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A BROADBAND WAVEGUIDE TRANSFER STANDARD FOR THE DISSEMINATION OF UK NATIONAL MICROWAVE POWER STANDARDS

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This memorandum describes the design, construction and use of an X-band transfer standard to calibrate reference standards in terms of the primary standard held at RSRE (Malvern). Sample results are quoted together with a rigorous analysis of the uncertainties involved.
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1 INTRODUCTION

The UK national standards for the measurement of microwave power in waveguide have been described in earlier reports\(^1\),(2)\) and the results of comparisons with the standards of other countries have also been reported\(^1\),\(^3\). The UK national standards consist of tuned barreter mounts calibrated at a nominal \(10 \text{ mW}\) level using a microcalorimeter\(^4\). These mounts, although portable, are not ideal for use as transfer standards because they have a limited dynamic range and are calibrated at a single frequency.

This paper describes a portable transfer standard which enables WG16 power meters to be calibrated in terms of the national standards over the range \(10 \text{ mW} \text{ to } 1 \text{ W}\) at frequencies of 8.2, 9.0, 10.0 and 12.4 GHz.

2 DESCRIPTION

2.1 BASIC PRINCIPLE

The transfer standard consists of a commercial high directivity WG16 20 dB coupler with a thermistor mount on the side-arm (Fig 1). The mount is contained within a temperature controlled enclosure and thermally and electrically isolated from the waveguide, to make it independent of the laboratory environment. The coupler feeds a fixed fraction of the microwave power in the waveguide to the thermistor where it is measured using a DC substitution technique. The sum of microwave and DC power dissipated in the thermistor is maintained constant by a self-balancing bridge of the type described by Larsen\(^5\). Using the known linearity of the thermistor mount with power\(^6\) the transfer standard can be used to measure any microwave power level between 10 mW and 1 W.

The Calibration Factor (CF) of the transfer standard is defined as the ratio of the microwave power present at the output port to the microwave power indicated by the bridge. This ratio is found by using the national standards to measure the power at the output port for each of the four frequencies.

2.2 BRIDGE

The Larsen self-balancing bridge in its most simple form consists of two amplifiers, a standard resistor and a thermistor (Fig 2). The circuit is arranged so that amplifier A is a voltage follower and applies the
voltage across the thermistor to the standard resistor, $R_s$. At the same
time amplifier B makes the currents through the two components equal.
Thus if the voltages across and the currents through the two components are
maintained equal, then the resistance of the standard resistor and the
thermistor are maintained equal. It can be seen that this method uses
4-wire connections which enables the thermistor to be distant from the
bridge.

The voltage across the thermistor (bridge voltage), is measured at
point V with respect to earth using a high impedance voltmeter. The
circuit used (Fig 3) includes a meter to give a visual indication of the
thermistor current, and external transistors as the output stages of the
amplifiers to reduce drift due to self heating. The two amplifiers are
completely independent and require independent power supplies.

2.3 TEMPERATURE CONTROLLER

Figure 4 shows the components used to control the temperature of the
thermistor mount. Protection from air temperature variations is achieved
by a perspex enclosure of 6 mm wall thickness. Thermal losses through
the waveguide are reduced by using a 10 mm length of thinwall waveguide
made of 0.003" electroformed copper supported by a 3 mm thickness of
fibreglass. The temperature of the thermistor mount is controlled above
ambient by a heater made from 30 turns of resistance wire wound around
the shoulders of two waveguide flanges soldered back to back (Fig 5).
The top flange has a hole in one side to accommodate a temperature sensor.

The electronic circuit shown in Figure 6 has three main sections.
The first is a bridge and amplifier which produces an error voltage
proportional to the difference between the temperature required and that
detected by the sensor. The second is a voltage generator which produces
a ramp with a peak value of 3.5 volts at a frequency of 3 Hz. The third
is a comparator and output stage which compares the error voltage with the
voltage ramp and turns the heater on while the error voltage is larger
than the ramp voltage (Fig 7). The resistance of the heater is such that
when it is on, about 20 watts are dissipated around the waveguide. The
large thermal inertia of the brass waveguide flanges smoothes out the
pulses of heat. This method combines the best features of proportional
control and "on-off" control in that the heat supplied is proportional to
the heat required yet the heater is either fully on or off which reduces waste heat in the output stage.

This controller stabilizes the temperature of the thermistor mount to within 10 m°C/hr when operated in an environment where the temperature excursions are less than 1°C.

3 OPERATION

3.1 USING THE TRANSFER STANDARD

The level of microwave power at the thermistor is calculated from the change in bridge voltage when that power is applied. If the bridge voltage is normally $V_1$ and this falls to $V_2$ when the power is present then the power supplied to the thermistor is given by

$$P_T = \frac{V_2^2 - V_1^2}{R_T}$$

(1a)

where $R_T$ is the resistance of the thermistor when the bridge is balanced.

Although the thermistor mount is temperature controlled, some drift may occur. The effect of this may be reduced by using the following measurement procedure.

1. With no microwave power present measure $V_{1A}$.
2. Apply microwave power and measure $V_{2A}$.
3. Remove microwave power and measure $V_{1B}$.

If the time intervals between each measurement are the same and any temperature drift is linear then $V_1$, the average of $V_{1A}$ and $V_{1B}$, is the bridge voltage that would have been present at the time when microwave power was applied.

For measuring low power (< 10 mW at the output port) the limiting factor is voltmeter resolution. Another digit of resolution can be obtained by using a sufficiently stable voltage reference $V$ to back-off the bridge output such that $V_1 - V$ is less than 1 volt. If we let $V_a$ be the measured voltage difference $V_1 - V$ and $V_b = V_2 - V$ then equation 1a
becomes

\[ P_T = \frac{2V(V_a - V_b) + V_a^2 - V_b^2}{R_T} \]  

(1b)

If \( P_{STD} \) is the power at the output port measured using the national standards, then the calibration faction is given by

\[ CF = \frac{P_{STD}R_T}{2V(V_a - V_b) + V_a^2 - V_b^2} \]  

(2)

The value of \( R_T \) is found by replacing the thermistor within a variable resistor which is adjusted until the bridge balances. The variable resistor is then removed and measured by conventional means.

3.2 ILLUSTRATIVE RESULTS

The measurements of Calibration Factor shown in Table 1 are taken from a comparison exercise between the bolometer power standards held at RSRE (Malvern) and the water calorimeters held at EQD (Aquila) carried out in 1980.

| Freq GHz | RSRE     |      |      | EQD (Aquila) |      |      |      |      |
|----------|----------|------|------|-------------|------|------|------|------|      |
|          | May      | Dec  | Mean | Cal A       | Cal B| Mean | Mean | % Difference in Means |
| 8.2      | 89.83    | 89.82| 89.83| 89.96       | 89.99| 89.98|      | -0.16 |
| 9.0      | 103.33   | 103.24| 103.29| 103.49      | 103.40| 103.44|      | -0.14 |
| 10.0     | 111.84   | 111.94| 111.89| 111.77      | 111.83| 111.80|      | +0.08 |
| 12.4     | 93.92    | 93.83| 93.87| 93.77       | 93.94| 93.85|      | +0.02 |

Table 1

4 UNCERTAINTY ANALYSIS

The uncertainty involved in calibrating the transfer standard against the RSRE bolometers and using it to calibrate a power meter at a level of 1 watt has four major components.
4.1 MISMATCH

Figure 3 shows the schematic diagram of the transfer standard. In terms of the scattering parameters \( S_{11} \) \( \ldots \) \( S_{33} \) the equations for the 3 port are

\[
\begin{align*}
  b_1 &= S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \quad (3a) \\
  b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \quad (3b) \\
  b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3 \quad (3c)
\end{align*}
\]

Letting \( \rho_i \) be the voltage reflection coefficient (vrc) of the device terminating port \( i \) these become

\[
\begin{align*}
  -S_{11}a_1 &= -b_1 + S_{12}b_2\rho_2 + S_{13}b_3\rho_3 \quad (4a) \\
  -S_{21}a_1 &= -b_2 + S_{22}b_2\rho_2 + S_{23}b_3\rho_3 \quad (4b) \\
  -S_{31}a_1 &= -b_3 + S_{32}b_2\rho_2 + S_{33}b_3\rho_3 \quad (4c)
\end{align*}
\]

The power absorbed by the device on port 2 is given by

\[
\frac{|b_2|^2}{R_0} \left( 1 - |\rho_2|^2 \right)
\]

and on port 3 by

\[
\frac{|b_3|^2}{R_0} \left( 1 - |\rho_3|^2 \right)
\]

where \( R_0 \) is the real characteristic impedance to which \( \rho_2 \) and \( \rho_3 \) are referred.

The ratio \( C \) of the powers detected at ports 2 and 3 when port 2 is terminated in a standard of vrc \( \rho_S \) is

\[
C = \left| \frac{b_3}{b_2} \right|^2 \cdot \left[ \frac{1 - |\rho_3|^2}{1 - |\rho_S|^2} \right]
\]

(5)
By solving equations 4 for $b_3$ and $b_2$

$$C = \frac{S_{31}^2}{S_{21}^2} \cdot \frac{1 - \rho_S \left( \frac{S_{22} - S_{21} S_{32}}{S_{31}} \right)^2}{1 - \rho_3 \left( \frac{S_{33} - S_{31} S_{23}}{S_{21}} \right)} \cdot \left[ \frac{1 - |\rho_3|^2}{1 - |\rho_S|^2} \right]$$

If we let

$$\Gamma_2 = S_{22} - \frac{S_{21} S_{32}}{S_{31}}$$

and

$$\Gamma_3 = S_{33} - \frac{S_{31} S_{23}}{S_{21}}$$

then

$$C = \frac{S_{31}^2}{S_{21}^2} \cdot \frac{1 - \rho_S \Gamma_2^2}{1 - \rho_3 \Gamma_3^2} \cdot \left[ \frac{1 - |\rho_3|^2}{1 - |\rho_S|^2} \right]$$

The uncertainty arises from not knowing the phase relationship between $\rho_S$ and $\Gamma_2$. Everything else in equation 7 is either known or common to both finding and using the Calibration Factor. Thus the rectangular limits of uncertainty are given by

$$e_f = \pm \frac{1}{2} \times \left[ \left( \frac{1 + |\rho_S \Gamma_2|}{1 - |\rho_S \Gamma_2|} \right)^2 - 1 \right] \times 100\%$$

In the same way the uncertainty in using the transfer standard to calibrate a power meter with a vrc of $\rho_U$ are

$$e_U = \pm \frac{1}{2} \times \left[ \left( \frac{1 + |\rho_U \Gamma_2|}{1 - |\rho_U \Gamma_2|} \right)^2 - 1 \right] \times 100\%$$
Therefore the limits of uncertainty in finding and using the Calibration Factor without phase information are

\[
e = \pm \frac{1}{2} \times \left[ \frac{\left( 1 + |p_U \Gamma_2| \right)^2}{1 - |p_U \Gamma_2|} + \frac{\left( 1 + |p_S \Gamma_2| \right)^2}{1 - |p_S \Gamma_2|} - 2 \right] \times 100\% \quad (10)
\]

As \( p_S \Gamma_2 \) and \( p_U \Gamma_2 \) are both much less than unity, this expression can be approximated by

\[
e = \pm 2 |\Gamma_2| \left[ |p_U| + |p_S| \right] \times 100\% \quad (11)
\]

Harris and Warner (7) indicate that the probability distribution governing a power measurement where phase information is not used is U shaped. Thus 95% Confidence Limits are given by

\[
e' = 3.071 |\Gamma_2| \left[ |p_U| + |p_S| \right] \times 100\% \quad (12)
\]

Table 2 gives the characteristics of the coupler in terms of measurements made with an ANA and the scattering parameters derived from those measurements.

| Frequency (GHz) | Gain in dB | \( |S_{22}| \) | \( |S_{21}| \) | \( |S_{32}| \) | \( |S_{31}| \) |
|----------------|------------|-----------|-----------|-----------|-----------|
| 8.2            | -0.21      | -68       | -19.63    | 0.012     | 0.976     | 3.89 x 10^{-4} | 0.104 |
| 9.0            | -0.09      | -70       | -20.26    | 0.005     | 0.989     | 3.16 x 10^{-4} | 0.097 |
| 10.0           | -0.08      | -75       | -20.52    | 0.005     | 0.991     | 1.78 x 10^{-4} | 0.094 |
| 12.4           | -0.17      | -70       | -19.80    | 0.005     | 0.981     | 3.16 x 15^{-4} | 0.102 |

**Table 2**

Substituting these values into equation 12 and taking \( |p_S| \) as 0.01 for various values of \( |p_U| \) gives the percentage mismatch uncertainties shown in Table 3.
The theory of the self-balancing bridge assumes that any amplifiers used are perfect; i.e., no currents flow in or out of the input and also that there is no voltage across the input. This section looks at the uncertainty introduced by this assumption.

Let the input bias currents be $I_3$, $I_4$, $I_5$, and $I_6$ and the input offset voltages be $E_1$ and $E_2$ (Fig 9). If the resistance of the thermistor under these conditions is $r$, then the voltage $V$ can be expressed in two ways depending on the current path chosen. is

$$V = E_2 + R_S(I - I_3)$$  \hfill (13)  

or

$$V = E_1 + r(I - I_3 - I_4 + I_6)$$ \hfill (14)

Thus

$$r = \frac{(V - E_1) \left( \frac{V - E_2}{R_S} + I_6 - I_4 \right)^{-1}}{I_3}$$ \hfill (15)
The bridge balance error arises from the variation of \( r \) with microwave power. If \( V \) and \( r \) change from \( V_1, r_1 \) to \( V_2, r_2 \) when microwave power is applied then the change in power dissipated in the thermistor due to the change in its resistance is

\[
\Delta P = \frac{r_2 - r_1}{\gamma}
\]  

(16)

where \( \gamma \) is the ohms/watt coefficient of the thermistor.

\[
\Delta P = \frac{(V_2 - E_1)}{\gamma} \left( \frac{(V_2 - E_2)}{R_s} + I_6 - I_4 \right)^{-1}
\]

\[
- \frac{(V_1 - E_1)}{\gamma} \left( \frac{(V_1 + E_2)}{R_s} + I_6 - I_4 \right)^{-1}
\]  

(17)

By considering only first order terms of small quantities

\[
\Delta P = (E_1 - E_2) \cdot R_s \cdot \left( \frac{V_2 - V_1}{\gamma V_2 V_1} \right)
\]

\[
+ (I_6 - I_4) \cdot R_s^2 \cdot \left( \frac{V_2 - V_1}{\gamma V_2 V_1} \right)
\]  

(18)

The microwave power supplied \( P_{RF} \) is given by the change in DC power in the thermistor minus \( \Delta P \)

\[
P_{RF} = P_1 - P_2 - \Delta P
\]  

(19)

From equation 15 the DC power dissipated in the thermistor with no microwave power is given by

\[
P_1 = \frac{(V_1 - E_1)^2}{r}
\]

\[
= (V_1 - E_1) \left( \frac{(V_1 - E_2)}{R_s} + I_6 - I_4 \right)
\]  

(20)
similarly when the microwave power is present the DC power is given by

\[ P_2 = (V_2 - E_1) \left( \frac{(V_2 - V_1)}{R_S} + I_6 - I_4 \right) \quad (21) \]

thus

\[ P_{RF} = \frac{V_1^2 - V_2^2}{R_S} \left[ 1 - \frac{E_1}{V_1 + V_2} \left( 1 + \frac{R_S^2}{\gamma V_1 V_2} \right) - \frac{E_2}{V_1 + V_2} \left( 1 - \frac{R_S^2}{\gamma V_1 V_2} \right) \right] \\
+ \frac{R_S (I_6 - I_4)}{V_1 + V_2} \left( 1 - \frac{R_S^2}{\gamma V_1 V_2} \right) \quad (22) \]

Thus the percentage error due to voltage offsets and input bias corrects is

\[ E = 100 \left[ \frac{E_1}{V_1 + V_2} \left( 1 + \frac{R_S^2}{\gamma V_1 V_2} \right) + \frac{E_2}{V_1 + V_2} \left( 1 - \frac{R_S^2}{\gamma V_1 V_2} \right) \right] \\\n- \frac{(I_6 - I_4)R_S}{V_1 + V_2} \left( 1 - \frac{R_S^2}{\gamma V_1 V_2} \right) \quad (23) \]

By assigning to \( E_1 \) and \( E_2 \) the values 100 \( \mu \)V, -100 \( \mu \)V (using nulling circuits), and to \( I_4 \) and \( I_6 \) the values 12 nA, -12 nA (taken from the manufacturers specifications) and to \( V_1 \) and \( V_2 \) values corresponding to a measurement taken at 1 W, the uncertainty due to amplifier imperfections is \( \pm 0.021\% \). These are rectangular limits which can be converted to 95% confidence limits by multiplying by \( 2/\sqrt{3} \) (8)

\[ E = 0.024\% \quad (95\% \, CL) \]

4.3 RESISTANCE AND VOLTAGE UNCERTAINTIES

The uncertainty in the resistance is given by the performance of the Maestro voltmeter when used for the resistance measurements and is less than 0.01\%. 

\[ \therefore \]

11
The effect of voltmeter uncertainty is quantified by examining the effect of each measurement on the calibration factor.

\[ CF = \frac{P_B}{P_L} \]

where

\[ P_B = V_2^2 - V_1^2 \quad \text{and} \quad P_L = 2V(V_a - V_b) + V_a^2 - V_b^2 \]

as in section 3.1, and let \( \delta P_L \) indicate the uncertainty in \( P_L \) etc then

\[ \delta P_B^2 = \left( \frac{\partial P_B}{\partial V_2} \cdot \delta V_2 \right)^2 + \left( \frac{\partial P_B}{\partial V_1} \cdot \delta V_1 \right)^2 \]

(24)

\[ = (2V_2 \cdot \delta V_2)^2 + (2V_1 \cdot \delta V_1)^2 \]

(25)

similarly

\[ \delta P_L^2 = \left( 2(V_a - V_b) \cdot \delta V \right)^2 + \left( 2(V + V_a) \cdot \delta V_a \right)^2 \]

\[ + \left( 2(V + V_b) \cdot \delta V_b \right)^2 \]

(26)

and as

\[ \left( \frac{\delta CF}{CF} \right)^2 = \left( \frac{\delta P_L}{P_L} \right)^2 + \left( \frac{\delta P_B}{P_B} \right)^2 \]

(27)

\[ \frac{\delta CF}{CF} = \frac{4 \left[ \left( \delta V(V_a - V_b) \right)^2 + \left( \delta V_a(V + V_a) \right)^2 + \left( \delta V_b(V + V_b) \right)^2 \right]}{\left( 2V(V_a - V_b) + V_a^2 + V_b^2 \right)^2} \]

\[ + \frac{4 \left[ (V_2 \cdot \delta V_2)^2 + (V_1 \cdot \delta V_1)^2 \right]}{\left( V_2^2 - V_1^2 \right)^2} \]

(28)

Assigning values for the various voltages and using the manufacturers quoted uncertainties for the different voltmeter ranges gives an uncertainty in \( CF \) of 0.056% (95% CL) when used to measure 10 mW. This uncertainty falls to 0.0054% (95% CL) when used to measure 1 watt.
4.4 LINEARITY

If the thermistor monitor is used in the range 0.1 to 10 mW, linearity of 0.05% (95% CL) can be assigned following the work of Abbott and Orford(6).

4.5 COMBINED UNCERTAINTY

The combined uncertainty $U$ in obtaining a calibration factor from the RSRE bolometers at a level of 10 mW and using it to calibrate a power meter at a level of 1 watt if the vrc of the power meter is known to be less than 0.01 is

$$U = \left[ \varepsilon_m^2 + \varepsilon_b^2 + \varepsilon_{vc}^2 + \varepsilon_{vu}^2 + \varepsilon_r^2 + \varepsilon_L^2 \right]^{1/2}$$

(29)

where

- $\varepsilon_m$ is mismatch uncertainty (Section 4.1) = 0.0961%
- $\varepsilon_b$ is the bridge balance uncertainty (Section 4.2) = 0.024%
- $\varepsilon_{vc}$ is voltmeter uncertainty at 10 mW (Section 4.3) = 0.0562%
- $\varepsilon_{vu}$ is voltmeter uncertainty at 1 W (Section 4.3) = 0.0054%
- $\varepsilon_r$ is resistance uncertainty (Section 4.3) = 0.01%
- $\varepsilon_L$ is linearity uncertainty (Section 4.4) = 0.05%

Thus

$$U = \left[ 0.0961^2 + 0.024^2 + 0.0562^2 + 0.0054^2 + 0.01^2 + 0.05^2 \right]^{1/2}$$

(30)

$$= 0.125\% \text{ (95\% CL)}$$

This must be combined with the uncertainty in the absolute calibration of the power standards used, normally 0.1\% for the RSRE bolometers.

$$U_T = (0.1^2 + 0.125^2)^{1/2}$$

$$= 0.16\%$$
5 CONCLUSIONS

The transfer standard is simple to make, easy to use and ideally suited for automated measurements. The results of the uncertainty analysis demonstrate that traceability to the national standards can be adequately established using this device.

6 ACKNOWLEDGEMENTS

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REFERENCES


FIG. 1.

FIG. 2.
FIG. 4.

FIG. 5.
FIG. 7.

FIG. 8.