FINAL SCIENTIFIC REPORT

SELECTED PROBLEMS IN SYSTEMS ENGINEERING
AFOSR 78-3500

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Introduction

Research conducted under AFOSR 78-3500 deals with two types of signal processing and control applications.

The first area of concentration is nonlinear filtering and signal extraction. In particular, polynomic filter theory has been extended to the point where optimal causal filters can be specialized for several signal extraction problems. A direct comparison of the power of nonlinear filters has been explored and some of the computational aspects of polynomic filters have been investigated.

The second area of research is that of multidimensional system theory. Such theories are central to image processing, robotics and would seem to impact artificial intelligence and other related sciences. In recent months a fundamental operator factorization problem has been solved and initial studies of rapid computation are completed. A specification of special purpose computer architectures for rapid on line processing of multidimensional signals is within reach although not yet in hand.

The research conducted under the grant has featured an operator theoretic approach. The concept of 'state' is secondary in most of our results. This is felt to have been a distinctive feature.

Results

The results established under the grant have been reported through a series of journal articles and conference papers. A listing
of these is given below. The articles have been (or will be) submitted to AFOSR using standard submission procedures. For the readers convenience, the abstracts of selected articles are given below.


Abstract--The paper considers the common ground shared by system parameter estimates, system adaptivity, and system sensitivity. A format is developed in which these objectives can be viewed in a unified way. The format is illustrated by applying some recent synthesis results to adaptive and sensitivity reducing compensators. A simple example is considered and partial results of a computer study presented.


Abstract--Let $\Gamma_i \subset \mathbb{R}^m$ denote pattern classes with means $\gamma_i$ covariances $\Sigma_i$, $i=1,\ldots,\ell$ respectively. The set $\{\psi_i\} \subset \mathbb{R}^k$ denotes signatures for the pattern classes. A function $f: \mathbb{R}^m \to \mathbb{R}^k$ is said to be a cluster function provided (i) $f(\gamma_i) = \psi_i$, $i=1,\ldots,\ell$ and (ii) $f$ minimizes a dispersion functional. This paper develops a complete theory for realization of polynomial cluster functions, including the linear case.


Abstract--The properties of polynomial functions on $\mathbb{R}^n$ are reviewed. Such functions are then used to establish a general synthesis procedure for arbitrary multivalued switching functions. The relationship with threshold logic and Boolean logic is also explored.


Abstract--Let $f$ be an arbitrary strictly causal polynomial map between Hilbert resolution spaces. Then $f$ has the form $f = T \tau$ where $\tau$ is a predetermined causal map and $T$ is linear and strictly causal. The maps $f$ and $T$ have state representations. The present paper develops the interrelationship between these.

Abstract--The minimum phase factorization of bounded Hermitian operators (covariance operators) is an essential ingredient in linear filtering and stochastic control. In a recent article the author studied an analogous class of polynomial filtering and stochastic control problems. The present study uses the concept of a Hilbert scale to establish a minimum phase factorization procedure for the polynomial setting. The linear theory is carried over in its entirety.


Abstract--Consider a system, $T$, which is dependent on a parameter tuplet $\lambda \in \mathbb{R}^n$. The map $\lambda \mapsto T(\lambda)$ is linearized about an a priori point $\lambda_0$. Parameter identification consists of using the observation $(u, T(\lambda)u)$ to invert the linearized $\lambda \mapsto T(\lambda)$ map. State variable realization of this inverse function and related topics are developed. Examples and a computer simulation demonstrate the synthesis procedure.


Abstract--The modeling of physical systems inherently involves constructing a mathematical approximation from observable data and/or a priori assumptions. This study refines some recent work on causal interpolation and causal approximation as system modeling techniques. Sufficient conditions for causal interpolators to approximate continuous causal systems are established. State realizations for minimal norm causal interpolators are also established.


Abstract--Let $(U_1, \ldots, U_n)$ denote stochastic processes with imbedded signals $(u_1, \ldots, u_n)$ respectively. A map $T$ is said to extract $(u_1, \ldots, u_n)$ without distortion provided
\[ \xi_i = Tu_i, \quad i = 1, \ldots, n \]

where \( \xi_i, \ i = 1, \ldots, n \) are chosen apriori. The map \( T \) is said to be optimal whenever it minimizes

\[
J(T) = E \sum_i \| \xi_i - Tu_i \|^2
\]

where \( \xi_i, \ i = 1, \ldots, n \) are suitable stochastic processes. The map \( T \) is called an optimal distortion free signal extractor if it also is causal.

This paper analyses polynomial optimal distortion free signal extractors.


Abstract--With \( H \) a Hilbert space and \( \{(x_i, y_i) : i = 1, \ldots, m\} \subset H \times H \) a basic problem is to determine the existence and uniqueness of causal functions, \( f \), on \( H \) satisfying \( y_i = fx_i \), \( i = 1, \ldots, m \). The present paper considers classes of polynomial functions which minimize an operator norm. The results include explicit necessary and sufficient conditions and an explicit synthesis procedure for realizing the resultant polynomial functions.


Abstract--Let \( K \) denote an arbitrary compact subset of real \( L_2 \). Let \( f \) be any causal continuous function on \( L_2 \). Then there is a linear differential system: \( \dot{z}(t) = A(t)z(t) + B(t)u(t) \), and a memoryless polynomial state to output map \( w(t) = \phi(z(t), t) \) such that the system, \( f \), thereby computed satisfies

\[
\sup_{u \in K} \| f(u) - \hat{f}(u) \| < \epsilon
\]

where \( \epsilon > 0 \) is arbitrary. This and other results are developed.


Abstract--Recent studies have introduced the concept of a partially ordered Hilbert resolution space as a means of modeling the behavior of multidimensional systems. The present study utilizes this framework.
to formulate and solve some typical optimization problems. The resultant solutions provide design criteria for multidimensional systems and underscore both the similarities and differences between the single dimensional and multidimensional cases.


Abstract--An operator factorization problem is stated and solved. The finite dimensional form of the problem includes as a particular case the classical Schur-Coleski triangular factorization. A connection between the main results and optimal digital image processing is illustrated.
This report deals with two types of signal processing and control applications. The first area of concentration is nonlinear filtering and signal extraction. In particular, polynomic filter theory has been extended to the point where optimal causal filters can be specialized for several signal extraction problems. A direct comparison of the power of nonlinear filters has been explored and some of the computational aspects of polynomic filters have been investigated. The second area of research is that of multidimensional (CONTINUED)
ITEM #20, CONTINUED: System theory. Such theories are central to image processing, robotics and would seem to impact artificial intelligence and other related sciences. In recent months a fundamental operator factorization problem has been solved and initial studies of rapid computation are completed. A specification of special purpose computer architectures for rapid on line processing of multidimensional signals is within reach although not yet in hand. The research conducted under the grant has featured an operator theoretic approach. The concept of 'state' is secondary in most of our results. This is felt to have been a distinctive feature.