Recoil Distributions in Some Proton Reactions

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**Abstract**

The distributions of recoil products from the two most important double decays of silicon excited by proton absorption are calculated by analytical and numerical methods. The nuclear reactions treated are Si(p,pp) and Si(p,2p) and it is assumed the decays are by evaporation. Conversion of the results to distributions of dose is discussed. The latter distributions are of interest in the soft upset of satellite electronic memories.
RECOIL DISTRIBUTIONS IN SOME PROTON REACTIONS

Single event upset in electronic memory devices has caused a great deal of concern in the last two or three years. One particular area of concern is upset produced in memories on board spacecraft which operate in the earth's radiation belts. Recent NRL calculations have shown that upset rates can be very high in spacecraft memories. Though protons do not deposit energy sufficiently fast to produce upsets directly, some of their secondary products are sufficiently heavy and energetic to do so and occur with sufficient frequency to contribute to observed rates. This paper calculates the energy distribution of the recoil nuclei from the two most important proton induced reactions, Si(p,α)Mg and Si(p,2p)Al. The dose density from the recoil is easily obtained from this and is briefly discussed. It is assumed for each case that the two light products are emitted essentially simultaneously so that the recoil velocities for both steps of the two step decay are calculated using the mass of the residual nucleus.
An excited nucleus with velocity $\mathbf{v}_0$ decays first with a recoil velocity distribution, $D_1(v_1)$, spherical relative to $\mathbf{v}_0$, and again with $D_2(v_2)$, also spherical relative to $\mathbf{v}_1$. In order to calculate the distribution of the energy deposited by the recoil, it is first necessary to obtain the combined distribution of the two steps in the frame of $\mathbf{v}_1$. Assuming the decays are by evaporation

$$D_1(E_i) = c_i (E_i - B_i) e^{-E_i/t_i}, \quad i = 1, 2, \quad E_i > B_i$$

(1)

where $B_i$ and $t_i$ are reaction constants and $c_i$ is determined by

$$1 = \int dE D_1(E).$$

(2)

The energy parameter is center of mass reaction energy so

$$E_i = \frac{1}{2} m_i v_i^2,$$

$$m_i = \frac{M_R}{M_i} (M_R + M_i), \quad M_1 = M_p, \quad M_2 = \left\{ \begin{array}{ll} M_{\alpha}, & M_R = \left\{ \begin{array}{ll} M(24M_5) & M(27Al) \end{array} \right. \end{array} \right.$$  

(3)

Carrying out the normalization, one obtains

$$c_i = \frac{-1}{E_i/t_i} e^{B_i/t_i}.$$  

(4)
Since it is our intention to apply the vector convolution theorem in velocity space, it is first necessary to change the given distributions from energy densities to velocity space densities. From (2)

\[ l = m \int D(v) dv \]

\[ = (4\pi)^{-1} m \int v^{-1} D(v) dv. \]  

Thus to convert \( D \) to a velocity space density one writes

\[ D'(v) = (4\pi)^{-1} m D(v). \]  

Assuming no correlation between \( D_1 \) and \( D_2 \), the combined distribution is given as a function of \( \vec{u} = \vec{v}_1 + \vec{v}_2 \) in the same frame as \( \vec{v}_1 \) by the convolution theorem.

\[ \begin{align*}
D'((\vec{u})) &= (4\pi)^{-2} m_1 m_2 \int \int v_1^{-1} v_2^{-1} D_1(v_1) D_2(v_2) \delta(\vec{u} - \vec{v}_1 - \vec{v}_2) d^3v_1 d^3v_2 \\
&= (4\pi)^{-2} m_1 m_2 \int \int v_1^{-1} v_2^{-1} D_1(v_1) D_2(v_2) d^3v_1, \quad v_2 = |\vec{u} - \vec{v}_1|, \\
&= (4\pi)^{-2} m_1 m_2 \int D_1 D_2 v_1 v_2^{-1} dv_1 d\mu_1 d\phi_1 \\
&= (8\pi)^{-1} m_1 m_2 \int D_1 D_2 v_1 v_2^{-1} dv_1 d\mu_1, \quad \mu_1 = \cos \theta_1. \quad \text{Note that } v_2 = 0 \text{ does not occur. Since}
\end{align*} \]
\[ v_2^2 = u^2 - 2uv_1u_1 + v_1^2, \]  

(8)

the entire \( \mu_1 \) dependence is in \( v_2^{-1} D_2 \). Also one has from (8), holding \( u \) and \( v_1 \) fixed,

\[ v_2 dv_2 = -uv_1 d\mu_1 \]

\[ v_2^{-1} d\mu_1 = -(uv_1)^{-1} dv_2. \]  

(9)

As \( \mu_1 \) goes from \(-1\) to \(1\), \( v_2 \) goes from \( v^+ = u+v_1 \) to \( v^- = |u-v_1| \).

For the \( \mu_1 \) integral then one has

\[
\int_{-1}^{1} v_2^{-1} D_2 d\mu_1 = -(uv_1)^{-1} \int_{v^+}^{v^-} D_2 dv_2
\]

\[
= -(uv_1)^{-1} \int_{v^+}^{v^-} \left( \frac{1}{2} m_2 v^2 - B_2 \right) e^{-\left( \frac{1}{2} m_2 v^2 - B_2 \right)/t_2} dv
\]

\[
= -(uv_1)^{-1} \left( 2 \right)^{\frac{1}{2}} \left( t_2 m_2 \right)^{-\frac{1}{2}} e^{B_2/t_2} \int_{w^+}^{w^-} (w^2 - B_2/t_2) e^{-w^2} dw
\]

\[
= (uv_1)^{-1} \left( 2 t_2 m_2 \right)^{-\frac{1}{2}} e^{B_2/t_2} \left[ \left. w^{-2} \right|_{w^+}^{w^-} + \left( \frac{1}{2} B_2^2/t_2 - 1 \right) \int_{w^+}^{w^-} e^{-w^2} dw \right]
\]

\[
= (uv_1)^{-1} \left( 2 t_2 m_2 \right)^{-\frac{1}{2}} e^{B_2/t_2} \left( w^{-2} + \frac{1}{2} \left( \pi \right)^{\frac{1}{2}} \left( 2 B_2^2/t_2 - 1 \right) \text{erf}(w) \right)|_{w^+}^{w^-}.
\]  

(10)
The point \( u = 0 \) will not be a problem after changing back to energy density below. Since there is no remaining angular dependence, \( D'(u) = D'(u) \) and,

\[
D'(u) = \frac{m_1}{8\pi u} \left( \frac{m_2}{2t_2} \right)^{\frac{1}{2}} e^{B_2/t_2} \int_{v_m}^{\infty} dv_1 D_1(v_1) F(v_1)
\]

where

\[
F(v_1) = \left( w e^{-w^2} + \frac{1}{2} (\pi)^{\frac{1}{2}} (2B_2/t_2 - 1) \text{erf}(w) \right) \left[ w^+ - \right. \left. w^- \right]
\]

and

\[
w^+ = \left( \frac{m_2}{2t_2} \right)^{\frac{1}{2}} (u + v_1), \quad w^- = \text{Max} \left\{ \left( \frac{m_2}{2t_2} \right)^{\frac{1}{2}} \left| u - v_1 \right|, \left( \frac{B_2}{t_2} \right)^{\frac{1}{2}} \right\}
\]

with

\[
v_m = \left( \frac{2B_1}{m_1} \right)^{\frac{1}{2}}.
\]

For the reactions of interest \( B_2 \geq B_1 \) and \( B_2/m_2 \geq B_1/m_1 \) so the lower limit for \( v_1 \) includes that for \( v_2 \). Also in evaluating (11), whenever \( (B_2/t_2)^{\frac{1}{2}} > w^+ \), \( F \), and thus the integrand, vanish at this particular \( v_1 \).

Of course at this point one has only the spherical distribution relative to \( \hat{v}_0 \). This is a convenient point at which to check normalization. One requires

\[
l = \int D'(u)d^3u
\]

\[
= 4\pi M^{-1} \int D'(E)udE
\]

so a simple check is just a sum over
on a grid linear in energy.

In the numerical evaluation it was convenient to program $D_1$ and $F$ as statement functions. The error function (erf) is a single precision library function and ought to be fairly fast. Still each call of $F$ is roughly equivalent to the evaluation of two one-dimensional integrals. Use of Simpson rule is made so one loop for evaluation and one for integration was written to allow printing one complete integrand for error checking. Another convenient error check was to require the sign of $F(v_1)$ to be positive. The constants are $B_1 = 2.4$, $B_2 = 4.6$, $t_1 = t_2 = 2.45$ in MeV. Units of the $D_1$ are MeV$^{-1}$ so writing the masses in MeV gives the velocities in units of light velocity, $c$. Since the integral is in velocity and the normalization check is in energy, the corresponding loops are on different grids designed to evaluate overlapping regions in $D_1(v_1)$ and $D'(u)$. Thus it was convenient to begin each loop with a unitless energy scale using $B_1$ as the unit. Then, for example, if a range in $E(u)$ from 0 to 10 (10 $B_1$ MeV) gives an adequate evaluation of $D_1$ the appropriate range for $v_1/v_m$ would be 1 to $10^{1.2}$. While in principle there is a serious complication of upper limits required by numerical evaluation, this is ignorable in practice because of the property of these distributions in having compact support. Thus any accuracy desired within computer precision may be attained simply by extending the upper limits sufficiently.

The program RECOIL (listed in the Appendix) has been written, the first half performing the preceding calculations, and the results from this part
are given in Tables I and II. Upper limits for the unitless energy grids were 49 for \((p,p')\) and 36 for \((p,2p)\), corresponding to evaporation energies of 117.6 and 86.4 MeV. For comparison the peak of the evaporation distribution is at \(B_{1+1'}\), or 7.05 and 4.85 MeV, respectively. The overall sums obtained were 0.996 and 0.989, respectively. It may be seen below that extending the energy range decreases this error still.

Consider now the transformation of these distributions to the frame of \(v_0\). Assume absorption of a 30 MeV proton in the initial excitation, forming a compound nucleus. One has

\[
M_{pp} = M_{vo}
\]

\[
\frac{1}{2} M_{pp}^2 = \frac{1}{2} M_{vo}^2 + E_x (14)
\]

and obtains for the excitation energy

\[
E_x = (1 - \frac{M}{M_c}) T, \quad T = \frac{1}{2} M_{pp}^2
\]  

(15)

Taking \(M_c = M(^{28}\text{Si} + p)\) one obtains \(E_x = 28.96\) MeV, \(E_c = \frac{1}{2} M_{vo}^2 = 1.04\), \(v_0 = 0.008945c\). Thus the transformation is in part accomplished by the substitution

\[
u^2 = v^2 - 2v_0 \cos \theta + v_0^2
\]  

(16)
Table I

PROGRAM RECOIL ON (P,PA)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

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OVERALL SUM 0.996306.
Table II

PROGRAM Recoil on (P,2p)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

| ENERGY DISTRIBUTION (PER MEV) | 0.0000 | 0.000000 | 0.0432 | 0.923879 | 0.0864 | 1.042541 | 0.1296 | 1.079965 | 0.1727 | 1.693790 | 0.2159 | 1.099337 | 0.2591 | 1.0102718 | 0.3023 | 1.105173 | 0.3455 | 1.049357 | 0.3887 | 1.030655 | 0.4319 | 1.099393 | 0.4790 | 1.087444 | 0.5182 | 1.063878 | 0.5614 | 1.027549 | 0.6046 | 0.979225 | 0.6478 | 0.929916 | 0.6910 | 0.855260 | 0.7342 | 0.785046 | 0.7774 | 0.728900 | 0.8205 | 0.641044 | 0.8637 | 0.571315 | 0.9069 | 0.505061 | 0.9501 | 0.443215 | 0.9933 | 0.386352 | 1.0365 | 0.334745 |
|-------------------------------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|
|                               | 1.0797 | 0.288428 | 1.1228 | 0.247268 | 1.1666 | 0.211004 | 1.2092 | 0.179097 | 1.2524 | 0.151765 | 1.2956 | 0.128065 | 1.3388 | 0.107606 | 1.3820 | 0.094052 | 1.4251 | 0.075367 | 1.4683 | 0.062821 | 1.5115 | 0.052236 | 1.5547 | 0.043337 | 1.5979 | 0.035878 | 1.6411 | 0.029644 | 1.6843 | 0.024443 | 1.7274 | 0.020128 | 1.7706 | 0.016544 | 1.8138 | 0.013578 | 1.8570 | 0.011127 | 1.9002 | 0.009107 | 1.9434 | 0.007444 | 1.9866 | 0.006077 | 2.0298 | 0.004955 | 2.0729 | 0.004036 |
|                               | 2.1161 | 0.063285 | 2.1593 | 0.062700 | 2.2025 | 0.061688 | 2.2457 | 0.061760 | 2.2889 | 0.061427 | 2.3321 | 0.061566 | 2.3752 | 0.060935 | 2.4184 | 0.060757 | 2.4616 | 0.060612 | 2.5048 | 0.060494 | 2.5480 | 0.060399 | 2.5912 | 0.060322 | 2.6344 | 0.060259 | 2.6775 | 0.060209 | 2.7207 | 0.060166 | 2.7639 | 0.060135 | 2.8071 | 0.060109 | 2.8503 | 0.060088 | 2.8935 | 0.060070 | 2.9367 | 0.060056 | 2.9798 | 0.060045 | 3.0230 | 0.060036 | 3.0662 | 0.060029 | 3.1094 | 0.060023 |

OVERALL SUM 0.988986
redefining $\theta$ as the angle between $\vec{v}_0$ and $\vec{v}$. Similarly defining $\theta_1$, as the angle between $\vec{v}_0$ and $\vec{u}$, one notes that (13) has the form

$$D(u) = \frac{dN}{dE_1d\Omega_1}$$

(17)

where $d\Omega_1 = d\mu_1 d\phi_1$, even though it lacks $\Omega_1$ dependence. The transformation is therefore completed by the Jacobian.

$$\frac{dN}{dE d\Omega} = \frac{\partial(E',\Omega_1)}{\partial(E,\Omega)} \frac{dN}{dE_1d\Omega_1}$$

(18)

$$\frac{\partial(E',\Omega_1)}{\partial(E,\Omega)} = \frac{v}{u} = \left(\frac{E}{E_1}\right)^{1/2}$$

(19)

Calculations of this transformation to the lab have been added to RECOIL, forming the second half. In addition lab solid angle $d\Omega$ is averaged to obtain $dN/dE$. Further the norm and average energy are obtained. The resulting distributions are in Tables III and IV and Figure 1. These tables show peaks near 1.35 and 0.97 MeV and average energies of 2.55 and 1.55 MeV for $(p,p\alpha)$ and $(p,2p)$ respectively.

Now notice that from (18) one easily obtains an energy deposition distribution, given the recoil energy loss $dE/dx$,

$$\frac{dN}{dx d\Omega} = \frac{dE}{dx} \frac{dN}{dE d\Omega}$$

(20)

If one assumes constant energy loss (a crude approximation) and sets $dE/dx = \epsilon$, then one recognizes that the multiplication of (18) by a constant is removed by renormalization. One then views (20) as nothing but a unit
Table III

PROGRAM REQUIRE ON (P,PA)

SPHERICAL ENERGY DISTRIBUTION:

LAB ENERGY DISTRIBUTION (PER MLV)

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Table IV

PROGRAM RECOIL ON (P,2P)

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POINTS: 129  EMAX: 5.5278  DU(EMAX): 0.000000  NORM: 0.988988

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LAB NORM: 9.85881E-01  EAVE: 1.5531
Fig. 1 — Recoil distributions

Absorbed proton energy 30 MeV

$\text{Si}(p,2p)\text{Al}$

$\text{Si}(p,p\alpha)\text{Mg}$
change and accepts the numerical distribution (18) as giving the
distribution per $\epsilon^{-1} \mu m$, the distance in which 1 MeV is lost. Instead of
assuming constant energy loss one may utilize the code E-DEP-1 to include
straggling effects. A minor change will be required however since the
energy-deposited report of E-DEP-1 does not include ionization loss, a
significant part of dose. In making this change it will be permissible to
ignore radiation loss, a few percent effect. Short of using E-DEP-1 but
better than assuming constant $dE/dx$, one can find expressions giving the
variation.

In fact $\epsilon = 1$ MeV/$\mu m$ is only about twenty percent low when averaged over
the first few $\mu m$. Thus the implication of Figure 1 is that significant
energy, deposited in a spherical region of radius 3 to 4 $\mu m$, should be added
to that previously considered.

Acknowledgments

The author wishes to thank E. L. Petersen for suggesting this problem
and P. Shapiro for helpful conversations and a critical reading.

References

1. C. S. Guenzer, R. G. Allas, A. B. Campbell, J. M. Kidd, E. L. Petersen,

5. Useful manipulations of vectorial distributions are given in J. B. Langworthy, Naval Research Laboratory Report 6391 (1966).

APPENDIX

SOURCE LISTING

ASC FAST FORTRAN COMPILER

CP OPTIONS = \( M,X \)

0001 PROGRAM RECOIL
C SET UP FOR SIG(P,P ALPHA). FOR (P,2P) SET B2=B1, MA=MP & CHANGE MR
C TRANSFORMATION OF SPHERICAL DISTRIBUTION TO LAB ADDED
0002 IMPPLICIT REAL(*)
0003 DIMENSION SG(241), DUC(22), EUC(22), DV(66), EV(66)
0004 DIMENSION DV2(23), EV2(23), DV3(23), EV3(23)
0005 EQUIVALENCE (DV2,DV(23)), (EV2,EV(23)), (DV3,DV(45)), (EV3,EV(45))
0006 DATA MC, MP, MR/931.5016, 938.7964, 0.02623, 98504/913.82, T
0007 #/2.44444, 2.44444, 0.0108/0.0, 0.0, 1.11, 1.11, 1.11, 1.11, 8241.66, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0008 E1(V)=V*V*Ml-B1)*RT
0009 ol(V)=RT*E1(V)*EXP(-E1(V))
0010 FWd(V)=V*EXP(-V*V)*FC*ERF(V)
0011 J2=J1-1
0012 DV1=RT*FLOAT(J2)
0013 MAS=MA*MC
0014 MR=MR*MC
0015 M1=M0*MR
0016 M2=MR*(MR+MA)*0.5/MA
0017 RT=1.0/VT
0018 VW=SQRT(M2/RT)
0019 F=ACOS(VW)*EXP(B2/RT)
0020 P=ATAN(1.0)
0021 FC=(2.0*B2/RT-1.0)*SQRT(P4)
0022 VW=SQRT(B1/M1)
0023 SI=2.*DV1*VU/3.*A
0024 VT=SQRT(B2/RT)
0025 PRINT 61
0026 61 FORMAT(*** PROGRAM RECOIL ON (P,PA)**)
0027 DUC(1)=0.
0028 EUC(1)=0.
0029 DO 1 I=2, 11
0030 U=SQRT(DF*(I-1))*VU
0031 EUC(I)=0.5*MR*U
0032 DO 2 J=I, 1
0033 VI=VU*(1.0+DV1*(J-1))
0034 SG(J)=0.
0035 VP=(VI+U)*VW
0036 IF(VP.LE.VT) GOTO 2
0037 VM=ARS(SV1-U)*VW
0038 IF(VM.LT.VT) VM=VT
0039 SG(J)=F*WV*WV
0040 IF(SG(J).LT.-1.0) GOTO 8
0041 PRINT 69, SG(J), VI, U, J, I
0042 69 FORMAT(***NEGATIVE INTEGRAND VII, U, JI="*P3E11.2, 2I4")
0043 8 SG(J)=F*DI(V1)*SG(J)
0044 2 CONTINUE
0045 S=S+2.*SG(J)+SG(J-1)
0046 DO 4 J=2, J2+2
0047 4 S=S+2.*SG(J)+SG(J-1)
CSN STATEMENT CP OPTIONS = (X,M)

0048 1 DOUC(I)=S*S1
0049  A=A+DOUC(I)
0060  A=A*EUC(2)
0061  PRINT 62,11*EUC(11),DU(I)*A
0062  FORMAT('" SPHERICAL ENERGY DISTRIBUTION:"' POINTS:*,I4,
0063  " EMAX:"*,F9.4," DUCEMAX:"*,F9.6," NORM:"*,F9.6)
0064  MT=M*N
0065  ME=M+MT
0066  EP=30.*NP/ME*MR/ME
0067  EV(I)=0.
0068  DVC(I)=0.
0069  DO 10 K=2,K1
0070  EVP(K)=0.1*(K-1)
0071  E2=2.0*SQRT(EV(K)*EP)
0072  S=0.
0073  DD 11 J=1,100
0074  IO=1
0075  MU=1.0-(2*J-1)/100.
0076  EO=EVP(K)+EP-E2*MU
0077  FI=SQRT(EV(K)/EO)
0078  DO 14 I=IO,II
0079  IF(EO.LT.EU(I)) GOTO 15
0080  CONTINUE ,
0081  GOTO 16
0082  CONTINUE ,
0083  CONTINUE ,
0084  CONTINUE ,
0085  CONTINUE ,
0086  CONTINUE ,
0087  PRINT 64
0088  FORMAT(/" LABR ENERGY DISTRIBUTION (PER MEV) ",
0089  " ENERGY DISTRIBUTION")
0090  PRINT 65*(EVC(K),DVC(K),EVC2(K),DVC2(K),EVC3(K),DVC3(K),K=1,22)
0091  FORMAT(3(F10.1,F10.6))
0092  S=0.
0093  A=0.
0094  DO 20 K=1,K1
0095  S=S+DVC(K)
0096  20 A=A+(EVC(K)+0.05)*DVC(K)
0097  A=A/S
0098  S=S+EVC(2)
0099  PRINT 66+S*A
0100  FORMAT(/" LAB NORM:\"*1PE13.5," EAVE:\"*0PF7.4)
0101  END