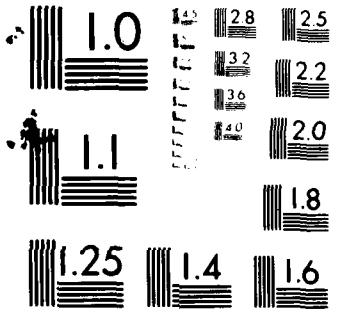
AD-A108 602 MARYLAND UNIV COLLEGE PARK COMPUTER VISION LAB F/G 14/5
ON CONNECTIVITY PROPERTIES OF GRAYSCALE PICTURES.(U)
SEP 81 A ROSENFELD AFOSR-77-3271
UNCLASSIFIED TR-1097 AFOSR-TR-81-0796 NL

END
DATE
FILMED
1 82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

AFOSR-TR- 81-0796

LEVEL II

AD A108602

TR-1097
AFOSR-77-3271

September 1981

ON CONNECTIVITY PROPERTIES OF GRAYSCALE PICTURES

Azriel Rosenfeld

Computer Vision Laboratory
Computer Science Center
University of Maryland
College Park, MD 20742

ABSTRACT

A grayscale digital picture is called "connected" if it has only one connected component of constant gray level that is maximal, i.e., not adjacent to any component of higher gray level. This note establishes some equivalent conditions for connectedness, and also defines a grayscale generalization of the genus in terms of sums of local property values.

DTIC
ELECTE
DEC 16 1981
S D
D

DTIC FILE COPY

The support of the U.S. Air Force Office of Scientific Research under Grant AFOSR-77-3271 is gratefully acknowledged, as is the help of Janet Salzman in preparing this paper.

Approved for public release;
distribution unlimited.

81 12 14 065

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 81-0796

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): ON CONNECTIVITY PROPERTIES OF GRAYSCALE PICTURES

5. TYPE OF REPORT & PERIOD COVERED: Technical

6. PERFORMING ORG. REPORT NUMBER: TR-1097

7. AUTHOR(s): Azriel Rosenfeld

8. CONTRACT OR GRANT NUMBER(s): AFOSR-77-3271

9. PERFORMING ORGANIZATION NAME AND ADDRESS:
Computer Vision Laboratory
Computer Science Center
University of Maryland
College Park, MD 20742

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F; 2304/A2

11. CONTROLLING OFFICE NAME AND ADDRESS:
Mathematical & Information Sciences Directorate
Air Force Office of Scientific Research
Bolling AFB DC 20332

12. REPORT DATE: September 1981

13. NUMBER OF PAGES: 16

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Image processing
Pattern recognition
Connectedness
Genus

Accession For	
NTIS GRA&I	X
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A grayscale digital picture is called "connected" if it has only one connected component of constant gray level that is maximal, i.e., not adjacent to any component of higher gray level. This note establishes some equivalent conditions for connectedness, and also defines a grayscale generalization of the genus in terms of sums of local property values.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 411074

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

1. Introduction

Some of the basic topological properties of subsets of a digital picture [1] were generalized in [2] to fuzzy subsets, i.e., to the case where the picture is multi-valued rather than two-valued. The purpose of this note is to present some additional results on the fuzzy case. In particular, some equivalent characterizations of fuzzy connectedness are established, and a generalization of the genus to the fuzzy case is proposed. A special class of digital pictures is defined for which these concepts have relatively simple interpretations.

In studying topological properties in the case where points have only two values (0 and 1), it is customary to use opposite types of connectedness for the two types of points, regarding diagonal neighbors as adjacent for the 1's but not for the 0's, or vice versa. For multivalued pictures the situation becomes more complicated, since there are many types of points. To avoid complications, we will deal primarily with pictures defined on a hexagonal grid, where a point has only one kind of neighbor.

Section 2 summarizes the basic concepts of digital topology that are needed in this paper, and also discusses the relationship of multivalued digital pictures with fuzzy sets. Section 3 establishes some equivalent conditions for fuzzy connectedness;

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DTIC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12.
Distribution is unlimited.
MATTHEW J. KERPER
Chief, Technical Information Division

Section 4 defines "coherent" digital pictures and gives a connectedness criterion in the coherent case; and Section 5 discusses fuzzy genus.

2. Background

Let Σ be a bounded regular grid of points in the plane. We will assume in most of this paper that the grid is hexagonal rather than Cartesian, so that each point of Σ has six neighbors. Let S be any subset of Σ, which we assume does not meet the border of Σ. We say that two points P,Q of S are connected in S if there exists a path $P=P_0,P_1,\ldots,P_n=Q$ of points of S such that P_i is a neighbor of P_{i-1}, $1\le i\le n$. "Connected" is an equivalence relation, and its equivalence classes are called the connected components of S; if there is only one component, we call S connected. The complement $\bar{S}$ of S also consists of connected components. One of these, called the background, contains the border of Σ; the others, if any, are called holes in S.

Let σ be a fuzzy subset of Σ, i.e., a mapping from Σ into [0,1]. For any $P\in\Sigma$, we call $\sigma(P)$ the degree of membership of P in σ. A subset $S\subseteq\Sigma$ can be regarded as a fuzzy subset in which σ is into {0,1}, and S is the preimage of 1. The fuzzy subset $1-\sigma$ is called the complement of σ.

Since Σ is finite, σ takes on only finitely many values on Σ. We are only interested in the relative size of these values, and can thus assume them to be rational numbers; hence there exists an α such that all these values are integer multiples of α. From now on we will regard σ as taking on integer values (dividing the original rational values by α), so that σ defines a digital picture on Σ whose gray level at P is

$\sigma(P)/\alpha \equiv g(P)$, where $0 \le g(P) \le M$ (say). We assume that σ has value 0 on the border of Σ.

A digital picture σ can be decomposed into connected components C of constant gray level - i.e., for some gray level ℓ, C is a connected component of the set σ_ℓ of points having gray level ℓ. C is called a <u>top</u> if the components adjacent to C all have lower gray level than C; a <u>bottom</u> is defined analogously. Evidently, for any point P, there is a monotonically nondescending (nonascending) path to a top (bottom). The gray level of a component C will be denoted by $|C|$.

In [1] we defined σ to be <u>connected</u> if for all P,Q in Σ there exists a path $P=P_0,P_1,\ldots,P_n=Q$ such that each $\sigma(P_i) \ge \min(\sigma(P),\sigma(Q))$, and we proved that σ is connected iff σ has a unique top. We will now establish some equivalent characterizations of connectedness.

3. Connectedness

For any $0 \le \ell \le M$, the set of points that have gray level ℓ will be denoted by σ_ℓ, and the set of points that have gray levels $>\ell$ will be denoted by $\sigma_{\ell+}$. For brevity, a connected component of σ_ℓ will be called an ℓ-component, and a connected component of $\sigma_{\ell+}$ will be called an ℓ^+-component.

Proposition 1. Any nonempty ℓ^+-component contains a top.

Proof: From any P in the component there is a monotonically nondescending path to a top, and this path evidently lies in the component. ||

Theorem 2. The following properties of σ are all equivalent:

a) σ has a unique top

b) For all ℓ, $\sigma_{\ell+}$ is connected

c) Every ℓ-component is adjacent to at most one ℓ^+-component

Proof: If some $\sigma_{\ell+}$ had two components, each of them would contain a top by Proposition 1, and these tops must be distinct; hence (a) implies (b), while (b) trivially implies (c). To see that (c) implies (a), suppose that σ had two tops U,V, and consider a sequence of components $U=C_0,C_1,\ldots,C_n=V$ such that C_i is adjacent to C_{i-1}, $1 \le i \le n$. Of all such sequences, pick one for which $\min|C_i|=\ell$ is as large as possible, and of all these, pick one for which the value ℓ is taken on as few times as possible. Let $|C_j|=\ell$; then evidently $0<j<n$. If C_{j-1} and C_{j+1} were in the same ℓ^+-component, the sequence could be diverted to avoid C_j by passing through a succession

of components of values $>\ell$; the diverted sequence would thus have fewer terms of value ℓ, contradiction. Hence C_j is adjacent to two ℓ^+-components, which completes the proof. Note that an ℓ-component is adjacent to no ℓ^+-components iff it is a top. ||

As in [1], if σ has the properties of Theorem 2, it is called connected. Note that by (b), σ is connected iff all its "level sets" are connected in the nonfuzzy sense. If we define $\sigma_{\ell-}$ and ℓ^--component in the obvious way, using gray levels $<\ell$, we have an analogous result with "+" replaced by "-" and "top" by "bottom"; if σ has these properties, it is called <u>hole-free</u>. If σ is both connected and hole-free, we call it <u>simply connected</u>.

It follows immediately from Proposition 1 that for any ℓ, if $\sigma_{\ell+}$ has k components, σ must have at least k tops of heights $>\ell$. Thus if σ has k tops, every $\sigma_{\ell+}$ has at most k components. It may be, however, that every $\sigma_{\ell+}$ has strictly fewer than k components; thus for $k>1$ we have no good generalization of the equivalence between (a) and (b) in Theorem 1.

We recall [3] that σ is called <u>convex</u> if for all P,Q in Σ, there exists a digital straight line segment ρ from P to Q such that for all R on ρ we have $\sigma(R)\geq\min(\sigma(P),\sigma(Q))$. It is easily seen that if σ is convex, it must be simply connected. Indeed, if σ had two tops, say with values $|U|\leq|V|$, a line segment between points of these tops would have to pass through

a point of value $<|U|$ adjacent to U. Similarly, if σ had a bottom B, other than the one containing the border of σ, there would exist points of value $>|B|$ adjacent to B such that any line segment joining them would pass through B. Evidently, σ is convex iff $\sigma_{\ell+}$ is convex for all ℓ. We are ignoring here the special problems involved in defining convexity on a discrete grid; see, e.g., [3].

4. Coherence

Any component C has an outer border along which it is adjacent to a component of $\bar{C}$ that surrounds it; and it may also have hole borders along which it is adjacent to components of $\bar{C}$ that it surrounds. In the two-valued case, if C is a component of (e.g.) 1's, there is in fact just one component of 0's adjacent to C along each of these borders; but in the multivalued case, many different ℓ-components (for various ℓ's) may be adjacent to C along each of its borders, the only restriction being that these components must have values different from that of C.

We call σ coherent if, for any component C, exactly one component is adjacent to C along each of its borders. This seems like a very strong requirement, but it turns out to be satisfied whenever σ is obtained by digitizing a bandlimited function f (of two variables) by sampling it sufficiently finely. Indeed, if f is bandlimited, the magnitude of its gradient is bounded; thus we can find a δ such that, e.g., for any two points P,Q within distance δ apart we have $|f(P)-f(Q)|<1$. If we sample f using a grid finer than δ, and quantize the result to integer values, then in the resulting digital picture g, any pair of neighboring points differ in value by at most 1. For such a g, suppose that two components of different values are adjacent to the ℓ-component C along the same border; then these values can only be $\ell+1$ and $\ell-1$. Moreover, at some point of the border these

components must meet, and we then have neighboring points that differ in value by 2, contradiction. Thus, g is coherent, and has the further property that adjacent components always differ in value by 1.

Note that if σ is coherent, any component C separates any two components that are adjacent to it. It follows that the adjacency graph of the components of σ has no cycles, hence is a tree, just as in the two-valued case.

<u>Proposition 3</u>. Let σ have the property that, along any of its borders, any C meets components that are either all higher or all lower than it in value; then σ is coherent.

<u>Proof</u>: Suppose C met D,E along the same border, where $|C|<|D|<|E|$; then along this border two such components must meet each other, so that the lower-valued one (say D) meets components having both lower and higher values (C and E) along one of its borders, contradiction. ||

<u>Proposition 4</u>. If σ is coherent, the conditions of Theorem 2 are also equivalent to

d) Every ℓ-component is adjacent to at most one ℓ'-component such that $\ell'>\ell$.

<u>Proof</u>: If C is adjacent to two ℓ^+-components, let P',P" be points of these components that are adjacent to C; then P',P" have values $\ell',\ell''>\ell$, so that C is adjacent to two components of values $>\ell$. Thus (d) implies (c) without assuming coherence. Conversely, if σ is coherent and C is adjacent to two components of values $>\ell$, it must be adjacent to them along different borders;

hence they belong to different ℓ^+-components (since they are contained in different components of $\overline{C}$), which proves that (c) implies (d). Note that an ℓ-component is adjacent to no ℓ'-component with $\ell'>\ell$ iff it is a top. Note also that a component that has no holes is adjacent to only one other component, and so must be either a top or a bottom. ||

If σ is convex, then any C, along any of its <u>hole</u> borders, can only meet components that are all higher than it in value. However, along its outer border, C can meet components that have both higher and lower values; e.g., the digital picture

1 2 (surrounded by 0's)

is convex. Thus a convex σ need not be coherent (compare Proposition 3).

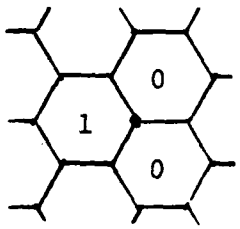
5. Genus

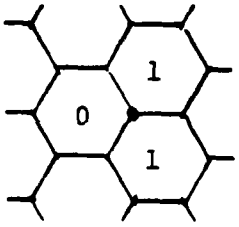
In the two-valued case, the genus G is defined as the number of components (of 1's) minus the number of holes (nonbackground components of 0's). It can be shown [4] that $G=\frac{1}{6}(I-II)$, where I is the number of triples of mutually adjacent points exactly one of which is 1, while II is the number of such triples of points exactly two of which are 1's. For example, if there is only a single isolated 1, there are six I's and no II's, so that the genus is 1. As another example, consider the two-valued picture

```
    1   1
1           1
    1   1
```

where the blanks are 0's; here there are 12 I's and 12 II's, so that the genus is 0.

One way of interpreting this formula for the genus is to regard I's as convex corners (of the set of 1's) and II's as concave corners:

(In 

and 

the dot indicates the position of the corner.) On the outer border of a component of 1's, there must be six more convex corners than concave corners, since the two types of corners represent 60° turns in opposite directions, and the net turn around the border must be 360°. Similarly, on a hole border, which is the outer border of a nonbackground component of 0's, there must be six more concave corners than convex ones. Hence each component of 1's contributes 1 ($= \frac{1}{6}\cdot 6$) to the formula, and each nonbackground component of 0's contributes -1, so that the formula does compute the genus.

We can generalize this formula to the multivalued case by considering all triples of mutually adjacent points, two of which have equal value. Let a be the value of the two equal points, and b the value of the third point; then the contribution of the triple is b-a. Readily, this reduces to the standard formula if the picture is two-valued.

If σ is coherent, any component C meets only one other component D along each of its borders, and it becomes easy to see what each border contributes to the generalized formula. Indeed, any border, if the surrounding component has value a and the surrounded component has value b, the contribution is b-a; note that this is positive if b>a (so that the border is the outer border of the higher-valued component), and negative if b<a (a hole border). Now since σ is coherent, the components form a directed tree, rooted at the background,

under the relationships of adjacency and surroundedness. Let the arc strength between a father and a son on this tree be b-a, where a is the father's value and b is the son's. Then the genus of σ is just the sum of these arc strengths.

For example, suppose σ consists of a set of tops of values t_i, all adjacent to the background; then the genus of σ is Σt_i, the sum of the top heights. Suppose σ is simply connected, so that it consists of a nested set of components of increasing value $0<a_1<\ldots<a_n$; then the genus of σ is $(a_1-0)+(a_2-a_1)+\ldots+(a_n-a_{n-1})=a_n$, the height of σ's top. In more general cases, however, the genus represents only the values of the components of σ relative to their surrounds, not their absolute values; for example, if there is a component of value v adjacent to the background, and a set of n tops of values t_i all surrounded by this component, the genus is $\Sigma(t_i-v)+(v-0)=\Sigma t_i-(n-1)v$. Note that if a component contains a set of holes, it still makes a positive contribution to the genus unless the sum of the relative depths of the holes exceeds its height relative to its surround.

6. Concluding remarks

This note has established some equivalent conditions for gray scale "connectedness", and has defined a grayscale generalization of the genus which relates global properties of the relative component heights to local properties summed over certain corners where triples of (hexagonal) pixels meet. It has introduced the notion of a "coherent" digital picture, and shown that the theory of these properties becomes much simpler in the coherent case.

References

1. A. Rosenfeld, Picture Languages, Academic Press, New York, 1979.

2. A. Rosenfeld, Fuzzy digital topology, Info. Control 40, 1979, 76-87.

3. J. Sklansky, Recognition of convex blobs, Pattern Recognition 2, 1970-3-10.

4. S. B. Gray, Local properties of binary images in two dimensions, IEEE Trans. Computers 20, 1971, 551-561.

DA
FIL