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LEVEL II

THE DESIGN OF MULTIACTIVITY MULTIFACILITY SYSTEMS

by

Richard M. Soland

Serial T-444
15 June 1981

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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1. Introduction

The objectives of this paper are to present and analyze a discrete facility location-allocation model that is more general than similar models previously considered. Although the model is a deterministic and static one, it is otherwise quite general and so offers modeling possibilities for problem situations that do not appear to have been formally treated heretofore. The spirit and approach of the paper are the same as those of an earlier one [Koss and Soland (1980)], but the facility location-allocation model considered herein is more general and more versatile.

To deal with similarities first, we feel that most locational decision problems, in both the public *and* private sectors, are really multiple criteria problems, and ought to be treated as such. We also feel that an interactive approach to multiple criteria decision making (MCDM) is in most cases the most appropriate one because it gets the decision maker (DM) involved with the criteria themselves and the

tradeoffs possible among them and forces him to directly insert his preferences into the decision procedure. In our earlier paper we made these points in connection with a fairly general discrete location-allocation model which we analyzed in detail with respect to multiple criteria considerations. We discussed at some length the various criteria of cost, service, and profit that are appropriate in public and private sector models, we presented an introductory look at MCDM and efficient solutions, and we outlined a finite interactive solution algorithm for the discrete model that generates only efficient solutions and finds them in a computationally efficient manner. A heuristic procedure based upon this algorithm was subsequently incorporated into a computer system utilizing a conversational command language designed for flexible user control [see Kultz, *et al.* (1980)].

The discrete location-allocation model of our previous paper, like many other such models, does not directly account for interactive effects among facilities or for the possibility that the facilities in the system may be hierarchically structured in different ways. But there are service/distribution systems, such as multi-echelon inventory systems [Cross, Pinkus, and Soland (1981)] and repairable item support systems [Cross and Pinkus (1979)] in which the choice of hierarchical structures used is of great importance and partially determines the facilities to be included. And there are service systems (e.g., health care systems) in which the *joint* effect of the facilities utilized determines such measures as average waiting time and average time in the system. In general, both the hierarchical structure and the interactive effects of the facilities included in it together determine some of the important criterion

measures for decision making. The location-allocation model to be presented below does allow for such generality as well as for the consideration of multiple criteria, and it includes the model of Ross and Soland (1980) as a special case. It is based on the use of configurations, a term we shall now define. A *configuration* is a specific subset of the set of facilities being considered, along with a specific strategy for the use of those facilities. Thus a configuration has a particular hierarchical structure and set of operating rules, so two configurations might conceivably include the same facilities and differ only in their structural arrangement or operating rules. A multi-echelon inventory system offers perhaps the best example of a configuration; the echelon structure and specific facilities at different levels in the structure, along with the inventory policies to be followed at the various facilities, determine the configuration. At the other extreme, a configuration may consist of a single facility along with a fairly obvious and straightforward rationale for using the facility to distribute the goods and/or services for which it is responsible.

The other concept we use here is that of an activity. An *activity* is an entity that is required to be "serviced" by some particular configuration in order for the system to function as intended. An activity is to be carried out at one or several facility locations according to the configuration to which it is assigned. Thus each activity is to be assigned to a unique configuration, and this assignment problem is what we call the design of a multiactivity multifacility system. Several examples will help to clarify the concept of an activity and the nature of the design problem. The activities may be different products and the

configurations multi-echelon inventory systems. Since different products need not be distributed according to the same multi-echelon structure, the problem [see Gross, Pinkus, and Soland (1981)] is to choose an appropriate configuration for each product. A similar problem arises in setting up a repairable item support system [see Gross and Pinkus (1979)]; each different repairable component constitutes an activity and each configuration is a subset of facilities at which to perform the repairs, along with rules to apportion the population of that component among the facilities in the given subset. An activity may be a particular type of health care or educational unit (such as a cardiac unit) or a particular type of emergency service vehicle (such as an ambulance or small fire engine) and the configurations are then different subsets of facilities at which to locate these units or vehicles.

Having indicated the similarities and differences between the present paper and our previous one, we remark that the reader is not expected to be familiar with the previous results. The presentation here is self-contained, but some aspects of it will be covered briefly in order to avoid unnecessary repetition.

In the remainder of this section we briefly discuss the different criteria which ought to be considered by those responsible for the design of multiactivity multifacility systems. In Section 2 we formulate a discrete choice model for the design of multiactivity multifacility systems and we quantify the various criteria previously discussed in terms of the variables and parameters of this model. We also show that our previous model, and hence a wide class of location-allocation models, is obtained as a special case of the present model. In Section 3 we briefly discuss

the main approaches to MCDM, present a general characterization of the *efficient* (or *nondominated*) solutions of a MCDM problem, and specialize it to the discrete model of Section 2 for the design of multiactivity multifacility systems. In Section 4 we introduce a 0-1 linear optimization model called the team generalized assignment problem (T-GAP) and show that parametric solution of a T-GAP will yield all efficient solutions of the multiactivity multifacility design problem with multiple criteria. We also discuss there the numerical solution of T-GAPs. Rather than attempting to find all efficient solutions of the design problem, however, we advocate an interactive approach and briefly describe in Section 5 an interactive branch-and-bound algorithm that solves a finite sequence of T-GAPs in order to determine a solution to the design problem that is most preferable to the DM. Some final remarks conclude the paper.

Now we turn to a discussion of the criteria to be considered. As indicated above, the criteria naturally fall into the categories of cost, service, and profit, but there exist several distinct criteria in each of these categories. Most of them have been discussed by Ross and Soland (1980, Sections 1 and 5), so we shall merely cite those here. With respect to costs, we deal with three classes: investment cost, operating cost, and discounted cost. Within each of these classes we differentiate between the *fixed cost*, the *incremental* (or *variable*) *cost*, and the *total cost*, which is the sum of the fixed and incremental components. The fixed cost, in each of the three classes, is assumed to be independent of the size of the facility concerned and the services offered there. The incremental cost at a particular facility, on the other hand, is completely dependent on the facility size and services offered.

But in the context of multiactivity multifacility systems there are also incremental costs that need not necessarily be allocated to a particular facility. For example, in a multi-echelon inventory system the *total* annual inventory cost for a particular product may be of more interest than the various inventory costs at the respective facilities. Indeed, the inventory policy followed by the system (for this particular product) may be chosen to minimize this quantity. Thus, while fixed costs will always be attributed to specific facilities, it will be convenient to deal with incremental costs that are attributed to configurations as well as those that are attributed to specific facilities.

We distinguish between investment and operating costs on the basis of when they occur--before or after the facilities are in operation. Total discounted cost is an appropriate weighted sum of investment and operating costs.

Now we turn to service criteria. These may differ significantly, depending on the context of the problem. For a system providing services to the public, the total demand served and the demand which can be served within a specified time (or distance) are important measures. Also important is the average response time or average distance traveled. The average time spent waiting for service and average total trip time are other important ones. For a multi-echelon inventory system the overall availability and mean turnaround time of orders may be the most appropriate service criteria, and similar ones may be used for repairable item support systems.

Criteria associated with profit are important in the private sector; both annual profit and total discounted profit are appropriate ones. Total annual revenue may also be of interest, and we include it as another criterion associated with profit.

Although certain criteria are to be minimized while others are to be maximized, it will hereafter be convenient to talk of *minimizing* all criteria. It will thus be necessary to use the *negatives* of certain criteria; e.g., the negative of demand served and the negative of profit.

7. A Discrete Location Model

The multiactivity multifacility design problem is to assign each activity to a specific configuration in a manner that respects capacity constraints at the facilities utilized; for obvious reasons we do not specify an objective at this point. We suppose there are N distinct activities to be assigned and L distinct sites, at some subset of which facilities will be (or already are) established. We number from 1 to M_j the configurations to which activity j may be assigned and note that configuration i for activity j need not bear any relation to configuration i for activity j'. We assume that configurations 1 to M_j are specified for each j $(j=1,\ldots,N)$ and do not deal here with the question of generating the sets of configurations to be considered. This is not a trivial problem, and we shall have some more to say about it in the final section.

To simplify our notation and terminology we let $M = \max_j M_j$ and suppose that each of the N activities is to be assigned to one of M configurations; as will be seen, it is a simple matter to prevent assignment of activity j to configuration i if i exceeds M_j.

For $h=1,\ldots,L$, $i=1,\ldots,M$, and $j=1,\ldots,N$, let

$z_h = 1$ if a facility is established at site h,

$= 0$ otherwise;

x_{ij} = 1 if activity j is assigned to configuration i,

= 0 otherwise;

d_{hij} = the usage of the facility at site h due to the assignment of activity j to configuration i;

a_h, b_h = the minimum and maximum, respectively, total usage permitted at site h.

Feasible solutions of the multiactivity multifacility design problem are then those choices of the z_h and x_{ij} which satisfy the following constraints:

$$a_h z_h \le \sum_{j=1}^{N} \sum_{i=1}^{M} d_{hij} x_{ij} \le b_h z_h , \quad \text{for all } h=1,\ldots,L , \tag{1}$$

$$\sum_{i=1}^{M} x_{ij} = 1 , \quad \text{for all } j=1,\ldots,N , \tag{2}$$

$$z_h, x_{ij} = 0 \text{ or } 1 , \quad \text{for all } h=1,\ldots,L;\ i=1,\ldots,M;\ j=1,\ldots,N . \tag{3}$$

Clearly, $0 < a_h < b_h$ for all h, but it is quite permissible to have $a_h = 0$ and/or $b_h = B$ (a very large number which represents $+\infty$) to indicate the lack of limitations on total usage at facility h. Each d_{hij} is nonnegative, but clearly $d_{hij} = 0$ if configuration i does *not* involve the use of facility h for activity j. Clearly, the d_{hij} must be so defined that the quantity $\sum_j \sum_i d_{hij} x_{ij}$ is a meaningful measure of the usage of facility h. Determination of d_{hij} (and of other parameters to be introduced) is not necessarily simple, and may indeed involve the solution of an optimization problem. For example, in the case of multi-echelon inventory systems [Gross, Pinkus, and Soland (1981)], configuration i represents a particular hierarchical structure along with the minimum cost inventory policy (for a specified

product) using that structure. To prevent the assignment of activity j to configuration i if $i > M_j$ we may set $d_{hij} = +\infty$ for all h.

Now we turn to the criterion functions to be used with this model. We use the index k to distinguish various criteria. For investment, operating or discounted cost, the quantity

$$\sum_{h=1}^{L} F_{hk} z_h \tag{4}$$

is the fixed cost, whereas we take

$$\sum_{j=1}^{N} \sum_{i=1}^{M} C_{ijk} x_{ij} \tag{5}$$

and

$$\sum_{h=1}^{L} F_{hk} z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} C_{ijk} x_{ij} \tag{6}$$

as the incremental cost and total cost, respectively. The F_{hk} and C_{ijk} are nonnegative constants whose determination, as noted before, may not be simple.

For service and profit criteria the exact forms of the criterion functions depend on the context as well as on the criterion, and the reader is referred to Ross and Soland (1980, Sections 2 and 5) for detailed treatment of a number of specific cases. What is important, however, is that almost all reasonable criterion functions can be written in the form

$$\sum_{h=1}^{L} f_{hk} z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} s_{ijk} x_{ij} , \tag{7}$$

where the f_{hk} and s_{ijk} are appropriately defined constants. In general we wish to *minimize* each of several expressions of the form (7), subject to the constraints (1) - (3).

The multiactivity multifacility design problem has not previously been considered in the generality presented here, especially with respect to multiple criteria, but its history goes back at least to the dissertation of Pinkus (1971) and the published version of Pinkus (1975). Pinkus, Gross, and Soland (1973) considered the problem in more generality than the inventory context and concentrated on a specialized branch-and-bound solution scheme. All three of these papers dealt with the uncapacitated case (all $a_h = 0$ and all $b_h = B$), as did the application of Gross and Pinkus (1979). The capacitated case was used as a model by Gross, Pinkus, and Soland (1981), and Chhabra and Soland (1980) and Chhabra (1981) have presented a non-LP branch-and-bound algorithm and computer code for this case. All the papers cited here dealt with a single criterion function of the form (7).

We indicated earlier that the present model includes that of Ross and Soland (1980) as a special case. To see this, let each activity represent a client group and let each configuration represent a single facility site to which client groups may be assigned. Thus, $M_j = M = L$ for all $j=1,\ldots,N$; we keep the natural correspondence between configurations and sites so that the index h is replaced by i and L is replaced by M . Also, d_{hij} is zero unless $h=i$, in which case we call it d_{ij} . Hence (1) reduces to

$$a_i z_i \leq \sum_{j=1}^{N} d_{ij} x_{ij} \leq b_i z_i \ , \quad \text{for all} \quad i=1,\ldots,M \ , \tag{8}$$

which is Equation (1) of Ross and Soland (1980). It is also easy to see that each criterion function of the general form (7) reduces to the form used in the earlier model.

3. Multiple Criteria Optimization and Efficient Solutions

Most techniques for deterministic MCDM involve some combination of these three approaches: construction and use of a value function, determination of the set of efficient solutions, and use of an interactive algorithm [see Soland (1979) for an overview of these approaches and a number of references]. In a general treatment of MCDM we take X as the set of feasible solutions and assume there are p criterion functions, $f_1,\ldots,f_p$, all to be minimized. We write $f(x)$ for the vector $(f_1(x),\ldots,f_p(x))$, and phrase the MCDM problem as

$$\text{"Minimize"} \quad f(x) \tag{9a}$$

$$\text{s.t.} \quad x \in X . \tag{9b}$$

Let $Y \equiv \{y \in R^p \mid y = f(x) \text{ for some } x \in X\}$ be the set of feasible criterion vectors.

The value function approach is based on the result that, under certain hypotheses concerning the preference structure of the DM, there exists an isotone decreasing value function v defined on a set containing Y such that the DM prefers $y^1 \in Y$ to $y^2 \in Y$ if and only if $v(y^1) > v(y^2)$. If the DM's value function v is given, a most desirable solution to his MCDM problem is clearly one that is optimal for problem (10):

$$\text{Maximize} \quad v(f(x)) \tag{10a}$$

$$\text{s.t.} \quad x \in X . \tag{10b}$$

The major difficulty in practice is the determination of v; see Keeney and Raiffa (1976) for an extensive treatment.

At the opposite extreme from the use of a value function is the determination of the set of efficient solutions of (9); this demands practically no preference specification on the part of the DM. We start with the definition of a dominated solution:

<u>Definition</u>: $x^0 \in X$ is a *dominated solution* of (9) if there exists $x \in X$ such that $f(x) \leqq f(x^0)$ but $f(x) \neq f(x^0)$.

An $x^0 \in X$ that is not dominated is said to be *nondominated* or *efficient*; we let X_E denote the set of efficient solutions of (9). A second approach to the resolution of (9) is to present to the DM the efficient set $Y_E = f(X_E)$ = the set of efficient solutions in criterion space. A result which, at least in theory, makes it easier to determine X_E and Y_E is based on the auxiliary problem $(P_{\lambda,\alpha})$:

$$\begin{aligned} &\text{Minimize} \quad \lambda f(x) = \sum_{k=1}^{p} \lambda_k f_k(x) \\ &\text{s.t.} \qquad x \in X , \\ &\qquad\qquad f(x) \leqq \alpha , \end{aligned} \qquad (P_{\lambda,\alpha})$$

where $\lambda > 0$ and $\alpha \in R^p$. Then we have

<u>Lemma 1</u> [Soland (1979)]: For arbitrary $\lambda > 0$, $x^0 \in X_E$ if and only if x^0 is optimal in $(P_{\lambda,\alpha})$ for some $\alpha \in R^p$.

Thus parametric solution of problem $(P_{\lambda,\alpha})$ will yield all elements of X_E and Y_E . Note that this result is independent of the nature of the constraint set X and the criterion functions f and so applies, e.g., to discrete problems. It will be used below to characterize the efficient solutions of our multiactivity multifacility design model

(1) - (3) and (7). But first we introduce the interactive approach to MCDM, with which we shall deal exclusively in Section 5.

In the interactive approach the DM works directly with an analyst and/or an interactive computer program. The DM supplies information which helps to reveal his preferences while the analyst and/or computer program is responsible for generating solutions for the DM to consider and evaluate. An interactive procedure should, after a reasonable amount of effort on the part of the DM, yield a solution with which he is "satisfied." It should be an efficient solution, and it is desirable that it be one that maximizes the DM's (unknown) value function. Most interactive procedures make certain convexity or linearity assumptions about the criterion functions and constraint set and/or the DM's value function. In Section 5 we describe an interactive algorithm that does not have these limitations and which we hope to be computationally efficient for our design problem.

Now we return to the multiactivity multifacility design problem of Section 2 and its set $\bar{X}_E$ of efficient solutions. We write (x,z) for the vector of x_{ij} and z_h values that identify a feasible solution of (1) - (3). We suppose that among the many possible criterion functions discussed exactly p, all having the algebraic form (7), are selected by the DM as relevant for the design decision. Then, by virtue of Lemma 1, we have the following result:

<u>Corollary 1.1</u>: For arbitrary $\lambda > 0$, $(x,z) \in \bar{X}_E$ if and only if (x,z) is optimal in the following auxiliary problem $(\bar{P}_{\lambda,\alpha})$ for some $\alpha \in R^p$:

$$\text{Minimize} \quad \sum_{k=1}^{p} \lambda_k \left\{ \sum_{h=1}^{L} f_{hk} z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} s_{ijk} x_{ij} \right\} \tag{11}$$

s.t. $(\bar{P}_{\lambda,\alpha})$

$$a_h z_h \leq \sum_{j=1}^{N} \sum_{i=1}^{M} d_{hij} x_{ij} \leq b_h z_h , \quad h=1,\ldots,L , \tag{1}$$

$$\sum_{i=1}^{M} x_{ij} = 1 , \quad j=1,\ldots,N , \tag{2}$$

$$\sum_{h=1}^{L} f_{hk} z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} s_{ijk} x_{ij} \leq \alpha_k , \quad k=1,\ldots,p , \tag{12}$$

$$z_h, x_{ij} = 0 \text{ or } 1; \; h=1,\ldots,L; \; i=1,\ldots,M; \; j=1,\ldots,N , \tag{3}$$

where $\lambda_k > 0$, $\alpha_k \in R^1$, and the f_{hk} and s_{ijk} are appropriately chosen constants. Note that the objective function (11) of $(\bar{P}_{\lambda,\alpha})$ can be written as

$$\sum_{h=1}^{L} f_h(\lambda) z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} s_{ij}(\lambda) x_{ij} , \tag{11'}$$

where $f_h(\lambda) \quad \sum_k \lambda_k f_{hk}$ and $s_{ij}(\lambda) = \sum_k \lambda_k s_{ijk}$. In the next section we shall relate problem $(\bar{P}_{\lambda,\alpha})$ to a discrete optimization model we call the team generalized assignment problem (T-GAP).

4. The Team Generalized Assignment Problem (T-GAP)

The generalized assignment problem (GAP) [see Ross and Soland (1975) and Ross, Soland, and Zoltners (1980)] is a discrete optimization model which may be interpreted as a problem of assigning tasks to agents at minimum cost, subject to upper and lower limits on the amount of a resource expended by each agent. We here generalize that model so

that tasks are assigned to teams, each team being composed of one or more agents and having a defined strategy for how the agents work together to perform a given task. A mathematical formulation of the T-GAP is:

$$\text{Minimize} \quad \sum_{j\in J} \sum_{i\in I} r_{0ij} x_{ij} \tag{13}$$

$$\text{s.t.} \quad a_h \leq \sum_{j\in J} \sum_{i\in I} r_{hij} x_{ij} \leq b_h \,, \quad \text{for all } h \in H \,, \tag{14}$$

$$\sum_{i\in I} x_{ij} = 1 \,, \quad \text{for all } j \in J \,, \tag{15}$$

$$x_{ij} = 0 \text{ or } 1 \,, \quad \text{for all } i \in I,\ j \in J \,. \tag{16}$$

Here $H \equiv \{1,2,\ldots,\ell\}$ is the set of agent indices, $I \equiv \{1,2,\ldots,m\}$ is the set of team indices, $J \equiv \{1,2,\ldots,n\}$ is the set of task indices, r_{0ij} is the cost incurred if task j is assigned to team i, $r_{hij} \geq 0$ is the amount of a resource required by agent h to help perform task j as a member of team i, and $a_h \geq 0$ and $b_h > 0$ are, respectively, the minimum and maximum amounts of the resource that may be expended by agent h. The decision variables are interpreted as

$$x_{ij} = 1 \quad \text{if task } j \text{ is assigned to team } i,$$
$$\phantom{x_{ij}} = 0 \quad \text{otherwise.}$$

We note that $r_{hij} = 0$ if agent h is not a member of team i, and we remark that one need not actually consider the same number or composition of teams for all the tasks (so a team is, intentionally, similar to a configuration). This generalization is dealt with as in Section 2.

We now proceed to show that problem $(\bar{P}_{\lambda,\alpha})$ can be written as a T-GAP of size $\ell = L+p$, $m = M$, and $n = N+L$. In the T-GAP formulation the x_{ij} are defined as in $(\bar{P}_{\lambda,\alpha})$ for $i=1,\ldots,M$ and $j=1,\ldots,N$.

The variable z_h of $(\bar{P}_{\lambda,\alpha})$ is represented by $x_{1,N+h}$, so we define

$$x_{1,N+h} = 1 \quad \text{if a facility is established at site } h,$$
$$= 0 \quad \text{otherwise:}$$

and

$$x_{2,N+h} = 0 \quad \text{if a facility is established at site } h,$$
$$= 1 \quad \text{otherwise.}$$

Thus $x_{1,N+h} + x_{2,N+h} = 1$ for $h=1,\ldots,L$. All other x_{ij} for $j > N$ must be zero. We now give a full stipulation of the T-GAP parameters. For $h \leq L$ the T-GAP parameters a_h and b_h are defined as in $(\bar{P}_{\lambda,\alpha})$. For $h = L+k$, $k=1,\ldots,p$, we distinguish two cases which correspond to whether the k*th* natural criterion function is one to be minimized (such as cost) or maximized (such as demand served). In the former case $f_{hk} \geq 0$ and $s_{ijk} \geq 0$ for all h,i,j and $\alpha_k \geq 0$ also. Then we set $a_h = 0$ and $b_h = \alpha_k$ for $h = L+k$. In the latter case $f_{hk} \leq 0$ and $s_{ijk} \leq 0$ for all h,i,j and $\alpha_k \leq 0$ also. Then for (12) we have the equivalent constraint

$$|\alpha_k| \leq \sum_{h=1}^{L} |f_{hk}| z_h + \sum_{j=1}^{N} \sum_{i=1}^{M} |s_{ijk}| x_{ij} , \tag{17}$$

so we set $a_h = |\alpha_k|$ and $b_h = B$ for $h = L+k$. We also have

$r_{0ij} = s_{ij}(\lambda)$, $\quad i=1,\ldots,M;\ j=1,\ldots,N$,

$r_{0,1,N+h} = f_h(\lambda) \quad r_{0,2,N+h} = 0$, $\quad h=1,\ldots,L$,

$r_{0,i,N+h} = \infty$, for $i > 2$ and $\quad h=1,\ldots,L$,

$r_{hij} = d_{hij}$ $\quad h=1,\ldots,L;\ i=1,\ldots,M;\ j=1,\ldots,N$,

$r_{h,2,N+h} = b_n$, $\quad h=1,\ldots,L$,

$r_{hij} = 0$ otherwise, $\quad h=1,\ldots,L;\ i=1,2;\ j=N+1,\ldots,N+L$,

$$r_{L+k,i,j} = |s_{ijk}| , \qquad i=1,\ldots,M;\ j=1,\ldots,N;\ k=1,\ldots,p ,$$

$$r_{L+k,1,N+h} = |f_{hk}| , \qquad h=1,\ldots,L;\ k=1,\ldots,p ,$$

$$r_{L+k,2,N+h} = 0 , \qquad h=1,\ldots,L;\ k=1,\ldots,p ,$$

$$r_{hij} = \infty \text{ otherwise.}$$

Thus problem $(\bar{P}_{\lambda,\alpha})$ is equivalent to a T-GAP, and hence by Corollary 1.1 all efficient solutions of the multiactivity multifacility design problem with multiple criteria can be found by parametric solution of this equivalent T-GAP. Rather than attempting to find all efficient solutions, however, we advocate an interactive approach that utilizes the equivalent T-GAP for computational efficiency. We will return to this point in the next section, but first we address the question of efficient numerical solution of the T-GAP.

The T-GAP is a somewhat specialized version of the multiple choice integer program (MCIP), the specialization being that the constraint coefficients r_{hij} in (14) are assumed to be nonnegative. The constraints (15) and (16) together are termed multiple choice constraints. Bean (1980) discusses the MCIP at length, provides references, and presents a new algorithm for its solution. He cites encouraging computational results for some problems with up to 400 variables. Another approach for solving the MCIP is given by Clover and Mulvey (1979).

It seems to us that a combination of approaches for the MCIP will eventually yield the most efficient algorithm for the T-GAP. Besides the additive approach of Bean (1980), some combination of Lagrangian relaxation and surrogate constraints [see Karwan and Rardin

(1979) and Dyer (1980)] will be important in generating tight lower bounds for a branch-and-bound algorithm. Lagrangian relaxation has already proved very successful in solving certain special classes of MCIPs that are location problems [see Fisher (1981) for a discussion]; both subgradient optimization [see Crowder (1976)] and multiplier adjustment methods [see Fisher (1981)] have proved successful in providing the tight Lagrangian bounds desired.

Chhabra (1981) and Chhabra and Soland (1980) have directly attacked the multiactivity multifacility design problem with a single criterion function. They treat the case with all $a_h = 0$ and use a combination of Lagrangian relaxation and the additive approach.

5. Interactive Solution of the Multiple Criteria Location Model

In Ross and Soland (1980, Section 4) we presented an interactive satisficing algorithm for MCDM that terminates after generating a finite number of efficient solutions. It does not generate any dominated solutions. In it the DM is asked to make some binary comparisons of vectors in the efficient set Y_E. He is also asked to vary the weights λ_k and to gradually decrease the "satisfaction levels" α_k that appear in problem $(P_{\lambda,\alpha})$. That algorithm may of course be applied in the current context without alteration, and in that case each of the finite number of optimization problems to be solved is a T-GAP; only λ and α change from one problem to the next.

Hultz, *et al.* (1980) subsequently incorporated a heuristic procedure based upon the above algorithm into an interactive computer system utilizing a conversational command language. That same computer

system, with minor alteration, could be modified to deal with the present multiple criteria location model, and would then be required to solve a T-GAP at each iteration.

The two interactive algorithms just mentioned may be criticized as being too demanding of the DM in that they require him to provide a sequence of λ and α vectors. Equally important, it is not clear how the λ and α vectors can be related to the DM's preference structure, and so the algorithms do not necessarily yield a decision choice that is in some sense "optimal" for the DM. In contrast, the interactive algorithm of Marcotte and Soland (1981), to be briefly described here, only requires the DM to make binary comparisons of vectors in criteria space and finally yields a decision choice that solves problem (10), i.e., maximizes the DM's (unknown) value function.

The algorithm of Marcotte and Soland (1981) uses a branch-and-bound approach to solve the following problem, which is equivalent to (10):

$$\text{Maximize} \quad v(y) \tag{18a}$$

$$\text{s.t.} \quad y \in Y . \tag{18b}$$

The feasible set Y, and subsequently subsets $Y_1, Y_2, \ldots$ of Y, are separated into subsets by appending to the requirement $y \in Y$ additional constraints of the form $y_k \leq \alpha_k$, one for each subset generated. The α_k may be interpreted as satisfaction levels, and their values are obtained from an efficient point $y \in Y_E$. We illustrate this separation process with the initial subset $Y_0 = Y$. An efficient point y^0 is found as the solution of the problem

$$\text{Minimize} \quad \sum_{k=1}^{p} c_k^0 y_k \tag{19a}$$

$$\text{s.t.} \quad y \in Y_0 \ , \tag{19b}$$

where the c_k^0 are positive constants; Y_0 is then separated into p subsets Y_k , $k=1,\ldots,p$, defined as

$$Y_k = Y_0 \cap \{y \mid y_k < y_k^1\} \ , \quad k=1,\ldots,p \ . \tag{20}$$

For discrete problems, such as those we deal with here, the strict inequality in (20) causes no difficulty.

In an ordinary branch-and-bound algorithm one would compute an upper bound on v over the subset Y_k for each $k=1,\ldots,p$. All subsets whose upper bounds did not exceed the value of the best known feasible solution (the incumbent solution y^*) would be discarded, and branching would continue from one of the remaining subsets. In this interactive algorithm an upper bound is the value of v associated with the *ideal vector* $\beta^k = (\beta_1^k,\ldots,\beta_p^k)$, where β_ℓ^k is the optimal value of the problem

$$\text{Minimize} \quad y_\ell \tag{21a}$$

$$\text{s.t.} \quad y \in Y_k \ , \tag{21b}$$

$\ell=1,\ldots,p$. Note that, in general, $\beta^k \notin Y_k$ and $\beta^k \notin Y$. At this point of the algorithm the incumbent solution is $y^* = y^0$, and it is necessary to compare $v(y^0)$ with the upper bounds $v(\beta^k)$, $k=1,\ldots,p$. As v is unknown, such comparisons are not possible. Instead, the DM is asked to compare the two vectors y^0 and β^k and to state which he prefers; this comparison is equivalent to one between $v(y^0)$ and $v(\beta^k)$.

There are other comparisons required by this branch-and-bound algorithm that are performed by the DM. He must compare y^* with other efficient solutions y^ℓ generated, and he must sometimes arrange the subsets Y_q generated at a separation according to decreasing preference of their ideal vectors β^q. This latter task can be done, if the DM so desires, through a series of binary comparisons.

Thus the algorithm only requires the DM to make binary comparisons of vectors in criteria space. It generates a finite sequence of feasible solutions, all of which are efficient, and it terminates after a finite number of separations. Assuming that the DM's responses are consistent with his preference structure, the incumbent solution at the termination of the algorithm solves problem (18), i.e., is a most preferred choice. For each separation the algorithm is required to solve one problem of the form (19) and (up to) p problems of the form (21). As we have seen in Section 4, these problems may be solved as T-GAPs; only the λ and α vectors change from one problem to another. With an efficient method for solving such T-GAPs the algorithm of Marcotte and Soland may become a practical vehicle for dealing with the wide class of locational decisions that may be modeled as multiactivity multifacility design problems with multiple criteria.

6. Concluding Remarks

The locational decision model presented here is more general than other static deterministic location models and allows for the incorporation of interactive effects among facilities and hierarchical structures of the facilities. For models of significant size, however,

fairly difficult 0-1 optimization problems result. These can be reduced to the form of T-GAPs, for which we hope that computationally efficient computer codes will become available. Work is definitely needed on efficient algorithms for the T-GAP.

The coefficients f_k, s_{ijk}, and d_{hij} of our model depend heavily upon the context of the problem and may be difficult to determine in some cases. Monte Carlo simulation may even be required to estimate them in certain applications. Clearly, then, good submodels are needed to generate the model coefficients for specific problems.

It is also necessary to generate "good" configurations to be considered for the various activities to be assigned. In most cases it will not be possible to consider all possible configurations, so it will partially fall upon the modeler/analyst to insure that the subset of configurations explicitly considered by the model is sufficiently "rich." Some suboptimization may inevitably result from the interaction of the capacity constraints and the inability to include all possible configurations; this difficulty is worthy of examination and may be surmountable in some cases.

We have stressed here an interactive approach to an MCDM problem, but we remark that the model and T-GAP equivalency can be useful in generating efficient solutions with guaranteed maximal values of the criterion functions even if no explicit interactive procedure is used. Even if one criterion is of paramount importance, the model can be used to impose constraints on a number of less important criterion functions, e.g., an upper limit may be set on the total number of facilities.

REFERENCES

[1] BEAN, J. C. (1980). An additive algorithm for the multiple choice integer program. Technical Report 80-26, Department of Operations Research, Stanford University, Stanford, California.

[2] CHHABRA, K. L. (1981). Solving multiactivity multifacility capacity-constrained 0-1 assignment problems. Technical Paper Serial T-441, Institute for Management Science and Engineering, The George Washington University.

[3] CHHABRA, K. L. and R. M. SOLAND (1980). Program description and user's guide for ZIPCAP--A zero-one integer program to solve multiactivity multifacility capacity-constrained assignment problems. Technical Paper Serial T-423, Institute for Management Science and Engineering, The George Washington University.

[4] CROWDER, H. (1976). Computational improvements for subgradient optimization. *Symposia Mathematica*, *19*, 357-372.

[5] DYER, M. E. (1980). Calculating surrogate constraints. *Mathematical Programming*, *19*, 255-278.

[6] FISHER, M. L. (1981). The Lagrangian relaxation method for solving integer programming problems. *Management Science*, *27*, 1-18.

[7] GLOVER, F. and J. M. MULVEY (1979). Network relaxations and penalties for multiple choice problems. Research Report No. EES-79-12, Princeton University, Princeton, New Jersey.

[8] GROSS, D. and C. E. PINKUS (1979). Designing a support system for repairable items. *Computers and Operations Research*, *6*, 59-68.

[9] GROSS, D., C. E. PINKUS, and R. M. SOLAND (1981). Designing a multi-product multi-echelon inventory system, in *Multi-level Production/Inventory Control Systems: Theory and Practice* (L. B. Schwarz, ed.), TIMS Studies in the Management Sciences, *16*, 11-49.

[10] HULTZ, J. W., D. D. KLINGMAN, G. T. ROSS, and R. M. SOLAND (1980). An interactive computer system for multicriteria facility location. Technical Paper Serial T-417, Institute for Management Science and Engineering, The George Washington University. To appear in *Computers and Operations Research*.

[11] KARWAN, M. H. and R. L. RARDIN (1979). Some relationships between Lagrangian and surrogate duality in integer programming. *Mathematical Programming*, *17*, 320-334.

[12] KEENEY, R. L. and H. RAIFFA (1976). *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Wiley, New York.

[13] MARCOTTE, O. and R. M. SOLAND (1981). An interactive branch-and-bound algorithm for multiple criteria optimization. Technical Paper Serial T-442, Institute for Management Science and Engineering, The George Washington University.

[14] PINKUS, C. E. (1971). The design of multi-product multi-echelon inventory systems using a branch-and-bound algorithm. Technical Paper Serial T-250, Institute for Management Science and Engineering, The George Washington University.

[15] PINKUS, C. E. (1975). Optimal design of multi-product multi-echelon inventory systems. *Decision Sciences*, *6*, 492-507.

[16] PINKUS, C. E., D. GROSS, and R. M. SOLAND (1973). Optimal design of multiactivity multifacility systems by branch and bound. *Operations Research*, *21*, 270-283.

[17] ROSS, G. T. and R. M. SOLAND (1975). A branch and bound algorithm for the generalized assignment problem. *Mathematical Programming*, *8*, 91-105.

[18] ROSS, G. T. and R. M. SOLAND (1980). A multicriteria approach to the location of public facilities. *European Journal of Operational Research*, *4*, 307-321.

[19] ROSS, G. T., R. M. SOLAND, and A. A. ZOLTNERS (1980). The bounded interval generalized assignment model. *Naval Research Logistics Quarterly*, *27*, 625-633.

[20] SOLAND, R. M. (1979). Multicriteria optimization: A general characterization of efficient solutions. *Decision Sciences*, *10*, 26-38.

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