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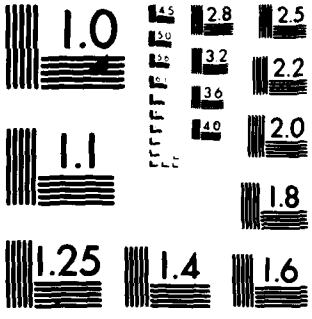

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Recently, a method for generating an ARMA spectral estimator model which possessed superresolution performance was developed, [2]. This method entailed minimizing a weighted quadratic functional of a set of "basic error terms". An issue which remained to be resolved at that time was the selection of the weighting matrix that characterized the functional being minimized. A weighting matrix selection procedure has recently been developed and is ~~herin~~ reported, [8]. This procedure has typically yielded an improvement in spectral estimation performance.

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A SUPERRESOLUTION METHOD OF ARMA SPECTRAL ESTIMATION*

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ABSTRACT

Recently, a method for generating an ARMA spectral estimator model which possessed super-resolution performance was developed [2]. This method entailed minimizing a weighted quadratic functional of a set of "basic error terms." An issue which remained to be resolved at that time was the selection of the weighting matrix that characterized the functional being minimized. A weighting matrix selection procedure has recently been developed and is herein reported [8]. This procedure has typically yielded an improvement in spectral estimation performance.

I. INTRODUCTION

In this paper we shall be concerned with the task of estimating the power spectral density of a *zero mean, wide sense stationary random time series* $\{x(n)\}$ from a finite set of observations. To this end, knowledge of the time series' underlying autocorrelation sequence as formally defined by

$$r_x(n) = E\{x(n+k)\ x^*(k)\} \tag{1}$$

conveys all the information required. Here, $E\{\ \}$ denotes the expected value operators and * denotes the operation of complex conjugation. The time series is characterized in the frequency domain by its power spectral density as given by

$$S_x(\omega) = \sum_{n=-\infty}^{\infty} r_x(n) e^{-j\omega n} \tag{2}$$

which is recognized as being the Fourier transform of the autocorrelation sequence.

Upon examination of (1) and (2) it is apparent that determination of the time series power spectral density entails complete knowledge of the generally infinite length autocorrelation sequence. Here, we will be concerned with extracting this information from the finite set of time series observations

$$x(1),\ x(2),\ \ldots,\ x(N) \tag{3}$$

Unless some constraints are imposed on the time series' basic nature, however, there exists a fundamental incompatibility in estimating the required statistical knowledge from the finite set of data. This dilemma is usually resolved by postulating a finite parameter linear model to represent the time series. In terms of parameter parsimony, the causal autoregressive moving average (ARMA) model of order (p,q) as specified by

$$x(n) + \sum_{i=1}^{p} a_i x(n-i) = \sum_{j=0}^{q} b_j \varepsilon(n-j) \tag{4}$$

is generally the most effective linear model [1]. In this model, the (unobserved) excitation process $\{\varepsilon(n)\}$ is assumed to be zero-mean, unit variance Gaussian white noise. It is important to note that the more specialized autoregressive model (i.e., $b_j \equiv 0$ for $j \neq 0$) generally requires a much higher model order p to achieve comparable spectral estimates. Conceptually, then, the more general ARMA model is the logical model choice.

It is well known that the power spectral density of a process $\{x(n)\}$ that satisfies (4) is given by:

$$S_x(\omega) = \frac{|b_0 + b_1 e^{-j\omega} + \ldots + b_q e^{-jq\omega}|^2}{|1 + a_1 e^{-j\omega} + \ldots + a_p e^{-jp\omega}|^2} = \frac{|B(\omega)|^2}{|A(\omega)|^2} \tag{5}$$

Thus the task of estimating the power spectral density of the time series can be accomplished by estimating the ARMA model parameters a_i and b_j.

Several procedures for estimating the a_i and b_j parameters have recently been developed [2-8]. Of these procedures, one developed by Cadzow [2] has been shown to be effective in a variety of cases. The crux of this procedure lies in obtaining the autoregressive parameters by minimizing a weighted quadratic function of a set of zero mean error elements. It was pointed out in [2] that the effectiveness of this procedure is dependent on a judicious selection of the weighting elements in the quadratic function. This paper develops an alternative weighting element selection to that used in [2] which results in improved spectral estimates.

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II. THE MODEL EQUATION ERROR SPECTRAL ESTIMATOR.

The spectral estimation procedure of this paper is predicated on the procedure in [2]. For completeness, this procedure is discussed below.

Of primary importance in spectral estimation is the method for estimating the autoregressive coefficients a_i in equation (4). An effective method for estimating these coefficients entails multiplying both sides of (4) by the term $x^*(n-m)$ to yield the "basic error terms"

$$e(m,n) = x(n)x^*(n-m) + \sum_{i=1}^{p} a_i x(n-i)x^*(n-m) \tag{6a}$$

$$= \sum_{j=0}^{q} b_j \varepsilon(n-j)x^*(n-m) \qquad \text{for } q+1 \le m \le N-1, \quad \max(p+1,m+1) \le n \le N \tag{6b}$$

where the range on the m and n variables is dictated by the time series observation range $1 \le k \le N$. If the time series is in fact an ARMA process of order less than or equal to (p,q) then the basic error terms are each zero mean random variables. Furthermore, the basic error terms are seen to be functions of the autoregressive coefficients $a_1, a_2, \ldots, a_p$. With these two properties in mind, a reasonable selection of the autoregressive coefficients is one that causes each of the e(m,n) terms to be as close to its mean value of zero as possible. This goal is achieved by minimizing a squared-error criterion of the form

$$f(\underline{a}) = \underline{e}^{+} W \underline{e} \tag{7}$$

where $\underline{e}$ is a vector of appropriately arranged e(m,n) terms, W is a positive semidefinite weighting matrix, and $^{+}$ denotes the operation of complex conjugate transposition.

A more specific format of the minimization criterion (7), and the one considered in [2], is

$$f(\underline{a}) = \sum_{m=q+1}^{N-1} w(m) \left| \sum_{n=s}^{N} e(m,n) \right|^2 \qquad s = \max(p+1,m+1) \tag{8}$$

In this expression the w(m) are nonnegative weighting elements.

Using standard calculus, it is readily shown that the set of autoregressive coefficients which minimize (8) are given by:

$$\hat{\underline{a}} = C^{-1}\underline{c} \tag{9}$$

where

$$\hat{\underline{a}} = [a_1 \quad a_2 \ldots a_p]^T \tag{10a}$$

$$C = \{c_{ik}\}_{p \times p} \tag{10b}$$

$$\underline{c} = -\{c_{10} \quad c_{20} \ldots c_{p0}\} \tag{10c}$$

and

$$c_{ik} = \sum_{m=q+1}^{N-1} \sum_{n=s}^{N} \sum_{\ell=s}^{N} w(m)x(n-k)x(\ell-m)x^*(n-m)x^*(\ell-i) \qquad i,k = 1,2,\ldots,p, \quad s = \max(m+1,p+1) \tag{10d}$$

An improvement in the above autoregressive coefficient estimation procedure can be realized by also considering the backward version of the time series $\{x(n)\}$. Also, an estimate of the numerator spectrum $B(\omega)$ in (5) must be obtained in order to arrive at the complete power spectral density estimate of $\{x(n)\}$. The details of these two tasks are presented in [2].

III. WEIGHTING ELEMENT SELECTION

In order to obtain autoregressive parameters using the above procedure, the elements w(m) in (8) must first be selected. In [2] the weights

$$w(m) = (N-m)^4 \qquad q+1 \le m \le N-1 \tag{11}$$

were employed.

A more prudent weight selection can be developed by considering the random terms

$$\left| \sum_{n=s}^{N} e(m,n) \right|^2 \qquad q+1 \le m \le N-1 \tag{12}$$

associated with the weights w(m) in (8) [8]. A logical selection for the w(m) weights is the inverse of the variances of the terms in (12), that is

$$w(m) = \left\{ \mathrm{var}\left[\left| \sum_{n=s}^{N} e(m,n) \right|^2 \right] \right\}^{-1} \qquad q+1 \le m \le N-1 \tag{13}$$

In this way, the terms in the minimization criterion (8) which have smaller variances from their mean value are weighted proportionately higher than those terms with larger variances from their mean.

It is easily shown that

$$\mathrm{var}\left[\left| \sum_{n=s}^{N} e(m,n) \right|^2 \right] = \sum_{n=s}^{N} \sum_{\ell=s}^{N} r_x(\ell-n)\, c(\ell-n) \qquad s = \max(m+1,p+1) \tag{14a}$$

where

$$c(m) = \begin{cases} \sum_{i=0}^{q-m} b_i b_{i+m}^* , & 0 \le m \le q \\ c^*(|m|) , & -1 \ge m \ge -q \\ 0 , & \text{otherwise} \end{cases} \tag{14b}$$

Unfortunately, the desired variances are seen to be dependent on the unknown parameters $b_0, b_1, \ldots, b_q$. However, an approximate expression for the inverse variance weights of (13) can be realized if a reasonable approximation of the $c(m)$ elements in (14b) can be found.

One can gain insight about the structure of the $c(m)$ elements by forming the polynomials $B(z)$ and $C(z)$ defined as

$$B(z) = b_0 z^q + b_1 z^{q-1} + \ldots + b_{q-1} z + b_q \tag{15a}$$

$$C(z) = c(-q)z^{-q} + c(-q+1)z^{-q+1} + \ldots + c(0) + c(1)z + \ldots + c(q)z^q \tag{15b}$$

It is easily shown that

$$C(z) = B(z)B(z^{-1}) \tag{16}$$

Furthermore, $B(z)$ can be factored as

$$B(z) = b_0 \prod_{i=1}^{q} (z - \beta_i) \tag{17}$$

where the β_i are zeroes of the polynomial $B(z)$. Applying (16) it is found that

$$C(z) = b_0^2 z^{-q} \prod_{i=1}^{q} (z - \beta_i)(1 - z\beta_i) \tag{18}$$

Thus, $C(z)$ can be found (to within the constant b_0^2) using (18) from knowledge of the q zeroes β_i of $B(z)$.

A reasonable approximation of $C(z)$ and therefore of the $c(m)$ elements can be found by approximating the location of the zeroes of $B(z)$. One such approximation is realized by assuming that each zero is a random variable uniformly distributed within the complex unit circle,[1] so that its probability density function is

$$f_{\beta_i}(\beta_i) = \begin{cases} \frac{1}{\pi}, & |\beta_i| \le 1 \\ 0, & \text{otherwise} \end{cases} \qquad i = 1,2,\ldots,q \tag{19}$$

If the time series $\{x(n)\}$ is a real process, then the zeroes of $B(z)$ must form complex conjugate pairs. For this case it is assumed that $q/2$ of the zeroes are uniformly distributed within the upper half of the complex unit circle, that is

$$f_{\beta_i}(\beta_i) = \begin{cases} \frac{2}{\pi}, & |\beta_i| \le 1 \text{ and } \mathrm{Im}[\beta_i] \ge 0 \\ 0 & \text{otherwise} \end{cases} \tag{20a}$$

$$i = 1,3,\ldots,q-1$$

$$\beta_i = \beta_{i-1}^* \qquad i = 2,4,6,\ldots,q \tag{20b}$$

where $\mathrm{Im}[\beta_i]$ denotes the imaginary part of β_i. If q is odd the unpaired zero is assumed to be uniformly distributed on the real axis inside the unit circle,

$$f_{\beta_q}(\beta q) = \begin{cases} \frac{1}{2}, & -1 \le \beta_q \le 1 \quad \mathrm{Im}[\beta_q] = 0 \\ 0, & \text{otherwise} \end{cases} \tag{20c}$$

Using these assumptions about the zeroes of $B(z)$, one can straightforwardly calculate the desired approximation to $C(z)$ by determining the expected value of expression (18)

$$\hat{C}(z) = E\{z^{-q} \prod_{i=0}^{q} (z - \beta_i)(1 - z\beta_i)\} \tag{21}$$

By carrying out this calculation, one finds that for a complex time series $\{x(n)\}$

$$\hat{C}_c(z) = 1 \tag{22a}$$

and for a real time series

$$\hat{C}_r(z) = z^{-q} \left[\frac{1}{2} z^4 + \frac{7}{3} z^2 + \frac{1}{2} \right]^k \tag{22b}$$

where

$$k = \begin{cases} \frac{q}{2}, & q \text{ even} \\ \frac{q-1}{2}, & q \text{ odd} \end{cases}$$

Thus, the approximate inverse variance weights are given by

$$w(m) = \left[\sum_{n=s}^{N} \sum_{\ell=s}^{N} r_x(\ell - n)\hat{c}(\ell - n) \right]^{-1} \qquad q+1 \le m \le N-1 \tag{23}$$

where $s = \max(m+1, p+1)$ and the $\hat{c}(m)$ elements are the coefficients corresponding to the z^m terms of the polynomials (22a) or (22b).

IV. NUMERICAL EXAMPLE

In order to compare the effectiveness of the new ARMA spectral estimator with the estimator in [2], the classical problem of resolving two closely spaced (in frequency) sinusoids in white noise will be considered. Specifically, the time series under study is specified by

$$x(n) = \sqrt{20}\cos(0.4\pi n) + \sqrt{2}\cos(0.426\pi n) + w(n) \tag{24}$$

where $\{w(n)\}$ is a white Gaussian noise process of zero mean and unit variance. The sinusoids of normalized frequencies 0.4 and 0.426 are readily calculated to have signal-to-noise ratios (SNR) of 10dB and 0dB, respectively. A sequence of length 512 defined over $0 \le n \le 511$ was next generated using this relationship. Furthermore, in order to provide a statistical basis for our comparison,

[1] If a zero of $B(z)$ is outside the unit circle, then the corresponding zero of $B(z^{-1})$ is inside the unit circle, and from equation (16) it is clear that $C(z)$ will not be affected.

this 512 length sequence was then decomposed into eight disjoint sequences each of length 64 defined on $0 \leq n \leq 63$, $64 \leq n \leq 127$, ..., $448 \leq n \leq 511$. An ensemble consisting of eight subsequences each of length 64 has thereby been generated with each subsequence having a different noise sample and a different initial phase between the two sinusoids. This latter condition is useful in revealing any potential sensitivity to initial phase that the new ARMA spectral estimation method may possess.

The spectral estimates which resulted when the $(N-m)^4$ weights and the new inverse variance weights were applied to the ARMA spectral estimator are displayed in Figures 1a and 1b, respectively. The ordinates are scaled from -20dB to 60dB for each individual plot. In both cases the spectral estimator order was (15,15). It is clear that the inverse variance weight estimator was able to resolve the two sinusoids in more cases than the $(N-m)^4$ weight estimator could. Moreover, the incidence of false peaks in the inverse variance weight estimates is smaller than that of the $(N-m)^4$ weight estimates.

V. CONCLUSIONS

An improved weight selection for a recently developed ARMA spectral estimation procedure was developed. The autoregressive parameters are found in this procedure by minimizing a weighted sum of squares of zero mean basic error terms. The new weight selection is chosen to provide more heavy weighting to those terms in the sum which possess lower variances. Empirical evidence indicates that this new weight selection provides superior spectral estimation performance when compared to the original ~~$(N-m)^4$~~ weight selection.

(N-m) to the 4th power

VI. REFERENCES

[1] Gutowski, P. R., E. A. Robinson, and S. Treitel, "Spectral Estimation, Fact or Fiction," IEEE Trans. Geoscience Electronics, Vol. GE-16, No.2, pp. 80-84, April 1978.

[2] Cadzow, J. A., "ARMA Spectral Estimation: A Model Equation Error Procedure," IEEE Trans. on Geoscience Electronics and Remote Sensing, Vol. GE-19, No.1, pp. 24-28, January 1981.

[3] Steiglitz, K., "On the Simultaneous Estimation of Poles and Zeros in Speech Analysis," IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. ASSP-25, No. 3, pp. 229-234, June 1977.

[4] Box, G. & G. Jenkins, TIME SERIES ANALYSIS: FORECASTING AND CONTROL, Revised Edition, Holden-Day, San Francisco, CA 1976.

[5] Kaveh, M., "High Resolution Spectral Estimation for Noisy Signals," IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. ASSP-27, No. 3, pp. 286-287, June 1979.

[6] Cadzow, J. A., "High Performance Spectral Estimation: A New ARMA Method," IEEE Trans. Acoustics, Speech and Signal Processing, Vol. ASSP-28, No. 5, pp. 524-528, Oct. 1980.

[7] Kay, S. M., "A New ARMA Spectral Estimator," IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. ASSP-28, No. 5, pp. 585-588, Oct. 1980.

[8] Moses, R. L., "An Iterative Algorithm for ARMA Spectral Estimation," Master's Thesis, Virginia Polytechnic Institute and State University, 1980.

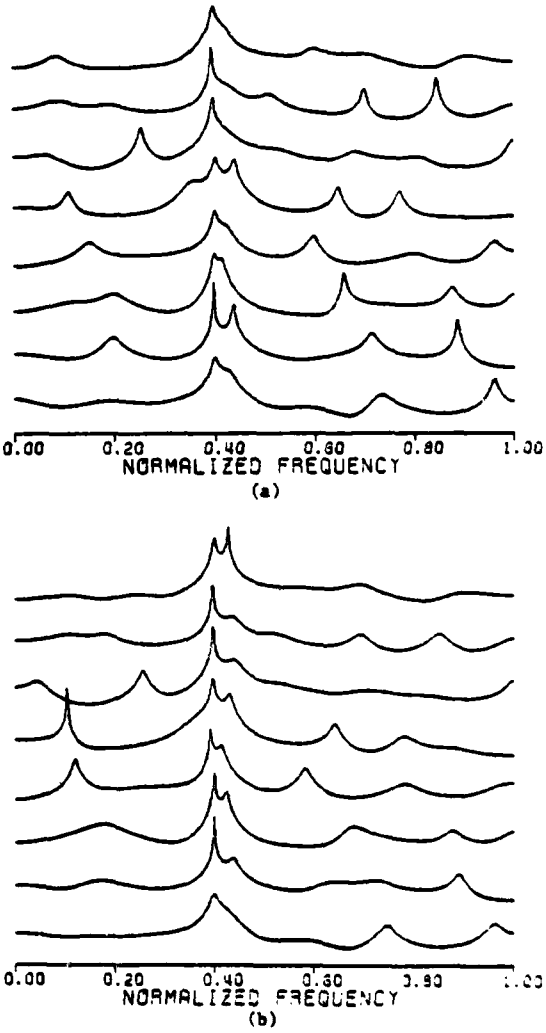

Figure 1. Spectral Estimates of Two Sinusoids at Normalized Frequencies of 0.400(10dB) and 0.426 (0dB) in Additive White Noise. a) $(N-m)^4$ weights used. b) inverse variance weights used.

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