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TABLES OF SIGNIFICANCE POINTS FOR THE VARIANCE-WEIGHTED KOLMOGO--ETC(U)
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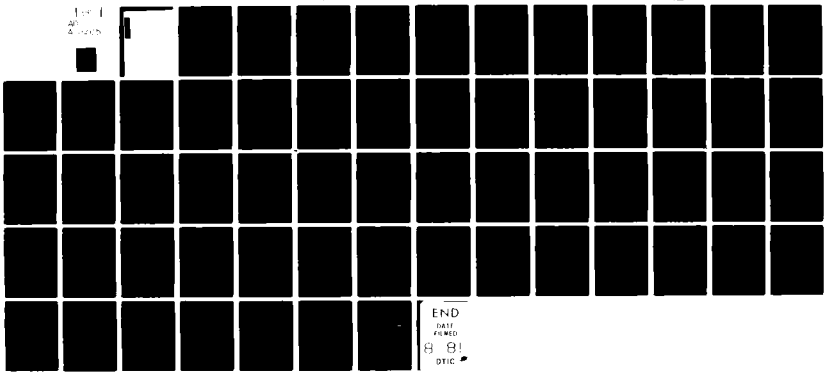

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TABLES OF SIGNIFICANCE POINTS FOR THE VARIANCE-WEIGHTED KOLMOGOROV-SMIRNOV STATISTICS

By

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Notations:

$\mathbb{Z}$ stands for the set of all integers,

$\mathbb{R}$ for the real numbers,

$\mathbb{N}_0 := \{n \in \mathbb{Z},\ n \geq 0\}$,

$\mathbb{N}_1 := \{n \in \mathbb{Z},\ n \geq 1\}$,

$x \wedge y := \min\{x,y\}$,

$x \vee y := \max\{x,y\}$,

$(x)_+ := \max\{0,x\}$,

$(x)_- := \min\{0,x\}$,

$\lceil x \rceil := \min\{i \in \mathbb{Z} \mid i \geq x\}$,

$\lfloor x \rfloor := \max\{i \in \mathbb{Z} \mid i \leq x\}$,

$\binom{x}{n} := \frac{x(x-1)\cdot\ldots\cdot(x-n+1)}{n!}$ for all $n \in \mathbb{N}_1$; $\binom{x}{0} := 1$; $\binom{x}{z} := 0$ for all $z \notin \mathbb{N}_0$,

For the values of a function $\nu\colon \mathbb{N}_0 \to \mathbb{R}$ we use both notations $\nu(i)$ and ν_i.

Tables of Significance Points for the Variance-Weighted

Kolmogorov-Smirnov Statistics

By

Heinrich Niederhausen

1. Introduction.

Let $X_1,\ldots,X_M$ be i.i.d. random variables with continuous distribution function F and empirical distribution function

$$F_X(x) = M^{-1} \sum_{i=1}^{M} 1_{(-\infty,x]}(X_i) .$$

The goodness-of-fit statistic

$$W_M^+ = \sup_{\theta_1 \le F(x) \le \theta_2} \frac{F_X(x)-F(x)}{\sqrt{F(x)(1-F(x))}}$$

has been shown to be asymptotically minimax (with respect to a certain loss function) by A.A. Borokov and N.M. Sycheva (1968). They also give some exact significance points and the asymptotic distribution of $\sqrt{M}\, W_M^+$. Beside W_M^+, we consider the following related statistics:

$$W_M = \sup_{\theta_1 \le F(x) \le \theta_2} \frac{|F_X(x)-F(x)|}{\sqrt{F(x)(1-F(x))}}$$

$$\tilde{W}_M^+ = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{F_X(x)-F(x)}{\sqrt{F(x)(1-F(x))}}$$

$$\tilde{W}_M = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{|F_X(x)-F(x)|}{\sqrt{F(x)(1-F(x))}}$$

$$W^+_{M,N} = \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$W_{M,N} = \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$\tilde{W}^+_{M,N} = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{F_X(x)-F_Y(x)}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$\tilde{W}_{M,N} = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}} ,$$

where $Y_1,\ldots,Y_N$ is a second independent sample with the same distribution function, and $V_1,\ldots,V_{M+N}$ is the combined sample. We call all these statistics variance-weighted Kolmogorov-Smirnov tests. In [10], we derived some methods to compute the exact distribution of such tests. Using those methods, we computed tables for the significance points of

(1) $\sqrt{M}\, W^+_M$, $\sqrt{M}\, W_M$, $\sqrt{M}\, \tilde{W}^+_M$, $\sqrt{M}\, \tilde{W}_M$, $\sqrt{MN/(M+N)}\, W^+_{M,N}$, $\sqrt{MN/(M+N)}\, W_{M,N}$, $\sqrt{MN/(M+N)}\, \tilde{W}^+_{M,N}$ and $\sqrt{MN/(M+N)}\, \tilde{W}_{M,N}$.

Let Z be any of the eight statistics in (1). Let

$$P(z) = P(Z \le z) .$$

For each $\alpha = .9$, $.95$ and $.99$ we try to find z_α such that $P(z_\alpha) = \alpha$. But the variance-weighted Kolmogorov-Smirnov distributions are discontinuous, even in the one-sample case. Therefore, we give $P(\underline{z}_\alpha)$ and $\underline{z}_\alpha$ in the tables, where $P(\underline{z}_\alpha)$ is smaller than α. After each $\underline{z}_\alpha$, a single digit D is printed. If the last digit of $\underline{z}_\alpha$ is increased by D, a $\overline{z}_\alpha$ is obtained, such that $P(\overline{z}_\alpha) \geq \alpha$. $P(\overline{z}_\alpha)$ is also listed. All numbers are rounded in the last digit.

In all the tables we chose $\theta_1 = 1-\theta_2 = \theta$ for $\theta = 0$, 0.01, 0.05, 0.1 and 0.25. In $\tilde{W}^+_M$, $\tilde{W}_M$, $\tilde{W}^+_{M,N}$ and $\tilde{W}_{M,N}$ we have to take the supremum over $\theta \leq F_X(x) \leq 1-\theta$. Thus, we have to replace θ by d/M, where the integer d is chosen such that d/M comes close to θ (see (3.2)). Analogously, replace θ by $d/(M+N)$ in $W^+_{M,N}$ and $W_{M,N}$.

In the two sample case, all tables are given for

$$M = 2,3,4,\ldots,10 \; ; \quad N = 2,3,4,\ldots,M$$

$$M = 15,20,25,\ldots,50; \quad N = M,M-1,M-2,\ldots,M-5$$

$$M = 100,500 \; ; \; N = M \; .$$

If for small M the table for a certain θ does not differ from the preceding table (with smaller θ), then this part of the table is omitted.

In the one sample, the same values of M are used, but the sample length $M = 500$ is omitted. The computer proved to be too slow for this case (and the desired accuracy).

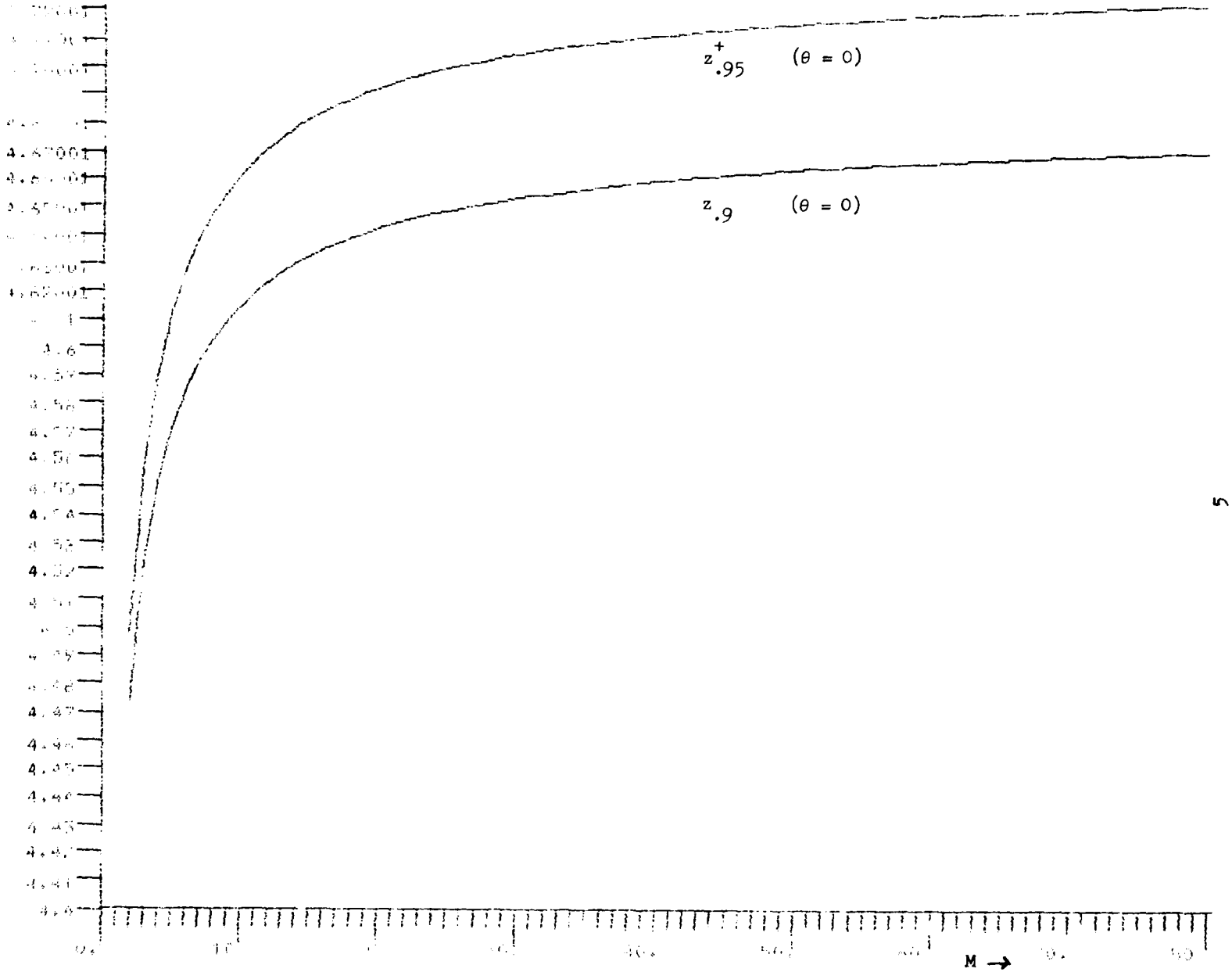
For large sample sizes, a significance value $z_{1-\gamma}$ of a two sided statistic can be approximated by $z^+_{1-\gamma/2}$ of the corresponding one sided statistic. The larger the θ, the better the approximation. Despite "bad" asymptotic behavior, this approximation is practically satisfying even for $\theta = 0$. For this case, the computer drawing on the next page shows $z_{.9}$ and $z^+_{.95}$ for $M = 2,\ldots,80$, where

$$P(\sqrt{M}\, W_M \leq z_{.9})_{\theta=0} \approx .9 \quad \text{and} \quad P(\sqrt{M}\, W^+_M \leq z^+_{.95})_{\theta=0} \approx .95 \ .$$

We demonstrate on page 6 what happens if $z^+_{.95}$ (which is much faster to compute) is used as an approximation for $z_{.9}$: We plotted M against $P(\sqrt{M}\, W_M \leq z^+_{.95})_{\theta=0}$ for $M = 2,\ldots,100$. To illustrate the effect of a larger θ, we plotted also $P(\sqrt{M}\, \tilde{W}_M \leq \tilde{z}^+_{.95})_{\theta=1}$. The jumps come from the approximation of θ by d/M.

We repeat that part of [10], which is necessary to understand the algorithms. Chapter 3 gives an overview over the significance points by small tables. The large tables are computed in the same way.

All computations are done on a pdp 11 computer, using 16 significant digits. The reader can compare the tables for $\sqrt{M}\, W^+_M$ with table 1 and 2 of A.A. Borokov and N.M. Sycheva (1968). With the expection of one printing error, their numbers differ at most by 1 in the last given digit. The case $\theta = 0$ for $\sqrt{M}\, W_M$ has been considered by M. Noé (1972). His method of computation is close to ours for all two-sided one sample statistics in (1), except for special cases, see 3.


$z^{+}_{.95}$ $(\theta = 0)$
$z_{.9}$ $(\theta = 0)$
M →

M
$P(\sqrt{M}\,\tilde{W}_M \le z^+_{.95})_{\theta=.1}$
$P(\sqrt{M}\,W_M \le z^+_{.95})_{\theta=0}$

1. One sample tests.

1.1. Sheffer polynomials for D.

Let μ and ν be monotone non-decreasing functions from $\mathbb{N}_0$ into $\mathbb{R}$, satisfying $0 \leq \nu_0 \leq \mu_0$ and $\nu_i < \mu_{i-1}$ $\forall i \in \mathbb{N}_1$. The following functions define a μ-Sheffer sequence (see (A.12)) for the derivative operator D: $p(x) \mapsto \frac{d}{dx} p(x)$

$$f_0(x) := \begin{cases} 1 & \text{if } x \leq 0 \\ 0 & \text{else} , \end{cases}$$

and

$$f_n(x) := \begin{cases} \int_{\nu(n)}^{x \wedge \mu(n-1)} \int_{\nu(n-1)}^{u_n \wedge \mu(n-2)} \cdots \int_{\nu(1)}^{u_2 \wedge \mu(0)} 1 du_1 \ldots du_n & \text{if } x \leq \mu_n \\ 0 & \text{else} \end{cases}$$

for all $n \in \mathbb{N}_1$. Obviously, $f_n(\nu_n) = \delta_{0,n}$, hence, (f_n) has roots in ν (see(A.14)).

Denote by $U_{(i)}$ the i-th order statistic of a size M random sample from U(0,1). If $\mu_{M-1} \leq 1$, then

$$\text{(1.1)} \qquad f_M(\mu_{M-1}) = P(\nu_i \leq U_{(i)} \leq \mu_{i-1} \quad \forall i = 1,\ldots,M)/M!$$
$$= f_M(\mu_M) .$$

1.2. Recursions.

For this section we assume $\mu_M = 1$ without loss of generality. With $q_{n-k}(x) = x^{n-k}/(n-k)!$ in (A.13), the algorithm A.1 was found by M. Nöe (1972). Given that μ_k is constant for $k = 0,\ldots,K$, say, it may be possible to use an explicit formula for $f_i(\mu_K)$ $(i = 0,\ldots,K)$. The same is true for the values p_i in the following application of (A.16): Define $p_0 := 1$ and

$$(1.2) \qquad p_i := \sum_{k=0}^{i-1} \binom{i}{k}(-1)^{i-k-1}(\mu_k-\nu_i)_+^{i-k}\, p_k \,.$$

Then

$$P(\nu_i \le U_{(i)} \le \mu_{i-1} \text{ for all } i = 1,\ldots,M) = p_M \,.$$

Alternatively, we get from corollary A.2:

$$p_M = \det\left(\binom{i}{j-1}(\mu_{j-1}-\nu_i)_+^{i-j+1}\right)_{i,j=1,\ldots,M} \,.$$

V.A. Epanechnikov (1968) found recursion (1.2) and G.P. Steck (1971) independently derived this determinant and many applications. See also E.J.G. Pitman (1972) for another proof.

Remark: Depending on the accuracy of the computer, (1.2) should be used only for small M because of the alternating summation. In algorithm A.1 the summation does not alternate, but compared with (1.2) the amount of

computations is approximately squared! In the computation of significance points it often occurs that $\nu_M \leq \mu_0$. The same is true, of course, in both one sided cases. Theorem A.2 (with $i := 0$) yields for any real function σ on $\mathbb{N}_0$:

$$(1.3) \qquad p_j = j!t_{j,0}(\sigma_j) - \sum_{k=0}^{j-1} \binom{j}{k}(\sigma_j-\mu_k)^{j-k}p_k \text{ for all } j = 1,\ldots,M .$$

From $\nu_M \leq \mu_0$ we see that $t_{j,0}$ equals for $j = 0,\ldots,M$ the Sheffer sequence for D with roots in ν. Given that $\sigma_j \geq \mu_{j-1}$ for all $j = 1,\ldots,M$, the summation in (1.3) is non-alternating. But how to compute $t_{j,0}(\sigma_j)$? In the simple case $\nu \equiv 0$ (one sided tests) we get $j!t_{j,0}(\sigma_j) = \sigma_j^j$. With $\sigma_j := \mu_{j-1}$ Steck's formula (1971, (2.3)) is obtained. See the previous section for other closed forms. We suggest the following procedure for general ν (with $\nu_M \leq \mu_0$):

Choose $\sigma \equiv 1$. Thus, for all $j = 1,\ldots,M$,

$$j!t_{j,0}(1) = P(\nu_i \leq U_{(i)} \text{ for all } i = 1,\ldots,j) = j!f_j^{(j)}(1) ,$$

if $(f_n^{(j)})$ is the $\mu^{(j)}$-Sheffer sequence for D with roots in 0, where $\mu_i^{(j)} = 1-\nu(j-i)$ for all $i = 0,\ldots,j$. Hence, (1.3) can be applied to compute $t_{j,0}(1)$ (we choose again $\sigma \equiv 1$):

$$p_0^{(j)} = 1 \text{ and } p_i^{(j)} = 1 - \sum_{k=0}^{i-1} \binom{i}{k}\nu(j-k)^{i-k}p_k^{(j)} \text{ for all } i = 1,\ldots,j .$$

Thus, $j!t_{j,0}(1) = p_j^{(j)}$ for all $j = 1,\ldots,M$. Finally, enter again (1.3) and compute p_M from $p_0 = 1$ and

$$p_j = p_j^{(j)} - \sum_{k=0}^{j-1} \binom{j}{k}(1-\mu_k)^{j-k}p_k \text{ for all } j = 1,\ldots,M .$$

1.3. Rényi type distributions.

In applications, the test distributions seldom occur in the form of (1.1). But if our method is applicable at all, they are easily transformed so that one of the following two lemmas can be used:

Lemma 1.1: Let f and g be monotone non-decreasing functions from $[0,1]$ into itself such that $f < g$ and

$$f(0) \leq a/M < b/M \leq g(1) = 1$$

for two fixed integers a and b. Then

$$P(f(F_U(x)) \leq x \leq g(F_U(x)) \quad \forall a/M \leq F_U(x) \leq b/M)$$
$$= P(\nu_i \leq U_{(i)} \leq \mu_{i-1} \quad \forall i = 1,\ldots,M)$$

if

$$\nu_i = \begin{cases} 0 & \text{for all } i = 0,\ldots,a-1 \\ f(i/M) & \text{for all } i = a,\ldots,b \\ f(b/M) & \text{for all } i > b, \end{cases}$$

and

$$\mu_i = \begin{cases} g(a/M) & \text{for all } i = 0,\ldots,a-1 \\ g(i/M) & \text{for all } i = a,\ldots,b \\ 1 & \text{for all } i > b\ . \end{cases}$$

The proof is obvious. The situation in the following lemma is much more complicated.

Lemma 1.2: Replace a/M and b/M in lemma 1.1 by any real α and β such that $0 \le \alpha < \beta \le 1$. Then, under the same assumptions about f and g,

$$P(f(F_U(x)) \le x \le g(F_U(x)) \quad \forall \alpha \le x \le \beta)$$

$$= P(\nu_i \le U_{(i)} \le \mu_{i-1} \quad \text{for all } i = 1,\ldots,M)$$

if

$$\nu_i = \begin{cases} 0 & \text{for all } i = 0,\ldots,\alpha_f \\ f(i/M) & \text{for all } i = \alpha_f+1,\ldots,\beta_f \\ \beta & \text{for all } i > \beta_f \end{cases}$$

and

$$\mu_i = \begin{cases} \alpha & \text{for all } i = 0,\ldots,\alpha_g-1 \\ g(i/M) & \text{for all } i = \alpha_g,\ldots,\beta_g-1 \\ 1 & \text{for all } i \ge \beta_g \end{cases}$$

where

$$\alpha_f = \max\{k \le M | f(k/M) \le \alpha\}, \ \beta_f = \max\{k \le M | f(k/M) \le \beta\}$$

$$\alpha_g = \min\{k \ge 0 | g(k/M) \ge \alpha\}, \ \beta_g = \min\{k \ge 0 | g(k/M) \ge \beta\}.$$

Proof. Denote by $[0,1]^{(M)}$ the set of all monotone non-decreasing ordered vectors $u \in [0,1]^M$:

$$u = (u_1,\ldots,u_M) \quad \text{such that} \quad 0 \le u_1 \le \cdots \le u_M \le 1 .$$

Define the subset A of $[0,1]^{(M)}$ by

$A := \{f(i/M) \leq x$ holds for all $i = 0,\ldots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$(u_0 := 0,\ u_{M+1} := 1)$. Then

$A = \{f(i/M) \leq x$ holds for all $i = \alpha_f+1,\ldots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$= \{f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\ldots,M$ such that $u_i \leq \beta\}$

$= \{u_{\beta_f+1} > \beta$, and $f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\ldots,\beta_f$ such that $u_i \leq \beta\}$

$= \{u_{\beta_f+1} > \beta$, and $f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\ldots,\beta_f\}$.

By interchanging the roles of f and g it follows analogously that

$B := \{x \leq g(i/M)$ holds for all $i = 0,\ldots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$= \{u_{\alpha_g} < \alpha$, and $u_i \leq g((i-1)/M)$ for all $i = \alpha_g+1,\ldots,\beta_g\}$.

$P(A \cap B) = P(f(F_U(x)) \leq x \leq g(F_U(x)) \quad \forall \alpha \leq x \leq \beta)$ finishes the proof. ∎

<u>Remark</u>: If $\nu_{i+1} < \mu_i$ for all $i = 0,\ldots,M-1$ in the lemmas above, look for the best applicable method in <u>1.2</u>. The probability is zero otherwise.

2. Two sample tests.

2.1. Sheffer polynomials for ∇.

Denote by $\mathfrak{J}(i,j)$ the set of all vectors T consisting of exactly i ones and j zeros. For each $T = (T_1,\ldots,T_{i+j}) \in \mathfrak{J}(i,j)$ define the path T'_ℓ of T by $T'_0 := 0$ and $T'_\ell := \Sigma_{k=1}^{\ell} T_k$ for all $\ell = 1,\ldots,i+j$. The set $\mathfrak{J}(i,j)$ is closely related to empirical distribution functions: Let $X_1,\ldots,X_M$, $Y_1,\ldots,Y_N$ be $M+N$ continuous and i.i.d. random variables. Denote the monotone non-decreasing ordered combined sample by $V_1,\ldots,V_{M+N}$. Define a.e.

$$(2.1) \qquad T_\ell = \begin{cases} 1 & \text{if } V_\ell = X_i \text{ for some } i,\ 1 \le i \le M \\ 0 & \text{if } V_\ell = Y_j \text{ for some } j,\ 1 \le j \le N\ . \end{cases}$$

Then $T'_\ell = MF_X(V_\ell)$ and $\ell - T'_\ell = NF_Y(V_\ell)$. Let μ and ν be integer valued function on $\mathbb{N}_0$, $-1 = \nu_0 \le \mu_0$ and

$$(2.2) \qquad \nu_{i-1}-1 \le \nu_i < \mu_{i-1} \le \mu_i \text{ for all } i \in \mathbb{N}_1\ .$$

Then $f_i(j) = \#\mathfrak{J}(i,j \mid \nu(T'_\ell) < \ell - T'_\ell \le \mu(T'_\ell)$ for all $\ell = 0,\ldots,i+j)$, if (f_n) is the μ-Sheffer sequence (with variables in $\mathbb{Z}$) for the backwards difference operator ∇ (see (A.6)) with roots in ν. Hence,

$$(2.3) \qquad P(\nu(T'_\ell) < \ell - T'_\ell \le \mu(T'_\ell) \text{ for all } \ell = 0,\ldots,M+N) = \binom{M+N}{M}^{-1} f_M(N)\ .$$

2.2. Recursions.

We assume $\mu(M) = N$ throughout this section. From the definition of a μ-Sheffer sequence (f_n) for ∇ with roots in ν we get the following two-dimensional recursion

$$(2.4) \qquad f_i(j) = \begin{cases} f_i(j-1)+f_{i-1}(j) & \text{for all } \nu_i < j \leq \mu_i \\ 0 & \text{else} , \end{cases}$$

with initial values $f_0(j) = 1$ for all $j \leq \mu_0$, and $f_i(\nu_i) = 0$ for all $i \in \mathbb{N}_1$. On a computer with unlimited integer precision, this algorithm may be slow but absolutely accurate!

The one-dimensional recursion (A.16) is left to the reader. From corollary A.2 one gets the determinantal solution

$$P(\nu(T'_\ell) < \ell - T'_\ell \leq \mu(T'_\ell) \text{ for all } \ell = 0,\ldots,M+N)$$

$$= \binom{M+N}{N}^{-1} = \det\left(\left(\binom{(\mu_{j-1}-\nu_i)_+}{i-j+1}\right)\right)_{i,j=1,\ldots,n} .$$

This determinant has been found independently by G. Kreweras (1965) and G.P. Steck (1969). See also S.G. Mohanty (1971) and E.J.G. Pitman (1972) for other proofs.

A close look on ν and μ may save some recursion steps. If $\nu(M) < \mu(0)$ the outside method allows non-alternating summation as described in 1.2.

2.3. Rényi type distributions.

Lemma 2.1. Define f, g, a and b as in lemma 1.1. Then

$$P(f(F_X(x)) \le F_V(x) \le g(F_X(x)) \text{ for all } a/M \le F_X(x) \le b/M)$$

$$= P(\nu(T'_\ell) < \ell - T'_\ell \le \mu(T'_\ell) \quad \text{for all } \ell = 0,\ldots,M+N),$$

if

$$\nu_i = \begin{cases} -1 & \text{for all } i = 0,\ldots,a-1 \\ \lceil (M+N)f(i/M) \rceil - i - 1 & \text{for all } i = a,\ldots,b \\ \nu_b & \text{for all } i > b\ , \end{cases}$$

and

$$\mu_i = \begin{cases} \mu_a & \text{for all } i = 0,\ldots,a-1 \\ \lfloor (M+N)g(i/M) \rfloor - i & \text{for all } i = a,\ldots,b \\ N & \text{for all } i > b\ . \end{cases}$$

The proof is obvious from 2.1 .

Lemma 2.2. Define f, g, a and b as in lemma 1.1, and α_f, β_f, α_g and β_g as in lemma 1.2 with $\alpha := a/(M+N)$ and $\beta := b/(M+N)$. Then

$$P(f(F_X(x)) \le F_V(x) \le g(F_X(x)) \text{ for all } a/(M+N) \le F_V(x) \le b/(M+N))$$

$$= P(\nu(T'_\ell) < \ell \le \mu(T'_\ell) \quad \text{for all } \ell = 0,\ldots,M+N)\ ,$$

if

$$\nu_i = \begin{cases} -1 & \text{for all } i = 0,\ldots,\alpha_f \\ \lceil (M+N)f(i/M) \rceil - i - 1 & \text{for all } i = \alpha_f+1,\ldots,\beta_f \\ b-\beta_f-1 & \text{for all } i > \beta_f\ , \end{cases}$$

and

$$\mu_i = \begin{cases} a-\alpha_g & \text{for all } i = 0,\ldots,\alpha_g-1 \\ \lfloor (M+N)g(i/M) \rfloor - i & \text{for all } i = \alpha_g,\ldots,\beta_g-1 \\ N & \text{for all } i \geq \beta_g \ . \end{cases}$$

The proof follows the same pattern as the proof of lemma 1.2 and is therefore omitted.

<u>Remark</u>: It may happen that ν or μ in lemma 2.1 or 2.2 violates the monotonicity conditions (2.2). In this case define the "monotone hulls" $\tilde{\nu}$ and $\hat{\mu}$ by

$$(2.5) \qquad \begin{aligned} \tilde{\nu}_0 &:= -1 \\ \tilde{\nu}_i &:= \max\{\nu_i, \tilde{\nu}_{i-1}\} \quad \text{for all } i = 1,\ldots,M \ , \end{aligned}$$

and

$$(2.6) \qquad \begin{aligned} \hat{\mu}_M &:= N \\ \hat{\mu}_i &:= \min\{\mu_i, \hat{\mu}_{i+1}\} \quad \text{for all } i = 0,\ldots,M-1 \ . \end{aligned}$$

If $\tilde{\nu}_{i+1} < \hat{\mu}_i$ for all $i = 0,\ldots,M-1$, look for the best applicable method in <u>2.2</u>. The probability is zero otherwise.

3. The variance-weighted Kolmogorv-Smirnov tests.

We defined W_M in the introduction. Let

$$h^{\pm}(i) = \frac{2i+s\pm[s^2+4si(1-i)]^{1/2}}{2(1+s)}$$

and

$$c^{\pm}(\gamma) = M(\gamma \pm[s\gamma(1-\gamma)]^{1/2} .$$

We get from lemma 1.2

$$P(W_M \leq s^{1/2}) = M!f_M(1) ,$$

if (f_n) is the μ-Sheffer sequence for D with roots in ν, where

(3.1) $$\nu_i = h^-(i/M) \text{ for all } i = \lfloor c^+(\theta_1)\rfloor \wedge M+1,\ldots,\lfloor c^+(\theta_2)\rfloor \wedge M,$$

and

(3.2) $$\mu_i = h^+(i/M) \text{ for all } i = \lceil c^-(\theta_1)\rceil_+,\ldots,\lceil c^-(\theta_2)\rceil_+-1 .$$

The following short tables of the percentage points of $M^{1/2}W_M$ are computed by algoritym A.1 and by the outside method (1.3) if applicable. We chose always $\theta_1 = \theta = 1-\theta_2$ for $\theta = 0$, .01, .05, .1 and .25. Let

$$P(z) = P(M^{1/2}W_M \leq z) .$$

We consider the significance probabilities $\alpha = 1-P(z_\alpha)$ for $\alpha = .1$, $.05$ and $.01$. Because of discontinuities, these levels can not always be attained. If the absolute difference between α and $1-P(z_\alpha)$ is less than $.000005$ this small discontinuity is not noted in the tables, and z_α is rounded to 4 digits after the decimal point. If

$$.000005 \leq |\alpha-1+P(z_\alpha)| < .005 ,$$

and α is greater(smaller) than $1-P(z_\alpha)$, then five digits are given and a bar is placed under (over) the last digit. This last digit is not rounded. Decreasing (increasing) it by one yields a probability greater (smaller) than α. Two bars indicate an absolute difference between $.005$ and $.013$. The asymptotic values of A.A. Borokov and N.M. Sycheva (1968, Theorem 3A) are given in the last row of table 2-5.

Table 1 is a confirmation of M. Noé's (1972) computations. In table 2 and 3 the results of P.L. Canner's (1975) simulation study are given in parentheses. In table 4 and 5 the rows marked by F contain the percentage points of $M^{1/2}W_M$ as the tables before. The rows marked by F_X refer to the correspondent statistic where the supremum is taken over $d/M \leq F_X(x) \leq 1-d/M$. The integer d is chosen such that d/M is closest to the desired θ:

$$d = \begin{cases} \lfloor M\theta \rfloor & \text{if } M\theta-\lfloor M\theta \rfloor < .5 \\ \lceil M\theta \rceil & \text{else .} \end{cases} \tag{3.3}$$

The F_X-row in table 4 equals for $M = 10$ the F-row and is therefore omitted.

M	α=.1	α=.05	α=.01
10	4.6146	6.4257	14.1863
20	4.6423	6.4398	14.1908
50	4.6631	6.4488	14.1929
100	4.6719	6.4519	14.1931

Table 1: θ = 0.

M	α=.1	α=.05	α= 01
10	3.2900	3.9829 (3.33)	6.03859 (5.70)
20	3.3962	4.04519 (3.76)	4.9094 (4.77)
50	3.2029	3.55334 (3.69)	4.5353 (4.40)
100	3.0640	3.4379	4.1899
∞	3.05	3.30	3.79

Table 2: θ = .01

M	α=.1	α=.05	α=.01
10	2.9218	3.4216 (3.33)	4.1705 (4.00)
20	2.9094	3.1831 (3.07)	4.10391 (3.80)
50	2.8616	3.1525 (3.15)	3.8289 (3.80)
100	2.8384	3.1417	3.7419
∞	2.89	3.15	3.67

Table 3: θ = .05

M		α=.1	α=.05	α=.01
10	F	2.7148	3.1336	3.8203
20	F	2.7130	2.9830	3.72677
	F_X	3.3938	4.0798	6.1725
50	F	2.7284	3.0071	3.6284
	F_X	2.9641	3.3533	4.2777
100	F	2.7362	3.0120	3.5960
	F_X	2.8562	3.1803	3.8839
∞		2.78	3.05	3.59

Table 4: θ = .1

M		α=.1	α=.05	α=.01
10	F	2.4383	2.6340	3.2863
	F_X	3.2747	3.9777	6.0714
20	F	2.4694	2.7236	3.2852
	F_X	2.7419	3.1507	4.0971
50	F	2.4890	2.77609	3.3414
50	F_X	2.6223	2.9530	3.6498
100	F	2.5159	2.7929	3.3568
	F_X	2.5657	2.8727	3.4969
∞		2.53	2.83	3.40

Table 5: θ = .25

We denote by $W_{M,N}$ the two sample version of W_M:

$$W_{M,N} := \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{[F_V(x)(1-F_V(x))]^{1/2}}$$

$(\theta_1 = a/(M+N)$ and $\theta_2 = b/(M+N)$; a and b integer) .

Now we get from lemma 2.2

$$P\left(\frac{N}{M+N} W_{M,N} \le s^{1/2}\right) = f_M(N)/\binom{M+N}{M} ,$$

if (f_n) is the $\hat{\mu}$-Sheffer sequence for ∇ with roots in $\tilde{\nu}$, where

$$\nu_i = \lceil (M+N)h^-(i/M) \rceil - i - 1 \quad \text{and} \quad \mu_i = \lfloor (M+N)h^+(i/M) \rfloor - i ,$$

with i in the same range as in (3.1) and (3.2). (See (2.5) and (2.6) for $\tilde{\nu}$ and $\hat{\mu}$.) The following tables of percentage points for $\left(\frac{MN}{M+N}\right)^{1/2} W_{M,N}$ are computed using only algorithm (2.4). Discontinuities occur at almost each entry. The bars are set following the same rules as above, but only four digits are given. The table for $\theta = 0$ equals the table for $\theta = .01$ and is therefore omitted. The numbers in parentheses are taken from P.L. Canner's (1975) simulation study (computed for $\theta = 0$). For $\theta = .05$, the rows $M = N = 10$, 20 and 50 are equal to those in table 6, and are omitted. Instead, we demonstrate the effect of slightly different, but large sample sizes. Again, the rows are marked by F_X, if the supremum is taken over all $a'/M \le F_X(x) \le b'/M$. In table 8 and 9 the rows are omitted which do not differ from table 6. The asymptotic values of table 2-5 may be used for comparison.

M=N	α=.1	α=.05	α=.01
10	2.3441	2.6832	3.1462
		(2.71)	(3.17)
20	2.5819	2.7603	3.2274
		(2.70)	(3.18)
50	2.7007	2.9488	3.4299
		(2.93)	(3.44)
100	2.7914	3.0249	3.5022
		(3.02)	(3.47)
500	2.9441	3.1863	3.6694

Table 6: θ = .01

M	N		α=.1	α=.05	α=.01
100	100	F_V	2.7795	3.0089	3.5022
		F_X	2.7369	2.9820	3.4724
100	99	F_V	2.7747	3.0157	3.5051
		F_X	2.7524	2.1893	3.4781
100	98	F_V	2.7607	3.0146	3.5112
		F_X	2.7470	2.9922	3.4686
100	95	F_V	2.7669	3.0239	3.5057
		F_X	2.7426	2.9939	3.4863
500	500	F_V	2.8423	3.0990	3.6136

Table 7: θ = .05

M=N	α=.1	α=.05	α=.01
50	2.6667	2.8968	3.4299
100	2.7003	2.9711	3.4720
500	2.7415	3.0139	3.5468

Table 8: θ = .1

M=N	α=.1	α=.05	α=.01
20	2.3631	2.6520	3.1870
50	2.4723	2.7357	3.2963
100	2.4829	2.7580	3.3131
500	2.5227	2.8028	3.3623

Table 9: θ = .25

For applications see K.A. Doksum and G.L. Sievers (1976): "Plotting with confidence: Graphical comparisons of two populations."

4. Tables

See the introduction for a description of the tables.

$\theta = 0 \;/\; .01 \;/\; .05$ $\qquad \sqrt{M}\, W_M^+$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					$\theta = 0$							
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					$\theta = .01$							
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					$\theta = .05$							
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

θ = .05 / .1 / .25 $\qquad \mathfrak{M}^{\prime} W_M^{+}$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					θ = .05 (continued)							
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	.9000	.9000	2.451235	[illegible]	[illegible]	[illegible]	2.5[illegible]5089	6	.9900	.9900	[illegible]	[illegible]
15	.9000	.9000	2.551383	5	.9500	.9500	2.6[illegible]5	6	.9900	.9930	[illegible]	[illegible]
20	.9000	.9000	2.493085	8	.9500	.9500	2.91[illegible]940	6	.9900	.9900	[illegible]	[illegible]
25	.8975	.9100	2.5[illegible]	[illegible]	.9500	.9500	[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
30	.9000	.9000	2.50[illegible]	[illegible]	.9500	.9500	2.8[illegible]1334	6	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	[illegible]87393	5	.9500	.9500	[illegible]	[illegible]
40	.9000	.9000	2.505[illegible]	8	.9500	.9500	2.8[illegible]432	6	.9900	[illegible]	[illegible]	[illegible]
50	.9500	.9000	2.[illegible]	[illegible]	.9500	.9500	2.844615	9	.9900	.9900	[illegible]	[illegible]
60	[illegible]	.9000	2.[illegible]	[illegible]	.9500	.9500	[illegible]4177	6	.9897	[illegible]	[illegible]	[illegible]
100	[illegible]	[illegible]	2.[illegible]	[illegible]	.9500	.9500	[illegible]172	7	.9900	[illegible]	[illegible]	[illegible]
					θ = .1							
[illegible]	[illegible]	[illegible]	[illegible]	9	.9500	.9500	2.635198	5	.9900	1	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	2.906673	[illegible]	.9900	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	2.437292	5	[illegible]	.9500	2.666662	5	.9900	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	2.235000	[illegible]	[illegible]	.9500	[illegible]	[illegible]	.9987	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]3595	5	.9900	.9900	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]	2.80[illegible]885	6	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	2.6[illegible]827	9	.9500	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	2.675036	5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	2.301[illegible]	8	[illegible]	.9500	2.715[illegible]96	6	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	[illegible]	5	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	.9500	[illegible]63	9	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	2.710359	9	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	8	[illegible]	[illegible]	[illegible]	9	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9530	[illegible]	6	.9900	[illegible]	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	[illegible]	9	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6	.9500	.9500	2.733831	6	.9900	[illegible]	[illegible]	[illegible]
					θ = .25							
2	.9000	.9000	2.079556	8	.9375	1	2.449487	5	.9375	1	[illegible]	[illegible]
3	[illegible]	.9000	1.850994	5	.9500	.9500	2.567877	9	.9842	1	[illegible]	[illegible]
4	[illegible]	[illegible]	2.079245	8	[illegible]	.9663	2.509400	5	[illegible]	.9900	[illegible]	[illegible]
5	.9000	.9000	1.965651	[illegible]	.9500	.9500	[illegible]	5	.9997	.9918	[illegible]	[illegible]
6	.9000	.9000	2.060188	8	.9387	.9607	2.570202	5	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.008056	7	.9500	.9500	2.564415	5	.9900	.9900	[illegible]	[illegible]
9	.9000	.9000	2.08[illegible]24	8	.9500	.9642	2.449489	5	.9900	.9900	[illegible]	[illegible]
9	.8995	.9173	2.11694[illegible]	9	.9500	.9500	2.5[illegible]254[illegible]	8	.9900	.9900	[illegible]	[illegible]
10	.9000	.9000	2.089627	7	.9500	.9500	2.473308	5	.9900	.9900	[illegible]	[illegible]
15	.9000	.9000	2.131[illegible]	9	.9500	.9500	2.464287	5	.9900	.9900	[illegible]	[illegible]
20	.9000	.9000	2.127442	9	.9500	.9500	2.469442	5	.9900	.9900	[illegible]	[illegible]
25	.9000	.9000	2.162197	[illegible]	.9500	.9500	2.474890	5	.9900	.9900	[illegible]	[illegible]
30	.9000	.9000	2.156960	7	.9500	.9500	2.471435	8	.9900	.9900	[illegible]	[illegible]
35	.9000	.9000	2.158485	7	.9500	.9500	2.470119	8	.9900	.9900	[illegible]	[illegible]
40	.9000	.9000	2.17[illegible]	9	.9500	.9500	2.495123	5	.9900	.9900	[illegible]	[illegible]
45	.9000	.9000	2.171818	7	.9500	.9500	2.493701	8	.9900	.9900	[illegible]	[illegible]
50	.9000	.9000	2.165977	7	.9500	.9500	2.489056	8	.9895	.9501	[illegible]	[illegible]
100	.9000	.9000	2.185550	5	.9500	.9500	2.515905	5	.9900	.9900	[illegible]	[illegible]

$\theta = 0 \,/\, .01 \,/\, .05$ $\qquad \sqrt{M}\, W_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					$\theta = 0$							
2	.9000	.9000	4.473473	9	.9500	.9500	6.333986	6	.9900	.9900	14.149906	7
3	.9000	.9000	4.522930	5	.9500	.9500	6.368907	7	.9900	.9900	14.164893	8
4	.9000	.9000	4.551500	5	.9500	.9500	6.387873	7	.9900	.9900	14.172502	8
5	.9000	.9000	4.570431	5	.9500	.9500	6.399866	7	.9900	.9900	14.177099	8
6	.9000	.9000	4.584036	5	.9500	.9500	6.408166	7	.9900	.9900	14.180173	8
7	.9000	.9000	4.594359	5	.9500	.9500	6.414264	7	.9900	.9900	14.182372	8
8	.9000	.9000	4.602491	5	.9500	.9500	6.418939	7	.9900	.9900	14.184019	8
9	.9000	.9000	4.609091	5	.9500	.9500	6.422640	7	.9900	.9900	14.185293	8
10	.9000	.9000	4.614573	5	.9500	.9500	6.425653	7	.9900	.9900	14.186308	8
15	.9000	.9000	4.632346	5	.9500	.9500	6.434438	7	.9900	.9900	14.189322	7
20	.9000	.9000	4.642249	5	.9500	.9500	6.439753	7	.9900	.9900	14.190768	8
25	.9000	.9000	4.648656	5	.9500	.9500	6.442707	7	.9900	.9900	14.191596	8
30	.9000	.9000	4.653187	5	.9500	.9500	6.444705	7	.9900	.9900	14.192107	8
35	.9000	.9000	4.656583	5	.9500	.9500	6.446151	7	.9900	.9900	14.192440	8
40	.9000	.9000	4.659236	5	.9500	.9500	6.447242	7	.9900	.9900	14.192668	8
45	.9000	.9000	4.661374	5	.9500	.9500	6.448091	7	.9900	.9900	14.192830	8
50	.9000	.9000	4.663134	5	.9500	.9500	6.448777	7	.9900	.9900	14.192936	8
100	.9000	.9000	4.674932	5	.9500	.9500	6.451909	7	.9900	.9900	14.193053	7
					$\theta = .01$							
2	.9000	.9000	4.473474	8	.9500	.9500	6.333983	5	.9586	.9969	6.964553	7
3	.9000	.9000	4.522930	5	.9358	.9904	5.628505	6	.9358	.9904	5.628507	7
4	.9000	.9000	4.551499	5	.9113	.9805	4.824180	5	.9900	.9900	5.753819	6
5	.8848	.9670	4.269927	8	.8848	.9670	4.269926	9	.9900	.9900	5.862998	6
6	.8564	.9496	3.856864	8	.9500	.9500	3.864508	8	.9900	.9900	5.933793	6
7	.8257	.9282	3.532774	7	.9500	.9500	3.905999	8	.9900	.9900	5.983517	6
8	.7926	.9026	3.289076	6	.9500	.9500	3.937582	8	.9900	.9900	6.020401	6
9	.9000	.9000	3.269144	7	.9500	.9500	3.962539	8	.9900	.9900	6.048878	6
10	.9000	.9000	3.289976	7	.9500	.9500	3.982821	9	.9898	.9935	6.038591	5
15	.9000	.9000	3.357700	6	.9500	.9500	4.046216	9	.9900	.9900	4.853348	5
20	.9000	.9000	3.376235	7	.9483	.9728	4.045195	6	.9900	.9900	4.908577	5
25	.9000	.9000	3.421915	7	.9500	.9500	3.610339	7	.9900	.9900	4.941915	5
30	.9000	.9000	3.139936	6	.9500	.9500	3.634806	8	.9899	.9948	4.954336	5
35	.9000	.9000	3.160673	7	.9500	.9500	3.653253	8	.9829	.9901	4.501876	9
40	.9000	.9000	3.177151	7	.9500	.9500	3.667791	8	.9900	.9900	4.511206	5
45	.9000	.9000	3.191185	7	.9500	.9500	3.679625	8	.9900	.9900	4.524516	5
50	.9000	.9000	3.202909	7	.9396	.9555	3.553339	6	.9900	.9900	4.535309	5
100	.9000	.9000	3.063968	6	.9500	.9500	3.437878	8	.9900	.9900	4.189906	7
					$\theta = .05$							
2	.7708	.9278	2.919985	6	.9500	.9500	3.263289	8	.9900	.9900	5.126814	6
3	.9000	.9000	2.908670	5	.9500	.9500	3.576910	7	.9839	.9973	4.900764	9
4	.9000	.9000	3.038705	6	.9500	.9500	3.721531	7	.9900	.9900	4.196344	9
5	.9000	.9000	3.117920	6	.9385	.9731	3.590922	6	.9900	.9900	4.391910	9
6	.9000	.9000	3.172380	7	.9500	.9500	3.230511	6	.9900	.9900	4.513449	9
7	.8527	.9077	2.861461	6	.9500	.9500	3.298479	7	.9900	.9960	4.595679	5
8	.9000	.9000	2.853163	9	.9500	.9500	3.349445	7	.9834	.9922	4.217750	8
9	.9000	.9000	2.890911	6	.9500	.9500	3.389348	7	.9900	.9900	4.119115	7
10	.9000	.9000	2.921795	5	.9500	.9500	3.421598	7	.9900	.9900	4.170525	9

θ = .05 / .1 / .25 $\qquad \sqrt{M}\, W_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					θ = .05 (continued)							
15	.9000	.9000	2.840416	6	.9500	.9500	3.265550	6	.9900	.9900	4.011451	8
20	.9000	.9000	2.909351	6	.9500	.9500	3.183144	5	.9900	.9931	4.103908	8
25	.9000	.9000	2.850340	5	.9500	.9500	3.236401	7	.9900	.9900	3.973842	7
30	.9000	.9000	2.888122	6	.9500	.9500	3.175929	5	.9900	.9900	3.886759	6
35	.9000	.9000	2.846932	5	.9500	.9500	3.208063	7	.9900	.9900	3.923989	8
40	.9000	.9000	2.872587	6	.9500	.9500	3.163645	5	.9900	.9900	3.856637	6
45	.9000	.9000	2.842328	9	.9500	.9500	3.186349	7	.9900	.9900	3.881739	8
50	.9000	.9000	2.861627	6	.9500	.9500	3.152505	5	.9900	.9900	3.828912	6
100	.9000	.9000	2.838427	5	.9500	.9500	3.141739	6	.9900	.9900	3.741942	8
					θ = .1							
2	.9000	.9000	2.635194	5	.9500	.9500	3.263297	6	.9800	.9999	4.242639	9
3	.9000	.9000	2.908674	6	.9322	.9790	3.271650	7	.9900	.9900	3.813667	8
4	.8482	.9201	2.666663	5	.9500	.9500	2.982600	6	.9900	.9900	4.196351	9
5	.9000	.9000	2.655507	9	.9500	.9500	3.134008	6	.9775	.9910	3.726774	8
6	.9000	.9000	2.743020	5	.9500	.9500	3.230508	6	.9900	.9900	3.845438	8
7	.9000	.9000	2.805585	6	.9500	.9500	2.965655	6	.9900	.9900	3.966865	8
8	.9000	.9000	2.624652	5	.9500	.9500	3.036217	6	.9846	.9917	3.771230	7
9	.9000	.9000	2.674590	5	.9500	.9500	3.090404	6	.9900	.9900	3.753606	6
10	.9000	.9000	2.714813	6	.9500	.9500	3.133631	6	.9900	.9900	3.820299	8
15	.9000	.9000	2.716642	5	.9415	.9537	3.012316	5	.9900	.9900	3.795932	6
20	.9000	.9000	2.712983	9	.9500	.9500	2.982984	5	.9893	.9918	3.726779	6
25	.9000	.9000	2.709516	9	.9444	.9516	2.999995	6	.9886	.9907	3.666663	6
30	.9000	.9000	2.706985	9	.9500	.9500	3.037533	6	.9889	.9906	3.651482	6
35	.9000	.9000	2.705311	9	.9500	.9500	3.026680	5	.9897	.9911	3.662328	8
40	.9000	.9000	2.704317	9	.9500	.9500	3.018418	5	.9900	.9900	3.656798	6
45	.9000	.9000	2.732219	6	.9500	.9500	3.012038	5	.9900	.9900	3.641355	6
50	.9000	.9000	2.728351	9	.9500	.9500	3.007049	5	.9900	.9900	3.628393	6
100	.9000	.9000	2.736163	5	.9500	.9500	3.012031	5	.9900	.9900	3.595980	6
					θ = .25							
2	.8750	.9999	2.449489	5	.8750	.9999	2.449486	6	.8750	.9999	2.449488	5
3	.9000	.9000	2.267873	9	.9500	.9500	2.694409	5	.9687	.9999	2.999998	6
4	.8611	.9325	2.309397	5	.9500	.9500	2.461594	5	.9900	.9900	3.323007	7
5	.9000	.9000	2.342701	5	.9500	.9500	2.722945	5	.9900	.9900	3.070340	6
6	.8773	.9213	2.357022	5	.9500	.9500	2.576874	9	.9884	.9960	3.299829	7
7	.9000	.9000	2.364412	5	.9500	.9500	2.715605	5	.9900	.9900	3.197025	5
8	.8999	.9285	2.449485	5	.9500	.9500	2.615492	9	.9877	.9937	3.265986	7
9	.9000	.9000	2.375411	8	.9500	.9500	2.707557	5	.9900	.9900	3.233146	5
10	.9000	.9000	2.438304	5	.9500	.9500	2.633963	9	.9886	.9931	3.286328	7
15	.9000	.9000	2.464260	5	.9500	.9500	2.696417	5	.9900	.9900	3.249258	6
20	.9000	.9000	2.469403	5	.9500	.9500	2.723559	6	.9900	.9900	3.285203	7
25	.9000	.9000	2.474835	5	.9500	.9500	2.732763	6	.9900	.9900	3.301083	7
30	.9000	.9000	2.471383	8	.9500	.9500	2.742865	6	.9900	.9900	3.309647	7
35	.9000	.9000	2.470067	8	.9500	.9500	2.772704	6	.9900	.9900	3.313743	7
40	.9000	.9000	2.495068	5	.9500	.9500	2.769853	9	.9900	.9900	3.317094	7
45	.9000	.9000	2.493637	8	.9500	.9500	2.768674	9	.9900	.9900	3.343975	7
50	.9000	.9000	2.489001	8	.9484	.9509	2.776085	6	.9900	.9900	3.341443	5
100	.9000	.9000	2.515856	5	.9500	.9500	2.792941	5	.9900	.9900	3.356768	6

$\theta = 0 \;/\; .01 \;/\; .05$ $\qquad \sqrt{M}\,\tilde{W}_M^+$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D

For $\theta = 0$ see $\sqrt{M}\,W_M^+$

$\theta = .01$

2	.9000	.9000	3.168531	8	.9500	.9500	4.497593	6	.9900	.9900	10.021762	9
3	.9000	.9000	3.246598	7	.9500	.9500	4.556056	5	.9900	.9900	10.047498	5
4	.9000	.9000	3.294047	7	.9500	.9500	4.589382	5	.9900	.9900	10.060775	5
5	.9000	.9000	3.326808	7	.9500	.9500	4.611263	5	.9900	.9900	10.068589	5
6	.9000	.9000	3.351192	7	.9500	.9500	4.626872	5	.9900	.9900	10.074367	5
7	.9000	.9000	3.370260	7	.9500	.9500	4.638649	5	.9900	.9900	10.078309	5
8	.9000	.9000	3.385706	7	.9500	.9500	4.647888	5	.9900	.9900	10.081[illegible]0	5
9	.9000	.9000	3.398569	7	.9500	.9500	4.655355	5	.9900	.9900	10.083014	5
10	.9000	.9000	3.409468	7	.9500	.9500	4.661[illegible]	5	.9900	.9900	10.085464	5
11	.9000	.9000	3.448859	7	.9500	.9500	4.6914[illegible]	5	.9900	.9900	10.[illegible]	9
[illegible]	[illegible]	[illegible]	3.4[illegible]	[illegible]	.9500	.9500	4.69[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.485[illegible]	7	.9500	.9500	4.6[illegible]	5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.49969[illegible]	[illegible]	.9500	.9500	4.7[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
4[illegible]	[illegible]	[illegible]	3.[illegible]048	[illegible]	.9500	.9500	4.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	8	.9500	.9500	4.[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	4.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]

$\theta = .05$

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.4[illegible]599	8	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7	.9500	.9500	[illegible]056054	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.29[illegible]	[illegible]	.9500	.9500	4.5[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3.3[illegible]	[illegible]	.9500	.9500	4.61[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	.9000	3511[illegible]	[illegible]	.9500	.9500	4.5[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3.70[illegible]	[illegible]	.9500	.9500	4.6[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.6[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	3.[illegible]	[illegible]	.9500	.9500	4.65[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	4.6[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	3.44[illegible]	[illegible]	.9500	.9500	4.6[illegible]	8	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.69[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.4[illegible]11	6	.9900	.9900	[illegible]	5
[illegible]	[illegible]	.9000	[illegible]	[illegible]	.9500	.9500	3.4[illegible]04	7	.9900	.9900	5.21[illegible]	5
[illegible]	[illegible]	.9000	[illegible]	[illegible]	.9500	.9500	3.46[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.49[illegible]	7	.9900	.9900	5.2[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	5	.9500	.9500	3.[illegible]15	6	.9900	.9900	5.40[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.025399	7	.9900	.9900	5.244087	[illegible]

$\theta = .1 / .25$ $\qquad \sqrt{M}\,\tilde{W}_M^+$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					$\theta = .1$							
2	.9000	.9000	3.168534	6	.9500	.9500	4.497596	8	.9900	.9900	10.021760	8
3	.9000	.9000	3.246999	7	.9500	.9500	4.556057	5	.9900	.9900	10.047475	5
4	.9000	.9000	3.294048	7	.9500	.9500	4.589383	5	.9900	.9900	10.061775	5
5	.9000	.9000	3.308034	7	.9500	.9500	4.606667	5	.9900	.9900	10.068841	5
6	.9000	.9000	3.340215	7	.9500	.9500	4.624908	5	.9900	.9900	10.074455	5
7	.9000	.9000	3.363255	7	.9500	.9500	4.637705	5	.9900	.9900	10.078357	5
8	.9000	.9000	3.380929	7	.9500	.9500	4.647391	5	.9900	.9900	10.081785	5
9	.9000	.9000	3.395137	7	.9500	.9500	4.655070	5	.9900	.9900	10.083616	5
10	.9000	.9000	3.406929	7	.9500	.9500	4.661357	5	.9900	.9900	10.085487	5
15	.9000	.9000	2.755779	6	.9500	.9500	3.366268	7	.9900	.9900	[illegible]	5
20	.9000	.9000	2.808701	6	.9500	.9500	3.408647	7	.9900	.9900	[illegible]	5
25	.9000	.9000	2.631816	5	.9500	.9500	3.113591	6	.9900	.9900	[illegible]	5
30	.9000	.9000	2.666856	5	.9500	.9500	3.143281	7	.9900	.9900	[illegible]	5
35	.9000	.9000	2.578749	9	.9500	.9500	3.008210	5	.9900	.9900	[illegible]	5
40	.9000	.9000	2.604833	5	.9500	.9500	3.039824	6	.9900	.9900	[illegible]	5
45	.9000	.9000	2.542712	9	.9500	.9500	2.956783	5	.9900	.9900	3.856056	5
50	.9000	.9000	2.570438	5	.9500	.9500	2.969184	6	.9900	.9900	[illegible]	5
100	.9000	.9000	2.508909	6	.9500	.9500	2.858875	7	.9900	.9900	[illegible]	5

					$\theta = .25$							
2	.9000	.9000	3.168537	6	.9500	.9500	4.497594	8	.9900	.9900	10.021762	8
3	.9000	.9000	3.160504	5	.9500	.9500	4.512855	5	.9900	.9900	10.041775	5
4	.9000	.9000	3.257602	7	.9500	.9500	4.576840	5	.9900	.9900	10.060309	5
5	.9000	.9000	3.308036	7	.9500	.9500	4.606666	5	.9900	.9900	10.068843	5
6	.9000	.9000	3.340214	7	.9500	.9500	4.624908	5	.9900	.9900	10.074561	5
7	.9000	.9000	2.529286	7	.9500	.9500	3.176183	6	.9900	.9900	4.966576	5
8	.9000	.9000	2.584677	5	.9500	.9500	3.233949	7	.9900	.9900	[illegible]	5
9	.9000	.9000	2.625684	5	.9500	.9500	3.258906	7	.9900	.9900	[illegible]	5
10	.9000	.9000	2.658176	5	.9500	.9500	3.265989	7	.9900	.9900	5.077296	5
15	.9000	.9000	2.328809	9	.9500	.9500	2.786214	5	.9900	.9900	3.837155	5
20	.9000	.9000	2.316667	8	.9500	.9500	2.744[illegible]	9	.9900	.9900	[illegible]	5
25	.9000	.9000	2.311667	7	.9500	.9500	2.717536	9	.9900	.9900	[illegible]	5
30	.9000	.9000	2.306731	7	.9500	.9500	2.699178	9	.9900	.9900	[illegible]	6
35	.9000	.9000	2.246283	8	.9500	.9500	2.626054	9	.9900	.9900	3.408[illegible]	6
40	.9000	.9000	2.251801	5	.9500	.9500	2.625135	9	.9900	.9900	3.368398	5
45	.9000	.9000	2.255946	5	.9500	.9500	2.624134	9	.9900	.9900	3.371874	5
50	.9000	.9000	2.259191	5	.9500	.9500	2.623170	9	.9900	.9900	3.358227	5
100	.8999	.9000	2.220834	5	.9500	.9500	2.566108	6	.9900	.9900	3.239695	7

$\theta = 0 \;/\; .01 \;/\; .05$ $\qquad\qquad \sqrt{M}\,\tilde{W}_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D

For $\theta = 0$ see $\sqrt{M}\,W_M$

$\theta = .01$

[illegible]	.9000	.9000	4.473473	9	.9500	.9500	6.335986	6	.9900	.9900	14.149953	7
[illegible]	.9000	.9000	4.522930	5	.9500	.9500	6.368915	7	.9900	.9900	14.164957	8
[illegible]	.9000	[illegible]	4.551500	5	.9500	.9500	6.387881	[illegible]	.9900	.9900	14.172[illegible]	[illegible]
[illegible]	.9000	.9000	4.570436	5	.9500	.9500	6.399873	7	.9900	.9900	14.17[illegible]03	[illegible]
[illegible]	.9000	.9000	4.584041	5	.9500	.9500	6.408173	7	.9900	.9900	14.18[illegible]24	[illegible]
[illegible]	.9000	.9000	4.594459	5	.9500	.9500	6.414272	7	.9900	.9900	14.182502	8
[illegible]	.9000	.9000	4.602496	5	.9500	.9500	6.418953	[illegible]	.9900	.9900	14.184180	8
[illegible]	.9000	.9000	4.609696	5	.9500	.9500	6.422654	7	.9900	.9900	14.1[illegible]471	8
[illegible]	.9000	.9000	4.614579	5	.9500	.9500	6.425667	7	.9900	.9900	14.186[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	6.4249[illegible]	[illegible]	.9900	.9900	14.1[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	5	.9500	.9500	6.43978[illegible]	[illegible]	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	6.442737	[illegible]	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	4.[illegible]	5	.9500	.9500	6.444135	[illegible]	.9900	.9900	[illegible]	8
[illegible]	.9000	[illegible]	[illegible]594	5	.9500	.9500	6.446180	7	.9900	.9900	[illegible]	8
[illegible]	.9000	[illegible]	4.65[illegible]4	5	.9500	.9500	6.447[illegible]79	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	4.[illegible]	[illegible]	.9500	.9500	6.448135	7	[illegible]	.9900	[illegible]	8
[illegible]	.9000	[illegible]	[illegible]14	5	.9500	.9500	6.448821	7	.9900	.9900	14.[illegible]	8
[illegible]	.9000	.9000	[illegible]056	6	.9500	.9500	6.452548	9	.9900	.9900	14.19[illegible]	[illegible]

$\theta = .05$

[illegible]	.9000	.9000	[illegible]	8	.9500	.9500	6.335987	5	.9900	.9900	14.1[illegible]	6
[illegible]	.9000	[illegible]	4.5329[illegible]	5	.9500	.9500	6.368914	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	.9500	6.387880	7	.9900	.9900	14.[illegible]	8
[illegible]	[illegible]	.9000	[illegible]570434	5	.9500	.9500	6.399873	7	.9900	.9900	14.17[illegible]	8
[illegible]	.9000	[illegible]	[illegible]582039	5	.9500	.9500	6.408173	7	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]59[illegible]	5	.9500	.9500	6.41[illegible]271	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]0,[illegible]24	5	.9500	.9500	6.41[illegible]953	7	.9900	.9900	1[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]0[illegible]5	[illegible]	.9500	.9500	6.4[illegible]654	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	5	.9500	.9500	6.425660	7	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.434761	7	[illegible]	.9900	14.[illegible]15	7
[illegible]	.9000	[illegible]	6.[illegible]	5	.9500	.9500	6.439782	7	.9900	.9900	14.191149	8
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.442737	7	.9900	.9900	14.19[illegible]	8
[illegible]	.9000	.9000	[illegible]39[illegible]	5	.9500	.9500	4.116722	9	.9900	.9900	[illegible]	6
[illegible]	.9000	[illegible]	[illegible]3.4[illegible]	7	.9500	.9500	4.12[illegible]014	9	.9900	.9900	[illegible]	7
[illegible]	.9000	.9000	[illegible]96[illegible]	7	.9500	.9500	4.136816	9	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	3.4[illegible]	[illegible]	.9500	.9500	4.143886	9	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	3.[illegible]	6	.9500	.9500	3.689012	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	3.[illegible]	4	.9500	.9500	3.13[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]

$\theta = .1 / .25$ $\qquad \sqrt{M}\,\tilde{W}_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					$\theta = .1$							
2	.9000	.9000	4.473473	8	.9500	.9500	6.333989	5	.9900	.9900	14.149956	6
3	.9000	.9000	4.522929	5	.9500	.9500	6.368911	7	.9900	.9900	14.16496[illegible]	7
4	.9000	.9000	4.551504	5	.9500	.9500	6.387877	7	.9900	.9900	14.172566	8
5	.9000	.9000	4.565463	5	.9500	.9500	6.399059	7	.9900	.9900	14.177200	8
6	.9000	.9000	4.581875	5	.9500	.9500	6.407950	7	.9900	.9900	14.180295	[illegible]
7	.9000	.9000	4.593308	5	.9500	.9500	6.414203	7	.9900	.9900	14.182[illegible]	8
8	.9000	.9000	4.601931	5	.9500	.9500	6.418928	7	.9900	.9900	14.184[illegible]	[illegible]
9	.9000	.9000	4.608778	5	.9500	.9500	6.422644	7	.9900	.9900	14.185[illegible]	[illegible]
10	.9000	.9000	4.614381	5	.9500	.9500	6.425664	7	.9900	.9900	14.186510	[illegible]
15	.9000	.9000	3.352703	6	.9500	.9500	4.045169	8	.9900	.9900	[illegible]	6
20	.9000	.9000	3.393782	7	.9500	.9500	4.079796	9	.9900	.9900	[illegible]	[illegible]
25	.9000	.9000	3.105391	6	.9500	.9500	3.608057	7	.9900	.9900	[illegible]	[illegible]
30	.9000	.9000	3.134453	7	.9500	.9500	3.633362	6	.9900	.9900	[illegible]	5
35	.9000	.9000	3.002223	5	.9500	.9500	3.431598	6	.9900	.9900	4.454[illegible]	[illegible]
40	.9000	.9000	3.024452	6	.9500	.9500	3.451278	7	.9900	.9900	4.5[illegible]	[illegible]
45	.9000	.9000	2.946202	5	.9500	.9500	3.337306	6	.9900	.9900	4.2[illegible]	8
50	.9000	.9000	2.964130	6	.9500	.9500	3.353256	7	.9900	.9900	4.[illegible]	[illegible]
100	.9000	.9000	2.85[illegible]	7	.9500	.9500	3.18[illegible]	8	.9900	[illegible]	3.88[illegible]	5
					$\theta = .25$							
2	.9000	.9000	4.473475	8	.9500	.9500	6.333989	5	.9900	.9900	14.149953	6
3	.9000	.9000	4.477701	5	.9500	.9500	6.349215	7	.9900	.9900	14.162796	8
4	.9000	.9000	4.538141	5	.9500	.9500	6.384351	7	.9900	.9900	14.172496	8
5	.9000	.9000	4.565459	5	.9500	.9500	6.399056	7	.9900	.9900	14.177196	8
6	.9000	.9000	4.581877	5	.9500	.9500	6.407948	7	.9900	.9900	14.180295	8
7	.9000	.9000	3.162184	6	.9500	.9500	3.883435	9	.9900	[illegible]	[illegible]80360	8
8	.9000	.9000	3.214065	7	.9500	.9500	3.924698	8	.9900	.9900	6.019261	6
9	.9000	.9000	3.245[illegible]	7	.9500	.9500	3.954623	8	.9900	.9900	[illegible]045438	6
10	.9000	.9000	3.274686	7	.9500	.9500	3.977673	8	.9900	.9900	6.071347	6
15	.9000	.9000	2.783164	5	.9500	.9500	3.232701	5	.9900	.9900	4.314097	9
20	.9000	.9000	2.741885	9	.9500	.9500	3.150680	5	.9900	.9900	4.097055	[illegible]
25	.9000	.9000	2.715561	9	.9500	.9500	3.099434	5	.9900	.9900	3.965390	7
30	.9000	.9000	2.697479	9	.9500	.9500	3.064553	5	.9900	.9900	3.877288	6
35	.9000	.9000	2.625000	9	.9500	.9500	2.974293	5	.9900	.9900	3.[illegible]26047	6
40	.9000	.9000	2.624162	9	.9500	.9500	2.965851	5	.9900	.9900	3.695134	6
45	.9000	.9000	2.623225	9	.9500	.9500	2.958838	5	.9900	.9900	3.670236	6
50	.9000	.9000	2.622306	9	.9500	.9500	2.952950	5	.9900	.9900	3.649772	6
100	.9000	.9000	2.565736	6	.9500	.9500	2.872725	6	.9900	.9900	3.496974	8

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.8333	1	1.999992	7	.8333	1	1.999992	8	.8333	1	1.999992	8
3	3	.7000	.9500	1.732050	6	.7000	.9500	1.732050	7	.9500	1	2.449488	5
3	2	.7000	.9000	1.490711	9	.9000	1	2.236060	8	.9000	1	2.236060	9
4	4	.8857	.9857	2.190888	9	.8857	.9857	2.190888	9	.9857	1	2.828421	7
4	3	.8857	.9714	1.984307	8	.8857	.9714	1.984307	8	.9714	1	2.645751	6
4	2	.8000	.9333	1.732050	9	.9333	1	2.449489	5	.9333	1	2.449489	5
5	5	.8571	.9603	2.070193	9	.8571	.9603	2.070193	8	.9603	.9960	2.581982	7
5	4	.8571	.9286	1.897360	7	.9286	.9603	2.371704	6	.9603	.9921	2.399999	5
5	3	.8571	.9286	2.108180	6	.9286	.9821	2.190888	8	.9821	1	2.828420	7
5	2	.8571	.9524	1.932181	5	.8571	.9524	1.932181	8	.9524	1	2.645748	6
6	6	.8301	.9286	1.999997	9	.9459	.9870	2.449489	7	.9870	.9989	2.927692	8
6	5	.8810	.9069	2.100524	8	.9459	.9762	2.288686	8	.9870	.9978	2.763848	8
6	4	.8238	.9095	1.936486	8	.9095	.9524	2.108183	7	.9762	.9952	2.581988	7
6	3	.8929	.9524	2.267783	6	.8929	.9524	2.267783	9	.9881	1	2.999999	7
6	2	.8929	.9643	2.108183	5	.8929	.9643	2.108183	9	.9643	1	2.828425	6
7	7	.8939	1	2.160237	8	.9312	.9735	2.366425	8	.9808	.9959	2.788864	8
7	6	.8858	.9266	2.133073	7	.9266	.9545	2.225391	8	.9808	.9924	2.638986	8
7	5	.8813	.9167	2.070194	7	.9356	.9672	2.366429	7	.9848	.9924	2.898274	8
7	4	.8636	.9394	2.068275	8	.9394	.9667	2.288686	8	.9848	.9970	2.746421	8
7	3	.8750	.9167	2.070190	8	.9167	.9667	2.415228	7	.9667	.9917	2.535456	9
7	2	.7500	.9167	1.984307	8	.9167	.9722	2.267783	7	.9722	1	2.999998	7
8	8	.8738	.9148	2.065584	8	.9148	.9565	2.309395	7	.9740	.9907	2.696797	8
8	7	.8631	.9037	2.070191	7	.9417	.9557	2.351447	7	.9883	.9935	2.927697	8
8	6	.8988	.9187	2.160244	8	.9187	.9537	2.256296	8	.9887	.9953	2.806241	7
8	5	.8361	.9176	1.944887	8	.9394	.9534	2.433745	6	.9899	.9953	3.040463	8
8	4	.8909	.9576	2.190883	9	.8909	.9576	2.190883	9	.9899	.9980	2.898269	7
8	3	.8485	.9030	1.854045	5	.9333	.9758	2.553136	5	.9758	.9939	2.686770	5
8	2	.7778	.9333	2.108179	8	.9333	.9778	2.415226	7	.9778	1	3.162275	7
9	9	.8968	1	2.171311	8	.9453	.9659	2.417466	7	.9832	.9906	2.846048	8
9	8	.8786	.9021	2.156283	7	.9436	.9607	2.425815	8	.9899	.9944	2.870958	8
9	7	.8816	.9003	2.095232	7	.9366	.9510	2.378347	7	.9895	.9926	2.984119	8
9	6	.8911	.9253	2.236060	8	.9397	.9694	2.371705	9	.9846	.9930	2.860385	8
9	5	.8871	.9136	2.078696	8	.9406	.9545	2.415229	7	.9860	.9930	2.788859	8
9	4	.8909	.9105	2.303541	7	.9105	.9692	2.306110	9	.9818	.9930	2.962261	7
9	3	.8818	.9227	1.999997	5	.9455	.9818	2.683276	6	.9818	.9955	2.828424	5
9	2	.8000	.9455	2.224853	7	.9455	.9818	2.553131	8	.9818	1	3.316624	8
10	10	.8774	1	2.108176	7	.9345	.9563	2.344031	8	.9870	.9938	2.927699	8
10	9	.8910	.9158	2.135413	8	.9472	.9580	2.471258	7	.9895	.9918	2.987748	8
10	8	.8805	.9170	2.171317	8	.9369	.9512	2.353388	8	.9894	.9940	2.941739	8
10	7	.8940	.9113	2.236725	7	.9245	.9566	2.281245	9	.9892	.9930	2.886172	7
10	6	.8930	.9188	2.088927	8	.9432	.9537	2.472273	7	.9890	.9955	2.981419	8
10	5	.8565	.9171	2.148343	8	.9171	.9557	2.236067	8	.9730	.9903	2.738606	7
10	4	.8951	.9091	2.366426	6	.9091	.9530	2.415229	9	.9860	.9950	3.089571	8
10	3	.8566	.9056	2.043145	5	.9371	.9545	2.497998	7	.9860	.9965	2.962258	9
10	2	.8182	.9545	2.335492	9	.8182	.9545	2.335492	5	.9848	1	3.464099	6
15	15	.8921	.9017	2.195774	6	.9436	.9549	2.477161	8	.9891	.9926	3.021654	8
15	14	.8888	.9070	2.229665	8	.9496	.9536	2.535291	8	.9882	.9903	2.936018	9
15	13	.8945	.9008	2.289230	7	.9496	.9553	2.572576	8	.9889	.9900	3.012077	9
15	12	.8984	.9138	2.215645	8	.9477	.9554	2.484233	8	.9882	.9900	2.928262	9
15	11	.8977	.9042	2.249427	7	.9485	.9606	2.538947	8	.9888	.9907	2.935798	9
15	10	.8878	.9154	2.261335	7	.9468	.9569	2.499995	8	.9899	.9943	3.061858	8
20	20	.8983	.9048	2.363054	7	.9481	.9524	2.581982	9	.9885	.9904	3.038214	9
20	19	.8970	.9027	2.298753	7	.9497	.9529	2.573541	8	.9894	.9901	3.064835	9
20	18	.8959	.9081	2.376356	8	.9489	.9532	2.558552	8	.9893	.9906	3.019933	9
20	17	.8934	.9090	2.297013	7	.9485	.9522	2.529584	8	.9897	.9903	3.095419	8
20	16	.8972	.9077	2.323286	8	.9499	.9517	2.599197	8	.9898	.9906	3.060678	9
20	15	.8989	.9039	2.261574	8	.9497	.9525	2.538369	8	.9899	.9916	3.107273	8

$\theta = 0 / .01 / .05 / .1 \sqrt{MN/(M+N)}\, W^{+}_{M,N}$ 25 - 500; 25 - 30

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = 0$ (continued)							
25	25	.8931	.9181	2.357018	8	.9467	.9564	2.611160	9	.9899	.9902	3.111263	9
25	24	.9000	.9027	2.338098	7	.9481	.9505	2.576814	9	.9896	.9902	3.072826	9
25	23	.8980	.9004	2.301713	7	.9487	.9501	2.588380	8	.9899	.9904	3.108690	9
25	22	.8985	.9020	2.271233	8	.9493	.9518	2.558434	8	.9899	.9905	3.095295	9
25	21	.8931	.9002	2.311036	8	.9499	.9513	2.600714	8	.9894	.9903	3.065762	9
25	20	.8913	.9052	2.342606	7	.9465	.9507	2.575179	9	.9894	.9900	3.085770	9
30	30	.8741	.9088	2.335491	8	.9497	.9517	2.587737	9	.9900	.9908	3.151443	9
30	29	.8989	.9014	2.375915	8	.9493	.9538	2.628604	9	.9896	.9900	3.130741	9
30	28	.8989	.9032	2.391197	7	.9486	.9509	2.652709	9	.9900	.9918	3.153222	9
30	27	.8991	.9030	2.373914	8	.9488	.9504	2.610013	9	.9899	.9903	3.123151	9
30	26	.8989	.9005	2.353645	7	.9497	.9509	2.628962	9	.9897	.9903	3.150536	9
30	25	.9000	.9035	2.324660	8	.9487	.9526	2.585344	8	.9896	.9904	3.127595	9
35	35	8979	.9023	2.390457	8	.9488	.9508	2.645749	9	.9899	.9905	3.174896	9
35	34	.8954	.9053	2.355665	8	.9496	.9505	2.617124	9	.9899	.9902	3.165484	9
35	33	.8914	.9004	2.392462	8	.9489	.9506	2.658255	9	.9898	.9904	3.154320	9
35	32	.8991	.9032	2.397930	7	.9497	.9510	2.662829	9	.9897	.9905	3.152208	9
35	31	.8999	.9064	2.417785	7	.9494	.9508	2.669152	9	.9900	.9903	3.181593	9
35	30	.8984	.9054	2.380306	8	.9494	.9508	2.609010	9	.9899	.9902	3.142135	9
40	40	.8977	.9001	2.387041	8	.9489	.9500	2.696795	9	.9896	.9906	3.184468	9
40	39	.8995	.9006	2.406446	8	.9495	.9514	2.664323	9	.9899	.9903	3.182487	9
40	38	.8996	.9030	2.399845	8	.9498	.9507	2.671394	9	.9899	.9901	3.187225	9
40	37	.8950	.9038	2.404320	8	.9498	.9510	2.656782	9	.9898	.9900	3.184932	9
40	36	.8980	.9043	2.421333	8	.9495	.9502	2.657180	9	.9896	.9903	3.183287	9
40	35	.8988	.9010	2.405030	8	.9494	.9504	2.680548	9	.9898	.9903	3.184000	9
45	45	.8998	.9020	2.452760	8	.9491	.9517	2.698643	9	.9898	.9902	3.187669	9
45	44	.8994	.9001	2.439492	7	.9492	.9501	2.692481	9	.9897	.9903	3.200002	9
45	43	.8995	.9004	2.402833	8	.9498	.9504	2.685186	9	.9899	.9902	3.215837	9
45	42	.8990	.9009	2.409033	8	.9499	.9505	2.672514	9	.9896	.9900	3.185898	9
45	41	.8995	.9006	2.421832	8	.9466	.9507	2.660681	9	.9898	.9901	3.210578	9
45	40	.8998	.9079	2.444698	8	.9494	.9531	2.694926	9	.9899	.9900	3.172098	9
50	50	.8948	.9008	2.445997	8	.9488	.9502	2.700650	9	.9899	.9902	3.223286	9
50	49	.8993	.9001	2.451808	8	.9493	.9519	2.716968	9	.9900	.9901	3.227428	9
50	48	.8996	.9013	2.439671	8	.9496	.9501	2.694433	9	.9900	.9902	3.223328	9
50	47	.8991	.9009	2.437650	8	.9497	.9503	2.685562	9	.9898	.9900	3.210537	9
50	46	.8977	.9046	2.426516	8	.9499	.9506	2.693739	9	.9899	.9903	3.243179	9
50	45	.8928	.9010	2.421603	8	.9490	.9501	2.684828	9	.9900	.9902	3.204624	9
100	100	.8998	.9014	2.537297	9	.9491	.9521	2.791446	9	.9899	.9900	3.312942	5
500	500	.8985	.9003	2.691516	5	.9500	.9500	2.963091	5	.9899	.9901	3.474396	5
						$\theta = .01$ (see $\theta = 0$ for smaller values of M)							
500	500	.8998	.9010	2.690857	5	.9500	.9500	2.949066	5	.9899	.9901	3.474400	5
						$\theta = .05$ (see $\theta = 0$ for smaller values of M)							
100	100	.8997	.9002	2.483005	9	.9496	.9505	2.779514	9	.9899	.9900	3.312943	5
500	500	.8998	.9001	2.547324	5	.9498	.9500	2.842456	9	.9900	.9900	3.407770	5
						$\theta = .1$ (see $\theta = 0$ for smaller values of M)							
25	25	.8931	.9181	2.357018	8	.9467	.9564	2.611160	9	.9899	.9902	3.111263	9
25	24	.9000	.9027	2.338098	7	.9481	.9505	2.576814	9	.9896	.9902	3.072826	9
25	23	.8980	.9004	2.301713	7	.9487	.9501	2.588380	8	.9899	.9904	3.108690	9
25	22	.8985	.9020	2.271233	8	.9493	.9518	2.558434	8	.9899	.9905	3.095295	9
25	21	.8931	.9002	2.311036	8	.9499	.9513	2.600714	8	.9894	.9903	3.065762	9
25	20	.8994	.9023	2.267224	8	.9465	.9507	2.575181	8	.9894	.9900	3.085772	9
30	30	.8937	.9088	2.335493	8	.9497	.9517	2.587739	9	.9900	.9908	3.151436	9
30	29	.8988	.9044	2.333530	8	.9493	.9538	2.628599	8	.9896	.9900	3.130745	9
30	28	.8995	.9006	2.350345	8	.9486	.9509	2.652711	8	.9900	.9918	3.153225	9
30	27	.8999	.9025	2.346262	8	.9488	.9504	2.610010	9	.9899	.9903	3.123157	9
30	26	.9000	.9041	2.348687	8	.9497	.9509	2.628958	9	.9897	.9903	3.150541	9
30	25	.8988	.9009	2.302945	8	.9487	.9526	2.585340	8	.9896	.9904	3.127591	9

$\theta = 0.1 / .25$ $\sqrt{MN/(M+N)}\,W^{+}_{M,N}$ 35 - 500; 6 - 9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = .1$ (continued)							
35	35	.8945	.9026	2.298923	8	.9488	.9508	2.645745	8	.9899	.9905	3.174900	9
35	34	.8989	.9003	2.323115	8	.9496	.9505	2.617124	8	.9899	.9902	3.165483	9
35	33	.8989	.9051	2.347889	8	.9489	.9506	2.658255	8	.9898	.9904	3.154320	9
35	32	.8987	.9015	2.344733	8	.9492	.9501	2.620799	9	.9897	.9905	3.152211	9
35	31	.8996	.9029	2.325303	8	.9486	.9521	2.633547	8	.9900	.9903	3.181589	9
35	30	.9000	.9026	2.316701	8	.9478	.9512	2.603200	8	.9899	.9902	3.142133	9
40	40	.8978	.9002	2.344032	8	.9498	.9517	2.677391	8	.9896	.9906	3.184465	9
40	39	.8994	.9008	2.359977	8	.9491	.9505	2.636461	9	.9899	.9903	3.182486	9
40	38	.8990	.9009	2.349532	8	.9488	.9511	2.623474	9	.9899	.9901	3.187226	9
40	37	.8985	.9033	2.360361	8	.9493	.9500	2.618856	9	.9898	.9900	3.184926	9
40	36	.8990	.9007	2.333967	8	.9490	.9510	2.652736	8	.9896	.9903	3.183289	9
40	35	.8996	.9005	2.360691	8	.9497	.9506	2.643608	9	.9898	.9903	3.184000	9
45	45	.8983	.9001	2.356540	8	.9485	.9500	2.658350	8	.9898	.9902	3.187677	9
45	44	.8996	.9013	2.375515	8	.9496	.9512	2.657539	9	.9897	.9903	3.200002	9
45	43	.8993	.9007	2.370768	8	.9498	.9507	2.632291	9	.9899	.9902	3.215834	9
45	42	.8989	.9020	2.360051	8	.9499	.9505	2.622553	9	.9896	.9900	3.185903	9
45	41	.8996	.9014	2.345000	8	.9497	.9507	2.632239	8	.9898	.9901	3.210579	9
45	40	.8999	.9009	2.334385	8	.9469	.9501	2.658854	8	.9899	.9900	3.172092	9
50	50	.8976	.9001	2.341463	8	.9461	.9510	2.666661	8	.9899	.9902	3.223289	9
50	49	.8979	.9005	2.348361	8	.9499	.9506	2.673242	8	.9900	.9901	3.227426	9
50	48	.8994	.9004	2.363009	8	.9491	.9503	2.644346	9	.9900	.9902	3.223327	9
50	47	.8998	.9029	2.353708	8	.9494	.9506	2.658956	8	.9900	.9901	3.204306	9
50	46	.8997	.9005	2.365886	8	.9498	.9503	2.647974	9	.9898	.9900	3.209180	9
50	45	.8998	.9010	2.347968	8	.9499	.9504	2.663857	8	.9899	.9901	3.192745	9
100	100	.8992	.9001	2.389756	9	.9494	.9502	2.700308	9	.9899	.9901	3.270271	9
500	500	.9000	.9001	2.439167	6	.9500	.9500	2.742827	9	.9900	.9900	3.329183	9
						$\theta = .25$ (see $\theta = 0$ for smaller values of M)							
6	6	.8301	.9286	1.999997	9	.9459	.9870	2.449489	7	.9870	.9989	2.927692	8
6	5	.8810	.9069	2.100524	8	.9459	.9762	2.288686	8	.9870	.9978	2.763848	8
6	4	.8524	.9095	1.844654	8	.9095	.9524	2.108179	6	.9762	.9952	2.581983	7
6	3	.8929	.9524	2.267785	6	.8929	.9524	2.267785	9	.9881	1	2.999994	7
6	2	.8929	.9643	2.108185	5	.8929	.9643	2.108185	9	.9643	1	2.828421	6
7	7	.8939	1	2.160234	8	.9312	.9735	2.366430	8	.9808	.9959	2.788861	8
7	6	.8858	.9266	2.137077	7	.9266	.9545	2.225389	8	.9808	.9924	2.638993	8
7	5	.8813	.9167	2.070192	7	.9356	.9672	2.366426	7	.9848	.9924	2.898270	8
7	4	.8939	.9394	2.013658	9	.9394	.9667	2.288688	7	.9848	.9970	2.746422	8
7	3	.8333	.9167	1.690306	5	.9167	.9667	2.070194	5	.9667	.9917	2.535461	7
7	2	.8333	.9167	1.792840	9	.9167	.9722	2.267781	6	.9722	1	2.999994	7
8	8	.8738	.9148	2.065590	8	.9148	.9565	2.309394	7	.9740	.9907	2.696796	8
8	7	.8738	.9037	1.901594	7	.9417	.9557	2.351450	6	.9883	.9935	2.977699	8
8	6	.8821	.9038	2.049386	8	.9304	.9537	2.160240	8	.9887	.9953	2.806242	7
8	5	.8827	.9176	1.935256	8	.9394	.9534	2.433743	6	.9899	.9953	3.040461	8
8	4	.8889	.9212	2.070195	9	.9212	.9576	2.165057	8	.9899	.9980	2.898274	6
8	3	.8788	.9394	1.854048	5	.9394	.9758	2.224855	6	.9758	.9939	2.686773	7
8	2	.8667	.9333	1.936489	9	.9333	1	2.415225	6	.9333	1	2.415225	5
9	9	.8476	1	1.620176	6	.9453	.9659	2.417463	9	.9832	.9906	2.846044	8
9	8	.8786	.9021	2.156280	7	.9436	.9607	2.425821	8	.9899	.9944	2.870956	8
9	7	.8843	.9156	2.036692	8	.9366	.9510	2.378349	7	.9895	.9926	2.984122	8
9	6	.8689	.9121	1.906924	9	.9121	.9526	2.236065	6	.9846	.9930	2.860383	7
9	5	.8996	.9271	2.078694	8	.9271	.9570	2.093193	8	.9860	.9930	2.788863	6
9	4	.8741	.9175	1.960291	8	.9399	.9692	2.303541	6	.9818	.9930	2.962259	7
9	3	.8273	.9091	1.924491	5	.9091	.9545	1.999999	7	.9818	.9955	2.828424	6
9	2	.8909	.9455	2.068271	5	.9455	1	2.553134	7	.9455	1	2.553134	5

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8121	.9999	1.622872	6	.9345	.9563	2.344033	9	.9870	.9938	2.927695	8
10	9	.8971	.9112	2.082578	8	.9472	.9580	2.471260	7	.9895	.9918	2.987741	8
10	8	.8842	.9071	1.897357	9	.9499	.9648	2.353391	6	.9894	.9940	2.941734	8
10	7	.8837	.9053	2.099489	8	.9417	.9566	2.265029	8	.9892	.9930	2.886173	7
10	6	.8917	.9105	2.065589	8	.9374	.9539	2.367454	7	.9890	.9955	2.981421	8
10	5	.8841	.9281	2.064300	9	.9281	.9697	2.236063	8	.9797	.9903	2.711086	7
10	4	.8701	.9061	1.940214	8	.9371	.9530	2.366431	6	.9850	.9950	2.732512	8
10	3	.8566	.9301	2.043138	8	.9301	.9650	2.133068	8	.9860	.9965	2.962259	6
10	2	.8485	.9091	1.833028	6	.9091	.9545	2.190889	6	.9545	.9999	2.683281	7
15	15	.8810	.9120	1.992044	7	.9489	.9642	2.477167	6	.9896	.9926	2.981417	8
15	14	.8993	.9074	2.107457	8	.9466	.9501	2.410496	7	.9882	.9903	2.936016	8
15	13	.8908	.9018	1.968990	8	.9464	.9575	2.406540	6	.9889	.9900	3.012084	8
15	12	.8915	.9062	2.073320	8	.9481	.9593	2.464747	7	.9871	.9900	2.921499	8
15	11	.8999	.9151	2.050950	8	.9433	.9502	2.327570	7	.9898	.9917	2.935798	7
15	10	.8912	.9041	2.041233	8	.9461	.9580	2.485336	7	.9872	.9909	2.909569	9
20	20	.8980	.9186	2.190884	7	.9421	.9508	2.363055	8	.9881	.9905	2.939387	8
20	19	.8920	.9041	2.106974	7	.9487	.9517	2.401993	7	.9886	.9900	3.028778	8
20	18	.8915	.9062	2.019270	8	.9497	.9577	2.407589	7	.9894	.9903	3.013133	8
20	17	.8977	.9049	2.126196	8	.9474	.9504	2.429543	7	.9900	.9910	3.014924	8
20	16	.8998	.9078	2.103499	8	.9491	.9538	2.371708	8	.9890	.9904	2.993446	8
20	15	.8989	.9121	2.091647	8	.9491	.9577	2.456167	7	.9900	.9911	3.014387	8
25	25	.8952	.9042	2.089776	8	.9498	.9557	2.425355	7	.9898	.9913	3.031689	8
25	24	.8978	.9001	2.138564	8	.9491	.9510	2.442405	8	.9899	.9906	3.025856	8
25	23	.8963	.9039	2.099483	8	.9485	.9509	2.424806	7	.9895	.9904	2.999824	8
25	22	.8993	.9023	2.151128	8	.9496	.9513	2.446070	8	.9898	.9904	3.041746	8
25	21	.8987	.9041	2.053628	8	.9451	.9528	2.374012	7	.9895	.9905	3.018807	8
25	20	.8987	.9042	2.132759	8	.9485	.9534	2.449482	7	.9896	.9907	3.061856	8
30	30	.8852	.9029	2.086996	9	.9462	.9502	2.389753	7	.9896	.9906	3.052801	8
30	29	.8994	.9029	2.107302	8	.9468	.9504	2.422487	7	.9895	.9901	3.010136	8
30	28	.8974	.9001	2.102146	8	.9495	.9520	2.428554	7	.9898	.9905	3.016190	8
30	27	.8973	.9002	2.105926	8	.9482	.9513	2.473088	7	.9896	.9901	3.042513	8
30	26	.8992	.9019	2.143558	8	.9478	.9517	2.398619	8	.9892	.9900	3.003676	8
30	25	.8926	.9014	2.091053	8	.9495	.9513	2.445553	7	.9898	.9903	3.044468	8
35	35	.8984	.9017	2.154931	8	.9470	.9530	2.418963	8	.9894	.9903	3.021655	8
35	34	.8974	.9001	2.110234	8	.9476	.9516	2.445857	7	.9895	.9901	3.039057	8
35	33	.8935	.9015	2.101327	8	.9499	.9512	2.434591	7	.9897	.9902	3.038500	8
35	32	.8982	.9013	2.124724	8	.9481	.9514	2.429496	7	.9900	.9903	3.052166	8
35	31	.8968	.9003	2.137621	8	.9495	.9513	2.441570	8	.9897	.9901	3.016970	8
35	30	.8920	.9015	2.088196	8	.9498	.9511	2.473611	7	.9900	.9904	3.071329	8
40	40	.8979	.9023	2.093161	8	.9500	.9519	2.479113	7	.9894	.9901	3.073744	8
40	39	.8941	.9003	2.135633	8	.9492	.9510	2.440245	8	.9898	.9901	3.059980	8
40	38	.8984	.9003	2.113754	8	.9496	.9529	2.460916	7	.9899	.9902	3.034827	8
40	37	.8985	.9001	2.149213	8	.9493	.9508	2.441905	8	.9899	.9901	3.075087	8
40	36	.8948	.9086	2.122192	8	.9499	.9506	2.484960	7	.9899	.9903	3.055428	8
40	35	.8993	.9018	2.141659	8	.9495	.9507	2.438203	8	.9898	.9902	3.076295	8
45	45	.8986	.9016	2.134163	8	.9497	.9519	2.440097	8	.9896	.9904	3.059410	8
45	44	.8976	.9016	2.116776	8	.9496	.9505	2.461484	7	.9900	.9902	3.079249	8
45	43	.8999	.9014	2.134053	8	.9494	.9504	2.458620	7	.9899	.9902	3.059142	8
45	42	.8992	.9015	2.133396	8	.9496	.9505	2.457687	7	.9899	.9902	3.078839	8
45	41	.8981	.9007	2.138024	8	.9480	.9504	2.421832	8	.9899	.9902	3.048291	8
45	40	.8997	.9015	2.129987	8	.9491	.9500	2.462647	7	.9897	.9904	3.082309	8
50	50	.8980	.9012	2.110997	8	.9490	.9506	2.472254	7	.9899	.9905	3.055048	8
50	49	.8996	.9048	2.140493	8	.9495	.9520	2.486336	7	.9896	.9902	3.065854	8
50	48	.8984	.9004	2.123150	8	.9496	.9504	2.435144	7	.9900	.9903	3.064531	8
50	47	.8996	.9011	2.139685	8	.9487	.9500	2.477804	7	.9899	.9902	3.083601	8
50	46	.8941	.9051	2.123161	8	.9499	.9509	2.455867	7	.9892	.9904	3.066792	8
50	45	.8999	.9019	2.134841	8	.9491	.9505	2.449911	7	.9900	.9902	3.078717	8
100	100	.8996	.9005	2.152650	5	.9500	.9505	2.483001	8	.9899	.9900	3.103202	8
500	500	.8994	.9005	2.190888	6	.9500	.9500	2.522719	8	.9900	.9900	3.136102	8

$\theta = 0$ $\qquad \sqrt{MN/(M+N)}\,W_{M,N}$

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.6667	1	1.999992	7	.6667	1	1.999992	7	.6667	1	1.999992	7
3	3	.7500	.9000	1.732051	6	.9000	1	2.449480	9	.9000	1	2.449480	9
3	2	.8000	1	2.236064	6	.8000	1	2.236064	6	.8000	1	2.236064	6
4	4	.7714	.9714	2.190887	6	.7714	.9714	2.190887	6	.9714	1	2.828422	5
4	3	.8571	.9429	1.984313	8	.9429	.9714	2.645745	6	.9714	1	2.645751	6
4	2	.8667	1	2.449480	9	.8667	1	2.449480	9	.8667	1	2.449480	9
5	5	.7143	.9206	2.070193	9	.9206	.9921	2.581984	7	.9206	.9921	2.581984	7
5	4	.8571	.9206	2.371699	9	.9206	.9841	2.399993	8	.9841	1	2.999999	7
5	3	.8571	.9643	2.190886	5	.8571	.9643	2.190886	5	.9643	1	2.828424	8
5	2	.7143	.9048	1.932183	8	.9048	1	2.645750	6	.9048	1	2.645750	6
6	6	.8918	.9524	2.449480	9	.8918	.9524	2.449480	9	.9740	.9978	2.927695	7
6	5	.8918	.9524	2.288684	5	.8918	.9524	2.288684	5	.9740	.9957	2.763848	9
6	4	.8190	.9048	2.108181	6	.9048	.9524	2.535458	5	.9524	.9905	2.581984	5
6	3	.7857	.9048	2.267783	7	.9048	.9762	2.371701	7	.9762	1	2.999994	5
6	2	.7857	.9286	2.108181	5	.9286	1	2.828422	8	.9286	1	2.828422	8
7	7	.8625	.9470	2.366431	5	.9470	.9615	2.672603	9	.9615	.9918	2.788861	9
7	6	.8571	.9091	2.225394	6	.9524	.9615	2.596289	5	.9883	.9918	3.078826	9
7	5	.8712	.9343	2.366427	6	.9343	.9697	2.474358	6	.9848	.9975	2.927695	5
7	4	.8788	.9333	2.288685	8	.9333	.9515	2.686766	6	.9697	.9939	2.746424	6
7	3	.8333	.9333	2.415220	9	.9333	.9833	2.535457	8	.9833	1	3.162272	7
7	2	.8333	.9444	2.267786	6	.9444	1	2.999999	5	.9444	1	2.999999	5
8	8	.8298	.9430	2.309397	6	.9429	.9814	2.696795	5	.9869	.9975	3.098383	9
8	7	.8634	.9114	2.351452	6	.9441	.9557	2.561731	6	.9869	.9953	2.958038	5
8	6	.8378	.9074	2.256299	7	.9394	.9554	2.650336	6	.9840	.9907	2.806243	6
8	5	.8788	.9068	2.433745	8	.9068	.9580	2.497996	7	.9798	.9907	3.040462	6
8	4	.7838	.9152	2.190889	5	.9152	.9515	2.449480	9	.9798	.9960	2.898269	7
8	3	.8667	.9515	2.553134	5	.8667	.9515	2.553134	5	.9879	.9939	3.316616	8
8	2	.8667	.9556	2.415227	7	.8667	.9556	2.415227	7	.9556	1	3.162272	6
9	9	.8906	.9317	2.417466	7	.9317	.9664	2.631173	6	.9812	.9905	2.999994	5
9	8	.8873	.9314	2.425820	7	.9441	.9552	2.671571	6	.9888	.9923	3.149589	5
9	7	.8733	.9021	2.378350	8	.9399	.9659	2.618610	7	.9851	.9925	3.057878	6
9	6	.8795	.9389	2.371699	9	.9389	.9580	2.581984	8	.9860	.9940	2.958034	7
9	5	.8811	.9091	2.415220	9	.9301	.9720	2.621583	5	.9860	.9940	3.174900	7
9	4	.8224	.9385	2.206117	6	.9385	.9636	2.596293	5	.9860	.9972	3.040460	9
9	3	.8909	.9636	2.683280	6	.8909	.9636	2.683280	6	.9636	.9909	2.828422	6
9	2	.8909	.9636	2.553138	9	.8909	.9636	2.553138	9	.9636	1	3.316618	7
10	10	.8690	.9126	2.344035	8	.9476	.9554	2.683277	7	.9877	.9935	3.146266	6
10	9	.8944	.9161	2.471263	8	.9426	.9562	2.595912	7	.9886	.9930	3.121472	6
10	8	.8738	.9025	2.353389	9	.9217	.9501	2.535458	8	.9880	.9930	3.027148	7
10	7	.8896	.9131	2.281241	5	.9445	.9574	2.671569	8	.9860	.9901	3.121778	7
10	6	.8865	.9075	2.472279	5	.9075	.9580	2.480694	5	.9780	.9910	2.981418	8
10	5	.8342	.9114	2.236064	6	.9461	.9807	2.738608	5	.9807	.9900	2.927695	9
10	4	.8192	.9061	2.415225	6	.9061	.9540	2.432075	6	.9720	.9900	3.089570	5
10	3	.8741	.9091	2.497991	8	.9091	.9720	2.836917	7	.9720	.9930	2.962258	6
10	2	.8515	.9091	2.535496	6	.9091	.9697	2.683280	5	.9697	1	3.464100	8
15	15	.8972	.9099	2.477166	5	.9399	.9564	2.738604	9	.9899	.9908	3.286331	9
15	14	.8993	.9072	2.535290	8	.9496	.9531	2.782793	9	.9893	.9909	3.195060	5
15	13	.8992	.9106	2.572578	7	.9432	.9505	2.654929	5	.9880	.9903	3.115546	9
15	12	.8954	.9107	2.484228	7	.9467	.9523	2.762508	9	.9896	.9910	3.240497	5
15	11	.8972	.9213	2.538949	8	.9434	.9500	2.688572	5	.9894	.9911	3.177120	9
15	10	.8936	.9139	2.499996	8	.9342	.9545	2.672609	5	.9886	.9913	3.105164	9
20	20	.8964	.9049	2.581982	7	.9431	.9505	2.760260	5	.9895	.9915	3.227481	5
20	19	.8995	.9059	2.573546	7	.9480	.9510	2.766439	5	.9893	.9906	3.254874	5
20	18	.8979	.9065	2.558548	8	.9449	.9507	2.778952	5	.9897	.9907	3.257878	5
20	17	.8971	.9044	2.529583	8	.9476	.9505	2.773016	9	.9888	.9909	3.186979	5
20	16	.8999	.9035	2.591198	7	.9473	.9506	2.774995	5	.9892	.9908	3.295764	5
20	15	.8994	.9051	2.538373	8	.9374	.9585	2.788865	9	.9900	.9918	3.237672	5

$\theta = 0 / .01 / .05$ $\sqrt{MN/(M+N)}\, W_{M,N}$ 25 - 500

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = 0$ (continued)							
25	25	.8936	.9129	2.611160	8	.9467	.9501	2.849009	5	.9889	.9903	3.311330	5
25	24	.8963	.9011	2.576818	8	.9494	.9517	2.805807	5	.9898	.9904	3.299827	5
25	23	.8976	.9004	2.588380	8	.9493	.9521	2.835793	5	.9900	.9908	3.268822	5
25	22	.8987	.9036	2.558433	8	.9489	.9531	2.813671	9	.9895	.9901	3.323179	5
25	21	.8998	.9028	2.600712	8	.9485	.9522	2.852145	5	.9898	.9906	3.276172	5
25	20	.8931	.9015	2.575178	8	.9476	.9507	2.794653	5	.9895	.9906	3.287310	5
30	30	.8995	.9035	2.587738	8	.9496	.9517	2.841766	5	.9899	.9905	3.358451	5
30	29	.8987	.9077	2.628599	8	.9489	.9507	2.849403	5	.9897	.9905	3.314543	5
30	28	.8973	.9020	2.652715	7	.9485	.9506	2.889139	5	.9897	.9901	3.355934	5
30	27	.8978	.9010	2.610014	8	.9488	.9504	2.831194	5	.9898	.9904	3.308399	5
30	26	.8996	.9020	2.628961	8	.9470	.9505	2.844097	5	.9894	.9903	3.299850	5
30	25	.8975	.9053	2.585348	8	.9449	.9506	2.842818	5	.9897	.9906	3.351719	5
35	35	.8977	.9018	2.645746	8	.9491	.9510	2.879140	5	.9893	.9901	3.380897	5
35	34	.8993	.9011	2.617128	8	.9492	.9502	2.887846	5	.9900	.9904	3.357211	5
35	33	.8979	.9014	2.658252	8	.9496	.9515	2.894682	5	.9900	.9904	3.351746	5
35	32	.8996	.9021	2.662826	8	.9486	.9517	2.882021	5	.9898	.9903	3.360711	5
35	31	.8989	.9017	2.669152	8	.9498	.9514	2.915909	5	.9896	.9900	3.363868	5
35	30	.8990	.9017	2.609014	8	.9500	.9515	2.861443	5	.9899	.9905	3.352086	5
40	40	.8980	.9002	2.696796	8	.9470	.9506	2.921908	5	.9895	.9909	3.380612	5
40	39	.8993	.9029	2.664318	8	.9487	.9503	2.894423	5	.9899	.9901	3.376525	5
40	38	.8999	.9016	2.671396	8	.9496	.9524	2.910080	5	.9899	.9901	3.393253	5
40	37	.8999	.9021	2.656786	8	.9484	.9523	2.885186	5	.9898	.9901	3.358492	5
40	36	.8992	.9007	2.657180	8	.9493	.9527	2.926911	5	.9898	.9902	3.377388	5
40	35	.8991	.9009	2.680551	8	.9496	.9505	2.903017	5	.9899	.9902	3.374005	5
45	45	.8984	.9037	2.698643	8	.9491	.9508	2.943310	5	.9897	.9902	3.383358	5
45	44	.8987	.9005	2.692477	8	.9493	.9501	2.930842	5	.9899	.9904	3.394387	5
45	43	.8998	.9011	2.685182	8	.9473	.9501	2.899806	5	.9899	.9900	3.409947	5
45	42	.8999	.9011	2.672515	8	.9497	.9504	2.915055	5	.9900	.9902	3.393191	5
45	41	.8935	.9015	2.660681	8	.9493	.9500	2.895261	5	.9897	.9900	3.390005	5
45	40	.8989	.9064	2.694930	8	.9495	.9501	2.916282	5	.9899	.9902	3.375119	5
50	50	.8978	.9007	2.700655	8	.9498	.9547	2.948834	5	.9899	.9903	3.429972	5
50	49	.8988	.9041	2.716969	8	.9496	.9503	2.935704	5	.9900	.9902	3.418823	5
50	48	.8994	.9004	2.694438	8	.9499	.9505	2.934915	5	.9898	.9901	3.407365	5
50	47	.8996	.9007	2.685570	8	.9498	.9505	2.944334	5	.9899	.9902	3.412756	5
50	46	.8991	.9001	2.687647	8	.9496	.9503	2.930365	5	.9898	.9900	3.428852	5
50	45	.8982	.9003	2.684827	8	.9497	.9503	2.929361	5	.9898	.9901	3.408348	5
100	100	.8936	.9044	2.791449	8	.9499	.9502	3.024948	5	.9899	.9900	3.502189	5
500	500	.8999	.9003	2.961849	5	.9500	.9500	3.186422	5	.9899	.9900	3.669355	5
						$\theta = .01$ (see $\theta = 0$ for smaller values of M)							
500	500	.8999	.9000	2.944097	5	.9500	.9500	3.186421	5	.9899	.9900	3.669354	5
						$\theta = .05$ (see $\theta = 0$ for smaller values of M)							
100	100	.8996	.9012	2.779509	8	.9497	.9501	3.008865	5	.9899	.9900	3.502189	5
500	500	.9000	.9003	2.842462	5	.9500	.9500	3.099087	5	.9900	.9900	3.613597	5

$\theta = 0.1 \;/\; .25$ $\qquad \sqrt{MN/(M+N)}\, W_{M,N}$ $\qquad$ 35 - 500 / 8 - 9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = .1$ (see $\theta = 0$ for smaller values of M)							
35	35	.8977	.9018	2.645746	8	.9491	.9510	2.879140	5	.9893	.9901	3.380897	5
35	34	.8993	.9011	2.617128	8	.9492	.9502	2.887846	5	.9900	.9904	3.357211	5
35	33	.8979	.9014	2.658252	8	.9496	.9515	2.894682	5	.9900	.9904	3.351746	5
35	32	.8985	.9003	2.620807	8	.9486	.9517	2.882018	5	.9898	.9903	3.360707	5
35	31	.8974	.9042	2.633544	8	.9498	.9514	2.915910	5	.9896	.9900	3.363869	5
35	30	.8957	.9024	2.603207	8	.9500	.9515	2.861439	5	.9899	.9905	3.352087	5
40	40	.8996	.9035	2.677397	8	.9470	.9506	2.921906	5	.9895	.9909	3.380615	5
40	39	.8983	.9011	2.636463	8	.9487	.9503	2.894420	5	.9899	.9901	3.376522	5
40	38	.8978	.9024	2.623478	8	.9496	.9524	2.910078	5	.9899	.9901	3.393251	5
40	37	.8988	.9002	2.618862	8	.9500	.9509	2.880481	5	.9898	.9901	3.358491	5
40	36	.8981	.9021	2.652736	8	.9493	.9519	2.897329	5	.9898	.9902	3.377389	5
40	35	.8996	.9014	2.643610	8	.9486	.9516	2.893184	5	.9899	.9902	3.374004	5
45	45	.8971	.9002	2.658346	8	.9493	.9525	2.908373	5	.9897	.9902	3.383359	5
45	44	.8993	.9024	2.657541	8	.9492	.9503	2.900927	5	.9899	.9904	3.394385	5
45	43	.8997	.9015	2.632293	8	.9491	.9503	2.871079	5	.9899	.9900	3.409946	5
45	42	.8999	.9012	2.622553	8	.9499	.9506	2.896045	5	.9900	.9902	3.393189	5
45	41	.8995	.9016	2.632237	8	.9487	.9507	2.894458	5	.9897	.9900	3.390004	5
45	40	.8940	.9003	2.658851	8	.9496	.9514	2.900130	5	.9899	.9902	3.375118	5
50	50	.8924	.9021	2.666666	8	.9496	.9515	2.896827	5	.9899	.9903	3.429970	5
50	49	.8999	.9013	2.673243	8	.9497	.9501	2.914944	5	.9900	.9902	3.418822	5
50	48	.8984	.9008	2.644348	8	.9492	.9503	2.905985	5	.9898	.9901	3.407367	5
50	47	.8989	.9014	2.658960	8	.9495	.9500	2.934078	5	.9899	.9902	3.412753	5
50	46	.8998	.9008	2.647982	8	.9497	.9509	2.918853	5	.9898	.9900	3.428854	5
50	45	.9000	.9009	2.663861	8	.9484	.9503	2.897344	5	.9898	.9901	3.408347	5
100	100	.9000	.9005	2.700306	8	.9500	.9503	2.971183	5	.9900	.9901	3.4[illegible]2094	5
500	500	.9000	.9001	2.721619	5	.9500	.9500	3.013864	5	.9900	.9900	3.546824	5
						$\theta = .25$ (see $\theta = 0$ for smaller values of M)							
8	8	.8798	.9130	2.309397	6	.9479	.9814	2.696795	5	.9869	.9975	3.098383	9
8	7	.8834	.9114	2.351452	6	.9441	.9557	2.561731	6	.9859	.9953	2.956038	5
8	6	.8608	.9074	2.160239	8	.9394	.9554	2.650336	6	.9840	.9907	2.806243	6
8	5	.8788	.9068	2.433745	8	.9068	.9580	2.497996	7	.9798	.9907	3.040462	6
8	4	.8788	.9152	2.165064	5	.9152	.9515	2.419480	9	.9798	.9960	2.893269	7
8	3	.8788	.9515	2.224858	6	.8788	.9515	2.224858	6	.9879	.9939	3.166516	8
8	2	.8667	1	2.415227	7	.8667	1	2.415227	7	.8667	1	2.415227	7
9	9	.8906	.9317	2.417466	7	.9317	.9664	2.631173	6	.9812	.9905	2.990994	9
9	8	.8873	.9214	2.425830	7	.9441	.9552	2.671571	6	.9888	.9923	3.149589	5
9	7	.8733	.9021	2.378350	8	.9399	.9659	2.618610	7	.9851	.9925	3.057878	6
9	6	.8242	.9053	2.236064	9	.9389	.9580	2.581984	8	.9860	.9940	2.953064	7
9	5	.8541	.9141	2.093198	5	.9441	.9720	2.577578	9	.9860	.9940	3.174900	7
9	4	.8797	.9385	2.303545	6	.9385	.9636	2.596293	5	.9860	.9972	3.040460	9
9	3	.8182	.9091	1.999996	8	.9091	.9636	2.366426	7	.9636	.9909	2.824422	6
9	2	.8909	1	2.553138	9	.8909	1	2.553138	9	.8909	1	2.553138	9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8690	.9126	2.344035	8	.9476	.9554	2.683277	7	.9877	.9935	3.146266	6
10	9	.8944	.9161	2.471263	8	.9426	.9562	2.595912	7	.9886	.9930	3.121472	6
10	8	.8998	.9296	2.353389	9	.9296	.9501	2.439517	8	.9880	.9930	3.027148	7
10	7	.8835	.9131	2.265026	5	.9445	.9574	2.671569	8	.9860	.9901	3.121778	7
10	6	.8749	.9078	2.367453	5	.9336	.9580	2.472279	5	.9780	.9910	2.981418	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9807	.9900	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	.9900	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	.9930	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8979	.9284	2.477166	5	.9472	.9611	2.652067	9	.9899	.9908	3.286329	9
15	14	.8932	.9003	2.410500	8	.9489	.9559	2.686729	9	.9893	.9909	3.195057	9
15	13	.8929	.9150	2.406542	8	.9467	.9542	2.654925	9	.9880	.9903	3.115552	9
15	12	.8962	.9185	2.464749	7	.9455	.9535	2.598076	5	.9896	.9910	3.240499	8
15	11	.8866	.9003	2.327574	8	.9497	.9565	2.688570	8	.9895	.9921	3.123849	9
15	10	.8923	.9160	2.485337	7	.9378	.9502	2.545871	5	.9886	.9913	3.105170	8
20	20	.8842	.9017	2.363055	8	.9428	.9520	2.651972	9	.9877	.9907	3.186960	9
20	19	.8975	.9034	2.401996	8	.9483	.9508	2.718804	9	.9892	.9906	3.171417	5
20	18	.8994	.9155	2.407591	8	.9467	.9511	2.648053	9	.9892	.9900	3.232690	9
20	17	.8948	.9008	2.429541	8	.9437	.9500	2.670118	9	.9889	.9902	3.180658	9
20	16	.8981	.9077	2.371702	8	.9491	.9574	2.662556	9	.9891	.9904	3.174900	9
20	15	.8981	.9155	2.456166	8	.9458	.9506	2.644655	9	.9899	.9909	3.231542	9
25	25	.8996	.9114	2.425349	8	.9472	.9537	2.686858	9	.9896	.9913	3.204932	9
25	24	.8981	.9021	2.442400	8	.9494	.9549	2.739560	9	.9892	.9900	3.253304	9
25	23	.8970	.9018	2.424800	8	.9473	.9502	2.686053	9	.9895	.9903	3.213438	9
25	22	.8992	.9026	2.446069	8	.9450	.9500	2.669670	9	.9894	.9908	3.211944	9
25	21	.8901	.9056	2.374012	8	.9490	.9551	2.672463	9	.9898	.9907	3.244117	9
25	20	.8971	.9068	2.449483	8	.9469	.9503	2.736085	9	.9896	.9905	3.203240	5
30	30	.8923	.9003	2.389747	8	.9427	.9532	2.683279	9	.9891	.9908	3.279558	9
30	29	.8936	.9007	2.422489	8	.9483	.9500	2.738250	9	.9900	.9905	3.262783	9
30	28	.8990	.9041	2.428550	8	.9485	.9507	2.689643	9	.9894	.9900	3.220249	9
30	27	.8963	.9026	2.473087	8	.9485	.9507	2.720708	9	.9896	.9902	3.293590	9
30	26	.8956	.9034	2.398621	8	.9492	.9512	2.724689	9	.9896	.9904	3.237465	9
30	25	.8990	.9027	2.445555	8	.9493	.9508	2.763533	9	.9896	.9902	3.226615	5
35	35	.8941	.9061	2.418958	8	.9481	.9560	2.734700	9	.9894	.9910	3.281647	9
35	34	.8953	.9032	2.445854	8	.9489	.9511	2.730650	9	.9895	.9901	3.261389	9
35	33	.8998	.9024	2.434595	8	.9492	.9508	2.716451	9	.9898	.9902	3.248118	9
35	32	.8963	.9027	2.429492	8	.9480	.9523	2.743250	9	.9898	.9903	3.296289	9
35	31	.8991	.9026	2.441569	8	.9489	.9506	2.712996	9	.9896	.9901	3.242127	9
35	30	.8996	.9022	2.473611	8	.9483	.9502	2.709942	9	.9900	.9903	3.301233	9
40	40	.9000	.9038	2.479115	8	.9479	.9517	2.717283	9	.9896	.9903	3.281650	9
40	39	.8984	.9019	2.440243	8	.9494	.9520	2.733965	9	.9896	.9900	3.273561	9
40	38	.8992	.9058	2.460918	8	.9489	.9527	2.726968	9	.9894	.9902	3.245753	9
40	37	.8986	.9015	2.441905	8	.9496	.9513	2.724457	9	.9900	.9903	3.296534	9
40	36	.8998	.9012	2.484957	8	.9491	.9506	2.732196	9	.9897	.9901	3.252646	9
40	35	.8991	.9014	2.438204	8	.9493	.9507	2.727643	9	.9897	.9901	3.273266	9
45	45	.8994	.9038	2.440091	8	.9496	.9502	2.740637	9	.9893	.9901	3.255910	9
45	44	.8993	.9010	2.461486	8	.9494	.9510	2.729247	9	.9899	.9902	3.297359	9
45	43	.8989	.9008	2.458619	8	.9499	.9506	2.764376	9	.9898	.9904	3.281685	9
45	42	.8992	.9009	2.457682	8	.9494	.9520	2.730768	9	.9898	.9902	3.286086	9
45	41	.8961	.9008	2.421829	8	.9472	.9508	2.727269	9	.9900	.9903	3.262100	9
45	40	.8983	.9001	2.462645	8	.9490	.9506	2.718667	9	.9895	.9900	3.297916	9
50	50	.8981	.9012	2.472254	8	.9481	.9523	2.735755	9	.9900	.9904	3.296338	9
50	49	.8990	.9041	2.486332	8	.9500	.9514	2.741131	9	.9900	.9901	3.315270	9
50	48	.8993	.9008	2.435141	8	.9496	.9508	2.726158	9	.9900	.9902	3.282599	9
50	47	.8974	.9000	2.477804	8	.9492	.9501	2.739743	9	.9897	.9902	3.304186	9
50	46	.8998	.9018	2.455865	8	.9498	.9509	2.744790	9	.9898	.9901	3.268825	9
50	45	.8982	.9010	2.449909	8	.9500	.9509	2.737624	9	.9899	.9903	3.284911	9
100	100	.8999	.9011	2.483005	9	.9486	.9506	2.757936	9	.9898	.9900	3.313039	9
500	500	.8999	.9000	2.522724	5	.9500	.9501	2.802861	9	.9900	.9900	3.362302	5

$\theta = 0 \;/\; .01 \;/\; .05 \quad \sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$ 10 - 40

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D

See $W^{+}_{M,N}$ for $\theta = 0$.

$\theta = .01$ (for smaller values of M see $\theta = 0$)

100	100	.8996	.9001	2.519758	9	.9490	.9503	2.789055	9	.9899	.9900	3.311330	5
500	500	.9000	.9000	2.661939	5	.9495	.9501	2.926526	5	.9900	.9900	3.463385	5

$\theta = .05$ (for smaller values of M see $\theta = 0$)

10	10	.8503	.9016	1.951796	9	.9406	.9672	2.344035	8	.9802	.9912	2.738606	6
10	9	.8869	.9107	2.132999	9	.9365	.9517	2.205804	9	.9865	.9912	2.809325	7
10	8	.8776	.9071	2.028367	5	.9351	.9529	2.347866	9	.9849	.9905	2.810920	7
10	7	.8837	.9107	2.099487	5	.9488	.9642	2.265026	5	.9838	.9913	2.671569	8
10	6	.8917	.9105	2.065590	6	.9374	.9580	2.367453	5	.9843	.9905	2.732514	9
10	5	.8841	.9281	2.064304	6	.9281	.9697	2.236064	6	.9797	.9930	2.711086	5
10	4	.8701	.9061	1.940216	8	.9371	.9530	2.366431	7	.9850	.9950	2.732519	6
10	3	.8566	.9301	2.043142	5	.9301	.9650	2.133068	9	.9860	1	2.962258	6
10	2	.8485	.9091	1.833026	7	.9091	.9545	2.190889	6	.9545	1	2.683280	5
15	15	.8771	.9019	2.158327	6	.9463	.9595	2.477168	8	.9884	.9913	2.981418	8
15	14	.8953	.9050	2.132762	8	.9458	.9504	2.419674	8	.9891	.9918	2.936014	8
15	13	.8969	.9017	2.244526	7	.9411	.9520	2.406536	8	.9895	.9906	3.012080	8
15	12	.8951	.9147	2.173700	8	.9413	.9515	2.464746	8	.9871	.9905	2.921498	8
15	11	.8967	.9054	2.132315	7	.9484	.9580	2.391650	8	.9898	.9917	2.935793	8
15	10	.8431	.9041	2.041238	8	.9461	.9580	2.485336	7	.9872	.9916	2.909568	9
20	20	.8953	.9024	2.238921	7	.9493	.9535	2.542560	8	.9895	.9914	3.038211	9
20	19	.8975	.9015	2.265293	7	.9483	.9516	2.491826	8	.9894	.9901	3.048044	8
20	18	.8967	.9000	2.260279	7	.9450	.9511	2.532288	8	.9897	.9909	3.016519	9
20	17	.8947	.9088	2.216761	7	.9433	.9504	2.467200	8	.9900	.9910	3.014925	8
20	16	.8983	.9017	2.250078	7	.9416	.9508	2.448275	8	.9890	.9904	2.993440	8
20	15	.8862	.9030	2.200980	7	.9465	.9501	2.484281	8	.9900	.9911	3.014382	8
25	25	.8946	.9078	2.314546	7	.9436	.9507	2.576730	8	.9893	.9903	3.086059	9
25	24	.8957	.9066	2.312022	7	.9477	.9531	2.561975	8	.9899	.9906	3.072562	9
25	23	.8985	.9035	2.282523	7	.9491	.9520	2.563548	8	.9899	.9907	3.101819	9
25	22	.8994	.9022	2.251872	8	.9470	.9528	2.532099	8	.9893	.9905	3.085049	8
25	21	.8972	.9022	2.311032	7	.9482	.9504	2.586718	8	.9895	.9902	3.061097	9
25	20	.8989	.9031	2.261332	8	.9486	.9505	2.533952	8	.9898	.9905	3.063967	8
30	30	.8910	.9189	2.355492	8	.9433	.9548	2.581987	9	.9898	.9904	3.105290	9
30	29	.8979	.9091	2.298007	8	.9445	.9501	2.540984	8	.9899	.9903	3.088230	8
30	28	.8882	.9000	2.259844	8	.9497	.9511	2.573535	8	.9900	.9904	3.126374	9
30	27	.8964	.9043	2.275928	8	.9457	.9500	2.547256	8	.9897	.9903	3.075512	9
30	26	.8977	.9019	2.318970	7	.9485	.9506	2.561227	9	.9898	.9903	3.073543	9
30	25	.8972	.9010	2.265053	8	.9500	.9518	2.547568	8	.9897	.9903	3.089812	9
35	35	.8943	.9048	2.320473	8	.9493	.9523	2.621588	8	.9898	.9904	3.154283	9
35	34	.8994	.9020	2.334302	8	.9486	.9500	2.583534	9	.9898	.9902	3.121389	9
35	33	.8999	.9015	2.305586	8	.9491	.9511	2.639673	8	.9896	.9903	3.127321	9
35	32	.8999	.9016	2.339790	8	.9497	.9517	2.594919	9	.9900	.9904	3.098438	9
35	31	.8964	.9004	2.319412	7	.9491	.9508	2.581090	8	.9900	.9905	3.156600	9
35	30	.8993	.9010	2.321045	8	.9467	.9504	2.593092	8	.9887	.9909	3.110074	9
40	40	.8977	.9001	2.344035	8	.9498	.9514	2.653296	8	.9893	.9903	3.150901	9
40	39	.8998	.9005	2.360296	8	.9500	.9515	2.636463	9	.9898	.9901	3.147721	9
40	38	.8992	.9003	2.361987	8	.9492	.9512	2.623473	9	.9897	.9900	3.162393	9
40	37	.8977	.9005	2.368448	8	.9494	.9506	2.635704	9	.9899	.9902	3.131933	9
40	36	.8997	.9029	2.344043	8	.9499	.9532	2.634218	8	.9898	.9904	3.181455	9
40	35	.8981	.9002	2.362273	8	.9494	.9502	2.603862	9	.9898	.9903	3.126280	9

θ = .05 / .1 $\sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$ 45 - 500 / 5 - 10

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .05 (continued)							
45	45	.8985	.9003	2.356540	8	.9489	.9505	2.658350	8	.9898	.9901	3.165403	9
45	44	.8990	.9006	2.375511	8	.9490	.9504	2.657534	9	.9900	.9902	3.169887	9
45	43	.8999	.9009	2.377790	8	.9499	.9514	2.648713	9	.9899	.9900	3.198830	9
45	42	.9000	.9051	2.381935	8	.9497	.9502	2.663688	9	.9898	.9901	3.170070	9
45	41	.8995	.9006	2.363058	8	.9480	.9513	2.634932	9	.9899	.9901	3.183469	9
45	40	.8996	.9013	2.377835	8	.9497	.9507	2.661448	9	.9900	.9906	3.170569	9
50	50	.8979	.9014	2.395955	8	.9494	.9512	2.666658	9	.9896	.9901	3.191678	9
50	49	.8989	.9002	2.363861	8	.9498	.9510	2.653292	9	.9896	.9901	3.174482	9
50	48	.8993	.9033	2.392378	8	.9498	.9510	2.644348	9	.9898	.9902	3.197502	9
50	47	.8994	.9004	2.374360	8	.9482	.9504	2.658960	9	.9899	.9902	3.177045	9
50	46	.8985	.9007	2.375526	8	.9469	.9501	2.635617	9	.9898	.9900	3.160956	9
50	45	.8990	.9000	2.372050	8	.9498	.9506	2.640609	9	.9899	.9900	3.179711	9
100	100	.8999	.9010	2.476525	9	.9497	.9500	2.736857	9	.9900	.9901	3.280524	9
500	500	.9000	.9000	2.546704	5	.9500	.9501	2.838951	9	.9900	.9900	3.403127	5
						θ = .1 (see θ = 0 for smaller values of M)							
5	5	.8571	.9167	1.897363	5	.9167	.9762	2.070193	9	.9762	1	2.581984	7
5	4	.8810	.9603	1.897366	5	.8810	.9603	1.897366	5	.9603	1	2.399993	8
5	3	.8214	.9286	1.697055	7	.9286	1	2.190886	5	.9286	1	2.190886	5
5	2	.8571	1	1.932183	8	.8571	1	1.932183	8	.8571	1	1.932183	8
6	6	.8896	.9437	1.999996	5	.9437	.9697	2.309397	9	.9697	.9924	2.449480	9
6	5	.8463	.9113	1.854047	6	.9113	.9545	2.100528	5	.9870	1	2.763848	9
6	4	.8524	.9286	1.844657	7	.9286	.9762	2.108181	6	.9762	1	2.581984	5
6	3	.8810	.9524	1.897361	9	.8810	.9524	1.897361	9	.9524	1	2.371701	7
6	2	.8929	1	2.108181	5	.8929	1	2.108181	5	.8929	1	2.108181	5
7	7	.8584	.9079	1.954012	6	.9079	.9569	2.160243	6	.9895	.9977	2.788861	9
7	6	.8613	.9021	1.935261	7	.9371	.9662	2.225394	6	.9837	.9959	2.638991	5
7	5	.8535	.9040	1.756613	9	.9419	.9735	2.276409	7	.9735	.9924	2.474358	6
7	4	.8939	.9545	2.013658	9	.8939	.9545	2.013658	9	.9848	1	2.746424	6
7	3	.8333	.9167	1.690308	6	.9167	.9667	2.070193	5	.9667	1	2.535457	8
7	2	.8333	.9167	1.792838	8	.9167	1	2.267786	6	.9167	1	2.267786	6
8	8	.8877	.9406	2.065590	7	.9406	.9623	2.309397	6	.9841	.9928	2.696795	5
8	7	.8834	.9145	1.901588	8	.9409	.9596	2.324172	7	.9879	.9944	2.835575	5
8	6	.8721	.9038	2.049383	8	.9371	.9620	2.160239	8	.9787	.9907	2.650336	6
8	5	.8982	.9371	1.935261	5	.9371	.9604	2.253461	8	.9837	.9953	2.638987	7
8	4	.8889	.9212	2.070193	5	.9212	.9697	2.165058	5	.9899	1	2.898269	7
8	3	.8788	.9394	1.854042	7	.9394	.9758	2.224858	6	.9758	1	2.686768	5
8	2	.8667	.9333	1.936482	9	.9333	1	2.415227	7	.9333	1	2.415227	7
9	9	.8691	.9218	1.999996	8	.9427	.9514	2.357021	7	.9855	.9944	2.846046	6
9	8	.8893	.9134	2.156285	8	.9492	.9650	2.265026	8	.9843	.9912	2.691467	6
9	7	.8920	.9240	2.036694	9	.9458	.9607	2.378350	8	.9858	.9926	2.732519	7
9	6	.8689	.9121	1.906921	5	.9121	.9580	2.236064	9	.9860	.9944	2.797154	7
9	5	.8996	.9271	2.078693	5	.9271	.9570	2.093198	5	.9895	.9970	2.788864	8
9	4	.8741	.9175	1.900286	7	.9399	.9790	2.303545	6	.9790	.9930	2.596293	5
9	3	.8273	.9091	1.924499	8	.9091	.9545	1.999996	8	.9818	1	2.828422	6
9	2	.8909	.9455	2.068277	5	.9455	1	2.553138	9	.9455	1	2.553138	9
10	10	.8503	.9016	1.951796	9	.9406	.9672	2.344035	8	.9802	.9912	2.738606	6
10	9	.8869	.9107	2.132999	9	.9365	.9517	2.205804	9	.9865	.9912	2.809325	7
10	8	.8776	.9071	2.028367	5	.9351	.9529	2.347866	9	.9849	.9905	2.810920	7
10	7	.8837	.9107	2.099487	5	.9488	.9642	2.265026	5	.9838	.9913	2.671569	8
10	6	.8917	.9105	2.065590	6	.9374	.9580	2.367453	5	.9843	.9905	2.732514	9
10	5	.8841	.9281	2.064304	6	.9281	.9697	2.236064	6	.9797	.9930	2.711088	5
10	4	.8701	.9061	1.940216	8	.9371	.9530	2.366431	7	.9850	.9950	2.732519	6
10	3	.8566	.9301	2.043142	5	.9301	.9650	2.133068	9	.9860	1	2.962258	6
10	2	.8485	.9091	1.833026	7	.9091	.9545	2.190889	6	.9545	1	2.683280	5

$\theta = .1$ (continued) $\sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$ 15 - 500

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
15	15	.8957	.9064	2.158327	6	.9464	.9559	2.449485	8	.9895	.9931	2.981417	8
15	14	.8968	.9047	2.107457	8	.9499	.9548	2.419674	7	.9882	.9903	2.861979	8
15	13	.8946	.9037	2.090051	7	.9484	.9551	2.306549	8	.9887	.9908	2.880877	7
15	12	.8935	.9031	2.078458	7	.9476	.9619	2.464746	7	.9887	.9912	2.857883	9
15	11	.8900	.9078	2.130648	7	.9460	.9528	2.342727	8	.9894	.9915	2.911020	8
15	10	.8766	.9122	2.041241	8	.9383	.9504	2.294150	7	.9886	.9924	2.891756	7
20	20	.8983	.9044	2.216363	7	.9432	.9523	2.478744	8	.9877	.9901	2.939383	8
20	19	.8961	.9021	2.189138	7	.9476	.9504	2.471597	8	.9898	.9906	3.048050	8
20	18	.8972	.9001	2.251189	8	.9484	.9522	2.493685	8	.9896	.9906	3.016349	8
20	17	.8895	.9019	2.202753	8	.9468	.9539	2.467197	8	.9889	.9905	2.945480	8
20	16	.8994	.9040	2.216962	7	.9449	.9533	2.399995	8	.9891	.9902	2.986033	8
20	15	.8557	.9021	2.091649	8	.9438	.9523	2.456161	7	.9897	.9908	2.981165	8
25	25	.8988	.9027	2.264553	7	.9491	.9502	2.545580	8	.9898	.9906	3.072548	9
25	24	.8955	.9012	2.256420	8	.9499	.9517	2.525612	8	.9895	.9907	3.029603	9
25	23	.8966	.9010	2.248475	7	.9486	.9507	2.504569	8	.9896	.9910	2.999823	8
25	22	.8948	.9081	2.218961	7	.9483	.9503	2.475999	8	.9895	.9901	3.041746	8
25	21	.8967	.9008	2.181648	8	.9496	.9518	2.466323	8	.9895	.9903	2.988611	8
25	20	.8893	.9009	2.171764	7	.9496	.9559	2.487461	8	.9900	.9911	3.061861	8
30	30	.8987	.9211	2.335496	8	.9479	.9560	2.581983	9	.9898	.9901	3.098385	9
30	29	.8982	.9022	2.277955	8	.9481	.9537	2.540984	8	.9899	.9903	3.072931	9
30	28	.8914	.9032	2.259842	8	.9496	.9515	2.545241	8	.9893	.9902	3.101778	8
30	27	.8996	.9047	2.250919	8	.9483	.9500	2.515931	8	.9895	.9901	3.071769	8
30	26	.8998	.9020	2.243121	8	.9492	.9506	2.548161	8	.9888	.9900	3.046899	9
30	25	.8969	.9017	2.262552	7	.9500	.9523	2.542743	8	.9897	.9902	3.065360	9
35	35	.8985	.9021	2.298919	8	.9464	.9516	2.561732	8	.9897	.9902	3.112678	9
35	34	.8911	.9022	2.288361	8	.9494	.9510	2.563600	8	.9898	.9901	3.107550	9
35	33	.8994	.9010	2.262083	8	.9492	.9503	2.530571	8	.9898	.9905	3.125223	8
35	32	.8996	.9007	2.294444	8	.9491	.9501	2.562941	8	.9898	.9906	3.083079	9
35	31	.8989	.9008	2.267177	8	.9483	.9503	2.546338	8	.9897	.9906	3.071009	9
35	30	.8996	.9012	2.281706	7	.9494	.9506	2.550909	8	.9899	.9903	3.100878	9
40	40	.8929	.9060	2.309393	8	.9491	.9516	2.581981	8	.9896	.9902	3.135714	9
40	39	.8997	.9022	2.305096	8	.9497	.9504	2.594047	8	.9898	.9901	3.120332	9
40	38	.8991	.9006	2.303047	8	.9484	.9512	2.588710	8	.9895	.9902	3.108892	9
40	37	.8989	.9001	2.323151	7	.9495	.9502	2.602662	8	.9895	.9902	3.105713	9
40	36	.8982	.9000	2.319509	7	.9495	.9504	2.574150	8	.9899	.9902	3.143799	9
40	35	.8980	.9059	2.314550	8	.9500	.9536	2.599129	8	.9899	.9901	3.105164	9
45	45	.9000	.9026	2.325646	8	.9489	.9507	2.588727	8	.9898	.9908	3.162276	9
45	44	.8985	.9006	2.305894	8	.9494	.9502	2.606466	8	.9896	.9902	3.128894	9
45	43	.8999	.9013	2.339338	7	.9487	.9507	2.611434	8	.9899	.9902	3.133795	9
45	42	.8993	.9003	2.312020	8	.9492	.9503	2.598805	8	.9898	.9900	3.147068	9
45	41	.8997	.9007	2.336587	8	.9495	.9502	2.581439	9	.9898	.9903	3.132746	9
45	40	.8985	.9003	2.305622	8	.9491	.9525	2.603961	8	.9899	.9901	3.124396	9
50	50	.8998	.9025	2.341460	8	.9494	.9504	2.625856	8	.9891	.9900	3.144848	9
50	49	.8978	.9012	2.345972	8	.9487	.9506	2.634603	8	.9899	.9904	3.149229	9
50	48	.8997	.9008	2.364355	8	.9497	.9504	2.624764	9	.9899	.9901	3.174269	9
50	47	.8973	.9008	2.353458	8	.9497	.9503	2.623077	9	.9900	.9901	3.160010	9
50	46	.8959	.9001	2.321761	8	.9498	.9503	2.635529	8	.9896	.9902	3.139321	9
50	45	.8985	.9026	2.345725	8	.9480	.9514	2.607892	9	.9899	.9901	3.161185	9
100	100	.9000	.9003	2.404278	9	.9498	.9501	2.693607	9	.9900	.9901	3.256759	9
500	500	.9000	.9001	2.440191	9	.9500	.9500	2.743759	9	.9900	.9900	3.326801	9

$\theta = .25$ $\sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$ 2 - 10

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.8333	1	1.999992	7	.8333	1	1.999992	7	.8333	1	1.999992	7
3	3	.8000	1	1.732044	6	.8000	1	1.732044	6	.8000	1	1.732044	6
3	2	.7000	1	1.490709	9	.7000	1	1.490709	9	.7000	1	1.490709	9
4	4	.7857	.9286	1.632990	9	.9286	1	2.190887	6	.9286	1	2.190887	6
4	3	.8857	1	1.984305	8	.8857	1	1.984305	8	.8857	1	1.984305	8
4	2	.8000	1	1.732044	6	.8000	1	1.732044	6	.8000	1	1.732044	6
5	5	.8571	.9167	1.897363	5	.9167	.9762	2.070193	9	.9762	1	2.581984	7
5	4	.8810	.9603	1.897366	5	.8810	.9603	1.897366	5	.9603	1	2.399993	8
5	3	.8214	.9286	1.697055	7	.9286	1	2.190886	5	.9286	1	2.190886	5
5	2	.8571	.9990	1.932183	8	.8571	1	1.932183	8	.8571	1	1.932183	8
6	6	.8896	.9437	1.999996	5	.9437	.9697	2.309397	9	.9697	.9924	2.449480	9
6	5	.8463	.9113	1.854047	6	.9113	.9545	2.100528	5	.9870	1	2.763848	9
6	4	.8524	.9286	1.844657	7	.9286	.9762	2.108181	6	.9762	1	2.581984	5
6	3	.8810	.9524	1.897361	9	.8810	.9524	1.897361	9	.9524	1	2.371701	7
6	2	.8929	1	2.108181	5	.8929	1	2.108181	5	.8929	1	2.108181	5
7	7	.8794	.9324	1.954012	6	.9324	.9650	2.160243	6	.9895	1	2.788861	9
7	6	.8409	.9021	1.828345	7	.9021	.9510	1.974462	7	.9837	1	2.638991	5
7	5	.8535	.9293	1.756613	9	.9293	.9735	2.070189	7	.9735	1	2.474358	6
7	4	.8939	.9545	1.895211	9	.8939	.9545	1.895211	9	.9545	1	2.288685	8
7	3	.8333	.9167	1.690308	6	.9167	1	2.070193	5	.9167	1	2.070193	5
7	2	.8333	1	1.792838	8	.8333	1	1.792838	8	.8333	1	1.792838	8
8	8	.8938	.9242	1.999992	7	.9242	.9503	2.065590	7	.9872	.9965	2.696795	5
8	7	.8965	.9313	1.901588	8	.9313	.9596	2.184653	7	.9814	.9944	2.561731	6
8	6	.8555	.9021	1.714925	5	.9371	.9720	2.160239	8	.9720	.9907	2.415222	7
8	5	.8982	.9565	1.935261	5	.8982	.9565	1.935261	5	.9837	1	2.638987	7
8	4	.8586	.9293	1.732044	6	.9293	.9697	2.070193	5	.9697	1	2.449480	9
8	3	.8788	.9394	1.854042	7	.9394	1	2.224858	6	.9394	1	2.224858	6
8	2	.8667	1	1.936482	9	.8667	1	1.936482	9	.8667	1	1.936482	9
9	9	.8121	.9077	1.897361	9	.9319	.9535	2.267783	7	.9899	.9955	2.846046	6
9	8	.8766	.9066	1.854160	9	.9319	.9550	2.176080	8	.9843	.9932	2.691467	6
9	7	.8684	.9014	1.968252	9	.9344	.9589	2.095237	9	.9895	.9969	2.732519	7
9	6	.8909	.9371	1.906921	5	.9371	.9580	2.236064	9	.9832	.9944	2.581984	8
9	5	.8996	.9271	2.078693	5	.9271	.9720	2.093198	5	.9895	1	2.788864	8
9	4	.8322	.9021	1.805277	7	.9021	.9510	1.900286	7	.9790	1	2.596293	5
9	3	.8409	.9091	1.690305	5	.9091	.9545	1.999996	8	.9545	1	2.366426	7
9	2	.8909	1	2.068277	5	.8909	1	2.068277	5	.8909	1	2.068277	5
10	10	.8887	.9115	1.951796	9	.9319	.9595	2.247325	8	.9874	.9928	2.738606	6
10	9	.8798	.9010	2.057129	9	.9437	.9597	2.205804	9	.9893	.9942	2.809325	7
10	8	.8897	.9201	2.028367	5	.9409	.9569	2.333446	9	.9838	.9905	2.683280	7
10	7	.8707	.9107	1.913509	6	.9377	.9571	2.226730	5	.9838	.9938	2.671569	8
10	6	.8605	.9073	1.866660	6	.9292	.9580	2.088931	6	.9895	.9965	2.732514	9
10	5	.8911	.9281	1.936484	7	.9281	.9814	2.236064	6	.9814	.9930	2.561732	5
10	4	.8701	.9301	1.940216	8	.9301	.9650	2.049389	8	.9850	1	2.732519	6
10	3	.8776	.9301	1.828347	5	.9301	.9650	2.133068	9	.9650	1	2.497991	8
10	2	.8485	.9091	1.833026	7	.9091	1	2.190889	6	.9091	1	2.190889	6

$\theta = .25$ (continued) $\sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
15	15	.8899	.9104	2.148342	6	.9474	.9540	2.323785	8	.9874	.9904	2.788863	8
15	14	.8974	.9232	2.081021	7	.9409	.9519	2.374630	7	.9865	.9901	2.795149	8
15	13	.8843	.9130	2.011077	7	.9416	.9558	2.296813	7	.9896	.9924	2.844091	7
15	12	.8983	.9146	2.078457	7	.9341	.9508	2.215639	8	.9889	.9921	2.762508	7
15	11	.8882	.9015	1.984786	7	.9429	.9546	2.327569	7	.9880	.9928	2.778701	8
15	10	.8970	.9194	2.041237	7	.9472	.9566	2.288019	7	.9892	.9923	2.796294	8
20	20	.8936	.9053	2.080623	7	.9452	.9561	2.390451	7	.9884	.9903	2.912872	8
20	19	.8913	.9035	2.087496	7	.9479	.9506	2.399391	7	.9881	.9907	2.846872	8
20	18	.8966	.9027	2.064013	7	.9447	.9501	2.299964	7	.9889	.9908	2.910970	7
20	17	.8948	.9016	2.111073	7	.9497	.9556	2.429547	7	.9893	.9906	2.897456	8
20	16	.8996	.9068	2.103502	7	.9476	.9520	2.371703	8	.9888	.9908	2.868548	8
20	15	.8813	.9178	2.091644	7	.9427	.9524	2.330456	8	.9886	.9915	2.840562	8
25	25	.8981	.9022	2.089784	7	.9495	.9521	2.425355	7	.9897	.9906	2.946277	8
25	24	.8995	.9028	2.146663	7	.9489	.9542	2.444417	8	.9891	.9903	3.001710	8
25	23	.8967	.9017	2.099480	8	.9487	.9525	2.452055	7	.9895	.9904	2.948499	8
25	22	.8967	.9037	2.149519	7	.9461	.9503	2.446064	8	.9900	.9910	2.915005	8
25	21	.8967	.9013	2.072035	7	.9473	.9550	2.407483	7	.9892	.9901	2.919569	8
25	20	.8985	.9070	2.131319	7	.9484	.9556	2.449484	7	.9892	.9902	2.939728	8
30	30	.8974	.9077	2.143197	8	.9487	.9505	2.435992	8	.9895	.9906	3.038212	8
30	29	.8972	.9000	2.176025	8	.9469	.9507	2.479770	8	.9896	.9901	2.994507	8
30	28	.8983	.9012	2.125330	8	.9458	.9501	2.427483	7	.9886	.9900	2.943007	8
30	27	.8910	.9004	2.130333	7	.9483	.9505	2.473093	7	.9892	.9900	3.025780	8
30	26	.8959	.9022	2.095519	8	.9485	.9513	2.398618	7	.9898	.9906	2.985170	8
30	25	.8993	.9022	2.134530	8	.9473	.9508	2.482947	7	.9898	.9914	2.994556	8
35	35	.8967	.9012	2.151483	8	.9490	.9509	2.418963	8	.9894	.9902	2.988069	8
35	34	.8994	.9045	2.097262	8	.9486	.9500	2.439960	7	.9898	.9902	3.018046	8
35	33	.8976	.9008	2.153479	8	.9469	.9508	2.428471	8	.9899	.9903	3.026474	8
35	32	.8988	.9013	2.097488	8	.9491	.9508	2.433118	7	.9894	.9900	3.030095	8
35	31	.8975	.9001	2.126106	8	.9486	.9506	2.419558	8	.9899	.9904	3.003704	8
35	30	.8989	.9037	2.095047	8	.9491	.9506	2.418671	7	.9894	.9904	2.992335	8
40	40	.8974	.9035	2.121453	8	.9500	.9521	2.479114	7	.9896	.9901	3.006366	8
40	39	.8998	.9017	2.148612	8	.9489	.9503	2.440239	8	.9897	.9901	3.044987	8
40	38	.8998	.9017	2.113754	8	.9496	.9518	2.475986	7	.9898	.9901	3.034829	8
40	37	.8994	.9012	2.149215	8	.9488	.9501	2.422312	8	.9898	.9903	3.044672	8
40	36	.8985	.9012	2.122190	8	.9496	.9508	2.448950	7	.9896	.9901	2.986234	8
40	35	.8978	.9007	2.122915	8	.9492	.9504	2.424494	7	.9897	.9900	3.039490	8
45	45	.8995	.9027	2.151651	8	.9497	.9510	2.452755	8	.9898	.9901	3.043797	8
45	44	.8997	.9031	2.158836	8	.9496	.9508	2.472516	8	.9899	.9901	3.073079	8
45	43	.8985	.9002	2.143591	8	.9483	.9505	2.455378	8	.9900	.9903	3.027972	8
45	42	.8999	.9023	2.166723	8	.9486	.9515	2.470125	8	.9898	.9901	3.064728	8
45	41	.8998	.9018	2.138025	8	.9495	.9504	2.459195	7	.9899	.9903	3.021163	8
45	40	.8990	.9006	2.133925	8	.9497	.9507	2.462650	7	.9898	.9901	3.052642	8
50	50	.8987	.9044	2.182175	8	.9496	.9506	2.477698	8	.9898	.9905	3.075342	8
50	49	.8990	.9006	2.146054	8	.9476	.9502	2.480402	7	.9899	.9901	3.079273	8
50	48	.8987	.9000	2.135232	8	.9489	.9500	2.449984	7	.9898	.9901	3.062879	8
50	47	.8982	.9032	2.148523	8	.9485	.9503	2.467554	8	.9900	.9902	3.058764	8
50	46	.8990	.9002	2.175885	8	.9473	.9503	2.438462	8	.9899	.9901	3.059521	8
50	45	.8997	.9011	2.142386	8	.9494	.9502	2.491106	7	.9899	.9901	3.078716	8
100	100	.8996	.9001	2.160246	5	.9495	.9501	2.483003	8	.9897	.9901	3.103203	8
500	500	.9000	.9001	2.192262	6	.9500	.9500	2.522723	8	.9900	.9900	3.137082	8

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D

See $W_{M,N}$ for $\theta = 0$.

$\theta = .01$ (see $\theta = 0$ for smaller values of M)

100	100	.8984	.9010	2.789056	8	.9495	.9502	3.011905	5	.9899	.9900	3.496298	5
500	500	.8996	.9007	2.926527	5	.9495	.9503	3.178208	5	.9900	.9900	3.662334	5

$\theta = .05$ (see $\theta = 0$ for smaller values of M)

10	10	.8813	.9343	2.344035	8	.9343	.9522	2.581984	7	.9893	.9961	3.146266	6
10	9	.8731	.9034	2.205804	9	.9376	.9584	2.556737	7	.9884	.9939	3.002796	6
10	8	.8701	.9058	2.347866	9	.9358	.9569	2.439517	8	.9899	.9950	3.027148	7
10	7	.8976	.9284	2.265026	5	.9459	.9675	2.608386	8	.9826	.9905	2.886168	8
10	6	.8749	.9161	2.367453	5	.9423	.9685	2.472279	5	.9810	.9930	2.933331	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9860	.9960	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	.9900	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	1	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8925	.9191	2.477166	5	.9373	.9504	2.652067	9	.9861	.9914	3.098386	9
15	14	.8917	.9007	2.419673	7	.9406	.9505	2.657223	9	.9900	.9916	3.195054	9
15	13	.8823	.9041	2.406539	8	.9467	.9542	2.654931	9	.9866	.9902	3.100610	9
15	12	.8857	.9031	2.464744	7	.9455	.9535	2.598076	5	.9878	.9904	3.065335	9
15	11	.8968	.9161	2.391645	7	.9497	.9565	2.688571	9	.9859	.9901	3.109498	9
15	10	.8923	.9160	2.485337	7	.9440	.9502	2.545875	5	.9900	.9926	3.105167	8
20	20	.8987	.9070	2.542563	7	.9476	.9553	2.760259	5	.9872	.9903	3.186959	5
20	19	.8967	.9033	2.491825	7	.9449	.9516	2.749011	9	.9899	.9909	3.219319	5
20	18	.8901	.9021	2.532286	7	.9491	.9532	2.757078	9	.9899	.9909	3.253447	5
20	17	.8868	.9007	2.467197	8	.9475	.9512	2.745368	9	.9889	.9902	3.180658	5
20	16	.8833	.9016	2.448279	8	.9462	.9505	2.683274	9	.9891	.9904	3.174892	9
20	15	.8930	.9003	2.484279	7	.9493	.9610	2.788862	9	.9899	.9909	3.231544	5
25	25	.8874	.9015	2.576729	8	.9432	.9503	2.828423	5	.9899	.9915	3.311327	5
25	24	.8955	.9062	2.561975	8	.9487	.9538	2.799997	5	.9896	.9901	3.283810	5
25	23	.8982	.9040	2.563553	8	.9472	.9504	2.749647	5	.9893	.9906	3.250980	9
25	22	.8942	.9057	2.532098	8	.9484	.9507	2.787050	9	.9898	.9904	3.323183	5
25	21	.8965	.9009	2.586718	8	.9479	.9509	2.790875	5	.9892	.9901	3.253275	5
25	20	.8972	.9011	2.533955	8	.9489	.9532	2.789934	9	.9900	.9914	3.287308	5
30	30	.8868	.9096	2.581985	8	.9482	.9545	2.820314	5	.9898	.9909	3.286335	5
30	29	.8892	.9002	2.540980	8	.9496	.9512	2.786832	9	.9897	.9902	3.288573	5
30	28	.8996	.9023	2.573542	8	.9479	.9501	2.777412	5	.9899	.9904	3.278530	5
30	27	.8916	.9002	2.547258	8	.9498	.9535	2.813650	9	.9896	.9901	3.301188	5
30	26	.8971	.9013	2.561233	8	.9446	.9517	2.784569	5	.9892	.9900	3.248184	5
30	25	.8957	.9000	2.542741	8	.9500	.9540	2.793098	9	.9897	.9905	3.316624	5
35	35	.8986	.9048	2.621583	8	.9469	.9505	2.846371	5	.9899	.9905	3.359000	5
35	34	.8973	.9001	2.583537	8	.9478	.9514	2.829075	5	.9898	.9903	3.328848	5
35	33	.8984	.9023	2.639676	8	.9496	.9507	2.867230	5	.9895	.9901	3.324781	5
35	32	.8995	.9034	2.594921	8	.9496	.9518	2.837435	5	.9898	.9904	3.305329	5
35	31	.8983	.9017	2.581086	8	.9492	.9512	2.828540	5	.9897	.9903	3.309601	5
35	30	.8935	.9008	2.593088	8	.9498	.9511	2.832462	5	.9897	.9901	3.322165	5
40	40	.8998	.9028	2.653298	8	.9479	.9503	2.889983	5	.9897	.9901	3.355147	5
40	39	.8987	.9001	2.629184	8	.9488	.9500	2.869374	5	.9896	.9901	3.341112	5
40	38	.8986	.9026	2.623474	8	.9495	.9504	2.872474	5	.9897	.9902	3.350037	5
40	37	.8989	.9014	2.635697	8	.9489	.9520	2.873666	5	.9896	.9903	3.335859	5
40	36	.8985	.9000	2.610223	8	.9499	.9512	2.855678	5	.9896	.9900	3.337409	5
40	35	.8988	.9004	2.603861	8	.9499	.9508	2.872336	5	.9899	.9902	3.339556	5
45	45	.8980	.9011	2.658352	8	.9489	.9527	2.908374	5	.9900	.9904	3.380613	5
45	44	.8981	.9010	2.657536	8	.9492	.9502	2.888121	5	.9899	.9902	3.368466	5
45	43	.8987	.9000	2.644794	8	.9494	.9506	2.871076	5	.9898	.9901	3.405520	5
45	42	.8996	.9005	2.663692	8	.9494	.9500	2.879867	5	.9899	.9901	3.359984	5
45	41	.8963	.9027	2.634930	8	.9493	.9502	2.891464	5	.9899	.9900	3.366925	5
45	40	.8995	.9015	2.661453	8	.9494	.9502	2.894621	5	.9899	.9902	3.370232	5

θ = 0.05 / .1 $\sqrt{MN/(M+N)}\,\tilde{W}_{M,N}$ 50 - 500 / 5 - 9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .05 (continued)							
50	50	.8991	.9025	2.666661	8	.9489	.9508	2.896826	5	.9897	.9902	3.379627	5
50	49	.8998	.9021	2.653292	8	.9499	.9503	2.914943	5	.9900	.9901	3.377394	5
50	48	.8998	.9021	2.644345	8	.9484	.9519	2.891825	5	.9899	.9901	3.385744	5
50	47	.8966	.9010	2.658960	8	.9495	.9507	2.873315	5	.9899	.9901	3.366967	5
50	46	.8941	.9004	2.635620	8	.9495	.9503	2.884155	5	.9900	.9901	3.360853	5
50	45	.8998	.9014	2.640611	8	.9497	.9514	2.897347	5	.9900	.9901	3.375884	5
100	100	.8997	.9003	2.736859	8	.9497	.9501	2.981947	5	.9899	.9900	3.472314	5
500	500	.9000	.9000	2.838107	5	.9499	.9500	3.098615	5	.9900	.9900	3.610451	5
						θ = .1 (see θ = 0 for smaller values of M)							
5	5	.8333	.9524	2.070193	9	.8333	.9524	2.070193	9	.9524	1	2.581984	7
5	4	.7619	.9206	1.897366	5	.9206	1	2.399993	8	.9206	1	2.399993	8
5	3	.8571	1	2.190886	5	.8571	1	2.190886	5	.8571	1	2.190886	5
5	2	.7143	1	1.932183	8	.7143	1	1.932183	8	.7143	1	1.932183	8
6	6	.8874	.9394	2.309397	9	.9394	.9848	2.449480	9	.9848	1	2.927695	7
6	5	.8225	.9091	2.100528	5	.9091	.9740	2.288684	5	.9740	1	2.763848	9
6	4	.8571	.9524	2.108181	6	.8571	.9524	2.108181	6	.9524	1	2.581984	5
6	3	.7619	.9048	1.897361	9	.9048	1	2.371701	7	.9048	1	2.371701	7
6	2	.7857	1	2.108181	5	.7857	1	2.108181	5	.7857	1	2.108181	5
7	7	.8159	.9138	2.160243	6	.9138	.9586	2.366431	5	.9790	.9953	2.788861	9
7	6	.8741	.9324	2.225394	6	.9324	.9674	2.489542	5	.9674	.9918	2.638991	5
7	5	.8838	.9470	2.276409	7	.9470	.9848	2.474358	6	.9848	1	2.927695	5
7	4	.7879	.9091	2.013658	9	.9091	.9697	2.288685	8	.9697	1	2.746424	6
7	3	.8333	.9333	2.070193	5	.9333	1	2.535457	8	.9333	1	2.535457	8
7	2	.8333	1	2.267786	6	.8333	1	2.267786	6	.8333	1	2.267786	6
8	8	.8812	.9246	2.309397	6	.9246	.9681	2.519758	6	.9855	.9930	2.999995	5
8	7	.8819	.9192	2.324172	7	.9192	.9518	2.351452	6	.9888	.9975	2.958038	5
8	6	.8741	.9241	2.160239	8	.9241	.9574	2.415222	7	.9814	.9953	2.806237	6
8	5	.8741	.9207	2.253461	8	.9207	.9674	2.433745	8	.9674	.9907	2.638987	7
8	4	.8424	.9394	2.165058	5	.9394	.9798	2.449480	9	.9798	1	2.898269	7
8	3	.8788	.9515	2.224858	6	.8788	.9515	2.224858	6	.9515	1	2.686768	5
8	2	.8667	1	2.415227	7	.8667	1	2.415227	7	.8667	1	2.415227	7
9	9	.8854	.9027	2.357021	7	.9027	.9531	2.417466	7	.9887	.9951	2.999994	5
9	8	.8984	.9301	2.265026	8	.9301	.9531	2.509241	7	.9825	.9917	2.870957	6
9	7	.8916	.9213	2.378350	8	.9497	.9717	2.519761	7	.9851	.9937	2.984126	6
9	6	.8242	.9161	2.236064	9	.9161	.9520	2.324167	9	.9888	.9972	2.958034	7
9	5	.8541	.9141	2.093198	5	.9441	.9790	2.577578	9	.9790	.9940	2.788864	8
9	4	.8797	.9580	2.303545	6	.8797	.9580	2.303545	6	.9860	1	3.040460	9
9	3	.8182	.9091	1.999996	8	.9091	.9636	2.366426	7	.9636	1	2.828422	6
9	2	.8909	1	2.553138	9	.8909	1	2.553138	9	.8909	1	2.553138	9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8813	.9343	2.344035	8	.9343	.9522	2.581984	7	.9893	.9961	3.146266	6
10	9	.8731	.9034	2.205804	9	.9376	.9584	2.556737	7	.9884	.9939	3.002796	6
10	8	.8701	.9058	2.347866	9	.9358	.9569	2.439517	8	.9899	.9950	3.027148	7
10	7	.8976	.9284	2.265026	5	.9459	.9675	2.608386	8	.9826	.9905	2.886168	8
10	6	.8749	.9161	2.367453	5	.9423	.9685	2.472279	5	.9810	.9930	2.933331	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9860	.9960	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	1	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	1	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8930	.9119	2.449489	5	.9424	.9592	2.652069	9	.9898	.9922	3.098378	9
15	14	.8998	.9095	2.419675	7	.9440	.9540	2.657224	9	.9897	.9923	3.174198	9
15	13	.8967	.9102	2.306544	8	.9477	.9539	2.652520	8	.9885	.9911	3.066356	9
15	12	.8952	.9237	2.464748	7	.9449	.9502	2.497636	5	.9895	.9922	3.065332	8
15	11	.8920	.9057	2.342722	7	.9435	.9529	2.650380	8	.9869	.9909	3.075460	9
15	10	.8767	.9008	2.294152	8	.9441	.9566	2.545867	8	.9894	.9920	3.061858	9
20	20	.8865	.9047	2.478742	7	.9404	.9502	2.656844	9	.9893	.9907	3.186955	9
20	19	.8952	.9009	2.471597	7	.9471	.9501	2.728069	9	.9893	.9914	3.171416	5
20	18	.8969	.9046	2.493681	7	.9444	.9521	2.714713	9	.9896	.9905	3.248922	9
20	17	.8938	.9078	2.467200	8	.9434	.9502	2.708933	9	.9897	.9910	3.179445	9
20	16	.8900	.9067	2.399992	8	.9458	.9561	2.662558	9	.9899	.9912	3.174892	9
20	15	.8876	.9047	2.456168	8	.9465	.9505	2.732840	9	.9899	.9915	3.155242	5
25	25	.8982	.9005	2.545583	7	.9456	.9526	2.777459	5	.9885	.9902	3.204940	5
25	24	.8999	.9036	2.525614	7	.9490	.9526	2.763904	9	.9897	.9908	3.253304	5
25	23	.8974	.9016	2.504570	8	.9490	.9548	2.745811	9	.9900	.9908	3.213438	5
25	22	.8967	.9007	2.475997	8	.9477	.9502	2.750869	9	.9898	.9908	3.170087	5
25	21	.8992	.9037	2.466327	7	.9490	.9524	2.675115	9	.9898	.9906	3.244110	9
25	20	.8993	.9119	2.487467	8	.9495	.9526	2.757707	9	.9896	.9905	3.203243	5
30	30	.8960	.9120	2.581987	8	.9494	.9532	2.817180	5	.9895	.9901	3.279564	5
30	29	.8964	.9074	2.540978	8	.9480	.9533	2.770841	5	.9896	.9901	3.262785	5
30	28	.8994	.9012	2.545237	8	.9497	.9519	2.777407	5	.9896	.9902	3.262735	5
30	27	.8967	.9002	2.515930	8	.9495	.9511	2.792786	9	.9898	.9904	3.293592	5
30	26	.8986	.9013	2.548157	8	.9473	.9564	2.784569	5	.9898	.9906	3.248184	5
30	25	.8971	.9000	2.533617	8	.9498	.9521	2.780729	9	.9895	.9901	3.226618	5
35	35	.8931	.9033	2.561736	8	.9491	.9521	2.799167	5	.9895	.9903	3.308106	5
35	34	.8989	.9022	2.563604	8	.9498	.9510	2.808865	5	.9897	.9901	3.294606	5
35	33	.8985	.9007	2.530567	8	.9493	.9511	2.819228	9	.9898	.9901	3.297518	5
35	32	.8984	.9004	2.562937	8	.9487	.9512	2.808550	5	.9897	.9901	3.288755	5
35	31	.8966	.9007	2.546339	8	.9495	.9509	2.793386	9	.9895	.9901	3.249343	5
35	30	.8990	.9012	2.550915	7	.9452	.9502	2.796340	9	.9898	.9902	3.305529	5
40	40	.8982	.9033	2.581983	8	.9497	.9517	2.864459	9	.9894	.9901	3.341867	5
40	39	.8995	.9009	2.594046	8	.9499	.9508	2.844281	5	.9898	.9901	3.326882	5
40	38	.8970	.9025	2.588708	8	.9495	.9518	2.837070	5	.9897	.9902	3.301735	5
40	37	.8991	.9004	2.602679	8	.9495	.9510	2.842478	5	.9898	.9905	3.323741	5
40	36	.8992	.9010	2.574154	8	.9492	.9507	2.826069	5	.9899	.9903	3.332631	5
40	35	.8989	.9001	2.598192	8	.9493	.9511	2.834733	5	.9898	.9902	3.314846	5

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = .1$ (continued)							
45	45	.8979	.9016	2.588728	8	.9495	.9517	2.860383	5	.9899	.9904	3.354102	5
45	44	.8989	.9007	2.606463	8	.9493	.9504	2.865445	5	.9895	.9901	3.318955	5
45	43	.8977	.9016	2.611430	8	.9497	.9506	2.852728	5	.9899	.9902	3.326269	5
45	42	.8986	.9007	2.598801	8	.9481	.9511	2.867537	5	.9899	.9902	3.351490	5
45	41	.8993	.9006	2.581444	8	.9494	.9502	2.829832	5	.9898	.9903	3.318489	5
45	40	.8983	.9052	2.603958	8	.9498	.9506	2.857039	5	.9898	.9901	3.331101	5
50	50	.8990	.9010	2.625854	8	.9495	.9511	2.881951	5	.9896	.9900	3.377599	5
50	49	.8977	.9014	2.634608	8	.9498	.9506	2.885253	5	.9895	.9900	3.340310	5
50	48	.8995	.9009	2.624761	8	.9498	.9508	2.867625	5	.9900	.9903	3.376226	5
50	47	.8997	.9009	2.623080	8	.9478	.9508	2.862856	5	.9900	.9902	3.357593	5
50	46	.8998	.9008	2.635529	8	.9492	.9505	2.861924	5	.9895	.9901	3.340571	5
50	45	.8963	.9030	2.607892	8	.9493	.9500	2.861921	5	.9897	.9901	3.355481	5
100	100	.8998	.9004	2.693605	8	.9500	.9512	2.964997	5	.9900	.9901	3.459950	5
500	500	.9000	.9000	2.742831	5	.9500	.9500	3.012909	5	.9900	.9900	3.545579	5
						$\theta = .25$							
2	2	.6667	1	1.999992	7	.6667	1	1.999992	7	.6667	1	1.999992	7
3	3	.6000	1	1.732044	6	.6000	1	1.732044	6	.6000	1	1.732044	6
3	2	.4000	1	1.490709	9	.4000	1	1.490709	9	.4000	1	1.490709	9
4	4	.8571	1	2.190887	6	.8571	1	2.190887	6	.8571	1	2.190887	6
4	3	.7714	1	1.984305	8	.7714	1	1.984305	8	.7714	1	1.984305	8
4	2	.6000	1	1.732044	6	.6000	1	1.732044	6	.6000	1	1.732044	6
5	5	.8333	.9524	2.070193	9	.8333	.9524	2.070193	9	.9524	1	2.581984	7
5	4	.7619	.9206	1.897366	5	.9206	1	2.399993	8	.9206	1	2.399993	8
5	3	.8571	1	2.190886	5	.8571	1	2.190886	5	.8571	1	2.190886	5
5	2	.7143	1	1.932183	8	.7143	1	1.932183	8	.7143	1	1.932183	8
6	6	.8874	.9394	2.309397	9	.9394	.9848	2.449480	9	.9848	1	2.927695	7
6	5	.8225	.9091	2.100528	5	.9091	.9740	2.288684	5	.9740	1	2.763848	9
6	4	.8571	.9524	2.108181	6	.8571	.9524	2.108181	6	.9524	1	2.581984	5
6	3	.7619	.9048	1.897361	9	.9048	1	2.371701	7	.9048	1	2.371701	7
6	2	.7857	1	2.108181	5	.7857	1	2.108181	5	.7857	1	2.108181	5
7	7	.8648	.9301	2.160243	6	.9301	.9790	2.366431	5	.9790	1	2.788861	9
7	6	.8042	.9021	1.974462	7	.9021	.9674	2.225394	6	.9674	1	2.638991	5
7	5	.8586	.9470	2.070189	7	.9470	1	2.474358	6	.9470	1	2.474358	6
7	4	.7879	.9091	1.895211	9	.9091	1	2.288685	8	.9091	1	2.288685	8
7	3	.8333	1	2.070193	5	.8333	1	2.070193	5	.8333	1	2.070193	5
7	2	.6667	1	1.792838	8	.6667	1	1.792838	8	.6667	1	1.792838	8
8	8	.8486	.9008	2.065590	7	.9464	.9744	2.519758	6	.9744	.9930	2.696795	5
8	7	.8628	.9192	2.184653	7	.9192	.9627	2.351452	6	.9888	1	2.958038	5
8	6	.8741	.9441	2.160339	8	.9441	.9814	2.415222	7	.9814	1	2.806237	6
8	5	.7964	.9130	1.935261	5	.9130	.9674	2.253461	8	.9674	1	2.638987	7
8	4	.8586	.9394	2.070193	5	.9394	1	2.449480	9	.9394	1	2.449480	9
8	3	.8788	1	2.224858	6	.8788	1	2.224858	6	.8788	1	2.224858	6
8	2	.7333	1	1.936482	9	.7333	1	1.936482	9	.7333	1	1.936482	9
9	9	.8640	.9070	2.267783	7	.9367	.9613	2.417466	7	.9798	.9910	2.846046	6
9	8	.8638	.9099	2.176080	8	.9445	.9687	2.509241	7	.9864	.9963	2.870957	6
9	7	.8689	.9178	2.095237	9	.9497	.9790	2.519761	7	.9790	.9937	2.732519	7
9	6	.8741	.9161	2.236064	9	.9161	.9664	2.324167	9	.9888	1	2.958034	7
9	5	.8541	.9441	2.093198	5	.9441	.9790	2.415220	9	.9790	1	2.788864	8
9	4	.8042	.9021	1.900286	7	.9021	.9580	2.225390	6	.9580	1	2.596293	5
9	3	.8182	.9091	1.999996	8	.9091	1	2.366426	7	.9091	1	2.366426	7
9	2	.7818	1	2.068277	5	.7818	1	2.068277	5	.7818	1	2.068277	5

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8638	.9191	2.247325	8	.9434	.9617	2.581984	7	.9856	.9927	2.927695	6
10	9	.8875	.9193	2.205804	9	.9420	.9631	2.518472	7	.9884	.9952	3.002796	6
10	8	.8818	.9138	2.333446	9	.9445	.9675	2.439517	8	.9809	.9925	2.846045	7
10	7	.8754	.9142	2.226730	5	.9488	.9675	2.547718	9	.9877	.9963	2.886168	8
10	6	.8585	.9161	2.088931	6	.9423	.9790	2.472279	5	.9790	.9930	2.732514	9
10	5	.8561	.9627	2.236064	6	.8561	.9627	2.236064	6	.9860	1	2.927695	9
10	4	.8601	.9301	2.049389	8	.9301	.9700	2.366431	7	.9700	1	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	1	2.497991	8	.9301	1	2.497991	8
10	2	.8182	1	2.190889	6	.8182	1	2.190889	6	.8182	1	2.190889	6
15	15	.8949	.9082	2.323786	6	.9398	.9507	2.556032	9	.9889	.9925	3.021657	9
15	14	.8821	.9038	2.374627	7	.9459	.9553	2.535284	9	.9893	.9928	2.992506	9
15	13	.8834	.9116	2.296810	7	.9399	.9520	2.572581	8	.9894	.9932	3.066351	9
15	12	.8686	.9016	2.215639	8	.9443	.9610	2.497632	8	.9842	.9902	2.928254	9
15	11	.8860	.9093	2.327573	7	.9442	.9582	2.450070	9	.9898	.9945	2.935794	8
15	10	.8945	.9134	2.288015	7	.9498	.9626	2.545869	8	.9846	.9912	2.800559	9
20	20	.8908	.9122	2.390453	6	.9487	.9533	2.651966	9	.9882	.9915	3.162272	9
20	19	.8959	.9013	2.399386	8	.9404	.9517	2.595445	9	.9882	.9902	3.092127	9
20	18	.8895	.9003	2.299963	8	.9498	.9556	2.599142	8	.9883	.9910	3.019933	9
20	17	.8994	.9112	2.429542	7	.9477	.9527	2.614971	9	.9893	.9916	3.121657	9
20	16	.8954	.9041	2.371707	8	.9427	.9512	2.624997	9	.9882	.9902	3.060679	9
20	15	.8857	.9049	2.330453	7	.9486	.9588	2.561737	9	.9884	.9908	3.014382	9
25	25	.8991	.9043	2.425355	7	.9482	.9526	2.649064	9	.9896	.9907	3.149702	9
25	24	.8981	.9086	2.444417	7	.9487	.9511	2.718331	9	.9890	.9901	3.112130	5
25	23	.8976	.9051	2.452055	7	.9486	.9515	2.686057	9	.9892	.9902	3.169415	9
25	22	.8924	.9008	2.446065	7	.9462	.9501	2.669665	9	.9894	.9909	3.129822	9
25	21	.8949	.9101	2.407484	7	.9456	.9533	2.633505	9	.9884	.9902	3.065758	9
25	20	.8969	.9112	2.449485	7	.9474	.9514	2.605796	9	.9896	.9910	3.134101	9
30	30	.8976	.9011	2.435991	8	.9490	.9510	2.738609	9	.9895	.9902	3.214797	5
30	29	.8939	.9016	2.479766	8	.9480	.9501	2.741594	9	.9900	.9908	3.213316	5
30	28	.8919	.9004	2.427485	8	.9481	.9514	2.694307	9	.9884	.9905	3.153223	9
30	27	.8967	.9010	2.473089	7	.9480	.9507	2.703231	9	.9900	.9907	3.187358	9
30	26	.8972	.9027	2.398616	8	.9471	.9500	2.647935	9	.9893	.9902	3.185199	9
30	25	.8948	.9017	2.482944	7	.9471	.9504	2.708011	9	.9898	.9906	3.155836	9
35	35	.8981	.9018	2.418961	8	.9487	.9504	2.700998	9	.9899	.9910	3.213693	9
35	34	.8973	.9001	2.439956	8	.9487	.9518	2.713050	9	.9896	.9901	3.235110	9
35	33	.8939	.9018	2.428476	8	.9497	.9517	2.669043	9	.9895	.9901	3.174093	9
35	32	.8984	.9017	2.433121	7	.9495	.9514	2.664516	9	.9897	.9904	3.221699	9
35	31	.8972	.9014	2.419553	8	.9493	.9515	2.688038	9	.9899	.9905	3.206271	9
35	30	.8982	.9014	2.418675	8	.9479	.9505	2.700287	9	.9897	.9906	3.182737	9
40	40	.8962	.9001	2.466618	8	.9494	.9524	2.727585	9	.9894	.9900	3.212875	9
40	39	.8980	.9008	2.440241	8	.9493	.9509	2.733963	9	.9897	.9901	3.273558	9
40	38	.8992	.9036	2.475984	8	.9489	.9503	2.723065	9	.9895	.9901	3.241824	9
40	37	.8978	.9002	2.422312	8	.9483	.9502	2.714061	9	.9899	.9907	3.260459	9
40	36	.8993	.9016	2.448949	8	.9495	.9510	2.736411	9	.9899	.9907	3.219251	9
40	35	.8984	.9010	2.424496	8	.9490	.9532	2.704213	9	.9895	.9900	3.192578	9
45	45	.8994	.9021	2.452761	8	.9495	.9515	2.751529	9	.9899	.9902	3.255902	9
45	44	.8992	.9016	2.472518	8	.9498	.9522	2.710952	9	.9897	.9900	3.279751	9
45	43	.8966	.9012	2.455377	8	.9491	.9502	2.744602	9	.9899	.9902	3.231605	5
45	42	.8972	.9031	2.470127	8	.9499	.9511	2.716575	9	.9897	.9901	3.262286	9
45	41	.8990	.9009	2.459192	8	.9496	.9507	2.741890	9	.9897	.9900	3.238484	9
45	40	.8995	.9016	2.462650	8	.9494	.9515	2.716326	9	.9897	.9901	3.259598	9
50	50	.8994	.9013	2.477696	8	.9490	.9502	2.735764	9	.9895	.9901	3.270847	9
50	49	.8953	.9005	2.480403	8	.9490	.9502	2.755113	9	.9896	.9901	3.268895	9
50	48	.8978	.9002	2.449985	8	.9496	.9507	2.746549	9	.9897	.9904	3.260541	9
50	47	.8971	.9006	2.467553	8	.9499	.9510	2.757227	9	.9900	.9903	3.304179	9
50	46	.8947	.9007	2.438465	8	.9495	.9505	2.744795	9	.9894	.9901	3.255123	9
50	45	.8989	.9006	2.491113	8	.9480	.9510	2.735209	9	.9897	.9903	3.264504	9
100	100	.8990	.9002	2.483005	9	.9495	.9502	2.775122	9	.9899	.9901	3.313039	9
500	500	.9000	.9001	2.522724	5	.9500	.9500	2.802861	9	.9900	.9900	3.365530	5

Appendix.

The purpose of this appendix is to summarize the algebraic results which we used in the preceding chapters. All proofs are straightforward verifications of the definitions. Hence, we give only some hints and leave the details to the reader. A more general approach including Eulerian polynomials can be found in [11]. The "Finite Operator Calculus" of G.-C. Rota, D. Kahaner and A. Odlyzko [14] is the fundament of the whole theory.

Let $\underset{\sim}{P}$ be the algebra of polynomials over a field K with characteristic zero. In our rank test applications K always equals $\mathbb{Z}$, for the order tests choose $K = \mathbb{R}$. We will deal with linear operators $\underset{\sim}{P} \to \underset{\sim}{P}$ only, and omit the word "linear" in the sequel. For all $a \in K$ the shift operator is denoted by E^a: $p(x) \mapsto p(x+a)$. An operator Q on $\underset{\sim}{P}$ is a delta operator, if

Q is shift invariant: $QE^a = E^aQ \quad \forall a \in K$, and

Qx is a non-zero constant.

The derivative operator D is a delta operator if $K = \mathbb{R}$, and the following properties show how Q generalizes D:

(A.1) $\qquad Qa = 0$ for every constant a [14,p.687]

(A.2) $\qquad \deg(Qp) = \deg(p)-1$ for each $p \in \underset{\sim}{P}$ with $\deg(p) \geq 1$ [14,p.687] .

Hence, the kernel of Q consists only of the constant polynomials. A sequence of polynomials $(s_n)_{n \in \mathbb{N}_0}$ is a Sheffer sequence for Q, if

(A.3) $\qquad s_0$ is a non-zero constant

(A.4) $\qquad Qs_n = s_{n-1}$ for all $n \geq 1$.

We make the convention $s_n = 0$ if $n < 0$. For instance, $(x^n/n!)$ is a Sheffer sequence for D.

<u>Lemma A.1</u>: If (s_n) is a Sheffer sequence for Q then $\deg(s_n) = n$.

<u>Proof</u>: (A.1)-(A.4) ■

<u>Lemma A.2</u>: If (s_n) and (t_n) are both Sheffer sequences for Q with the property

$$s_n(\nu_n) = t_n(\nu_n)$$

for a given sequence (ν_n) in K, then the two sequences are equal.

<u>Proof</u>: Induction over n. Use $\ker(Q)$ = constant functions ■

(s_n) has <u>roots</u> in ν: $\mathbb{N}_0 \to K$, say, if $s_n(\nu_n) = \delta_{0,n}$ for all $n \in \mathbb{N}_0$. The Sheffer sequence for Q with roots in 0 is called the <u>basic sequence</u> for Q and always denoted by (q_n). Obviously,

(A.5) $\qquad (x^n/n!)$ is the basic sequence for D .

It is easy to verify that

(A.6) $\left(\binom{x+n-1}{n}\right)_{n\in\mathbb{N}_0}$ is the basic sequence for $\nabla = I-E^{-1}$.

More examples can be found in [14].

Immediately from the shift-invariance follows: If (s_n) is a Sheffer sequence for Q with roots in ν, then $(E^a s_n)$ is a Sheffer sequence for Q with roots in ν-a.

Deeper than all the other results in this appendix is the following

<u>Lemma A.3</u>: If $\nu(n) = an+b$ $(a,b\in K)$, then

$$s_n(x) := (x-an-b)(x-b)^{-1}q_n(x-b)$$

defines the Sheffer sequence for Q with roots in ν.
(For $n = 0$ we have to define $\frac{0}{0} = 1$.)
<u>Proof</u>: See [14, p. 702] ∎

Now we come to a representation theorem for Sheffer sequences with roots in

$$\nu(i) := \begin{cases} \varphi(i) & \forall 0 \le i \le L \\ ci+d & \forall i > L , \end{cases}$$

where $L \in \mathbb{N}_0$; $c,d\in K$ and $\varphi: \mathbb{N}_0 \to R$ arbitrary.

<u>Theorem A.1:</u> If (s_n) is the Sheffer sequence for Q with roots in ν as above, then

(A.7) $$s_n(x) = \sum_{i=0}^{L} s_i(ci+d)(x-cn-d)(x-ci-d)^{-1}q_{n-i}(x-ci-d) \quad \forall n \in \mathbb{N}_0 .$$

Proof: Check recurrence and side conditions, using lemma A.3 ∎

Corollary A.1: (Binomial Theorem). If (s_n) is the Sheffer- and (q_n) the basic sequence for Q, then

$$s_n(x+y) = \sum_{i=0}^{n} s_i(y)q_{n-i}(x) \quad \forall n \in \mathbb{N}_0 .$$

Proof: Choose $c = 0$, $d = y$ and $L = \infty$ in (A.7) ∎

Avoiding alternating summation in (A.7), it may be sometimes preferable to use the "outside method" (a term, introduced by J.L. Hodges (1957)):

$$\text{(A.8)} \qquad s_n(x) = r_n(x) - \sum_{i=L+1}^{n} r_i(ci+d)(x-cn-d)(x-ci-d)^{-1}q_{n-i}(x-ci-d)$$

where (r_n) is the Sheffer sequence for Q with roots in φ (follows by summation over all $i = 0,\ldots,n$ in (A.7)).

Repeated use of (A.7) yields a representation of the Sheffer sequence (s_n) for Q with roots in the piecewise affine function

$$\nu(i) := ia_j+b_j \quad \forall L_j < i \leq L_{j+1} ,$$

where $-1 = L_0 < L_1 < \cdots$, each L_j integer, and $a_j,b_j \in K$ for all $j \in \mathbb{N}_0$. Then for all $L_j < n \leq L_{j+1}$

$$\text{(A.9)} \qquad s_n(x) = \sum_{k_j=0}^{L_j} \cdots \sum_{k_1=0}^{L_1} p_j(x)p_{j-1}(\nu_j(k_j))\cdot\ldots\cdot p_0(\nu_1(k_1)) ,$$

if

$$p_i(x) = \frac{x-\nu_i(k_{i+1})}{x-\nu_i(k_i)}\, q_{k_{i+1}-k_i}(x-\nu_i(k_i)) ,$$

where $k_0 := 0$ and $k_{j+1} := n$. Because of its importance we explicitly write down the special case of (A.9) where

$$\nu(i) = \begin{cases} ia+b & \forall i = 0,\dots,L \\ ic+d & \forall i > L . \end{cases}$$

Then

$$\text{(A.10)} \qquad s_n(x) = \sum_{i=0}^{L} \frac{i(c-a)+d-b}{ic+d-b}\, q_i(ci+d-b)\, \frac{x-cn-d}{x-ci-d}\, q_{n-i}(x-ic-d) .$$

For $n \le L$, the r.h.s. equals $(x-an-b)(x-b)^{-1}q_n(x-b)$ by lemma A.3.

Now we assume that K is completely ordered. Let $\mu\colon \mathbb{N}_0 \to K$ be a non decreasing function and $(t_{n,i})_{n,i \in \mathbb{N}_0}$ be a double sequence in $\underset{\sim}{P}$ with the properties

$$\text{(A.11)} \qquad \begin{aligned} t_{n,i}(\mu_i) &= t_{n,i+1}(\mu_i) && \forall 0 \le i \le r(n) := \min\{m \in \mathbb{N}_0 \mid \mu(m)=\mu(n)\}, \\ t_{n,i} &= 0 && \forall i > r(n) . \end{aligned}$$

Define an associated sequence (f_n) to $(t_{n,i})$ by

$$\text{(A.12)} \qquad f_n(x) := t_{n,i}(x) \quad \forall \mu_{i-1} < x \le \mu_i \quad (\mu_{-1} := -\infty) .$$

We call (f_n) a <u>μ-Sheffer sequence</u>, if $\left(t_{m+n,r(m)}\right)_{n \in \mathbb{N}_0}$ is a Sheffer

sequence for all $m \in \mathbb{N}_0$. From corollary A.1 we get a first representation of $f_n(x)$:

$$\text{(A.13)} \quad f_n(x) = \sum_{k=i}^{n} f_k(y) q_{n-k}(x-y) \quad \text{if} \quad x,y \in [\mu_{i-1}, \mu_i] .$$

If (f_n) has roots in ν, i.e.

$$\text{(A.14)} \qquad f_n(\nu_n) = \delta_{0,n} \quad \forall n \in \mathbb{N}_0 ,$$

then any value $f_n(z)$ can be computed from (A.13) by stepping through all the intervals $[\mu_j, \mu_{j+1}]$ until z is enclosed. We give only a brief description of this trivial algorithm:

<u>Algorithm A.1</u>. Assume $f_{r(j)}(\mu_j), \ldots, f_i(\mu_j)$ are already computed such that $j \leq n$ and $\nu_{i+1} > \mu_j$

a) If $\nu_{i+1} < \mu_{j+1}$ then define $x := \nu_{i+1}$, $y := \mu_j$, and compute $f_{r(j)}(\nu_{i+1}), \ldots, f_i(\nu_{i+1})$ from (A.13). Of course, $f_{i+1}(\nu_{i+1}) = 0$. Therefore, the i-index increased by one, and it increases again if ν_{i+2} lies also in the same interval (define $x = \nu_{i+2}$ and $y = \nu_{i+1}$). Finally a k is reached such that $\mu(j) < \nu_k < \mu_{j+1} < \nu_{k+1}$ (the case $\nu_k = \mu_{j+1}$ is left to the reader). Then choose $x := \mu_{j+1}$, $y := \nu_k$, and compute $f_{r(j)}(\mu_{j+1}, \ldots, f_k(\mu_{j+1})$ from (A.13). Now we are in the same situation as in the beginning.

b) If $\nu_{i+1} > \mu_{j+1} > \mu_j$ then define $x := \mu_{j+1}$, $y := \mu_j$, and compute $f_{r(j)}(\mu_{j+1}, \ldots, f_k(\mu_{j+1})$ from (A.13). Again, we are in the same situation as in the beginning.

c) If $\mu_j = \mu_{j+1}$ increase j by one.

In special cases this algorithm can be simplified.

A one dimensional recursion can be obtained from

Theorem A.2: Let (f_n) be a μ-Sheffer sequence for Q (with basic sequence (q_n)). If (f_n) is associated to $(t_{n,i})$ then

$$\text{(A.15)} \qquad t_{n,i}(x) = \sum_{k=i}^{n} f_k(\mu_k) q_{n-k}(x-\mu_k) \quad \text{for all } n \in \mathbb{N}_0 \text{ and } i = 0,\ldots,n .$$

Proof: Verify side conditions (A.11).

See [26, theorem 4.1] for a general version of this theorem. The announced one dimensional recursion follows, when we write (A.15) as

$$\text{(A.16)} \qquad f_n(x) = \Sigma' f_k(\mu_k) q_{n-k}(x-\mu_k) \quad \text{for all } n \in \mathbb{N}_0 ,$$

where the summation runs over all k such that $\mu_k > x$. Thus, a system of equations for the unknown $f_k(\mu_k)$ is obtained, if only one value $f_n(\nu_n)$ with $\nu_n \leq \mu_n$ is known for each n. By Cramer's rule, $f_n(\mu_n)$ can be expressed as a determinant:

Corollary A.2: If (f_n) is a μ-Sheffer- and (q_n) the basic sequence for Q, then

$$f_n(\mu_n) = \det(\alpha_{i,j})_{i,j=1,\ldots,n+1} , \quad \text{where}$$

$$\alpha_{i,j} = \begin{cases} q_{i-j}((\nu_{i-1}-\mu_{j-1})_-) & \forall j = 1,\ldots,n \\ f_{i-1}(\nu_{i-1}) & \text{if } j = n+1 , \end{cases}$$

for any $\nu \leq \mu$. If, in addition, (f_n) has roots in ν, then

$$\text{(A.17)} \qquad f_n(\mu_n) = (-1)^n \det(q_{i+1-j}((\nu_i-\mu_{j-1})_-))_{i,j=1,\ldots,n} .$$

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A Kolmogorov-Smirnov statistic with the weight factor $F(x)(1-F(x))$ is called "variance-weighted". We give tables of the exact significance points for various one and two sample cases.

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