49676956E-12,3.640488738E-13,-2.343175479E-12,
     -1.17408002E-12,-4.738453526E-13,-1.956924747E-13,-8.802831499E-14,-4.324526662E-14,
 Q(1,3) = -2.362473067E-12,3.640488738E-13,8.384986211E-12,2.517128094E-13,-2.339273365E-12,
 -1.180883408E-12,-4.725728139E-13,-1.956924747E-13,-8.830662389E-14,
 Q(1,4) = -1.182179011E-12,-2.343175479E-12,2.51712809E-13,8.563588812E-12,3.484107527E-13,
   -2.362765444E-12,-1.180883408E-12,-4.738453526E-13,-1.97169935E-13,
 Q(1,5) = -4.737248319E-13,-1.174080021E-12,-2.339273365E-12,3.484107527E-13,7.467745672E-12,
    3.484107527E-13,-2.339273365E-12,-1.174080021E-12,-4.737248319E-13,
 Q(1,6) = -1.97169935E-13,-4.738453526E-13,-1.180883408E-12,-2.362765444E-12,3.484107527E-13,
    8.563588812E-12,2.51712809E-13,-2.343175479E-12,-1.182179011E-12,
 Q(1,7) = -8.830662389E-14,-1.956924747E-13,-4.725728139E-13,-1.180883408E-12,-2.339273365E-12,
    2.517128094E-13,8.384986211E-12,3.640488738E-13,-2.362473067E-12,
 Q(1,8) = -4.324526662E-14,-8.802831499E-14,-1.956924747E-13,-4.738453526E-13,-1.174080021E-12,
    -2.343175479E-12,3.640488738E-13,7.49676956E-12,3.293967558E-13,
 Q(1,9) = -2.291560193E-14,-4.324526662E-14,-8.830662389E-14,-1.97169935E-13,-4.737248319E-13,
    -1.182179011E-12,-2.362473067E-12,3.293967558E-13,8.706679319E-12,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 2

Outputs:

```
$OUT
N       = 27,
NCON    = 0,
NUNC    = 9,
NINEQ   = 18,
NEQ     = 0,
NINV    = 200,
ITMAX   = 200,
ITER    = 12,
IERR    = 0,
C       = .10657777E+04,
VALUE   = .7597274307163E+01,
$END
```

Q

.8707E-11	.3294E-12	-.2362E-11	-.1182E-11	-.4737E-12	-.1972E-12	-.8831E-13	-.4325E-13	-.2292E-13
.3294E-12	.7497E-11	.3640E-12	-.2343E-11	-.1174E-11	-.4738E-12	-.1957E-12	-.8803E-13	-.4325E-13
-.2362E-11	.3640E-12	.8385E-11	.2517E-12	-.2339E-11	-.1181E-11	-.4726E-12	-.1957E-12	-.8831E-13
-.1182E-11	-.2343E-11	.2517E-12	.8564E-11	.3484E-12	-.2363E-11	-.1181E-11	-.4738E-12	-.1972E-12
-.4737E-12	-.1174E-11	-.2339E-11	.3484E-12	.7468E-11	.3484E-12	-.2339E-11	-.1174E-11	-.4737E-12
-.1972E-12	-.4738E-12	-.1181E-11	-.2363E-11	.3484E-12	.8564E-11	.2517E-12	-.2343E-11	-.1182E-11
-.8831E-13	-.1957E-12	-.4726E-12	-.1181E-11	-.2339E-11	.2517E-12	.8385E-11	.3640E-12	-.2362E-11
-.4325E-13	-.8803E-13	-.1957E-12	-.4738E-12	-.1174E-11	-.2343E-11	.3640E-12	.7497E-11	.3294E-12
-.2292E-13	-.4325E-13	-.8831E-13	-.1972E-12	-.4737E-12	-.1182E-11	-.2362E-11	.3294E-12	.8707E-11

P

.2293E-04	.2766E-05	-.2179E-04	-.3785E-04	-.3739E-04	-.4533E-04	-.4154E-04	-.1864E-04	.1287E-04

A1*X	B1	A1							
.2112E+07	.3166E+07	.1000E+01	0.	0.	0.	0.	0.	0.	0.
.5657E+07	.8485E+07	0.	.1000E+01	0.	0.	0.	0.	0.	0.
.8707E+07	.1306E+08	0.	0.	.1000E+01	0.	0.	0.	0.	0.
.1084E+08	.1626E+08	0.	0.	0.	.1000E+01	0.	0.	0.	0.
.1274E+08	.1910E+08	0.	0.	0.	0.	.1000E+01	0.	0.	0.
.1185E+08	.1778E+08	0.	0.	0.	0.	0.	.1000E+01	0.	0.
.1105E+08	.1657E+08	0.	0.	0.	0.	0.	0.	.1000E+01	0.
.8470E+07	.1275E+08	0.	0.	0.	0.	0.	0.	0.	.1000E+01
.3869E+07	.5803E+07	0.	0.	0.	0.	0.	0.	0.	0.
-.2112E+07	-.1051E+07	-.1000E+01	0.	0.	0.	0.	0.	0.	0.
-.5657E+07	-.2828E+07	0.	-.1000E+01	0.	0.	0.	0.	0.	0.
-.8707E+07	-.4353E+07	0.	0.	-.1000E+01	0.	0.	0.	0.	0.
-.1084E+08	-.5419E+07	0.	0.	0.	-.1000E+01	0.	0.	0.	0.
-.1274E+08	-.6368E+07	0.	0.	0.	0.	-.1000E+01	0.	0.	0.
-.1185E+08	-.5927E+07	0.	0.	0.	0.	0.	-.1000E+01	0.	0.
-.1105E+08	-.5523E+07	0.	0.	0.	0.	0.	0.	-.1000E+01	0.
-.8470E+07	-.4235E+07	0.	0.	0.	0.	0.	0.	0.	-.1000E+01
.3869E+07	-.1934E+07	0.	0.	0.	0.	0.	0.	0.	0.

Example 4, Case 2
Outputs Cont.:

```
0.
0.
0.
0.
0.
0.
0.
0.
 .1000E+01
0.
0.
0.
0.
0.
0.
0.
0.
-.1000E+01

X
.2112E+07   .5657E+07   .8707E+07   .1084E+08   .1274E+08   .1185E+08   .1105E+08   .8470E+07   .3869E+07
```

Example 4, Case 3

Inputs:

```
$IN
 EPS = 1.E-10,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E+1,2.765742302E+0,-2.179388535E+1,-3.785483192E+1,
    -3.739012681E+1,-4.532612771E+1,-4.154215575E+1,-1.863912166E+1,
    1.286575818E+1,3.16621E+0,8.48499E+0,13.06003E+0,1.62557E+1,
    1.91037E+1,1.77811E+1,1.65675E+1,12.75075E+0,5.80300E+0,
    -1.05051E+0,-2.82833E+0,-4.35334E+0,-.54186E+1,-.63678E+1,-.59270E+1,
    -.55225E+1,-4.23525E+0,-1.93433E+0,
 Q = 8.706679319E+0,3.293967558E-1,-2.362473067E+0,-1.182179011E+0,
    -4.737248319E-1,-1.97169935E-1,-8.830662389E-2,-4.324526662E-2,-2.291560193E-2,
 Q(1,2) = 3.293967558E-1,7.49676956E+0,3.640488738E-1,-2.343175479E+0,
     -1.174080021E+0,-4.738453526E-1,-1.956924747E-1,-8.802831499E-2,-4.324526662E-2,
 Q(1,3) = -2.362473067E+0,3.640488738E-1,8.384986211E+0,2.517128094E-1,-2.339273365E+0,
 -1.180883408E+0,-4.725728139E-1,-1.956924747E-1,-8.830662389E-2,
 Q(1,4) = -1.182179011E+0,-2.343175479E+0,2.517128094E-1,8.563588812E+0,3.484107527E-1,
   -2.362765444E+0,-1.180883408E+0,-4.738453526E-1,-1.97169935E-1,
 Q(1,5) = -4.737248319E-1,-1.174080021E+0,-2.339273365E+0,3.484107527E-1,7.467745672E+0,
    3.484107527E-1,-2.339273365E+0,-1.174080021E+0,-4.737248319E-1,
 Q(1,6) = -1.97169935E-1,-4.738453526E-1,-1.180883408E+0,-2.362765444E+0,3.484107527E-1,
    8.563588812E+0,2.517128094E-1,-2.343175479E+0,-1.182179011E+0,
 Q(1,7) = -8.830662389E-2,-1.956924747E-1,-4.725728139E-1,-1.180883408E+0,-2.339273365E+0,
    2.517128094E-1,8.384986211E+0,3.640488738E-1,-2.362473067E+0,
 Q(1,8) = -4.324526662E-2,-8.802831499E-2,-1.956924747E-1,-4.738453526E-1,-1.174080021E+0,
    -2.343175479E+0,3.640488738E-1,7.49676956E+0,3.293967558E-1,
 Q(1,9) = -2.291560193E-2,-4.324526662E-2,-8.830662389E-2,-1.97169935E-1,-4.737248319E-1,
    -1.182179011E+0,-2.362473067E+0,3.293967558E-1,8.706679319E+0,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 3

Outputs:

```
$OUT
N       = 27,
NCON    = 0,
NUNC    = 9,
NINEQ   = 18,
NEQ     = 0,
NINV    = 200,
ITMAX   = 200,
ITER    = 9,
IERR    = 0,
C       = .10657777E+04,
VALUE   = .75972743071738E+01,
$END
```

Q

.8707E+01	.3294E+00	-.2362E+01	-.1182E+01	-.4737E+00	-.1972E+00	-.8831E-01	-.4325E-01	-.2292E-01
.3294E+00	.7497E+01	.3640E+00	-.2343E+01	-.1174E+01	-.4738E+00	-.1957E+00	-.8803E-01	-.4325E-01
-.2362E+01	.3640E+00	.8385E+01	.2517E+00	-.2339E+01	-.1181E+01	-.4726E+00	-.1957E+00	-.8831E-01
-.1182E+01	-.2343E+01	.2517E+00	.8564E+01	.3484E+00	-.2363E+01	-.1181E+01	-.4738E+00	-.1972E+00
-.4737E+00	-.1174E+01	-.2339E+01	.3484E+00	.7468E+01	.3484E+00	-.2339E+01	-.1174E+01	-.4737E+00
-.1972E+00	-.4738E+00	-.1181E+01	-.2363E+01	.3484E+00	.8564E+01	.2517E+00	-.2343E+01	-.1182E+01
-.8831E-01	-.1957E+00	-.4726E+00	-.1181E+01	-.2339E+01	.2517E+00	.8385E+01	.3640E+00	-.2362E+01
-.4325E-01	-.8803E-01	-.1957E+00	-.4738E+00	-.1174E+01	-.2343E+01	.3640E+00	.7497E+01	.3294E+00
-.2292E-01	-.4325E-01	-.8831E-01	-.1972E+00	-.4737E+00	-.1182E+01	-.2362E+01	.3294E+00	.8707E+01

P

.2293E+02	.2766E+01	-.2179E+02	-.3785E+02	-.3739E+02	-.4533E+02	-.4154E+02	-.1864E+02	.1287E+02

A1*X	B1	A1							
.2112E+01	.3166E+01	.1000E+01	0.	0.	0.	0.	0.	0.	0.
.5657E+01	.8485E+01	0.	.1000E+01	0.	0.	0.	0.	0.	0.
.8707E+01	.1306E+02	0.	0.	.1000E+01	0.	0.	0.	0.	0.
.1084E+02	.1626E+02	0.	0.	0.	.1000E+01	0.	0.	0.	0.
.1274E+02	.1910E+02	0.	0.	0.	0.	.1000E+01	0.	0.	0.
.1185E+02	.1778E+02	0.	0.	0.	0.	0.	.1000E+01	0.	0.
.1105E+02	.1657E+02	0.	0.	0.	0.	0.	0.	.1000E+01	0.
.8470E+01	.1275E+02	0.	0.	0.	0.	0.	0.	0.	.1000E+01
.3869E+01	.5803E+01	0.	0.	0.	0.	0.	0.	0.	0.
-.2112E+01	-.1051E+01	-.1000E+01	0.	0.	0.	0.	0.	0.	0.
-.5657E+01	-.2828E+01	0.	-.1000E+01	0.	0.	0.	0.	0.	0.
-.8707E+01	-.4353E+01	0.	0.	-.1000E+01	0.	0.	0.	0.	0.
-.1084E+02	-.5419E+01	0.	0.	0.	-.1000E+01	0.	0.	0.	0.
-.1274E+02	-.6368E+01	0.	0.	0.	0.	-.1000E+01	0.	0.	0.
-.1185E+02	-.5927E+01	0.	0.	0.	0.	0.	-.1000E+01	0.	0.
-.1105E+02	-.5523E+01	0.	0.	0.	0.	0.	0.	-.1000E+01	0.
-.8470E+01	-.4235E+01	0.	0.	0.	0.	0.	0.	0.	-.1000E+01
-.3869E+01	-.1934E+01	0.	0.	0.	0.	0.	0.	0.	0.

Example 4, Case 3
Outputs Cont.:

```
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                     .1000E+01
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    0.
                    -.1000E+01

        X
.2112E+01   .5657E+01   .8707E+01   .1084E+02   .1274E+02   .1185E+02   .1105E+02   .8470E+01   .3869E+01
```

REFERENCES

1. J. W. Wingate, LINOPT, A FORTRAN Routine for Solving Linear Programming Problems, NSWC TR 80-413

2. P. Wolfe, The Simplex Method for Quadratic Programming, Econometrica v.27 pp. 382-298 (1959)

3. G. Zoutendijk, Methods of Feasible Directions, Elsevier, Amsterdam, 1960

APPENDIX A

LISTING

```
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      SUBROUTINE WOLFQP(Q,A1,A2,BB3,IND,IPASS,C,VALUE,SCR,Y,FLAG,EPS1)
C-----------------------------------------------------------------------
C
C     QUADRATIC PROGRAMMING BY WOLFE'S METHOD
C
C          DETERMINE A VECTOR OF UNKNOWNS X WHICH
C
C
C                                  T                   T
C          MINIMIZE   1/2. * X * Q * X  +  P * X + C
C
C
C          SUCH THAT
C
C                A1 * X <=  B1
C
C                A2 * X = B2
C
C        ELEMENTS OF THE VECTOR X CAN HAVE NON-NEGATIVITY CONSTRAINTS
C        IMPLICITLY HANDLED USING IND, NCON, AND NUNC(SEE TEXT
C        FOLLOWING).  THUS,
C
C           X(IND(I)) >= 0.      I = 1,...,NCON
C
C           X(IND(I)) UNCONSTRAINED IN SIGN     I = NCON+1,...,NCON+NUNC
C
C           WHERE NCON IS THE NUMBER OF SIGN CONSTRAINED VARIABLES AND
C           NUNC IS THE NUMBER OF FREE VARIABLES.
C
C                        ------------
C
C
C      INPUTS--THE FOLLOWING VARIABLES AND ARRAYS MUST BE DEFINED ON
C              ENTRY
C
C        IPASS        INTEGER ARRAY CONTAINING SCALAR VARIABLES DEFINING
C                     THE PROBLEM.
C
C        IPASS(1)     N  THE DIMENSION OF THE PROBLEM (DIMENSION(A) +
C                        + DIMENSION(B1) + DIMENSION(B2)).
C
C        IPASS(2)     ITMAX  THE MAXIMUM NUMBER OF ITERATIONS DESIRED.
C
C        IPASS(5)     NCON   THE NUMBER OF SIGN CONSTRAINED VARIABLES.
C
C        IPASS(6)     NUNC   THE NUMBER OF UNCONSTRAINED IN SIGN VARIABLE
C
C        IPASS(7)     NINEQ   THE NUMBER OF INEQUALITY CONSTRAINTS.
C
C        IPASS(8)     NINVT  THE NUMBER OF ITERATIONS BETWEEN
C                     INVERSIONS TO CONTROL ROUND-OFF.
C
C        IPASS(9)     NQD    DECLARED ROW DIMENSION OF Q.
C
C        IPASS(10)    N1D    DECLARED ROW DIMENSION OF A1.
C
```

```
C                                                                       WOLFQP     57
C       IPASS(11)   NED   DECLARED ROW DIMENSION OF A2.                 WOLFQP     58
C                                                                       WOLFQP     59
C       Q           REAL ARRAY CONTAINING A POSITIVE SEMI-DEFINITE      WOLFQP     60
C                   MATRIX CORRESPONDING TO THE QUADRATIC               WOLFQP     61
C                   PORTION OF THE OBJECTIVE FUNCTION.                  WOLFQP     62
C                                                                       WOLFQP     63
C       A1          REAL ARRAY CONTAINING THE INEQUALITY CONSTRAINTS.   WOLFQP     64
C                                                                       WOLFQP     65
C       A2          REAL ARRAY CONTAINING THE EQUALITY CONTSTRAINTS.    WOLFQP     66
C                                                                       WOLFQP     67
C       OBJ         REAL ARRAY CONTAINING THE OBJECTIVE                 WOLFQP     68
C                   FUNCTION OF THE CORRESPONDING LINEAR                WOLFQP     69
C                   PROGRAMMING PROBLEM.                                WOLFQP     70
C                                T   T   T                              WOLFQP     71
C                   OBJ = (P ,B1 ,B2 )                                  WOLFQP     72
C                                                                       WOLFQP     73
C       IND         INTEGER ARRAY USED TO ORDER X INTO SIGN             WOLFQP     74
C                   CONSTRAINED AND SIGN UNCONSTRAINED PARTITIONS.      WOLFQP     75
C                   ALSO USED TO INDEX WHICH ROWS OF THE CONSTRAINT     WOLFQP     76
C                   MATRICES (A1, A2) ARE TO BE USED (IF ANY).          WOLFQP     77
C                   THUS                                                WOLFQP     78
C                                                                       WOLFQP     79
C                      X(IND(I)) >= 0.       I = 1,... NCON             WOLFQP     80
C                      X(IND(I))   FREE      I = NCON+1,... NCON+NUNC   WOLFQP     81
C                      SUM OVER I    A1(IND(J+K),I)*X(I) <= B1(J)      WOLFQP     82
C                         K = NCON + NUNCI      J = 1, ...NINEQ         WOLFQP     83
C                      SUM OVER I    A2(IND(J+K),I)*X(I) = B2(J)       WOLFQP     84
C                         K = NCON + NUNC + NINEQI     J = 1,...N-K    WOLFQP     85
C                                                                       WOLFQP     86
C       C           REAL SCALAR CONTAINING THE CONSTANT TERM            WOLFQP     87
C                   OF THE QUADRATIC PROBLEM.                           WOLFQP     88
C                                                                       WOLFQP     89
C       VALUE       ON INPUT IF QUADRATIC OBJECTIVE FUNCTION            WOLFQP     90
C                   IS TO BE EVALUATED VALUE SHOULD BE SET NOT EQUAL    WOLFQP     91
C                   TO ZERO.  IF VALUE IS ZERO THE                      WOLFQP     92
C                   QUADRATIC OBJECTIVE FUNCTION IS                     WOLFQP     93
C                   NOT EVALUATED.                                      WOLFQP     94
C                                                                       WOLFQP     95
C       FLAG        LOGICAL VARIABLE.  ON INITIAL CALL TO WOLFQP THIS   WOLFQP     96
C                   VARIABLE MUST BE FALSE.  IT IS USED IF WOLFQP IS    WOLFQP     97
C                   RECALLED AND THE TABLEAU IS NOT TO BE RESET.  IF    WOLFQP     98
C                   THE ITERATION LIMIT HAS BEEN REACHED PREVIOUSLY     WOLFQP     99
C                   WOLFQP CAN BE RECALLED WITH THE FLAG SET TO TRUE.   WOLFQP    100
C                   NO OTHER CHANGE IS NECESSARY OR ADVISED.            WOLFQP    101
C                   COMPUTATION BEGINS WHERE PREVIOUS ITERATIONS        WOLFQP    102
C                   STOPPED. THE ITERATION COUNTER IS RESET TO ZERO.    WOLFQP    103
C                                                                       WOLFQP    104
C       EPSI        REAL SCALAR. THIS VARIABLE REPRESENTS ZERO          WOLFQP    105
C                   TOLERANCE.  CONSTRAINT VIOLATIONS ON TABLEAU        WOLFQP    106
C                   ENTRIES LESS THAN OR EQUAL TO EPS ARE IGNORED.      WOLFQP    107
C                                                                       WOLFQP    108
C                                                                       WOLFQP    109
C     OUTPUTS--THE FOLLOWING VARIABLES AND ARRAYS ARE DEFINED           WOLFQP    110
C              OR REDEFINED ON EXIT.                                    WOLFQP    111
C                                                                       WOLFQP    112
C       IPASS(3)    ITER NUMBER OF ITERATIONS COMPLETED DURING          WOLFQP    113
```

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```
C                    THE CURRENT CALL TO WOLFQP.                          WOLFQP   114
C                                                                         WOLFQP   115
C        IPASS(4)    IERR   ERROR INDICATOR                               WOLFQP   116
C                       0  --OPTIMUM FOUND                                WOLFQP   117
C                       1  --INCONSISTENT CONSTRAINTS                     WOLFQP   118
C                       2  --ITERATION LIMIT REACHED                      WOLFQP   119
C                       3  --INVERSION FAILED(BAD INITIAL BASIS)          WOLFQP   120
C                       4+L--FAILED TO SOLVE QUADRATIC PROBLEM            WOLFQP   121
C                            REASON FOR FAILURE IS DUE TO ERROR L         WOLFQP   122
C                                                                         WOLFQP   123
C                                                                         WOLFQP   124
C        VALUE       IF ON INPUT VALUE WAS NOT ZERO VALUE RETURNS         WOLFQP   125
C                    THE VALUE OF THE QUADRATIC OBJECTIVE FUNCTION.       WOLFQP   126
C                                                                         WOLFQP   127
C        Y           REAL ARRAY CONTAINING THE SOLUTION X.                WOLFQP   128
C                    THE SOLUTION HAS BEEN                                WOLFQP   129
C                    UNSCRAMBLED BY THE IND ARRAY AND THUS                WOLFQP   130
C                    CORRESPONDS TO THE ORIGINAL ORDERING.                WOLFQP   131
C                                                                         WOLFQP   132
C        SCR         PORTIONS OF SCR HAVE BEEN USED                       WOLFQP   133
C                    BY THE PROGRAM.  THESE WILL NOT BE OF                WOLFQP   134
C                    GENERAL USE BUT THEIR DEFINITIONS.                   WOLFQP   135
C                    ARE GIVEN BELOW.                                     WOLFQP   136
C                                                                         WOLFQP   137
C        SCR(1 TO 2*N+1)                  BL   ARRAY OF LOWER BOUNDS      WOLFQP   138
C        SCR(2*N+2 TO 4*N+2)              BU   ARRAY OF UPPER BOUNDS      WOLFQP   139
C        SCR(4*N+3 TO 6*N+3)              X    PRIMAL SOLUTION ARRAY      WOLFQP   140
C        SCR(6*N+4 TO 8*N+4)              U    DUAL SOLUTION ARRAY        WOLFQP   141
C        SCR(8*N+5 TO 10*N+5)             BASIC FLAGS USED INTERNALLY     WOLFQP   142
C        SCR(10*N+6 TO 11*N+5)            AROW  SCRATCH SPACE             WOLFQP   143
C        SCR(11*N+6 TO N**2+11*N+5)       L    THE INVERSE MATRIX         WOLFQP   144
C        SCR(N**2+11*N+6 TO N**2+13*N+6)  K    BASIC SOLUTION KEY         WOLFQP   145
C                                                                         WOLFQP   146
C                                                                         WOLFQP   147
C        MINIMUM DECLARED ARRAY SIZES                                     WOLFQP   148
C                                                                         WOLFQP   149
C           Q(NCON+NUNC,NCON+NUNC)                                        WOLFQP   150
C           A1(NINEQ,NCON+NUNC)                                           WOLFQP   151
C           A2(N-(NCON+NUNC+NINEQ),NCON+NUNC)                             WOLFQP   152
C           OBJ(N)                                                        WOLFQP   153
C           SCR(N**2+13*N+6)                                              WOLFQP   154
C           Y(NCON+NUNC)                                                  WOLFQP   155
C           IND(N)                                                        WOLFQP   156
C           IPASS(11)                                                     WOLFQP   157
C                                                                         WOLFQP   158
C                                                                         WOLFQP   159
C                                                                         WOLFQP   160
C        THE FOLLOWING ARE FURTHER DESCRIPTIONS OF INTERNAL VARIABLES.    WOLFQP   161
C                                                                         WOLFQP   162
C                                                                         WOLFQP   163
C        BL          ARRAY OF LOWER BOUNDS.                       (REAL)  WOLFQP   164
C        BU          ARRAY OF UPPER BOUNDS.                       (REAL)  WOLFQP   165
C                                                                         WOLFQP   166
C        K           BASIC SOLUTION KEY.                       (INTEGER)  WOLFQP   167
C                    K, IN CONJUNCTION WITH X, SPECIFIES A                WOLFQP   168
C                    PARTICULAR BASIC SOLUTION.  THE EQUATIONS            WOLFQP   169
C                    RELATING  X(N+I), I = 1,....M  TO                    WOLFQP   170
```

```
C                 X(J), J = 1,...,N  (THE CONSTRAINT EQUATIONS)        WOLFQP    171
C                 CAN BE SOLVED FOR VARIOUS COMBINATIONS OF            WOLFQP    172
C                 M  VARIABLES (SECONDARY VARIABLES) IN TERMS          WOLFQP    173
C                 OF THE REMAINING  N  VARIABLES (PRIMARY              WOLFQP    174
C                 VARIABLES).  K, A PERMUTATION OF  (1,...,N+M),       WOLFQP    175
C                 SPECIFIES SUCH A PARTITION INTO PRIMARY AND          WOLFQP    176
C                 SECONDARY VARIABLES.  K(1),...,K(N)  ARE THE         WOLFQP    177
C                 INDICES OF THE PRIMARY VARIABLES.  K(N+1),...,       WOLFQP    178
C                 K(N+M)  ARE THE INDICES OF THE SECONDARY             WOLFQP    179
C                 VARIABLES.  FOR THE DUAL VARIABLES U(J),             WOLFQP    180
C                 J = 1,...,N+M, PRIMARY AND SECONDARY INDICES         WOLFQP    181
C                 SWITCH ROLES.  U(K(N+1)),...,U(K(N+M)) BEING         WOLFQP    182
C                 PRIMARY.  A BASIC SOLUTION IS SPECIFIED BY           WOLFQP    183
C                 SETTING EACH PRIMAL PRIMARY VARIABLE TO              WOLFQP    184
C                 EITHER OF ITS BOUNDS AND EACH DUAL PRIMARY           WOLFQP    185
C                 VARIABLE TO ZERO.  THE INPUT VALUES OF THE           WOLFQP    186
C                 PRIMAL PRIMARY VARIABLES ARE SWITCHED TO THE         WOLFQP    187
C                 OPPOSITE BOUND IF NECESSARY IN ORDER TO CREATE       WOLFQP    188
C                 A DUAL-FEASIBLE BASIC SOLUTION.                      WOLFQP    189
C                                                                      WOLFQP    190
C        X        PRIMAL SOLUTION ARRAY.                     (REAL)    WOLFQP    191
C                 X(K(J))  MUST BE SET TO EITHER  BL(K(J)) OR          WOLFQP    192
C                 BU(K(J)), J = 1,...,N. THESE ARE DEFAULT             WOLFQP    193
C                 VALUES TO BE USED WHEN A VANISHING  U(K(J))          WOLFQP    194
C                 MAKES  X(K(J))  INDETERMINATE IN SETTING UP          WOLFQP    195
C                 A DUAL-FEASIBLE SOLUTION.                            WOLFQP    196
C                                                                      WOLFQP    197
C        K        BASIC SOLUTION KEY.                        (INTEGER) WOLFQP    198
C                 SET FOR THE CURRENT BASIS.                           WOLFQP    199
C                                                                      WOLFQP    200
C        X        PRIMAL SOLUTION ARRAY.                     (REAL)    WOLFQP    201
C                                                                      WOLFQP    202
C        U        DUAL SOLUTION ARRAY.                       (REAL)    WOLFQP    203
C                 U(J)  IS THE DUAL VARIABLE (LAGRANGE                 WOLFQP    204
C                 MULTIPLIER) FOR THE CONSTRAINTS ON  X(J).            WOLFQP    205
C                 IT IS POSITIVE IF THE UPPER BOUND IS ACTIVE.         WOLFQP    206
C                 NEGATIVE IF THE LOWER BOUND IS ACTIVE.               WOLFQP    207
C                                                                      WOLFQP    208
C        E        INVERSE MATRIX ARRAY.                      (REAL)    WOLFQP    209
C                 X(I)  =  SUM (J = 1,...,N)  E(I,J) * X(K(J)),        WOLFQP    210
C                                                 I = 1,...,N.         WOLFQP    211
C                                                                      WOLFQP    212
C                     ------------                                     WOLFQP    213
C                                                                      WOLFQP    214
C                                                                      LINE        2
C-----------------------------------------------------------------------LINE        3
C                                                                      LINE        4
      DIMENSION U(1), OBJ(1), IPASS(1), SCR(1), Y(1), IND(1)           WOLFQP    216
      LOGICAL FLAG                                                     WOLFQP    217
C                                                                      WOLFQP    218
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                /XXXQP/     2
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21      /XXXQP/     3
      COMMON /XXXQP/ BIGM, NINVT, PHASE1                               /XXXQP/     4
      LOGICAL NEGV, PHASE1                                             /XXXQP/     5
C                                                                      LINE        2
C-----------------------------------------------------------------------LINE        3
C                                                                      LINE        4
```

```
      C     THE VARIABLE  BIGM  REPRESENTS A VERY LARGE NUMBER.  THE DEFAULT   WOLFQP    221
      C     VALUE IS 1.E100.  THE USER MAY RESET THIS VALUE IF SO DESIRED.     WOLFQP    222
      C     BIGM OR -BIGM MAY BE USED TO FILL IN MISSING UPPER OR LOWER        WOLFQP    223
      C     BOUNDS.  THIS CONSTANT IS MACHINE DEPENDENT.                       WOLFQP    224
      C                                                                        WOLFQP    225
            DATA BIGM /1.E+100/                                                WOLFQP    226
      C                                                                        LINE        2
      C------------------------------------------------------------------------ LINE        3
      C                                                                        LINE        4
            IF(FLAG) GO TO 105                                                 WOLFQP    228
            EPS = EPS1                                                         WOLFQP    229
            N = IPASS(1)                                                       WOLFQP    230
            N1 = IPASS(5) + IPASS(6)                                           WOLFQP    231
            N11 = N1 + 1                                                       WOLFQP    232
            N2 = IPASS(7) + N1                                                 WOLFQP    233
            N21 = N2 + 1                                                       WOLFQP    234
            NQD = IPASS(9)                                                     WOLFQP    235
            N1D = IPASS(10)                                                    WOLFQP    236
            N2D = IPASS(11)                                                    WOLFQP    237
            M = N + 1                                                          WOLFQP    238
            IOBJ = M + N                                                       WOLFQP    239
            PHASE1 = .TRUE.                                                    WOLFQP    240
            NP1 = N + 1                                                        WOLFQP    241
            NPM = N + M                                                        WOLFQP    242
            IP1 = NPM + 1                                                      WOLFQP    243
            IP2 = 2*NPM + 1                                                    WOLFQP    244
            IP3 = 3*NPM + 1                                                    WOLFQP    245
            IP4 = 4*NPM + 1                                                    WOLFQP    246
            IP5 = 5*NPM + 1                                                    WOLFQP    247
            IP6 = 5*NPM + N + 1                                                WOLFQP    248
            IP7 = N**2 + 11*N + 6                                              WOLFQP    249
      C     CALL SETUP ROUTINE                                                 WOLFQP    250
        100 CALL SETQP(SCR(IP7), OBJ, SCR(1), SCR(IP1), SCR(IP2), SCR(IP3),    WOLFQP    251
     *                SCR(IP4),SCR(IP6),IPASS(5),IND)                          WOLFQP    252
            CALL PSOL(Q,A1,A2,OBJ,IND,SCR(IP7),SCR(IP2),SCR(IP6),NQD,N1D,N2D)  WOLFQP    253
        105 ITER = 0                                                           WOLFQP    254
            ITMAX = IPASS(2)                                                   WOLFQP    255
            NINVT = IPASS(8)                                                   WOLFQP    256
            CALL DSIMP(Q,A1,A2,OBJ,IND,SCR(1),SCR(IP1),SCR(IP7),SCR(IP2),      WOLFQP    257
     *                SCR(IP3),SCR(IP6),SCR(IP5),SCR(IP4),NQD,N1D,N2D)         WOLFQP    258
      C                                                                        WOLFQP    259
      C                                                                        WOLFQP    260
            IF(IERR .NE. 0) GO TO 200                                          WOLFQP    261
            IF(PHASE1) 110, 120                                                WOLFQP    262
      C     THEN CHECK FOR CONSISTENT CONSTRAINTS                              WOLFQP    263
        110    IF(SCR(IP2+IOBJ-1) .LE. 0.) PHASE1 = .FALSE.                    WOLFQP    264
               IF(.NOT.PHASE1) GO TO 100                                       WOLFQP    265
      C        ELSE                                                            WOLFQP    266
                   IERR = 1                                                    WOLFQP    267
                   GO TO 200                                                   WOLFQP    268
      C        END                                                             WOLFQP    269
      C     ELSE                                                               WOLFQP    270
        120    IF(SCR(IP2+IOBJ-1) .NE. 0.) IERR = 4 + IERR                     WOLFQP    271
               IF(VALUE .EQ. 0.) GO TO 200                                     WOLFQP    272
      C        ELSE                                                            WOLFQP    273
                   VALUE = 0                                                   WOLFQP    274
                   DO 140 I = 1, N1                                            WOLFQP    275
```

```
SUBROUTINE WOLFQP      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      6

                          TEMP = 0.                                                  WOLFQP    276
                          DO 130 J = 1, N1                                           WOLFQP    277
                             TEMP = TEMP + Q((J-1)*NQD+I)*SCR(IP3-1+J+N)             WOLFQP    278
            130           CONTINUE                                                   WOLFQP    279
290                       VALUE = VALUE + SCR(IP3-1+I+N)*(TEMP/2. + OBJ(I))          WOLFQP    280
            140         CONTINUE                                                     WOLFQP    281
          C         END                                                              WOLFQP    282
          C      END                                                                 WOLFQP    283
            200 CONTINUE                                                             WOLFQP    284
295             DO 210 J = 1, N1                                                     WOLFQP    285
                   Y(J) = SCR(IP3-1+J+N)                                             WOLFQP    286
            210 CONTINUE                                                             WOLFQP    287
            220 CONTINUE                                                             WOLFQP    288
                IPASS(4) = IERR                                                      WOLFQP    289
300             IPASS(3) = ITER                                                      WOLFQP    290
                RETURN                                                               WOLFQP    291
                END                                                                  WOLFQP    292
```

SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS		DEF LINE	REFERENCES
4	WOLFQP	1	301

VARIABLES		SN	TYPE		RELOCATION								
0	A1		REAL		F.P.	REFS	263	267	DEFINED	1			
0	A2		REAL		F.P.	REFS	263	267	DEFINED	1			
20	BIGM		REAL		XXXQP	REFS	224	DEFINED	234				
0	C		REAL		F.P.	REFS	284	DEFINED	1				
4	EPS		REAL		XXXQP	REFS	222	DEFINED	239				
0	EPS1		REAL		F.P.	REFS	239	DEFINED	1				
0	FLAG		LOGICAL		F.P.	REFS	220	238	DEFINED	1			
337	I		INTEGER			REFS	288	2*290	DEFINED	285			
6	IERR		INTEGER		XXXQP	REFS	222	271	281	299	DEFINED	277	281
0	IND		INTEGER	ARRAY	F.P.	REFS	219	261	263	267	DEFINED	1	
0	IOBJ		INTEGER		XXXQP	REFS	222	274	281	DEFINED	249		
0	IPASS		INTEGER	ARRAY	F.P.	REFS	219	240	2*241	243	245	246	247
						261	265	266	DEFINED	1	299	300	
11	IPIV		INTEGER		XXXQP	REFS	223						
330	IP1		INTEGER			REFS	261	267	DEFINED	253			
331	IP2		INTEGER			REFS	261	263	267	274	281		
						DEFINED	254						
332	IP3		INTEGER			REFS	261	267	288	290	296		
						DEFINED	255						
333	IP4		INTEGER			REFS	261	267	DEFINED	256			
334	IP5		INTEGER			REFS	267	DEFINED	257				
335	IP6		INTEGER			REFS	261	263	267	DEFINED	258		
336	IP7		INTEGER			REFS	261	263	267	DEFINED	259		
5	ITER		INTEGER		XXXQP	REFS	222	300	DEFINED	264			
3	ITMAX		INTEGER		XXXQP	REFS	222	DEFINED	265				
341	J		INTEGER			REFS	2*288	2*296	DEFINED	287	295		
12	JPIV		INTEGER		XXXQP	REFS	223						
1	M		INTEGER		XXXQP	REFS	222	249	252	DEFINED	248		
2	N		INTEGER		XXXQP	REFS	222	248	249	251	252	258	2*259

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      SUBROUTINE WOLFOP      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      7

VARIABLES      SN  TYPE            RELOCATION
                                                          288        290        296    DEFINED      240
   13  NEGV       LOGICAL              XXXOP            REFS        223        225
   21  NINVT      INTEGER              XXXOP            REFS        224    DEFINED      266
   10  NPM        INTEGER              XXXOP            REFS        223        253        254        255        256        257        258
                                                       DEFINED        252
    7  NP1        INTEGER              XXXOP            REFS        223    DEFINED      251
  325  NUD        INTEGER                               REFS        263        267        288    DEFINED      245
   14  N1         INTEGER              XXXOP            REFS        223        242        243        285        287        295
                                                       DEFINED        241
  326  N1D        INTEGER                               REFS        263        267    DEFINED      246
   15  N11        INTEGER              XXXOP            REFS        223    DEFINED      242
   16  N2         INTEGER              XXXOP            REFS        223        244    DEFINED      243
  327  N2D        INTEGER                               REFS        263        267    DEFINED      247
   17  N21        INTEGER              XXXOP            REFS        223    DEFINED      244
    0  OBJ        REAL        ARRAY     F.P.            REFS        219        261        263        267        290
                                                       DEFINED          1
   22  PHASE1     LOGICAL              XXXOP            REFS        224        225        272        275    DEFINED      250        274
    0  Q          REAL        ARRAY     F.P.            REFS        219        263        267        288    DEFINED        1
    0  SCR        REAL        ARRAY     F.P.            REFS        219      7*261      3*263      8*267        274        281        288
                                                        290        296    DEFINED        1
  340  TEMP       REAL                                  REFS        288        290    DEFINED      286        288
    0  VALUE      REAL                   F.P.           REFS        282        290    DEFINED        1        284        290
    0  Y          REAL        ARRAY     F.P.            REFS        219    DEFINED        1        296

EXTERNALS          TYPE    ARGS        REFERENCES
       DSIMP                16             267
       PSOL                 11             263
       SETUP                10             261

STATEMENT LABELS              DEF LINE    REFERENCES
   54  100                       261        275
  115  105                       264        238
    0  110          INACTIVE     274        272
  164  120                       281        272
    0  130                       289        287
    0  140                       291        285
  232  200                       294        271        278        282
    0  210                       297        29[illegible]
    0  220          INACTIVE     298

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
  176  140     * I          285 291       33H                NOT INNER
  212  130       J          287 289       4H      INSTACK
  243  210       J          295 297       2H      INSTACK

COMMON BLOCKS    LENGTH
       XXXOP        19

STATISTICS
  PROGRAM LENGTH                  273H      251
  CM LABELED COMMON LENGTH         23H       19
          52000H CM USED
```

```
      SUBROUTINE SETUP(K,OBJ,BL,BU,X,U,BASIC,E,NCON,IND)                SETUP     2
C                                                                       SETUP     3
C                                                                       LINE      2
C-----------------------------------------------------------------------LINE      3
C                                                                       LINE      4
      DIMENSION K(1), OBJ(1), BL(1), BU(1), X(1), U(1)                  SETUP     5
      DIMENSION BASIC(1), E(1), IND(1)                                  SETUP     6
      INTEGER BASIC                                                     SETUP     7
C                                                                       LINE      2
C-----------------------------------------------------------------------LINE      3
C                                                                       LINE      4
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                 /XXXQP/   2
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21       /XXXQP/   3
      COMMON /XXXQP/ BIGM, NINVT, PHASE1                                /XXXQP/   4
      LOGICAL NEGV, PHASE1                                              /XXXQP/   5
C                                                                       LINE      2
C-----------------------------------------------------------------------LINE      3
C                                                                       LINE      4
C                                                                       SETUP    11
C     INTERNAL COMPOSITION OF FLAG WORD BASIC                           SETUP    12
C                                                                       SETUP    13
C        BASIC(*) =    (COMPLEMENTARY INDEX)  ABCDEFG                   SETUP    14
C                                                                       SETUP    15
C        COMPLEMENTARY INDEX - BITS(8-60) THIS CONTAINS THE INDEX       SETUP    16
C                              OF THE COMPLEMENTARY VARIABLE IF         SETUP    17
C                              COMPLEMENTARY SLACKNESS APPLIES TO       SETUP    18
C                              THIS VARIABLE.                           SETUP    19
C                                                                       SETUP    20
C        A  - BIT(7)  1 - WHEN VARIABLE BECOMES NON-BASIC CHANGE        SETUP    21
C                       LOWER BOUND TO ZERO, SET PRIMAL VARIABLE        SETUP    22
C                       TO ZERO, AND SET BIT A TO ZERO IN THIS          SETUP    23
C                       WORD AND IN ITS COMPLEMENT FLAG WORD.           SETUP    24
C                     0 - NO CHANGE WHEN VARIABLE BECOMES NON-BASIC     SETUP    25
C                                                                       SETUP    26
C        B  - BIT(6)  1 - WHEN VARIABLE BECOMES NON-BASIC MAINTAIN      SETUP    27
C                       UPPER BOUND                                     SETUP    28
C                     0 - CHANGE UPPER BOUND TO +INFINITY WHEN          SETUP    29
C                       VARIABLE BECOMES NON-BASIC.                     SETUP    30
C                                                                       SETUP    31
C        C  - BIT(5)  1 - WHEN VARIABLE BECOMES NON-BASIC MAINTAIN      SETUP    32
C                       LOWER BOUND.                                    SETUP    33
C                     0 - WHEN VARIABLE BECOMES NON-BASIC CHANGE        SETUP    34
C                       LOWER BOUND TO -INFINITY.                       SETUP    35
C                                                                       SETUP    36
C        D  - BIT(4)  1 - VARIABLE IS BASIC                             SETUP    37
C                     0 - VARIABLE IS NON-BASIC                         SETUP    38
C                                                                       SETUP    39
C        E  - BIT(3)  1 - COMPLEMENTARY SLACKNESS APPLIES TO VARIBALE   SETUP    40
C                     0 - COMPLEMENTARY SLACKNESS DOES NOT APPLY        SETUP    41
C                                                                       SETUP    42
C        F  - BIT(2)  1 - CHECK UPPER BOUND FOR CONSTRAINT VIOLATIONS   SETUP    43
C                     0 - DON'T CHECK UPPER BOUND                       SETUP    44
C                                                                       SETUP    45
C        G  - BIT(1)  1 - CHECK LOWER BOUND FOR CONSTRAINT VIOLATIONS   SETUP    46
C                     0 - DON'T CHECK LOWER BOUND                       SETUP    47
C                                                                       LINE      2
C-----------------------------------------------------------------------LINE      3
```

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      C                                                                 LINE      4
            DATA MASK1 / 77B/, MASK2 / 66B/, MASK3 / 73B/, MASK4 / 63B/ SETQP    49
            DATA MASK5 / 76B/, MASK6 / 64B/, MASK7 /70B/, MASK8 /60B/   SETQP    50
            DATA MASK9 / 1B/, MASK10 / 2B/, MASK11 / 10B/, MASK12 / 37B/ SETQP   51
            DATA MASK13 / 117B/, MASK14 / 13B/, MASK15 / 100B/          SETQP    52
      C                                                                 LINE      2
      C-----------------------------------------------------------------LINE      3
      C                                                                 LINE      4
            IF(.NOT.PHASE1) GO TO 65                                    SETQP    54
      C                                                                 SETQP    55
      C        PHASE I                                                  SETQP    56
      C                                                                 SETQP    57
      C                                                                 SETQP    58
      C     SET E TO THE IDENTITY                                       SETQP    59
      C                                                                 SETQP    60
            JJ = 0                                                      SETQP    61
            DO 15 J = 1, N                                              SETQP    62
               DO 10 I = 1, N                                           SETQP    63
                  E(I+JJ) = 0.                                          SETQP    64
         10       CONTINUE                                              SETQP    65
                  E(J+JJ) = 1.                                          SETQP    66
                  JJ = JJ + N                                           SETQP    67
         15 CONTINUE                                                    SETQP    68
      C                                                                 SETQP    69
      C     SET INTERNAL FLAGS(BASIC), UPPER AND LOWER BOUNDS           SETQP    70
      C     AND INITIAL PRIMAL AND DUAL SOLUTIONS                       SETQP    71
      C                                                                 SETQP    72
            DO 50 J = 1, N                                              SETQP    73
               K(J) = J                                                 SETQP    74
               IF(J .GT. NCON) GO TO 16                                 SETQP    75
      C       ELSE                                                      SETQP    76
      C       J   - DUAL TO ARTIFICIAL OR SLACK ON CONSTRAINED VARIABLE SETQP    77
      C       J+N - DUAL TO CONSTRAINED VARIABLE                        SETQP    78
      C                                                                 SETQP    79
                  BASIC(IND(J)) = OR(MASK1,SHIFT(IND(J)+N,7))           SETQP    80
                  BASIC(IND(J)+N) = OR(MASK2,SHIFT(IND(J),7))           SETQP    81
                  BL(IND(J)) = 0.                                       SETQP    82
                  BU(IND(J)) = 0.                                       SETQP    83
                  BL(IND(J)+N) = -BIGM                                  SETQP    84
                  BU(IND(J)+N) = 0.                                     SETQP    85
                  X(IND(J)) = 0.                                        SETQP    86
                  GO TO 19                                              SETQP    87
      C       END                                                       SETQP    88
         16    IF(J .GT. N1) GO TO 17                                   SETQP    89
      C       ELSE                                                      SETQP    90
      C       J   - DUAL TO ARTIFICIAL ON UNCONSTRAINED VARIABLE        SETQP    91
      C       J+N - DUAL TO UNCONSTRAINED VARIABLE                      SETQP    92
      C                                                                 SETQP    93
                  BASIC(IND(J)) = MASK3                                 SETQP    94
                  BASIC(IND(J)+N) = MASK4                               SETQP    95
                  BU(IND(J)) = 0.                                       SETQP    96
                  BL(IND(J)) = 0.                                       SETQP    97
                  BL(IND(J)+N) = 0.                                     SETQP    98
                  BU(IND(J)+N) = 0.                                     SETQP    99
                  X(IND(J)) = 0.                                        SETQP   100
                  GO TO 19                                              SETQP   101
      C       END                                                       SETQP   102
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   17    IF(J .GT. N2) GO TO 18                                    SETUP   103
C       ELSE                                                       SETUP   104
C       J   - DUAL TO ARTIFICIAL OR SLACK ON INEQUALITY CONSTRAINTS SETUP   105
C       J+N - DUAL TO LAGRANGE MULT ON INEQUALITY CONSTRAINTS      SETUP   106
C                                                                  SETUP   107
            BASIC(J) = OR(MASK5,SHIFT(J+N,7))                      SETUP   108
            BASIC(J+N) = MASK6                                     SETUP   109
            BL(J) = - BIGM                                         SETUP   110
            BU(J) = 0.                                             SETUP   111
            BL(J+N) = -BIGM                                        SETUP   112
            BU(J+N) = 0.                                           SETUP   113
            X(J) = 0.                                              SETUP   114
            GO TO 19                                               SETUP   115
C       END                                                        SETUP   116
C       J   - DUAL TO ARTIFICIAL ON EQUALITY CONSTRAINTS           SETUP   117
C       J+N - DUAL TO LAGRANGE MULT ON EQUALITY CONSTRAINTS        SETUP   118
C                                                                  SETUP   119
   18    BASIC(J) = MASK7                                          SETUP   120
         BASIC(J+N) = MASK8                                        SETUP   121
         BL(J) = -BIGM                                             SETUP   122
         BU(J) = BIGM                                              SETUP   123
         IF(OBJ(J) .GT. 0.) BASIC(J) = OR(BASIC(J),MASK10)         SETUP   124
         IF(OBJ(J) .LE. 0.) BASIC(J) = OR(BASIC(J),MASK9)          SETUP   125
         BL(J+N) = 0.                                              SETUP   126
         BU(J+N) = 0.                                              SETUP   127
         X(J) = BIGM                                               SETUP   128
   19    CONTINUE                                                  SETUP   129
         KJ = K(J)                                                 SETUP   130
         IF(J .LE. N1) KJ = IND(J)                                 SETUP   131
         U(KJ) = OBJ(KJ)                                           SETUP   132
         IF(J .LE. N1) U(KJ) = -U(KJ)                              SETUP   133
         IF (U(KJ))  20, 40, 30                                    SETUP   134
C        NEGATIVE                                                  SETUP   135
   20       CONTINUE                                               SETUP   136
            X(KJ) = BL(KJ)                                         SETUP   137
            GO TO 40                                               SETUP   138
C        POSITIVE                                                  SETUP   139
   30       CONTINUE                                               SETUP   140
            X(KJ) = BU(KJ)                                         SETUP   141
   40    CONTINUE                                                  SETUP   142
   50    CONTINUE                                                  SETUP   143
      DO 60 J = NP1, NPM                                           SETUP   144
         K(J) = J                                                  SETUP   145
         U(K(J)) = 0.                                              SETUP   146
   60    CONTINUE                                                  SETUP   147
      BL(IOBJ) = -BIGM                                             SETUP   148
      BU(IOBJ) = BIGM                                              SETUP   149
      BASIC(IOBJ) = MASK8                                          SETUP   150
      RETURN                                                       SETUP   151
C                                                                  SETUP   152
C                                                                  SETUP   153
C          SETUP FOR PHASE II                                      SETUP   154
C                                                                  SETUP   155
C                                                                  SETUP   156
   65 DO 110 J = 1, N1                                             SETUP   157
         IF(J .GT. N(ON) GO TO 90                                  SETUP   158
C       ELSE                                                       SETUP   159
```

```
SUBROUTINE SETOP      73/74   OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE     4

      C       CONSTRAINED VARIABLE AND COMPLEMENT                                  SETOP     160
      C                                                                            SETOP     161
                  BU(IND(J)) = 1.                                                  SETOP     162
                  L2 = SHIFT(IND(J)+N,7)                                           SETOP     163
                  L1 = AND(BASIC(IND(J)+N),MASK11)                                 SETOP     164
                  IF(L1 .EQ. 0) 70, 80                                             SETOP     165
      C          THEN                                                              SETOP     166
      C          VARIABLE IS ARTIFICIAL                                            SETOP     167
         70          BL(IND(J)) = 0.                                               SETOP     168
                     BASIC(IND(J)) = OR(MASK12,L2)                                 SETOP     169
                     GO TO 100                                                     SETOP     170
      C          ELSE                                                              SETOP     171
      C          VARIABLE IS SLACK                                                 SETOP     172
         80          BL(IND(J)) = -1.                                              SETOP     173
                     BASIC(IND(J)) = OR(MASK13,L2)                                 SETOP     174
                     BASIC(IND(J)+N) = OR(BASIC(IND(J)+N),MASK15)                  SETOP     175
                     GO TO 100                                                     SETOP     176
      C          END                                                               SETOP     177
      C       END                                                                  SETOP     178
      C       UNCONSTRAINED VARIABLES                                              SETOP     179
      C                                                                            SETOP     180
         90    BL(IND(J)) = -1.                                                    SETOP     181
               BU(IND(J)) = 1.                                                     SETOP     182
               BASIC(IND(J)) = MASK14                                              SETOP     183
      C                                                                            SETOP     184
      C        SET PRIMAL VARIABLE WITH NEW BOUNDS                                 SETOP     185
      C                                                                            SETOP     186
        100    X(IND(J)) = BL(IND(J))                                              SETOP     187
               IF(U(IND(J)) .GT. 0.) X(IND(J)) = BU(IND(J))                        SETOP     188
        110 CONTINUE                                                               SETOP     189
      C                                                                            SETOP     190
      C     SET FLAGS FOR COMPLEMENTS OF EQUALITY CONSTRAINTS                      SETOP     191
      C                                                                            SETOP     192
            IF(N11 .GT. N2) GO TO 125                                              SETOP     193
            DO 120 J = N11, N2                                                     SETOP     194
               BASIC(J+N) = OR(SHIFT(J,7),MASK2)                                   SETOP     195
        120 CONTINUE                                                               SETOP     196
        125 IF(N21 .GT. N) GO TO 140                                               SETOP     197
            DO 130 J = N21, N                                                      SETOP     198
               BASIC(J+N) = MASK4                                                  SETOP     199
        130 CONTINUE                                                               SETOP     200
        140 CONTINUE                                                               SETOP     201
            RETURN                                                                 SETOP     202
            END                                                                    SETOP     203


SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  SETOP          1          163        214

VARIABLES      SN  TYPE              RELOCATION
    0  BASIC      INTEGER     ARRAY      F.P.        REFS        7        8       136       137       176       187
                                                    DEFINED       1       92        93       106       107       120       121
```

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      SUBROUTINE SETUP      73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48       PAGE     5

VARIABLES      SN  TYPE            RELOCATION
                                                   132       133       136       137       162       181       186       187
                                                   195       207       211
   20  BIGM      REAL                 XXXUP        REFS        14        96       122       124       134       135       140
                                                   160       161
    0  BL        REAL      ARRAY       F.P.        REFS         6       149       199   DEFINED         1        94        96
                                                   109       110       122       124       134       138       160       180
                                                   185       193
    0  BU        REAL      ARRAY       F.P.        REFS         6       153       200   DEFINED         1        95        97
                                                   108       111       123       125       135       139       161       174
                                                   194
    0  E         REAL      ARRAY       F.P.        REFS         7   DEFINED         1        76        78
    4  EPS       REAL                 XXXUP        REFS        12
  314  I         INTEGER                           REFS        76   DEFINED        75
    6  IERR      INTEGER              XXXUP        REFS        12
    0  IND       INTEGER   ARRAY       F.P.        REFS         7      2*92      2*93        94        95        96        97
                                                    98       106       107       108       109       110       111       112
                                                   143       174       175       176       180       181       185       186
                                                  2*187       193       194       195     2*199     3*200
                                                DEFINED         1
    0  IDHJ      INTEGER              XXXUP        REFS        12       160       161       162
   11  IPIV      INTEGER              XXXUP        REFS        13
    5  ITER      INTEGER              XXXUP        REFS        12
    3  ITMAX     INTEGER              XXXUP        REFS        12
  313  J         INTEGER                           REFS        78      2*86        87      2*92      2*93        94        95
                                                    96        97        98       101       106       107       108       109
                                                   110       111       112       115     2*120       121       122       123
                                                   124       125       126       132       133       134       135     3*136
                                                  3*137       138       139       140       142     2*143       145     2*157
                                                   158       170       174       175       176       180       181       185
                                                   186     2*187       193       194       195     2*199     3*200     2*207
                                                   211   DEFINED        74        85       156       169       206       210
  312  JJ        INTEGER                           REFS        76        78        79   DEFINED        73        79
   12  JPIV      INTEGER              XXXUP        REFS        13
    0  K         INTEGER   ARRAY       F.P.        REFS         6       142       158   DEFINED         1        86       157
  315  KJ        INTEGER                           REFS     2*144     2*145       146     2*149     2*153
                                                DEFINED       142       143
  317  L1        INTEGER                           REFS       177   DEFINED       176
  316  L2        INTEGER                           REFS       181       186   DEFINED       175
    1  M         INTEGER              XXXUP        REFS        12
  271  MASK1     INTEGER                           REFS        92   DEFINED        59
  302  MASK10    INTEGER                           REFS       136   DEFINED        61
  303  MASK11    INTEGER                           REFS       176   DEFINED        61
  304  MASK12    INTEGER                           REFS       181   DEFINED        61
  305  MASK13    INTEGER                           REFS       186   DEFINED        62
  306  MASK14    INTEGER                           REFS       195   DEFINED        62
  307  MASK15    INTEGER                           REFS       187   DEFINED        62
  272  MASK2     INTEGER                           REFS        93       207   DEFINED        59
  273  MASK3     INTEGER                           REFS       106   DEFINED        59
  274  MASK4     INTEGER                           REFS       107       211   DEFINED        59
  275  MASK5     INTEGER                           REFS       120   DEFINED        60
  276  MASK6     INTEGER                           REFS       121   DEFINED        60
  277  MASK7     INTEGER                           REFS       132   DEFINED        60
  300  MASK8     INTEGER                           REFS       133       162   DEFINED        60
  301  MASK9     INTEGER                           REFS       137   DEFINED        61
    2  N         INTEGER              XXXUP        REFS        12        74        75        79        85        92        93
                                                    96        97       107       110       111       120       121       124
                                                   125       133       138       139       175       176     2*187       207
```

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      SUBROUTINE SETUP      73/74   OPT=1                          FTN 4.6+452        82/01/28. 14.34.48       PAGE     6

VARIABLES      SN  TYPE            RELOCATION
                                                        209       210       211
    0  NCON       INTEGER              F.P.        REFS       87       170    DEFINED       1
   13  NEGV       LOGICAL             XXXUP        REFS       13        15
   21  NINVT      INTEGER             XXXUP        REFS       14
   10  NPM        INTEGER             XXXUP        REFS       13       156
    7  NP1        INTEGER             XXXUP        REFS       13       156
   14  N1         INTEGER             XXXUP        REFS       13       101       143       145       169
   15  N11        INTEGER             XXXUP        REFS       13       205       206
   16  N2         INTEGER             XXXUP        REFS       13       115       205       206
   17  N21        INTEGER             XXXUP        REFS       13       209       210
    0  OBJ        REAL        ARRAY    F.P.        REFS        6       136       137       144    DEFINED       1
   22  PHASE1     LOGICAL             XXXUP        REFS       14        15        66
    0  U          REAL        ARRAY    F.P.        REFS        6       145       146       200    DEFINED       1       144
                                                     145       158
    0  X          REAL        ARRAY    F.P.        REFS        6    DEFINED       1        98       112       126       140
                                                     149       153       199       200

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
       AND        NO TYPE    0  INTRIN               176
       OR *       NO TYPE    0  INTRIN                92        93       120       136       137       181       186       187
                                                     207
       SHIFT      NO TYPE    2  INTRIN                92        93       120       175       207

STATEMENT LABELS                DEF LINE    REFERENCES
    0  10                          77          75
    0  15                          80          74
   67  16                         101          87
   76  17                         115         101
  106  18                         132         115
  122  19                         141          99       113       127
    0  20            INACTIVE     148         146
  140  30                         152         146
  142  40                         154         146       150
    0  50                         155          85
    0  60                         159         156
  167  65                         169          66
    0  70            INACTIVE     180         177
  212  80                         185         177
  220  90                         193         170
  224  100                        199         182       188
    0  110                        201         169
    0  120                        208         206
  251  125                        209         205
    0  130                        212         210
  266  140                        213         209

LOOPS  LABEL     INDEX      FROM-TO     LENGTH    PROPERTIES
   23  15      * J           74 80        20H                NOT INNER
   32  10        I           75 77        2H      INSTACK
   56  50        J           85 155       66H       OPT
  154  60        J          156 159       2H      INSTACK
  200  110       J          169 201       31H       OPT
  244  120       J          206 208       3H      INSTACK
  262  130       J          210 212       2H      INSTACK
```

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      SUBROUTINE SETUP       73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE     7

COMMON BLOCKS   LENGTH
      XXXUP        19

STATISTICS
  PROGRAM LENGTH                  362B      242
  CM LABELED COMMON LENGTH         23B       19
        52000B CM USED
```

```
SUBROUTINE DSIMP      73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE     1

      SUBROUTINE DSIMP(Q,A1,A2,OBJ,IND,BL,BU,K,X,U,E,AROW,BASIC,NQD,N1D,
     *                 N2D)
C
C-----------------------------------------------------------------------
C
C     DUAL SIMPLEX ALGORITHM
C
C-----------------------------------------------------------------------
C
      DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1), BASIC(1)
      INTEGER BASIC
C
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21
      COMMON /XXXQP/ BIGM, NINVT, PHASE1
      LOGICAL NEGV, PHASE1
C
C-----------------------------------------------------------------------
C
      DATA MASK1 /10B/, MASK2 /7B/, MASK3 /60B/, MASK4 /100B/
C
      IERR = 2
C     WHEN ITMAX.LT.1 THE LOOP IS PARTIALLY EXECUTED
      DO 100 II = 1, ITMAX
         CALL PIVROW(BL,BU,K,X,U,BASIC)
         IF (IPIV.GT.0)  GO TO 10
C           NO PIVOT ROW INDICATES THAT X IS OPTIMAL
            IERR = 0
            RETURN
   10    CONTINUE
         KROW = K(IPIV)
         CALL GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,N1D,N2D)
         CALL PIVCOL(BL,BU,K,X,U,AROW)
         IF (JPIV.GT.0)  GO TO 40
C           NO PIVOT COLUMN INDICATES THAT THE CONSTRAINTS
C           ARE INCONSISTENT
            IERR = 1
            RETURN
   40    CONTINUE
         IF (ITMAX.LT.1)  RETURN
C        NEW SOLUTION KEY
         KK = K(JPIV)
C
C        KROW IS NEW BASIC VARIABLE  KK IS EXITING VARIABLE
C        SET BITS IN BASIC FLAG WORDS APPROPRIATELY
C
         BASIC(KROW) = OR(BASIC(KROW),MASK1)
         BASIC(KK) = AND(BASIC(KK),COMPL(MASK1))
C
C        CHECK FOR CHANGES IN BOUNDS FOR EXITING VARIABLE
C        MAINT = 3 INDICATES NO CHANGE
C
         MAINT = AND(SHIFT(BASIC(KK),-4),MASK2)
         IF(MAINT .EQ. 3) GO TO 50
C       ELSE
C
C           MAINT LESS THAN 3 INDICATES BOUNDS CHANGE
```

```
SUBROUTINE DSIMP      73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

C          TO + OR - INFINITY                                                DSIMP      50
C                                                                            DSIMP      51
                IF(MAINT .LT. 3) GO TO 45                                    DSIMP      52
C          ELSE                                                              DSIMP      53
C                                                                            DSIMP      54
C              MAINT GREATER THAN 3 INDICATES LOWER BOUND                    DSIMP      55
C              CHANGES TO ZERO.  CHANGE BOUND, RESET VARIABLE                DSIMP      56
C              AND TURN FLAG OFF IN THIS WORD AND IN THE                     DSIMP      57
C              COMPLEMENTARY FLAG WORD.                                      DSIMP      58
C                                                                            DSIMP      59
               JJ = KK                                                       DSIMP      60
               ICOMP = SHIFT(BASIC(KK),-7)                                   DSIMP      61
               IF(MAINT .NE. 4) JJ = ICOMP                                   DSIMP      62
               BASIC(KK) = AND(BASIC(KK),COMPL(MASK4))                       DSIMP      63
               BASIC(ICOMP) = AND(BASIC(ICOMP),COMPL(MASK4))                 DSIMP      64
               BL(JJ) = 0.                                                   DSIMP      65
               IF(X(JJ) .EQ. -1.) X(JJ) = 0.                                 DSIMP      66
               IF(MAINT .NE. 4) GO TO 50                                     DSIMP      67
               MAINT = 1                                                     DSIMP      68
C          END                                                               DSIMP      69
   45       CONTINUE                                                         DSIMP      70
            BASIC(KK) = OR(AND(BASIC(KK),MAINT),MASK3)                       DSIMP      71
            IF(MAINT .LT. 2) BU(KK) = BIGM                                   DSIMP      72
            IF(MAINT .NE. 1) BL(KK) = -BIGM                                  DSIMP      73
C        END                                                                 DSIMP      74
   50    CONTINUE                                                            DSIMP      75
         K(IPIV) = KK                                                        DSIMP      76
         K(JPIV) = KROW                                                      DSIMP      77
         CALL NEWINV(E,AROW)                                                 DSIMP      78
C        NEW DUAL SOLUTION                                                   DSIMP      79
         CALL GETROW(Q,A1,A2,OBJ,IND,E,IOBJ,AROW,NQD,N1D,N2D)                DSIMP      80
         DO 70 J = 1, N                                                      DSIMP      81
            U(K(J)) = AROW(J)                                                DSIMP      82
   70       CONTINUE                                                         DSIMP      83
         U(K(IPIV)) = 0.                                                     DSIMP      84
C        NEW PRIMAL SOLUTION                                                 DSIMP      85
         X(KROW) = BU(KROW)                                                  DSIMP      86
         IF (NEGV)  X(KROW) = BL(KROW)                                       DSIMP      87
         CALL PSOL(Q,A1,A2,OBJ,IND,K,X,E,NQD,N1D,N2D)                        DSIMP      88
         ITER = ITER + 1                                                     DSIMP      89
         IF(MOD(ITER,NINVT).EQ.0)                                            DSIMP      90
     *            CALL SETINV(Q,A1,A2,OBJ,IND,K,E,AROW,NQD,N1D,N2D)          DSIMP      91
         IF(IERR .EQ. 3) RETURN                                              DSIMP      92
  100    CONTINUE                                                            DSIMP      93
      RETURN                                                                 DSIMP      94
      END                                                                    DSIMP      95


SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
   4  DSIMP          1          29        38        40        100       102
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```
      SUBROUTINE DSIMP        73/74    OPT=1                          FTN 4.6+452          82/01/28. 14.34.48        PAGE      3

VARIABLES      SN  TYPE              RELOCATION
    0  ARO#        REAL        ARRAY      F.P.      REFS        10        32        33        86        88        90        98
                                                   DEFINED      1
    0  A1          REAL                   F.P.      REFS        32        88        96        98    DEFINED        1
    0  A2          REAL                   F.P.      REFS        32        88        96        98    DEFINED        1
    0  BASIC       INTEGER     ARRAY      F.P.      REFS        10        11        25        47        48        53        69
                                                       71        72        79    DEFINED        1        47        48        71
                                                       72        79
   20  BIGM        REAL                   XXXUP     REFS        15        80        81
    0  BL          REAL        ARRAY      F.P.      REFS        10        25        33        95    DEFINED        1        73
                                                       81
    0  BU          REAL        ARRAY      F.P.      REFS        10        25        33        94    DEFINED        1        80
    0  E           REAL                   F.P.      REFS        32        86        88        96        98
                                                   DEFINED      1
    4  EPS         REAL                   XXXUP     REFS        13
  362  ICOMP       INTEGER                          REFS        70      2*72    DEFINED        69
    6  IERR        INTEGER                XXXUP     REFS        13       100    DEFINED        22        28        37
  355  II        * INTEGER                          DEFINED     24
    0  IND         INTEGER                F.P.      REFS        32        88        96        98    DEFINED        1
    0  IOBJ        INTEGER                XXXUP     REFS        13        88
   11  IPIV        INTEGER                XXXUP     REFS        14        26        31        84        92
    5  ITER        INTEGER                XXXUP     REFS        13        97        98    DEFINED        97
    3  ITMAX       INTEGER                XXXUP     REFS        13        24        40
  363  J           INTEGER                          REFS      2*90    DEFINED        89
  361  JJ          INTEGER                          REFS        73      2*74    DEFINED        68        70
   12  JPIV        INTEGER                XXXUP     REFS        14        34        42        85
    0  K           INTEGER     ARRAY      F.P.      REFS        10        25        31        33        42        90        92
                                                       96        98    DEFINED         1        84        85
  357  KK          INTEGER                          REFS      2*48        53        68        69      2*71      2*79        80
                                                       81        84    DEFINED        42
  356  KROW        INTEGER                          REFS        32      2*47        85      2*94      2*95
                                                   DEFINED     31
    1  M           INTEGER                XXXUP     REFS        13
  360  MAINT       INTEGER                          REFS        54        60        70        75        79        80        81
                                                   DEFINED     53        76
  347  MASK1       INTEGER                          REFS        47        48    DEFINED        20
  350  MASK2       INTEGER                          REFS        53    DEFINED        20
  351  MASK3       INTEGER                          REFS        79    DEFINED        20
  352  MASK4       INTEGER                          REFS        71        72    DEFINED        20
    2  N           INTEGER                XXXUP     REFS        13        89
   13  NEGV        LOGICAL                XXXUP     REFS        14        16        95
   21  NINVT       INTEGER                XXXUP     REFS        15        98
   10  NPM         INTEGER                XXXUP     REFS        14
    7  NP1         INTEGER                XXXUP     REFS        14
    0  NUD         INTEGER                F.P.      REFS        32        88        96        98    DEFINED        1
   14  N1          INTEGER                XXXUP     REFS        14
    0  N1D         INTEGER                F.P.      REFS        32        88        96        98    DEFINED        1
   15  N11         INTEGER                XXXUP     REFS        14
   16  N2          INTEGER                XXXUP     REFS        14
    0  N2D         INTEGER                F.P.      REFS        32        88        96        98    DEFINED        1
   17  N21         INTEGER                XXXUP     REFS        14
    0  OBJ         REAL                   F.P.      REFS        32        88        96        98    DEFINED        1
   22  PHASE1      LOGICAL                XXXUP     REFS        15        16
    0  Q           REAL                   F.P.      REFS        32        88        96        98    DEFINED        1
    0  U           REAL        ARRAY      F.P.      REFS        10        25        33    DEFINED         1        90        92
    0  X           REAL        ARRAY      F.P.      REFS        10        25        33        74        96
                                                   DEFINED      1        74        94        95
```

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      SUBROUTINE DSIMP      73/74   OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      4

EXTERNALS           TYPE    ARGS        REFERENCES
      GETROW                11              32        88
      NEWINV                 2              86
      PIVCOL                 6              33
      PIVROW                 6              35
      PSOL                  11              96
      SETINV                11              98

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE  REFERENCES
      AND        NO TYPE     0  INTRIN                48        53        71        72        79
      COMPL      NO TYPE     1  INTRIN                48        71        72
      MOD        INTEGER     2  INTRIN                98
      OR         NO TYPE     0  INTRIN                47        79
      SHIFT      NO TYPE     2  INTRIN                53        69

STATEMENT LABELS              DEF LINE    REFERENCES
   36  10                        30         26
   70  40                        39         34
  122  45                        78         60
  136  50                        83         54        75
    0  70                        91         89
    0  100                      101         24

LOOPS  LABEL    INDEX      FROM-TO    LENGTH    PROPERTIES
   22  100     * II          24 101     2234              EXT REFS  NOT INNER
  167  70        J           89 91        38     INSTACK

COMMON BLOCKS   LENGTH
      XXXQP        19

STATISTICS
  PROGRAM LENGTH                 5228      338
  CM LABELED COMMON LENGTH        238       19
          52000B CM USED
```

```
SUBROUTINE PIVROW      73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

      SUBROUTINE PIVROW(BL,BU,K,X,U,BASIC)                                  PIVROW      2
C                                                                           LINE        2
C-----------------------------------------------------------------------    LINE        3
C                                                                           LINE        4
C     PIVOT ROW SELECTION                                                   PIVROW      4
C                                                                           LINE        2
C-----------------------------------------------------------------------    LINE        3
C                                                                           LINE        4
      DIMENSION BL(1), BU(1), K(1), X(1)                                    PIVROW      6
      DIMENSION BASIC(1)                                                    PIVROW      7
      INTEGER BASIC                                                         PIVROW      8
      DIMENSION U(1)                                                        PIVROW      9
C                                                                           PIVROW     10
      COMMON /XXXOP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                     /XXXOP/     2
      COMMON /XXXOP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21           /XXXOP/     3
      COMMON /XXXOP/ BIGM, NINVT, PHASE1                                    /XXXOP/     4
      LOGICAL NEGV, PHASE1                                                  /XXXOP/     5
C                                                                           PIVROW     12
      LOGICAL FLAGU, FLAGL, FLAGB, FLAGC                                    PIVROW     13
C                                                                           PIVROW     14
C        THIS ROUTINE HAS BEEN MODIFIED TO HANDLE COMPLEMENTARY             PIVROW     15
C        SLACKNESS.  IF COMPLEMENTARY SLACKNESS APPLIES, THE                PIVROW     16
C        PIVOT ROW IS REJECTED IF ITS COMPLEMENT IS BASIC AND               PIVROW     17
C        THE PRIMAL VARIABLE IS ZERO.                                       PIVROW     18
C                                                                           PIVROW     19
C        FLAGU - CHECK UPPER BOUND FOR CONSTRAINT VIOLATIONS                PIVROW     20
C                                                                           PIVROW     21
C        FLAGL - CHECK LOWER BOUND FOR CONSTRAINT VIOLATIONS                PIVROW     22
C                                                                           PIVROW     23
C        FLAGC - VARIABLE HAS COMPLEMENTARY SLACKNESS CONDITION             PIVROW     24
C                                                                           PIVROW     25
C        FLAGB - COMPLEMENTARY VARIABLE IS BASIC                            PIVROW     26
C                                                                           LINE        2
C-----------------------------------------------------------------------    LINE        3
C                                                                           LINE        4
      DATA MASK1 /3B/, MASK2 /4B/, MASK3 /10B/                              PIVROW     28
C                                                                           PIVROW     29
      IPIV = 0                                                              PIVROW     30
      IF (NPM.LT.NP1)  RETURN                                               PIVROW     31
      VIOL = 0.                                                             PIVROW     32
      DO 50 II = NP1, NPM                                                   PIVROW     33
         I = K(II)                                                          PIVROW     34
         L = AND(BASIC(I),MASK1)                                            PIVROW     35
         IF(L .EQ. 0) GO TO 50                                              PIVROW     36
C       ELSE                                                                PIVROW     37
            FLAGU = .FALSE.                                                 PIVROW     38
            FLAGL = .FALSE.                                                 PIVROW     39
            IF(L .GT. 1) FLAGU = .TRUE.                                     PIVROW     40
            IF(L .NE. 2) FLAGL = .TRUE.                                     PIVROW     41
            FLAGC = .FALSE.                                                 PIVROW     42
            IF(PHASE1) GO TO 1                                              PIVROW     43
            FLAGB = .FALSE.                                                 PIVROW     44
            L1 = AND(BASIC(I),MASK2)                                        PIVROW     45
            IF(L1 .EQ. 0) GO TO 1                                           PIVROW     46
C           ELSE                                                            PIVROW     47
               FLAGC = .TRUE.                                               PIVROW     48
               ICOMP = SHIFT(BASIC(I),-7)                                   PIVROW     49
```

```
SUBROUTINE PIVROW      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

                          LI = AND(BASIC(ICOMP),MASK3)                                PIVROW     50
                          IF(LI .NE. 0) FLAGB = .TRUE.                                PIVROW     51
C            END                                                                      PIVROW     52
C         END                                                                         PIVROW     53
      1    CONTINUE                                                                   PIVROW     54
C          CHECK CONSTRAINTS ON X(I)                                                  PIVROW     55
             IF(.NOT.FLAGL) GO TO 10                                                  PIVROW     56
             D = X(I) - BL(I)                                                         PIVROW     57
             IF (D.GE.-EPS)  GO TO 10                                                 PIVROW     58
                D = -D                                                                PIVROW     59
                IF (VIOL.GT.D)  GO TO 40                                              PIVROW     60
                IF(.NOT.FLAGC) GO TO 5                                                PIVROW     61
C              ELSE                                                                   PIVROW     62
                   IF(FLAGB.AND.X(ICOMP).EQ.0.) GO TO 40                              PIVROW     63
C              END                                                                    PIVROW     64
      5         CONTINUE                                                              PIVROW     65
                VIOL = D                                                              PIVROW     66
                IPIV = II                                                             PIVROW     67
                NEGV = .TRUE.                                                         PIVROW     68
                GO TO 40                                                              PIVROW     69
     10      CONTINUE                                                                 PIVROW     70
             IF(.NOT.FLAGU) GO TO 40                                                  PIVROW     71
             D = X(I) - BU(I)                                                         PIVROW     72
             IF (D.LE.EPS)  GO TO 30                                                  PIVROW     73
                IF (VIOL.GT.D)  GO TO 40                                              PIVROW     74
                IF(.NOT.FLAGC) GO TO 20                                               PIVROW     75
C              ELSE                                                                   PIVROW     76
                   IF(FLAGB.AND.X(ICOMP).EQ.0.) GO TO 40                              PIVROW     77
C              END                                                                    PIVROW     78
     20         CONTINUE                                                              PIVROW     79
                VIOL = D                                                              PIVROW     80
                IPIV = II                                                             PIVROW     81
                NEGV =.FALSE.                                                         PIVROW     82
     30      CONTINUE                                                                 PIVROW     83
     40   CONTINUE                                                                    PIVROW     84
     50   CONTINUE                                                                    PIVROW     85
          RETURN                                                                      PIVROW     86
          END                                                                         PIVROW     87


SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  PIVROW          1          39         94

VARIABLES      SN  TYPE            RELOCATION
    0  BASIC       INTEGER     ARRAY      F.P.        REFS        10        11        43        53        57        58
                                                     DEFINED       1
   20  BIGM        REAL                   XXXUP       REFS        16
    0  BL          REAL        ARRAY      F.P.        REFS         9        65    DEFINED        1
    0  BU          REAL        ARRAY      F.P.        REFS         9        80    DEFINED        1
  122  D           REAL                               REFS        66        67        68        74        81        82        88
                                                     DEFINED       65        67        80
    4  EPS         REAL                   XXXUP       REFS        14        66        81
```

```
      SUBROUTINE PIVROW      73/74    OPT-1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      3

VARIABLES      SN  TYPE            RELOCATION
  112  FLAGB     LOGICAL                          REFS        19        71        85    DEFINED        52        59
  113  FLAGC     LOGICAL                          REFS        19        69        83    DEFINED        50        56
  111  FLAGL     LOGICAL                          REFS        19        64    DEFINED        47        49
  110  FLAGD     LOGICAL                          REFS        19        79    DEFINED        46        48
  116  I         INTEGER                          REFS        43        53        57      2*65      2*80
                                                DEFINED        42
  121  ICOMP     INTEGER                          REFS        58        71        85    DEFINED        57
    6  IERR      INTEGER            XXXUP         REFS        14
  115  II        INTEGER                          REFS        42        75        89    DEFINED        41
    0  IOBJ      INTEGER            XXXUP         REFS        14
   11  IPIV      INTEGER            XXXUP         REFS        15    DEFINED        38        75        89
    5  ITER      INTEGER            XXXUP         REFS        14
    3  ITMAX     INTEGER            XXXUP         REFS        14
   12  JPIV      INTEGER            XXXUP         REFS        15
    0  K         INTEGER    ARRAY     F.P.        REFS         9        42    DEFINED         1
  117  L         INTEGER                          REFS        44        48        49    DEFINED        43
  120  L1        INTEGER                          REFS        54        59    DEFINED        53        58
    1  M         INTEGER            XXXUP         REFS        14
  105  MASK1     INTEGER                          REFS        43    DEFINED        36
  106  MASK2     INTEGER                          REFS        53    DEFINED        36
  107  MASK3     INTEGER                          REFS        58    DEFINED        36
    2  N         INTEGER            XXXUP         REFS        14
   13  NEGV      LOGICAL            XXXUP         REFS        15        17    DEFINED        76        90
   21  NINVT     INTEGER            XXXUP         REFS        16
   10  NPM       INTEGER            XXXUP         REFS        15        39        41
    7  NP1       INTEGER            XXXUP         REFS        15        39        41
   14  N1        INTEGER            XXXUP         REFS        15
   15  N11       INTEGER            XXXUP         REFS        15
   16  N2        INTEGER            XXXUP         REFS        15
   17  N21       INTEGER            XXXUP         REFS        15
   22  PHASE1    LOGICAL            XXXUP         REFS        16        17        51
    0  U         REAL       ARRAY     F.P.        REFS        12    DEFINED         1
  114  VIOL      REAL                             REFS        68        82    DEFINED        40        74        88
    0  X         REAL       ARRAY     F.P.        REFS         9        65        71        80        85
                                                DEFINED         1

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
       AND        NO TYPE    0  INTRIN                43        53        58
       SHIFT      NO TYPE    2  INTRIN                57

STATEMENT LABELS                DEF LINE   REFERENCES
   46  1                           62        51        54
   61  5                           73        69
   65  10                          78        64        66
   77  20                          87        83
  102  30                          91        81
  102  40                          92        58        71        77        79        82        85
  102  50                          93        41        44

LOOPS  LABEL     INDEX     FROM-TO     LENGTH     PROPERTIES
   23  50        II         41 93        608        OPT

COMMON BLOCKS    LENGTH
       XXXUP        19
```

```
      SUBROUTINE PIVROW      73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE     4

STATISTICS
  PROGRAM LENGTH                  123B       83
  CM LABELED COMMON LENGTH         23B       19
        52000B CM USED
```

```
SUBROUTINE PIVCOL      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

                 SUBROUTINE PIVCOL(BL,BU,K,X,U,AROW)                                  PIVCOL      2
            C                                                                         LINE        2
            C-----------------------------------------------------------------------  LINE        3
            C                                                                         LINE        4
            C    PIVOT COLUMN SELECTION                                               PIVCOL      4
            C                                                                         LINE        2
            C-----------------------------------------------------------------------  LINE        3
            C                                                                         LINE        4
                 DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1)                    PIVCOL      6
            C                                                                         PIVCOL      7
                 COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                    /XXXQP/     2
                 COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21          /XXXQP/     3
                 COMMON /XXXQP/ BIGM, NINVT, PHASE1                                   /XXXQP/     4
                 LOGICAL NEGV, PHASE1                                                 /XXXQP/     5
            C                                                                         LINE        2
            C-----------------------------------------------------------------------  LINE        3
            C                                                                         LINE        4
                 JPIV = 0                                                             PIVCOL     10
                 W = BIGM                                                             PIVCOL     11
                 DO 30 JJ = 1, N                                                      PIVCOL     12
                    J = K(JJ)                                                         PIVCOL     13
                    AA = AROW(JJ)                                                     PIVCOL     14
                    IF (NEGV)  AA = -AA                                               PIVCOL     15
                    IF (AA.GE.0. .AND. X(J).EQ.BL(J))  GO TO 20                       PIVCOL     16
                    IF (AA.LE.0. .AND. X(J).EQ.BU(J))  GO TO 20                       PIVCOL     17
                       R = U(J)/AA                                                    PIVCOL     18
                       IF (R.GT.W)  GO TO 10                                          PIVCOL     19
                       IF (R.EQ.W .AND. RANF(AA).GT.0.5)  GO TO 10                    PIVCOL     20
                          JPIV = JJ                                                   PIVCOL     21
                          W = R                                                       PIVCOL     22
                 10       CONTINUE                                                    PIVCOL     23
                 20    CONTINUE                                                       PIVCOL     24
                 30    CONTINUE                                                       PIVCOL     25
                    RETURN                                                            PIVCOL     26
                    END                                                               PIVCOL     27


      SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  PIVCOL        1            34

VARIABLES      SN  TYPE            RELOCATION
   66  AA          REAL                                  REFS        23        24        25        26        28
                                                        DEFINED      22        23
    0  AROW        REAL        ARRAY      F.P.           REFS         9        22    DEFINED        1
   20  BIGM        REAL                   XXXQP          REFS        13        19
    0  BL          REAL        ARRAY      F.P.           REFS         9        24    DEFINED        1
    0  BU          REAL        ARRAY      F.P.           REFS         9        25    DEFINED        1
    4  EPS         REAL                   XXXQP          REFS        11
    6  IERR        INTEGER                XXXQP          REFS        11
    0  IOBJ        INTEGER                XXXQP          REFS        11
   11  IPIV        INTEGER                XXXQP          REFS        12
    5  ITER        INTEGER                XXXQP          REFS        11
```

```
                                                                                  FTN 4.6+652        82/01/28. 14.34.48        PAGE     2

      SUBROUTINE PIVCOL      73/74    OPT=1

                                   RELOCATION
VARIABLES        SN  TYPE              XXXQP        REFS        11        2*25       26     DEFINED      21
   3  ITMAX          INTEGER                         REFS      2*24        22         29     DEFINED      20
  65  J              INTEGER                         REFS        21        DEFINED    18        29
  64  JJ             INTEGER                         REFS        12        21       DEFINED     1
  12  JPIV           INTEGER          XXXQP          REFS         9
   0  K              INTEGER   ARRAY   F.P.          REFS        11        20
   1  M              INTEGER          XXXQP          REFS        11        14         23
   2  N              INTEGER          XXXQP          REFS        12
  13  NEGV           LOGICAL          XXXQP          REFS        13
  21  NINVT          INTEGER          XXXQP          REFS        12
  10  NPM            INTEGER          XXXQP          REFS        12
   7  NP1            INTEGER          XXXQP          REFS        12
  14  N1             INTEGER          XXXQP          REFS        12
  15  N11            INTEGER          XXXQP          REFS        12
  16  N2             INTEGER          XXXQP          REFS        12
  17  N21            INTEGER          XXXQP          REFS        12
  22  PHASE1         LOGICAL                         REFS        13        14
  67  R              REAL                            REFS        27        28         30     DEFINED      26
   0  U              REAL      ARRAY   F.P.          REFS         9        26     DEFINED      1
  63  W              REAL                            REFS        27        28     DEFINED      19        30
   0  X              REAL      ARRAY   F.P.          REFS         9        24         25     DEFINED      1

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
      RANF          REAL     1   INTRIN                28

STATEMENT LABELS                DEF LINE   REFERENCES
   56  10                          31         27         28
   56  20                          32         24         25
    0  30                          33         20

LOOPS  LABEL     INDEX     FROM-TO     LENGTH     PROPERTIES
   33  30        JJ         20 33        24H         OPT

COMMON BLOCKS    LENGTH
      XXXQP        19

STATISTICS
  PROGRAM LENGTH                    104B       68
  CM LABELLED COMMON LENGTH          23B       19
        52000B CM USED
```

```
SUBROUTINE GETROW      73/74    OPT=1                          FTN 4.6+452          82/01/28. 14.34.48          PAGE      1

      SUBROUTINE GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,N1D,N2D)
C
C-----------------------------------------------------------------------
C
C     GENERATION OF CONSTRAINT COEFFICIENTS FOR THE CURRENT BASIS
C
C-----------------------------------------------------------------------
C
      DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), E(1), AROW(1), OBJ(1)
      DIMENSION IND(1)
C
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21
      COMMON /XXXQP/ BIGM, NINVT, PHASE1
      LOGICAL NEGV, PHASE1
C
C-----------------------------------------------------------------------
C
      IF (KROW.GT.N)  GO TO 20
C        ORIGINAL INDEPENDENT VARIABLE.  GET ROW KROW OF THE INVERSE.
         JJ = 0
         DO 10 J = 1, N
            AROW(J) = E(KROW+JJ)
            IF (ABS(AROW(J)).LE.EPS)  AROW(J) = 0.
            JJ = JJ + N
   10       CONTINUE
         GO TO 140
   20 CONTINUE
C        ORIGINAL DEPENDENT VARIABLE.
C        MULTIPLY ORIGINAL ROW BY THE INVERSE.
         KK = KROW - N
         L = IND(KK)
         JJ = 0
         DO 130 J = 1, N
            AROW(J) = 0.
            IF(KK .NE. M) GO TO 40
C          ELSE
               DO 30 I = 1, N1
                  AROW(J) = AROW(J) - OBJ(I)*E(I+JJ)
   30          CONTINUE
               IF(N11 .GT. N) GO TO 120
               DO 34 I = N11, N
                  AROW(J) = AROW(J) + OBJ(I)*E(I+JJ)
   34          CONTINUE
               GO TO 120
C          END
   40       IF(KK .GT. N1) GO TO 80
C          ELSE
               DO 50 I = 1, N1
                  AROW(J) = AROW(J) + Q(KK,I)*E(I+JJ)
   50          CONTINUE
               IF(N11 .GT. N2) GO TO 65
               DO 60 I = N11, N2
                  AROW(J) = AROW(J) + A1(IND(I),KK)*E(I+JJ)
   60          CONTINUE
   65          IF(N21 .GT. N) GO TO 120
               DO 70 I = N21, N
```

```
SUBROUTINE GETROW      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

                          AROW(J) = AROW(J) + A2(IND(I),KK)*E(I+JJ)                GETROW     50
                 70     CONTINUE                                                   GETROW     51
                        GO TO 120                                                  GETROW     52
            C         END                                                          GETROW     53
                 80     IF(KK .GT. N2) GO TO 100                                   GETROW     54
            C         ELSE                                                         GETROW     55
                        DO 90 I = 1, N1                                            GETROW     56
                          AROW(J) = AROW(J) + A1(L,I)*E(I+JJ)                      GETROW     57
                 90     CONTINUE                                                   GETROW     58
                        GO TO 120                                                  GETROW     59
            C         END                                                          GETROW     60
                100     DO 110 I = 1, N1                                           GETROW     61
                          AROW(J) = AROW(J) + A2(L,I)*E(I+JJ)                      GETROW     62
                110     CONTINUE                                                   GETROW     63
                120     CONTINUE                                                   GETROW     64
                        IF(ABS(AROW(J)) .LE. EPS) AROW(J) = 0.                     GETROW     65
                        JJ = JJ + N                                                GETROW     66
                130   CONTINUE                                                     GETROW     67
                140 CONTINUE                                                       GETROW     68
                    RETURN                                                         GETROW     69
                    END                                                            GETROW     70
```

SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS		DEF LINE	REFERENCES
4	GETROW	1	77

VARIABLES		SN	TYPE		RELOCATION								
0	AROW		REAL	ARRAY	F.P.	REFS	9	24	39	43	50	54	58
						65	70	73	DEFINED	1	23	24	35
						39	43	50	54	58	65	70	73
0	A1		REAL	ARRAY	F.P.	REFS	9	54	65	DEFINED	1		
0	A2		REAL	ARRAY	F.P.	REFS	9	58	70	DEFINED	1		
20	BIGM		REAL		XXXUP	REFS	14						
0	E		REAL	ARRAY	F.P.	REFS	9	23	39	43	50	54	58
						65	70	DEFINED	1				
4	EPS		REAL		XXXUP	REFS	12	24	73				
271	I		INTEGER			REFS	2*39	2*43	2*50	2*54	2*58	2*65	2*70
						DEFINED	38	42	49	53	57	64	69
6	IERR		INTEGER		XXXUP	REFS	12						
0	IND		INTEGER	ARRAY	F.P.	REFS	10	32	54	58	DEFINED	1	
0	IOBJ		INTEGER		XXXUP	REFS	12						
11	IPIV		INTEGER		XXXUP	REFS	13						
5	ITER		INTEGER		XXXUP	REFS	12						
3	ITMAX		INTEGER		XXXUP	REFS	12						
266	J		INTEGER			REFS	23	2*24	35	2*39	2*43	2*50	2*54
						2*58	2*65	2*70	2*73	DEFINED	22	34	
265	JJ		INTEGER			REFS	23	25	39	43	50	54	58
						65	70	74	DEFINED	21	25	33	74
12	JPIV		INTEGER		XXXUP	REFS	13						
267	KK		INTEGER			REFS	32	36	47	50	54	58	62
						DEFINED	31						
0	KROW		INTEGER		F.P.	REFS	19	23	31	DEFINED	1		

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      SUBROUTINE GETROW      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      3

VARIABLES      SN  TYPE            RELOCATION
  270  L          INTEGER                          REFS        65        70    DEFINED       32
    1  M          INTEGER              XXXQP       REFS        12        36
    2  N          INTEGER              XXXQP       REFS        12        19        22        25        31        34        41
                                                      42        56        57        74
   13  NEGV       LOGICAL              XXXQP       REFS        13        15
   21  NINVT      INTEGER              XXXQP       REFS        14
   10  NPM        INTEGER              XXXQP       REFS        13
    7  NP1        INTEGER              XXXQP       REFS        13
    0  NQD        INTEGER               F.P.       REFS         9    DEFINED        1
   14  N1         INTEGER              XXXQP       REFS        13        38        47        49        64        69
    0  N1D        INTEGER               F.P.       REFS         9    DEFINED        1
   15  N11        INTEGER              XXXQP       REFS        13        41        42        52        53
   16  N2         INTEGER              XXXQP       REFS        13        52        53        62
    0  N2D        INTEGER               F.P.       REFS         9    DEFINED        1
   17  N21        INTEGER              XXXQP       REFS        13        56        57
    0  OBJ        REAL       ARRAY      F.P.       REFS         9        39        43    DEFINED        1
   22  PHASE1     LOGICAL              XXXQP       REFS        14        15
    0  Q          REAL       ARRAY      F.P.       REFS         9        50    DEFINED        1

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
      ABS.         REAL       1  INTRIN                 24        73

STATEMENT LABELS                DEF LINE    REFERENCES
    0  10                          26          22
   52  20                          28          19
    0  30                          40          38
    0  34                          44          42
  117  40                          47          46
    0  50                          51          49
    0  60                          55          53
  162  65                          56          52
    0  70                          59          57
  206  80                          62          47
    0  90                          66          64
  230  100                         69          62
    0  110                         71          69
  246  120                         72          41        45        56        60        67
    0  130                         75          44
  257  140                         76          27

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
   40  10        J           22 26        7H       INSTACK
   60  130     * J           34 75      176H                NOT INNER
   73  30        I           38 40        3H       INSTACK
  111  34        I           42 44        3H       INSTACK
  133  50        I           49 51        3H       INSTACK
  154  60        I           53 55        4H       INSTACK
  177  70        I           57 59        4H       INSTACK
  222  90        I           64 66        3H       INSTACK
  241  110       I           69 71        3H       INSTACK

COMMON BLOCKS    LENGTH
      XXXQP        19

STATISTICS
  PROGRAM LENGTH                  342B      226
  CM LABELED COMMON LENGTH         23B       19
```

```
SUBROUTINE GETROW      73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE     4

STATISTICS
          52000B CM USED
```

```
SUBROUTINE PSOL        73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

      SUBROUTINE PSOL(Q,A1,A2,OBJ,IND,K,X,E,NQD,N1D,N2D)                PSOL        2
C                                                                       LINE        2
C-----------------------------------------------------------------------LINE        3
C                                                                       LINE        4
C     PRIMAL SOLUTION                                                   PSOL        4
C                                                                       LINE        2
C-----------------------------------------------------------------------LINE        3
C                                                                       LINE        4
      DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), K(1), X(1), E(1)         PSOL        6
      DIMENSION OBJ(1), IND(1)                                          PSOL        7
C                                                                       PSOL        8
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                 /XXXQP/     2
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21       /XXXQP/     3
      COMMON /XXXQP/ BIGM, NINVT, PHASE1                                /XXXQP/     4
      LOGICAL NEGV, PHASE1                                              /XXXQP/     5
C                                                                       LINE        2
C-----------------------------------------------------------------------LINE        3
C                                                                       LINE        4
      DO 30 I = NP1, NPM                                                PSOL       11
         KI = K(I)                                                      PSOL       12
         IF (KI.GT.N)  GO TO 20                                         PSOL       13
            X(KI) = 0.                                                  PSOL       14
            JJ = 0                                                      PSOL       15
            DO 10 J = 1, N                                              PSOL       16
               X(KI) = X(KI) + E(KI+JJ) * X(K(J))                       PSOL       17
               JJ = JJ + N                                              PSOL       18
   10          CONTINUE                                                 PSOL       19
   20    CONTINUE                                                       PSOL       20
   30    CONTINUE                                                       PSOL       21
      DO 130 I = NP1, NPM                                               PSOL       22
         KI = K(I)                                                      PSOL       23
         IF(KI .LE. N) GO TO 130                                        PSOL       24
            X(KI) = 0.                                                  PSOL       25
            KK = KI - N                                                 PSOL       26
            L = IND(KK)                                                 PSOL       27
            IF(KK .GT. N1) GO TO 70                                     PSOL       28
C          ELSE                                                         PSOL       29
               DO 40 J = 1, N1                                          PSOL       30
                  X(KI) = X(KI) + Q(KK,J)*X(J)                          PSOL       31
   40          CONTINUE                                                 PSOL       32
               IF(N11 .GT. N2) GO TO 55                                 PSOL       33
               DO 50 J = N11, N2                                        PSOL       34
                  X(KI) = X(KI) + A1(IND(J),KK)*X(J)                    PSOL       35
   50          CONTINUE                                                 PSOL       36
   55          IF(N21 .GT. N) GO TO 130                                 PSOL       37
               DO 60 J = N21, N                                         PSOL       38
                  X(KI) = X(KI) + A2(IND(J),KK)*X(J)                    PSOL       39
   60          CONTINUE                                                 PSOL       40
               GO TO 130                                                PSOL       41
C          END                                                          PSOL       42
   70       IF(KK .GT. N2) GO TO 90                                     PSOL       43
C          ELSE                                                         PSOL       44
               DO 80 J = 1, N1                                          PSOL       45
                  X(KI) = X(KI) + A1(L,J)*X(J)                          PSOL       46
   80          CONTINUE                                                 PSOL       47
               GO TO 130                                                PSOL       48
C          END                                                          PSOL       49
```

```
SUBROUTINE PSOL        73/74   OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

                 90       IF(KK .GT. N) GO TO 110                                    PSOL        50
            C            ELSE                                                        PSOL        51
                             DO 100 J = 1, N1                                        PSOL        52
                                X(KI) = X(KI) + A2(L,J)*X(J)                         PSOL        53
                100          CONTINUE                                                PSOL        54
                             GO TO 130                                               PSOL        55
            C            END                                                         PSOL        56
                110       DO 120 J = 1, N1                                           PSOL        57
                             X(KI) = X(KI) - OBJ(J)*X(J)                             PSOL        58
                120       CONTINUE                                                   PSOL        59
                          IF(N11 .GT. N) GO TO 130                                   PSOL        60
                          DO 124 J = N11, N                                          PSOL        61
                             X(KI) = X(KI) + OBJ(J)*X(J)                             PSOL        62
                124       CONTINUE                                                   PSOL        63
            C        END                                                             PSOL        64
                130 CONTINUE                                                         PSOL        65
                    RETURN                                                           PSOL        66
                    END                                                              PSOL        67


SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  PSOL           1          74

VARIABLES      SN  TYPE            RELOCATION
    0  A1         REAL       ARRAY      F.P.      REFS        9       43       54    DEFINED       1
    0  A2         REAL       ARRAY      F.P.      REFS        9       47       61    DEFINED       1
   20  RIGM       REAL                  XXXUP     REFS       14
    0  E          REAL       ARRAY      F.P.      REFS        9       25    DEFINED       1
    4  EPS        REAL                  XXXUP     REFS       12
  254  I          INTEGER                          REFS       20       31    DEFINED      19       30
    6  IERR       INTEGER               XXXUP     REFS       12
    0  IND        INTEGER    ARRAY      F.P.      REFS       10       35       43       47    DEFINED       1
    0  IOBJ       INTEGER               XXXUP     REFS       12
   11  IPIV       INTEGER               XXXUP     REFS       13
    5  ITER       INTEGER               XXXUP     REFS       12
    3  ITMAX      INTEGER               XXXUP     REFS       12
  257  J          INTEGER                          REFS       25     2*39     2*43     2*47     2*54     2*61     2*66
                                                    2*70  DEFINED       24       38       42       46       53       60
                                                      65       69
  256  JJ         INTEGER                          REFS       25       26    DEFINED      23       26
   12  JPIV       INTEGER               XXXUP     REFS       13
    0  K          INTEGER    ARRAY      F.P.      REFS        9       20       25       31    DEFINED       1
  255  KI         INTEGER                          REFS       21       22     3*25       32       33       34     2*39
                                                    2*43     2*47     2*54     2*61     2*66     2*70
                                                 DEFINED       20       31
  260  KK         INTEGER                          REFS       35       36       39       43       47       51       58
                                                 DEFINED       34
  261  L          INTEGER                          REFS       54       61    DEFINED      35
    1  M          INTEGER               XXXUP     REFS       12
    2  N          INTEGER               XXXUP     REFS       12       21       24       26       32       34       45
                                                      46       58       68       69
   13  NEGV       LOGICAL               XXXUP     REFS       13       15
```

```
      SUBROUTINE PSOL        73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      3

VARIABLES      SN  TYPE            RELOCATION
   21  NINVT       INTEGER              XXXUP        REFS        14
   10  NPM         INTEGER              XXXUP        REFS        13        19        30
    7  NP1         INTEGER              XXXUP        REFS        13        19        30
    0  NUD         INTEGER               F.P.        REFS         9   DEFINED         1
   14  N1          INTEGER              XXXUP        REFS        13        36        38        53        60        65
    0  N1D         INTEGER               F.P.        REFS         9   DEFINED         1
   15  N11         INTEGER              XXXUP        REFS        13        41        42        68        69
   16  N2          INTEGER              XXXUP        REFS        13        41        42        51
    0  N2D         INTEGER               F.P.        REFS         9   DEFINED         1
   17  N21         INTEGER              XXXUP        REFS        13        45        46
    0  OBJ         REAL        ARRAY     F.P.        REFS        10        66        70   DEFINED         1
   22  PHASE1      LOGICAL              XXXUP        REFS        14        15
    0  U           REAL        ARRAY     F.P.        REFS         9        39   DEFINED         1
    0  X           REAL        ARRAY     F.P.        REFS         9      2*25      2*39      2*43      2*47      2*54      2*61
                                                      2*66      2*70   DEFINED         1        22        25        33        39
                                                        43        47        54        61        66        70

STATEMENT LABELS              DEF LINE    REFERENCES
    0  10                        27         24
   52  20                        28         21
    0  30                        29         19
    0  40                        40         38
    0  50                        44         42
  126  55                        45         41
    0  60                        48         46
  151  70                        51         36
    0  80                        55         53
  172  90                        58         51
    0  100                       62         60
  213  110                       65         58
    0  120                       67         65
    0  124                       71         69
  243  130                       73         30        32        45        49        56        63        68

LOOPS  LABEL     INDEX     FROM-TO     LENGTH    PROPERTIES
   26  30      * I          19 29        27H              NOT INNER
   43  10        J          24 27         5H     INSTACK
   57  130     * I          30 73       167H              NOT INNER
  100  40        J          38 40         3H     INSTACK
  120  50        J          42 44         4H     INSTACK
  142  60        J          46 48         4H     INSTACK
  164  80        J          53 55         3H     INSTACK
  205  100       J          60 62         3H     INSTACK
  221  120       J          65 67         3H     INSTACK
  236  124       J          69 71         3H     INSTACK

COMMON BLOCKS   LENGTH
       XXXUP       19

STATISTICS
  PROGRAM LENGTH                 340H      224
  CM LABELED COMMON LENGTH        23H       19
          52000B CM USED
```

```
SUBROUTINE SETINV      73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

      SUBROUTINE SETINV(Q,A1,A2,OBJ,IND,K,E,AROW,NQD,N1D,N2D)          SETINV     2
C                                                                      LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                      LINE       4
C     INITIAL INVERSE                                                  SETINV     4
C                                                                      LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                      LINE       4
      DIMENSION K(1), E(1), AROW(1)                                    SETINV     6
C                                                                      SETINV     7
      COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, IIER, IERR                /XXXQP/    2
      COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21      /XXXQP/    3
      COMMON /XXXQP/ MIGM, NINVT, PHASE1                               /XXXQP/    4
      LOGICAL NEGV, PHASE1                                             /XXXQP/    5
C                                                                      LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                      LINE       4
C     SET E TO THE IDENTITY                                            SETINV    10
      JJ = 0                                                           SETINV    11
      DO 20 J = 1, N                                                   SETINV    12
         DO 10 I = 1, N                                                SETINV    13
            E(I+JJ) = 0.                                               SETINV    14
   10       CONTINUE                                                   SETINV    15
         E(J+JJ) = 1.                                                  SETINV    16
         JJ = JJ + N                                                   SETINV    17
   20    CONTINUE                                                      SETINV    18
C     GENERATE INITIAL INVERSE                                         SETINV    19
      DO 30 J = 1, N                                                   SETINV    20
         K(J) = -K(J)                                                  SETINV    21
   30    CONTINUE                                                      SETINV    22
      DO 90 JJ = 1, N                                                  SETINV    23
         DO 40 J = 1, N                                                SETINV    24
            IF (K(J).LT.0)  GO TO 50                                   SETINV    25
   40       CONTINUE                                                   SETINV    26
   50    CONTINUE                                                      SETINV    27
         KROW = -K(J)                                                  SETINV    28
         CALL GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,N1D,N2D)          SETINV    29
         ROWMAX = 0.                                                   SETINV    30
         DO 70 L = 1, N                                                SETINV    31
            TEST = ABS(AROW(L))                                        SETINV    32
            IF (K(L).GT.0 .OR. TEST.LT.ROWMAX)  GO TO 60               SETINV    33
               ROWMAX = TEST                                           SETINV    34
               JPIV = L                                                SETINV    35
   60       CONTINUE                                                   SETINV    36
   70       CONTINUE                                                   SETINV    37
         IF (ROWMAX.GT.0.)  GO TO 80                                   SETINV    38
            IERR = 3                                                   SETINV    39
            RETURN                                                     SETINV    40
   80    CONTINUE                                                      SETINV    41
         K(J) = K(JPIV)                                                SETINV    42
         K(JPIV) = KROW                                                SETINV    43
         CALL NEWINV(E,AROW)                                           SETINV    44
   90    CONTINUE                                                      SETINV    45
      IIER = 0                                                         SETINV    46
      RETURN                                                           SETINV    47
      END                                                              SETINV    48
```

```
      SUBROUTINE SETINV      74/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

      SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  SETINV          1          48         55

VARIABLES      SN  TYPE            RELOCATION
    0  ARGW        REAL        ARRAY      F.P.        REFS        9        37        40        52    DEFINED        1
    0  A1          REAL                   F.P.        REFS       37    DEFINED         1
    0  A2          REAL                   F.P.        REFS       37    DEFINED         1
   20  BIGM        REAL                   XXXUP       REFS       13
    0  E           REAL        ARRAY      F.P.        REFS        9        37        52    DEFINED        1        22        24
    4  EPS         REAL                   XXXUP       REFS       11
  164  I           INTEGER                            REFS       22    DEFINED        21
    6  IERR        INTEGER                XXXUP       REFS       11    DEFINED        47
    0  IND         INTEGER                F.P.        REFS       37    DEFINED         1
    0  IOBJ        INTEGER                XXXUP       REFS       11
   11  IPIV        INTEGER                XXXUP       REFS       12
    5  ITER        INTEGER                XXXUP       REFS       11    DEFINED        54
    3  ITMAX       INTEGER                XXXUP       REFS       11
  163  J           INTEGER                            REFS       24      2*29        33        36        50
                                                   DEFINED       26        28        32
  162  JJ          INTEGER                            REFS       22        24        25    DEFINED        19        25        31
   12  JPIV        INTEGER                XXXUP       REFS       12        50        51    DEFINED        43
    0  K           INTEGER     ARRAY      F.P.        REFS        9        29        33        36        41        50
                                                   DEFINED        1        29        50        51
  165  KROW        INTEGER                            REFS       37        51    DEFINED        36
  167  L           INTEGER                            REFS       40        41        43    DEFINED        39
    1  M           INTEGER                XXXUP       REFS       11
    2  N           INTEGER                XXXUP       REFS       11        20        21        25        28        31        32
                                                                 19
   13  NEGV        LOGICAL                XXXUP       REFS       12        14
   21  NINVT       INTEGER                XXXUP       REFS       13
   10  NPM         INTEGER                XXXUP       REFS       12
    7  NP1         INTEGER                XXXUP       REFS       12
    0  NUD         INTEGER                F.P.        REFS       37    DEFINED         1
   14  N1          INTEGER                XXXUP       REFS       12
    0  N1D         INTEGER                F.P.        REFS       37    DEFINED         1
   15  N11         INTEGER                XXXUP       REFS       12
   16  N2          INTEGER                XXXUP       REFS       12
    0  N2D         INTEGER                F.P.        REFS       37    DEFINED         1
   17  N21         INTEGER                XXXUP       REFS       12
    0  OBJ         REAL                   F.P.        REFS       37    DEFINED         1
   22  PHASE1      LOGICAL                XXXUP       REFS       13        14
    0  W           REAL                   F.P.        REFS       37    DEFINED         1
  166  ROWMAX      REAL                               REFS       41        46    DEFINED        38        42
  170  TEST        REAL                               REFS       41        42    DEFINED        40

EXTERNALS          TYPE    ARGS        REFERENCES
       GETROW                1              37
       NEWINV                2              52

INLINE FUNCTIONS   TYPE    ARGS        DEF LINE   REFERENCES
       ABS         REAL      1   INTRIN                 40
```

```
      SUBROUTINE SETINV      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      3

STATEMENT LABELS                DEF LINE    REFERENCES
    0  10                          23          21
    0  20                          26          20
    0  30                          30          28
    0  40                          34          32
   65  50                          35          33
  117  60                          44          41
    0  70                          45          19
  126  80                          49          46
    0  90                          53          31

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
   22  20      * J           20 26        20B                NOT INNER
   31  10        I           21 23        2B      INSTACK
   50  30        J           28 30        2B      INSTACK
   55  90      * JJ          31 53        63B                EXT REFS  NOT INNER
   57  40      * J           32 34        5B      INSTACK     EXITS
  111  70        L           39 45        7B      INSTACK

COMMON BLOCKS    LENGTH
       XXXGP        19

STATISTICS
  PROGRAM LENGTH                 230B      152
  CM LABELED COMMON LENGTH        23B       19
          52000B CM USED
```

```
SUBROUTINE NEWINV      73/74*  (OPT=1)                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

                 SUBROUTINE NEWINV(E,AROW)                                          NEWINV      2
           C                                                                        LINE        2
           C-----------------------------------------------------------------------LINE        3
           C                                                                        LINE        4
           C     INVERSE UPDATE BY COLUMN OPERATIONS                                NEWINV      4
           C                                                                        LINE        2
           C-----------------------------------------------------------------------LINE        3
           C                                                                        LINE        4
                 DIMENSION E(1), AROW(1)                                            NEWINV      6
           C                                                                        NEWINV      7
                 COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                  /XXXQP/     2
                 COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21        /XXXQP/     3
                 COMMON /XXXQP/ BIGM, NINVT, PHASE1                                 /XXXQP/     4
                 LOGICAL NEGV, PHASE1                                               /XXXQP/     5
           C                                                                        LINE        2
           C-----------------------------------------------------------------------LINE        3
           C                                                                        LINE        4
                 JJPIV = (JPIV-1)*N                                                 NEWINV     10
                 DO 20 I = 1, N                                                     NEWINV     11
                    EPIV = E(I+JJPIV)/AROW(JPIV)                                    NEWINV     12
                    IF(ABS(EPIV)-EPS) 15, 20, 15                                    NEWINV     13
              15    JJ = 0                                                          NEWINV     14
                    DO 10 J = 1, N                                                  NEWINV     15
                       E(I+JJ) = E(I+JJ) - EPIV*AROW(J)                             NEWINV     16
                       JJ = JJ + N                                                  NEWINV     17
              10    CONTINUE                                                        NEWINV     18
                    E(I+JJPIV) = EPIV                                               NEWINV     19
              20    CONTINUE                                                        NEWINV     20
                 RETURN                                                             NEWINV     21
                 END                                                                NEWINV     22
```

SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS		DEF LINE	REFERENCES
4	NEWINV	1	29

VARIABLES		SN	TYPE		RELOCATION								
0	AROW		REAL	ARRAY	F.P.	REFS	9	20	24	DEFINED	1		
20	BIGM		REAL		XXXQP	REFS	13						
0	E		REAL	ARRAY	F.P.	REFS	9	20	24	DEFINED	1	24	27
60	EPIV		REAL			REFS	21	24	27	DEFINED	20		
4	EPS		REAL		XXXQP	REFS	11	21					
57	I		INTEGER			REFS	20	2*24	27	DEFINED	19		
6	IERR		INTEGER		XXXQP	REFS	11						
0	IOBJ		INTEGER		XXXQP	REFS	11						
11	IPIV		INTEGER		XXXQP	REFS	12						
5	ITER		INTEGER		XXXQP	REFS	11						
3	ITMAX		INTEGER		XXXQP	REFS	11						
62	J		INTEGER			REFS	24	DEFINED	23				
61	JJ		INTEGER			REFS	2*24	25	DEFINED	22	25		
56	JJPIV		INTEGER			REFS	20	27	DEFINED	18			
12	JPIV		INTEGER		XXXQP	REFS	12	18	20				
1	M		INTEGER		XXXQP	REFS	11						

```
SUBROUTINE NEWINV      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

VARIABLES       SN  TYPE            RELOCATION
    2  N            INTEGER             XXXQP        REFS        11        18        19        23        25
   13  NEGV         LOGICAL             XXXQP        REFS        12        14
   21  NINVT        INTEGER             XXXQP        REFS        13
   10  NPM          INTEGER             XXXQP        REFS        12
    7  NP1          INTEGER             XXXQP        REFS        12
   14  N1           INTEGER             XXXQP        REFS        12
   15  N11          INTEGER             XXXQP        REFS        12
   16  N2           INTEGER             XXXQP        REFS        12
   17  N21          INTEGER             XXXQP        REFS        12
   22  PHASE1       LOGICAL             XXXQP        REFS        13        14

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE  REFERENCES
       ABS          REAL       1  INTRIN               21

STATEMENT LABELS                DEF LINE    REFERENCES
    0  10                          26          23
    0  15            INACTIVE      22         2*21
   52  20                          28          19         21

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
   23  20      * I            19 28        32B                NOT INNER
   41  10        J            23 26         5B      INSTACK

COMMON BLOCKS    LENGTH
       XXXQP        19

STATISTICS
  PROGRAM LENGTH                  72B        5B
  CM LABELLED COMMON LENGTH       23B       19
          52000B CM USED
```

```
SUBROUTINE QPTAB      73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

      SUBROUTINE QPTAB(IOUT,Q,A1,A2,P,IND,IPASS,C,VALUE,Y,                QPTAB      2
     *                 NQD,N1D,N2D)                                       QPTAB      3
C                                                                         LINE       2
C-----------------------------------------------------------------------  LINE       3
C                                                                         LINE       4
C      OUTPUT QUADRATIC PROGRAM TABLEAU                                   QPTAB      5
C      WOLFQP MUST HAVE PREVIOUSLY BEEN CALLED TO                         QPTAB      6
C      RECEIVE THE COMPLETE OUTPUT                                        QPTAB      7
C                                                                         QPTAB      8
      DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), IPASS(11)                 QPTAB      9
      DIMENSION P(1), IND(1), Y(1)                                        QPTAB     10
C                                                                         QPTAB     11
C                                                                         QPTAB     12
      NAMELIST /OUT/                                                      QPTAB     13
     1  N, NCON, NUNC, NINEQ, NEQ, NINV,                                  QPTAB     14
     2  ITMAX, ITER, IERR, C, VALUE                                       QPTAB     15
C                                                                         QPTAB     16
C                                                                         QPTAB     17
    1 FORMAT(1H0,9X,1HQ,/)                                                QPTAB     18
    2 FORMAT(1H0,9X,1HP,/)                                                QPTAB     19
    3 FORMAT(1X,10(2X,E10.4))                                             QPTAB     20
    4 FORMAT(1H0,4X,4HA1*X,9X,2HB1,15X,2HA1,/)                            QPTAB     21
    5 FORMAT(1H0,4X,4HA2*X,9X,2HB2,15X,2HA2,/)                            QPTAB     22
    6 FORMAT(1H0)                                                         QPTAB     23
    7 FORMAT(1H ,24X,8(2X,E10.4))                                         QPTAB     24
    8 FORMAT(1H0,9X,1HX,/)                                                QPTAB     25
C                                                                         LINE       2
C-----------------------------------------------------------------------  LINE       3
C                                                                         LINE       4
      N = IPASS(1)                                                        QPTAB     27
      NCON = IPASS(5)                                                     QPTAB     28
      NUNC = IPASS(6)                                                     QPTAB     29
      NINEQ = IPASS(7)                                                    QPTAB     30
      NEQ = N - (NCON + NUNC + NINEQ)                                     QPTAB     31
      NINV = IPASS(8)                                                     QPTAB     32
      ITMAX = IPASS(2)                                                    QPTAB     33
      ITER = IPASS(3)                                                     QPTAB     34
      IERR = IPASS(4)                                                     QPTAB     35
      NN = NCON + NUNC                                                    QPTAB     36
      NN1 = NCON + NUNC + NINEQ                                           QPTAB     37
      WRITE(IOUT,OUT)                                                     QPTAB     38
      WRITE(IOUT,1)                                                       QPTAB     39
      DO 110 I = 1, NN, 10                                                QPTAB     40
         N1 = MIN0(I+9,NN)                                                QPTAB     41
         DO 100 J = 1, NN                                                 QPTAB     42
            WRITE(IOUT,3)(Q(J,L),L=I,N1)                                  QPTAB     43
  100    CONTINUE                                                         QPTAB     44
         WRITE(IOUT,6)                                                    QPTAB     45
  110 CONTINUE                                                            QPTAB     46
      WRITE(IOUT,2)                                                       QPTAB     47
      DO 120 I = 1, NN, 10                                                QPTAB     48
         N1 = MIN0(I+9,NN)                                                QPTAB     49
         WRITE(IOUT,3)(P(L),L=I,N1)                                       QPTAB     50
  120 CONTINUE                                                            QPTAB     51
      IF(NINEQ .LE. 0) GO TO 160                                          QPTAB     52
      WRITE(IOUT,4)                                                       QPTAB     53
      DO 150 I = 1, NN, 8                                                 QPTAB     54
```

```
SUBROUTINE QPTAB      73/74   OPT=1                    FTN 4.6+452        82/01/28. 14.34.48       PAGE      2

                 N1 = MINO(I+7,NN)                                          QPTAB      55
                 DO 140 L = 1, NINEQ                                        QPTAB      56
                    LL = IND(L+NN)                                          QPTAB      57
                    IF(I .GT. 1) GO TO 135                                  QPTAB      58
          C       ELSE                                                      QPTAB      59
                       SUM = 0.                                             QPTAB      60
                       DO 130 K = 1, NN                                     QPTAB      61
                          SUM = SUM + A1(LL,K)*Y(K)                         QPTAB      62
            130        CONTINUE                                             QPTAB      63
                       WRITE(IOUT,3) SUM, P(NN+L), (A1(LL,K),K=I,N1)        QPTAB      64
                       GO TO 140                                            QPTAB      65
          C       END                                                       QPTAB      66
            135      WRITE(IOUT,7)(A1(LL,K),K=I,N1)                         QPTAB      67
            140   CONTINUE                                                  QPTAB      68
                  WRITE(IOUT,6)                                             QPTAB      69
            150 CONTINUE                                                    QPTAB      70
            160 IF(NEQ .LE. 0) GO TO 200                                    QPTAB      71
                WRITE(IOUT,5)                                               QPTAB      72
                DO 190 I = 1, NN, 8                                         QPTAB      73
                   N1 = MINO(I+7,NN)                                        QPTAB      74
                   DO 180 L = 1, NEQ                                        QPTAB      75
                      LL = IND(L+NN1)                                       QPTAB      76
                      IF(I .GT. 1) GO TO 175                                QPTAB      77
          C       ELSE                                                      QPTAB      78
                       SUM = 0.                                             QPTAB      79
                       DO 170 K = 1, NN                                     QPTAB      80
                          SUM = SUM + A2(LL,K)*Y(K)                         QPTAB      81
            170        CONTINUE                                             QPTAB      82
                       WRITE(IOUT,3) SUM, P(NN1+L), (A2(LL,K),K=I,N1)       QPTAB      83
                       GO TO 180                                            QPTAB      84
          C       END                                                       QPTAB      85
            175      WRITE(IOUT,7)(A2(LL,K),K=I,N1)                         QPTAB      86
            180   CONTINUE                                                  QPTAB      87
                  WRITE(IOUT,6)                                             QPTAB      88
            190 CONTINUE                                                    QPTAB      89
            200 WRITE(IOUT,8)                                               QPTAB      90
                DO 210 I = 1, NN, 10                                        QPTAB      91
                   N1 = MINO(I+9,NN)                                        QPTAB      92
                   WRITE(IOUT,3)(Y(L),L=I,N1)                               QPTAB      93
            210 CONTINUE                                                    QPTAB      94
                RETURN                                                      QPTAB      95
                END                                                         QPTAB      96


      SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  QPTAB          1          98

VARIABLES      SN  TYPE            RELOCATION
    0  A1          REAL       ARRAY      F.P.       REFS       10        65        67        70    DEFINED        1
    0  A2          REAL       ARRAY      F.P.       REFS       10        84        86        89    DEFINED        1
    0  C           REAL                  F.P.       REFS       14    DEFINED        1
  572  I           INTEGER                          REFS       44        46        52        53        58        61        67
```

```
      SUBROUTINE OPTAB      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.46        PAGE     3

VARIABLES      SN  TYPE            RELOCATION
                                                        70        77        80        86        89        95        96
                                                   DEFINED        43        51        57        76        94
  567  IERR       INTEGER                             REFS        14   DEFINED        38
    0  IND        INTEGER    ARRAY      F.P.         REFS        11        60        79   DEFINED         1
    0  IOUT       INTEGER               F.P.      DEFINED         1   I/O REFS        41        42        46        48        50
                                                       53        56        67        70        72        75        86        89
                                                       91        93        96
    0  IPASS      INTEGER    ARRAY      F.P.         REFS        10        30        31        32        33        35        36
                                                       37        38   DEFINED         1
  566  ITER       INTEGER                             REFS        14   DEFINED        37
  565  ITMAX      INTEGER                             REFS        14   DEFINED        36
  574  J          INTEGER                             REFS        46   DEFINED        45
  600  K          INTEGER                             REFS      2*65        67        70      2*84        86        89
                                                   DEFINED        64        67        70        83        86        89
  575  L          INTEGER                             REFS        46        53        60        67        79        86        96
                                                   DEFINED        46        53        59        78        96
  576  LL         INTEGER                             REFS        65        67        70        84        86        89
                                                   DEFINED        60        79
  557  N          INTEGER                             REFS        14        34   DEFINED        30
  560  NCON       INTEGER                             REFS        14        34        39        40   DEFINED        31
  563  NEQ        INTEGER                             REFS        14        74        78   DEFINED        34
  562  NINEQ      INTEGER                             REFS        14        34        40        55        59
                                                   DEFINED        33
  564  NINV       INTEGER                             REFS        14   DEFINED        35
  570  NN         INTEGER                             REFS        43        44        45        51        52        57        58
                                                       60        64        67        76        77        83        94        95
                                                   DEFINED        39
  571  NN1        INTEGER                             REFS        79        86   DEFINED        40
    0  NOD        INTEGER               F.P.         REFS        10   DEFINED         1
  561  NUNC       INTEGER                             REFS        14        34        39        40   DEFINED        32
  573  N1         INTEGER                             REFS        46        53        67        70        86        89        96
                                                   DEFINED        44        52        58        77        95
    0  N1D        INTEGER               F.P.         REFS        10   DEFINED         1
    0  N2D        INTEGER               F.P.         REFS        10   DEFINED         1
    0  P          REAL       ARRAY      F.P.         REFS        11        53        67        86   DEFINED         1
    0  Q          REAL       ARRAY      F.P.         REFS        10        46   DEFINED         1
  577  SUM        REAL                                REFS        65        67        84        86   DEFINED        63        65
                                                       82        84
    0  VALUE      REAL                  F.P.         REFS        14   DEFINED         1
    0  Y          REAL       ARRAY      F.P.         REFS        11        65        84        96   DEFINED         1
       VARIABLES USED AS FILE NAMES. SEE ABOVE

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
       MIN0        INTEGER    0  INTRIN                44        52        58        77        95

NAMELISTS          DEF LINE      REFERENCES
       OUT             14          41

STATEMENT LABELS                DEF LINE    REFERENCES
  421  1          FMT               19          42
  424  2          FMT               20          50
  427  3          FMT               21          46        53        67        86        96
  432  4          FMT               22          56
  436  5          FMT               23          75
  442  6          FMT               24          48        72        91
  444  7          FMT               25          70        89
  447  8          FMT               26          93
```

```
      SUBROUTINE OPTAB       73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      4

STATEMENT LABELS                DEF LINE    REFERENCES
    0  100                         47          45
    0  110                         49          43
    0  120                         54          51
    0  130                         66          64
  204  135                         70          61
  223  140                         71          59          68
    0  150                         73          57
  234  160                         74          55
    0  170                         85          83
  314  175                         89          80
  333  180                         90          78          87
    0  190                         92          76
  344  200                         93          74
    0  210                         97          94

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
   47  110     * I            43 49       34B                EXT REFS  NOT INNER
   54  100     * J            45 47       22B                EXT REFS  NOT INNER
   60          * L            46 46       11B                EXT REFS
  106  120     * I            51 54       15B                EXT REFS
  130  150     * I            57 73      103B                EXT REFS  NOT INNER
  135  140     * L            59 71       71B                EXT REFS  NOT INNER
  152  130       K            64 66        4B      INSTACK
  167          * K            67 67       11B                EXT REFS
  210          * K            70 70       11B                EXT REFS
  240  190     * I            76 92      103B                EXT REFS  NOT INNER
  245  180     * L            78 90       71B                EXT REFS  NOT INNER
  262  170       K            83 85        4B      INSTACK
  277          * K            86 86       11B                EXT REFS
  320          * K            89 89       11B                EXT REFS
  347  210     * L            94 97       15B                EXT REFS

STATISTICS
  PROGRAM LENGTH                 644B      420
          520000B CM USED
```

```
PROGRAM TEST        73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

                 PROGRAM TEST(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                    TEST        2
           C                                                                            LINE        2
           C------------------------------------------------------------------------    LINE        3
           C                                                                            LINE        4
           C     TEST PROGRAM FOR WOLFOP                                                TEST        4
           C                                                                            LINE        2
           C------------------------------------------------------------------------    LINE        3
           C                                                                            LINE        4
                 DIMENSION Q(30,30), A1(10,30), A2(10,30), OBJ(50), IND(50)             TEST        6
                 DIMENSION SCR(3156), Y(30), IPASS(11)                                  TEST        7
                 LOGICAL FLAG                                                           TEST        8
           C                                                                            TEST        9
           C                                                                            LINE        2
           C------------------------------------------------------------------------    LINE        3
           C                                                                            LINE        4
                 NAMELIST /IN/                                                          TEST       11
                1  N, NCON, NUNC, NINEQ, NINV, FLAG, EPS,                               TEST       12
                2  ITMAX, Q, A1, A2, OBJ,IND, C, VALUE                                  TEST       13
           C                                                                            LINE        2
           C------------------------------------------------------------------------    LINE        3
           C                                                                            LINE        4
              10 READ(5,IN)                                                             TEST       15
                 IF(EOF(5)) 999, 20                                                     TEST       16
              20 NQD = 30                                                               TEST       17
                 N1D = 10                                                               TEST       18
                 N2D = 10                                                               TEST       19
                  IPASS(1) = N                                                          TEST       20
                 IPASS(2) = ITMAX                                                       TEST       21
                 IPASS(5) = NCON                                                        TEST       22
                 IPASS(6) = NUNC                                                        TEST       23
                 IPASS(7) = NINEQ                                                       TEST       24
                 IPASS(8) = NINV                                                        TEST       25
                 IPASS(9) = NQD                                                         TEST       26
                 IPASS(10) = N1D                                                        TEST       27
                 IPASS(11) = N2D                                                        TEST       28
                 CALL WOLFOP(Q,A1,A2,OBJ,IND,IPASS,C,VALUE,SCR,Y,FLAG,EPS)              TEST       29
                 CALL OPTAB(6,Q,A1,A2,OBJ,IND,IPASS,C,VALUE,Y,NQD,N1D,N2D)              TEST       30
                 GO TO 10                                                               TEST       31
             999 CONTINUE                                                               TEST       32
                 STOP                                                                   TEST       33
                 END                                                                    TEST       34


      SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
 4137  TEST            1

VARIABLES      SN  TYPE            RELOCATION
 6121  A1         REAL        ARRAY                 REFS        9        16        36        37
 6575  A2         REAL        ARRAY                 REFS        9        16        36        37
 4310  C          REAL                              REFS       16        36        37
 4306  EPS        REAL                              REFS       16        36
 4300  FLAG       LOGICAL                           REFS       11        16        36
```

```
          PROGRAM TEST        73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

VARIABLES       SN  TYPE            RELOCATION
 7333  IND         INTEGER    ARRAY                  REFS        9        16        36        37
15577  IPASS       INTEGER    ARRAY                  REFS       10        36        37   DEFINED        27        28        29
                                                        30        31        32        33        34        35
 4307  ITMAX       INTEGER                           REFS       16        28
 4301  N           INTEGER                           REFS       16        27
 4302  NCON        INTEGER                           REFS       16        29
 4304  NINEQ       INTEGER                           REFS       16        31
 4305  NINV        INTEGER                           REFS       16        32
 4312  NOD         INTEGER                           REFS       33        37   DEFINED        24
 4303  NUNC        INTEGER                           REFS       16        30
 4313  NID         INTEGER                           REFS       34        37   DEFINED        25
 4314  N2D         INTEGER                           REFS       35        37   DEFINED        26
 7251  OBJ         REAL       ARRAY                  REFS        9        16        36        37
 4315  Q           REAL       ARRAY                  REFS        9        16        36        37
 7415  SCR         REAL       ARRAY                  REFS       10        36
 4311  VALUE       REAL                              REFS       16        36        37
15541  Y           REAL       ARRAY                  REFS       10        36        37

FILE NAMES        MODE
    0  INPUT
 2054  OUTPUT
    0  TAPE5       NAME                    READS       22
 2054  TAPE6

EXTERNALS           TYPE    ARGS       REFERENCES
       EOF         REAL       1              23
       OPTAR                 13              37
       WOLFOP                12              36

NAMELISTS         DEF LINE      REFERENCES
       IN             16          22

STATEMENT LABELS                DEF LINE    REFERENCES
 4141  10                         22          38
    0  20            INACTIVE     24          23
 4170  999                        39          23

STATISTICS
  PROGRAM LENGTH                 11462B      4914
  BUFFER LENGTH                   4130B      2136
          52000B CM USED
```

APPENDIX B

MODIFICATION FOR PROJECTION PROBLEMS

WOLFQP as written will solve a projection problem. For large problems space may be reduced by modification to the program.

In the projection problem the Q matrix reduces to the identity. This need not be stored. The specific modifications are:

WOLFQP

Delete the Do Loop with 130
Replace with
TEMP = TEMP + SCR(IP3 + I + N)

GETROW

Delete Do Loop with 50
Replace with
AROW (J) = AROW(J) + E(KK + JJ)

PSOL

Delete Do Loop with 40
Replace with
X(KI) = X(KI) + X(KK)

By then substituting a dummy argument for Q in the calling Sequence the user has removed the necessity of storing the identity matrix. Of course the user may then remove Q and NQD completely from the calling sequence if desired.

APPENDIX C

MACHINE DEPENDENCIES

BIGM - Defined by Data Statement in WOLFQP. BIGM represents machine infinity. For CDC Machines 10^{100} is used. This constant clearly depends on the exponent range available.

BASIC - An array used to store internal flags. To reduce space 7 bits of each BASIC word are used to store flags the remaining bits are required to store an integer. If the word size of the machine is M bits this integer must be less than $2^{M-7}-1$. For CDC machines this becomes $2^{53}-1$. This number $2^{M-7}-1$ represents the largest problem which can be solved with this encoding of the algorithm.

Since bit operations are performed on the Basic array certain bit functions must be utilized.

Octal constants are used to set certain bit patterns in SETQP, DSIMP and PIVROW. They are either used to set the bit patterns or mask certain bits from the given word. The octal constants appear only in Data Statements in the appropriate routines. They appear as integers followed by the letter B. These are numbers written in Base 8 and must be written with the appropriate notation and in the appropriate base for the machine.

i.e. 7B = 7 Hex
10B = 8 Hex
60B = 30 Hex

The Machine functions utilized are:

OR(A,B) - Perform bit by bit logical or on A,B. The truth table is

	0	1
0	0	1
1	1	1

AND(A,B) - Perform bit by bit logical and on A,B. The truth table is

	0	1
0	0	0
1	0	1

SHIFT(A,I) - SHIFT the word I bits. If I is positive SHIFT left I positions. If I is negative shift right I positions.

i.e. if A contains the following bit pattern

A = 0001011010

then

SHIFT(A,2) = 0101101000
SHIFT(A,-2)= 0000010110

COMPL(A) - form bit by bit Boolean complement of A.

i.e. if A = 10110

COMPL(A)= 01001

The following library function is required in PIVCOL

RANF(X) - returns a uniform random number, the argument is ignored, in the function.
The random number is used in an anti-cycling procedure. Some other method of preventing cycling could be used instead.

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