SCALING UNDERWATER EXPLOSION SHOCK WAVES FOR DIFFERENCES IN AMBIENT SOUND SPEED AND DENSITY

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NAVAL SURFACE WEAPONS CENTER

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Scaling Underwater Explosion Shock Waves for Differences in Ambient Sound Speed and Density

The underwater explosion shock wave similitude equations which describe the variation of peak pressure, decay constant, impulse and energy flux density with distance are rederived to include the effects of ambient sound speed and density. The resulting equations are found to be in good agreement with experimental measurements.
FOREWORD

Significant differences in underwater shock wave peak pressure and decay constant have been measured when identical explosive charges have been fired at different test sites. This "Key West Effect" has resulted in uncertainties in interpreting explosion test data. This report presents a method for quantitatively describing these and other changes in underwater explosion shock wave parameters in terms of the sound speed and density of the ambient water.

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J. F. PROCTOR
By direction
1. INTRODUCTION

In the measurement of underwater explosion shock waves it has generally been assumed that local ambient water conditions have a negligible effect on the recorded pressure signature. The exception to this is when distortions due to refraction occur. A recent set of data shows that even with no refraction, the shock wave pressure signature may be strongly dependent on the temperature and salinity of the water.

In an underwater explosions field test conducted by the Naval Surface Weapons Center (NSWC) the test site had to be moved from one location to another in mid-program. After a number of shots had been fired in the Potomac River at Dahlgren, Virginia, in January of 1979, the operation was suspended due to environmental considerations. The remaining shots were fired in the Atlantic Ocean near Key West, Florida, in June of 1979.

The program consisted of a number of different explosives fired in 100-lb charges. Duplicate shots of four different charges were fired at the two sites. There was a consistent pattern of variation in measured pressures between the two sites. This is shown by the pairs of curves on the left hand side of Figures 1.1 through 1.4. These curves are the familiar power law fits made by the method of Cole, Robert H., 1948, "Underwater Explosions," Princeton University Press.
FIGURE 1.1 GENERALIZED SIMILITUDE EQUATION SCALING OF PEAK PRESSURE
FIGURE 1.2 GENERALIZED SIMILITUDE EQUATION SCALING OF DECAY CONSTANT
FIGURE 1.3 GENERALIZED SIMILITUDE EQUATION SCALING OF IMPULSE
FIGURE 1.4 GENERALIZED SIMILITUDE EQUATION SCALING OF ENERGY FLUX DENSITY

SCALE TO:
$\rho_0 = 1\ \text{GM/CC}$

$\gamma_0 = 5000\ \text{FT/SEC}$

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DAHLGREN

KEY WEST
least squares to the experimental data for peak pressure, decay constant, impulse and energy flux density from four different types of explosive charges--each fired at the two sites. They show that in moving from Dahlgren (solid curves) to Key West (dashed curves) the peak pressure increased about 12%, the decay constant decreased about 14%, the impulse decreased about 4%, and the energy flux density was not significantly changed. Differences between the Key West and Dahlgren test conditions are summarized in Table 1.1.

Gaspin and Lehto\(^2\) have shown that this "Key West Effect" is real and that it can be approximately accounted for by differences between the two sites in the sound speed, \(c_0\), and the bulk modulus, \(K_0 = \rho_0 c_0^2\), where \(c_0\) is the ambient water density. From an approximate physical model they estimated the effect on peak overpressure, \(P_{MAX}\), and on decay constant, \(\varepsilon\), at a fixed distance from the charge to be

\[
\frac{P_{MAX_2}}{P_{MAX_1}} = \left(\frac{K_{0_2}}{K_{0_1}}\right)^{0.56} (14,500 > P_{MAX} > 1450 \text{ psi}) \tag{1.1}
\]

\[
\frac{\varepsilon_2}{\varepsilon_1} = \frac{c_{0_1}}{c_{0_2}} \tag{1.2}
\]

Equation 1.1 is in good agreement with the experimental data shown in Figure 1.1, while Equation 1.2 partially corrects for the change in decay constant shown in Figure 1.2. Lehto and Gaspin do not give results for the effects on impulse and energy flux density.

The purpose of this note is to demonstrate the possibility of accounting for changes in the peak pressure, decay constant, impulse, and energy flux density.

Table 1.1  Key West vs Dahlgren Differences
(Average Values for 2 Sites)

<table>
<thead>
<tr>
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<th>Dahlgren</th>
<th>Key West</th>
<th>Change</th>
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<tbody>
<tr>
<td>Water Temperature (Deg. C)</td>
<td>2.3</td>
<td>27.8</td>
<td>+ 25.5</td>
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<tr>
<td>Salinity (%)</td>
<td>7.6</td>
<td>36.5</td>
<td>+ 29.9</td>
</tr>
<tr>
<td>Sound Speed (ft/sec)</td>
<td>4668</td>
<td>5048</td>
<td>+ 8.1%</td>
</tr>
<tr>
<td>Water Density (gm/cc)</td>
<td>1.006</td>
<td>1.024</td>
<td>+ 1.8%</td>
</tr>
<tr>
<td>Bulk Modulus (lb/ft²)</td>
<td>42.54 E6</td>
<td>50.66 E6</td>
<td>+19.1%</td>
</tr>
<tr>
<td>Peak Pressure</td>
<td>—</td>
<td>—</td>
<td>+12%</td>
</tr>
<tr>
<td>Decay Constant</td>
<td>—</td>
<td>—</td>
<td>-14%</td>
</tr>
<tr>
<td>Impulse</td>
<td>—</td>
<td>—</td>
<td>-4%</td>
</tr>
<tr>
<td>Energy Flux Density</td>
<td>—</td>
<td>—</td>
<td>0%</td>
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(PMAX, ε, I, and E) due to variations in c₀ and c₀ by simply rederiving the
dimensionless variables used to formulate the underwater explosion shock wave
similitude equations—and not discarding c₀ and c₀. The pairs of curves shown on
the right-hand side of Figures 1.1 through 1.4 show the results of this effort.

2. SIMILITUDE EQUATIONS

The similitude equations for the underwater explosion shock wave

\[ \text{PMAX} = k \left( \frac{W^{1/3}}{R} \right)^a \]
\[ \varepsilon = w^{1/3} \left( \frac{R}{W^{1/3}} \right)^b \]
\[ I = n^{1/3} \left( \frac{W^{1/3}}{R} \right)^c \]
\[ E = m^{1/3} \left( \frac{W^{1/3}}{R} \right)^d \]

are specific expressions (power curve fits, \( y = ax^b \)) of a general scaling expressed
by

\[ p(R, t) = F \left( \frac{R}{W^{1/3}} , \frac{t}{W^{1/3}} \right) \]

We now proceed to rederive Equation 2.5 from dimensional considerations but
retaining the dependence on c₀ and c₀. The significant variables for the problem
are:

- \( p = p(t) = \text{shock wave overpressure} \)
- \( R = \text{radial distance from center of charge} \)
- \( t = \text{time} \)
- \( Y_0 = \text{total chemical energy released upon detonation} \)
- \( c_0 = \text{ambient water density} \)
- \( c_0 = \text{ambient sound speed} \).
Thus, we are in effect assuming that the overpressure, \( p \), is a function of \( R \), \( t \), \( Y_0 \), \( \rho_0 \), and \( c_0 \) or that

\[
\frac{p}{\rho_0 c_0^2} = \frac{p}{K_0} = 0
\]

(2.6)

where \( f \) is some unknown function. These six variables in Equation 2.6 can be expressed in terms of three dimensions—mass, length, and time. By Buckingham's \( \pi \)-theorem\(^3\), the problem can be reformulated in terms of three dimensionless groups or \( \pi \)'s and the solution must be of the form

\[
F (\pi_1, \pi_2, \pi_3) = 0
\]

(2.7)

A convenient set of \( \pi \)'s for considering this problem is the following\(^\ast\)

\[
\pi_1 = \frac{p}{\rho_0 c_0^2} = \frac{p}{K_0}
\]

(2.8)

\[
\pi_2 = \frac{(\rho_0 c_0^2)^{1/3} R}{Y_0^{1/3}} = \frac{K_0^{1/3} R}{Y_0^{1/3}}
\]

(2.9)

\[
\pi_3 = \frac{c_0 (\rho_0 c_0^2)^{1/3} t}{Y_0^{1/3}} = \frac{c_0 K_0^{1/3} t}{Y_0^{1/3}}
\]

(2.10)

where, \( K_0 = \rho_0 c_0^2 \), which is the bulk modulus of the ambient water.

Making use of this set of \( \pi \)'s Equation 2.7 can be written

\[
\frac{p}{K_0} = G \left( \frac{K_0^{1/3} R}{Y_0^{1/3}}, \frac{c_0 K_0^{1/3} t}{Y_0^{1/3}} \right)
\]

(2.11)

where \( G \) is some unknown function.


\(^\ast\)The reader is referred to References 3 and 4 for the method of determining convenient sets of \( \pi \)'s.

If we restrict our consideration to a single explosive, we can replace $\gamma_0$ in Equation 2.11 by $W$, the weight of explosive, and obtain

$$\frac{\rho(R,t)}{K_0} = H\left(\frac{K_0^{1/3}R}{W^{1/3}}, \frac{c_0K_0^{1/3}t}{W^{1/3}}\right)$$

(2.12)

which is similar to Equation 2.5 except we have retained the desired dependence on $c_o$ and $\rho_0$. Note, if we assume $K_0$ and $c_0$ are constants, then Equation 2.12 is equivalent to the familiar shock wave scaling expressed by Equation 2.5.

**Generalized Similitude Equations.** Note that the similitude equations, Equations 2.1 through 2.4, are simply power curve fits to experimental data using dimensionless variables. Thus, our generalized similitude equations are simply

$$\frac{P_{\text{MAX}}}{K_0} = k^*\left(\frac{K_0^{1/3}R}{W^{1/3}}\right)^{-\alpha}$$

(2.13)

$$\frac{c_0K_0^{1/3}}{W^{1/3}} = \xi^*\left(\frac{K_0^{1/3}R}{W^{1/3}}\right)^{\beta}$$

(2.14)

$$\frac{c_0I}{K_0^{2/3}W^{1/3}} = m^*\left(\frac{K_0^{1/3}R}{W^{1/3}}\right)^{-\gamma}$$

(2.15)

$$\frac{E}{K_0^{2/3}W^{1/3}} = n^*\left(\frac{K_0^{1/3}R}{W^{1/3}}\right)^{-\delta}$$

(2.16)

The exponents--$\alpha$, $\beta$, $\gamma$, and $\delta$--are the same as in Equations 2.1 through 2.4. The coefficients--$k^*$, $\xi^*$, $m^*$, and $n^*$--can be calculated from $k$, $\xi$, $m$, and $n$, respectively, by transforming Equations 2.1 through 2.4 to the same form as 2.13 through 2.16, respectively. Comparison then shows that
Equations 2.17 through 2.20 can be used to compare data fits made using Equations 2.13 through 2.16 to those made using Equations 2.1 through 2.4.

By substitution into Equations 2.13 through 2.16, we determine the changes in $\text{PMAX}$, $\theta$, $I$, and $E$ at constant range due to different ambient conditions. They are as follows:

\[
\frac{\text{PMAX}_2}{\text{PMAX}_1} = \left( \frac{K_{o_2}}{K_{o_1}} \right)^{1 - \frac{3}{\gamma}}
\]

\[
\frac{\theta_2}{\theta_1} = \frac{c_{o_1}}{c_{o_2}} \left( \frac{K_{o_1}}{K_{o_2}} \right)^{1 + \frac{8}{3}}
\]

\[
\frac{I_2}{I_1} = \frac{c_{o_1}}{c_{o_2}} \left( \frac{K_{o_2}}{K_{o_1}} \right)^{\frac{2}{3} - \frac{2}{3}}
\]

\[
\frac{E_2}{E_1} = \left( \frac{K_{o_1}}{K_{o_2}} \right)^{\frac{5}{3} - \frac{2}{3}}
\]
Changes in the similitude coefficients -- k, ℓ, m, n -- from one ambient state to another can be computed from Equations 2.17 through 2.20. They are

\[
\frac{k_2}{k_1} = \left( \frac{k_{o_2}}{k_{o_1}} \right)^{1 - \frac{\alpha}{3}}
\]

(2.25)

\[
\frac{\ell_2}{\ell_1} = \frac{c_{o_1}}{c_{o_2}} \left( \frac{k_{o_1}}{k_{o_2}} \right)^{\frac{1-\kappa}{3}}
\]

(2.26)

\[
\frac{m_2}{m_1} = \frac{c_{o_1}}{c_{o_2}} \left( \frac{k_{o_1}}{k_{o_2}} \right)^{\frac{2-\gamma}{3}}
\]

(2.27)

\[
\frac{n_2}{n_1} = \left( \frac{k_{o_1}}{k_{o_2}} \right)^{\frac{\delta - 2}{3}}
\]

(2.28)

3. RESULTS

The pairs of curves shown on the right-hand side of Figures 1.1 through 1.4 have been calculated from the corresponding curves shown on the left using Equations 2.13 through 2.20 and the values for ambient water density and sound speed measured on the respective shots. They have been scaled to the same ambient water condition, \( \rho_0 = 1 \) gm/cc and \( c_0 = 5000 \) ft/sec. For these tests the generalized similitude equations, Equations 2.13 through 2.16, do a good job of accounting for the observed changes in all of the shock wave parameters, PMAX, θ, I, and E. The calculated change in energy flux density from Dahlgren to Key West is a decrease of about 0.1% which is too small to detect experimentally or to observe on Figure 1.4.
4. DISCUSSION

Inserting current NSWC values\textsuperscript{5}, $\alpha = 1.19$, $\beta = 0.26$, $\gamma = 0.90$, $\varepsilon = 2.10$, for the similitude equation exponents for cast 50/50 pentolite into Equations 2.21 through 2.24, we get

$$\frac{P_{\text{MAX}_2}}{P_{\text{MAX}_1}} = \left(\frac{K_{03}}{K_{01}}\right)^{0.60} \tag{4.1}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{c_{01}}{c_{02}} \left(\frac{K_{01}}{K_{02}}\right)^{0.25} \tag{4.2}$$

$$\frac{I_2}{I_1} = \frac{c_{01}}{c_{02}} \left(\frac{K_{01}}{K_{02}}\right)^{0.37} \tag{4.3}$$

$$\frac{E_2}{E_1} = \left(\frac{K_{01}}{K_{02}}\right)^{0.03} \tag{4.4}$$

The result for peak pressure is essentially the same as Lehto and Gaspin's result (Equation 1.1). The result for decay constant is apparently more complete than their result (Equation 1.2). The results for impulse and energy flux density are new and are in agreement with experimental measurements.

\textsuperscript{5}Price, Robert S., NSWC limited distribution report
In the final analysis, the value of these generalized similitude equations will only be demonstrated by extensive use. This should include incorporation into the shock wave data analysis programs used by the Naval Surface Weapons Center. The equations needed to do this are derived in the Appendix.
APPENDIX A
POWER LAW FITS TO MEASURED DATA

Since self-consistent units of measurement are not used in underwater explosion research in this country, it is not recommended to make power law fits to experimental measurements using Equations 2.13 through 2.16. Instead, we can make use of the dimensionless variables implied in Equations 2.13 through 2.16 and scale our measured experimental quantities--R, PMAX, \( \theta \), I, E--to some standard condition, e.g., fresh water at some standard temperature and pressure*. Thus, using Equations 2.13 through 2.16

\[
R^* = \frac{K_0^{1/3}R}{W^{1/3}} = \frac{K_s^{1/3}R_s}{W^{1/3}} \quad (A.1)
\]

\[
PMAX^* = \frac{PMAX}{K_0} = \frac{PMAX_s}{K_s} \quad (A.2)
\]

\[
\theta^* = \frac{c_0K_0^{1/3}\theta}{W^{1/3}} = \frac{c_sK_s^{1/3}\theta_s}{W^{1/3}} \quad (A.3)
\]

\[
I^* = \frac{c_0I}{K_0^{2/3}W^{1/3}} = \frac{c_sI_s}{K_s^{2/3}W^{1/3}} \quad (A.4)
\]

\[
E^* = \frac{E}{K_0^{2/3}W^{1/3}} = \frac{E_s}{K_s^{2/3}W^{1/3}} \quad (A.5)
\]

*The author recommends that a standard condition be adopted for reporting measured values of similitude coefficients and suggests that fresh water at 15°C and 1 atmosphere pressure would be suitable. With this choice \( \rho_s = 0.99913 \) gm/cc and \( c_s = 4809.3 \) ft/sec.6


A-1
where "*" denotes a dimensionless variable and subscript "s" denotes standard condition.

Solving Equations A.1 through A.5 for \( R_s, \ PMAX_s, \ \theta_s, \ I_s, \ E_s \), respectively, we get

\[
R_s = \left( \frac{K_0}{K_s} \right)^{1/3} R \tag{A.6}
\]

\[
P_{\text{MAX}_s} = \left( \frac{K_s}{K_0} \right) P_{\text{MAX}} \tag{A.7}
\]

\[
\theta_s = \left( \frac{c_0}{c_s} \right) \left( \frac{K_0}{K_s} \right)^{1/3} \theta \tag{A.8}
\]

\[
I_s = \left( \frac{c_0}{c_s} \right) \left( \frac{K_s}{K_0} \right)^{2/3} I \tag{A.9}
\]

\[
E_s = \left( \frac{K_s}{K_0} \right)^{2/3} E \tag{A.10}
\]

The derived quantities \( R_s, \ P_{\text{MAX}_s}, \ \theta_s, \ I_s \) and \( E_s \) can be used to make the power law fits indicated by the ordinary similitude equations (Equations 2.1 through 2.4); and thereby obtain coefficients--\( k_s, \ l_s, \ m_s, \ n_s \)--and exponents--\( \alpha_s, \ \beta_s, \ \gamma_s, \ \delta_s \)--scaled to the desired standard condition. This can be verified by substituting Equations A.6 through A.10 into Equations 2.13 through 2.16 to get

\[
P_{\text{MAX}_s} = k^* \ K_s^{1-\frac{\alpha}{3}} \left( \frac{R_s}{W^{1/3}} \right)^{-\alpha} \tag{A.11}
\]

\[
\frac{\theta_s}{W^{1/3}} = \frac{\theta^*}{c_s K_s^{(1-\beta)/3}} \left( \frac{R_s}{W^{1/3}} \right)^{\beta} \tag{A.12}
\]
Substituting Equations 2.17 through 2.20 into Equations A.11 through A.14, respectively, we get:

\[ P_{\text{MAX}} = k_s \left( \frac{W^{1/3}}{R_s} \right)^\alpha \]  

\[ \frac{\Theta_s}{W^{1/3}} = \epsilon_s \left( \frac{R_s}{W^{1/3}} \right)^\beta \]  

\[ \frac{I_s}{W^{1/3}} = m_s \left( \frac{W^{1/3}}{R_s} \right)^\gamma \]  

\[ \frac{E_s}{W^{1/3}} = n_s \left( \frac{W^{1/3}}{R_s} \right)^\delta \]  

which is our desired result.
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