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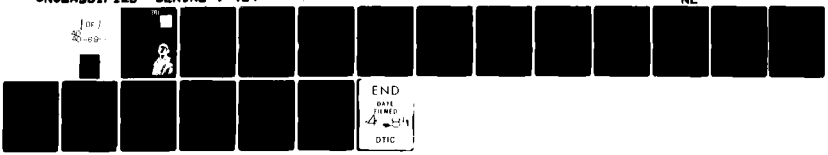

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by

Mahesh Chandra
Tertius De Wet
Nozer D. Singpurwalla

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ON THE SAMPLE REDUNDANCY AND A TEST FOR EXPONENTIALITY

by

Mahesh Chandra*#
Tertius De Wet†
Nozer D. Singpurwalla§#

In this paper we obtain the sampling properties of a scale-free estimator of "redundancy," an information theoretic measure, which is used by economists and communications engineers. We then propose a new test of exponentiality based upon the sample redundancy. This test is unbiased against a large class of alternatives, and it performs at least as well as another recently proposed test with respect to power and asymptotic relative efficiency.

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ON THE SAMPLE REDUNDANCY AND A TEST FOR EXPONENTIALITY

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1. Introduction and Summary

Economists have used the traditional information theory measures such as "entropy" and "redundancy" to measure the extent to which business is concentrated in the control of giant firms. Hart (1971) has compared these measures with the classical statistical measures of dispersion, and other measures of business concentration derived from the "Lorenz curve." More recently, the sample entropy has been used to develop a test of goodness of fit for normality (Vasicek, 1976). The redundancy has also been used as a measure for ordering any two distributions within the class of "star ordered"* distributions (Chandra and Singpurwalla, 1980). This ordering is useful for discussing the unbiasedness of certain goodness of fit tests; see, for example, Sections 3.1 and 3.2.

In this paper, we consider an estimator of the redundancy, and obtain its sampling properties. We show that the estimator is almost

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*Let F_1 and F_2 be two continuous distributions on $[0,\infty)$; then F_1 is said to be *star* (*anti-star*) ordered with respect to F_2 , if $F_2^{-1}F_1(x)/x$ is nondecreasing (nonincreasing) in x for $0 \leq x \leq F_1^{-1}(1)$.

sure consistent, and is asymptotically normal. We use these properties to propose a scale-free test for exponentiality based on the sample redundancy, and obtain the asymptotic relative efficiency of this test compared to tests based on the maximum likelihood estimates. Percentage points of the test statistic for finite samples are obtained via a Monte Carlo experiment. The power of this test against various alternatives is compared with a recently proposed test for exponentiality based on the Gini index (or equivalently, the total time on test statistic) using a Monte Carlo experiment.

Our conclusion is that a test for exponentiality based on the sample redundancy is unbiased in the sense of Lehman and Scheffé [cf. Ferguson (1967, p. 224)], and that it performs as well as a test for exponentiality based on the Gini index for the gamma, the Weibull, the uniform, and the Pareto alternatives. These distributions are either star or anti-star ordered with respect to the exponential distribution. The same conclusion also holds for the lognormal alternative, when the parameters of this distribution are so chosen that it is either star or anti-star ordered with respect to the exponential distribution. When the lognormal alternative fails to be either of the above, then neither the test based on the sample redundancy nor the test based on the Gini index should be used; this is because it is not possible to claim unbiasedness of these tests. However, should we decide to go ahead and use these tests, then we find that the test based on redundancy appears to have a slight advantage over that based on the Gini index, with respect to power: both the tests have low power. Finally, the asymptotic relative efficiency of the test based on redundancy for the gamma and the Weibull alternatives is also comparable to that based on the Gini index.

We have no compelling reason, other than ease of computation and the slight advantage of power, to recommend a test for exponentiality based on redundancy over other available tests. The main purpose of this paper, then, is to point out the possibility of using the sample redundancy as another means for testing for exponentiality, and to

describe the circumstances under which the tests based on the redundancy and the Gini index should *not* be used.

2. Redundancy and Its Estimation

For a nonnegative random variable X with a distribution function F and probability density function f, the *redundancy* R_F is defined [see, for example, Thiel (1967, p. 96)] as

$$R_F = E\left(\frac{X}{\mu} \log \frac{X}{\mu}\right) , \tag{2.1}$$

where $\mu = E(X)$ is assumed to be finite.

Let $X_1,\ldots,X_n$ be a random sample from F, and let $Y_i = X_i/\sum_1^n X_i$, $i=1,\ldots,n$. The *sample entropy* of $Y_1,\ldots,Y_n$ is defined by Hart (1971) as

$$H(Y) = -\sum_i^n Y_i \log Y_i . \tag{2.2}$$

Note that $H(Y)$ attains its maximum value $\log n$ when $Y_i = 1/n$, for $i=1,\ldots,n$. Vasicek (1976) proposes a test for normality based on the sample entropy of f.

The difference between the sample entropy $H(Y)$ and its maximum value $\log n$ is called the *sample redundancy*, R_n. Thus,

$$\begin{aligned} R_n &= \log n - H(Y) \\ &= \sum_i^n Y_i \log(nY_i) , \quad \text{since } \sum^n Y_i = 1 . \end{aligned}$$

Some further simplifications lead us to write

$$R_n = \frac{1}{n\bar{X}} \sum_i^n X_i \log X_i - \log \bar{X} , \tag{2.3}$$

where $\bar{X} = \sum_1^n X_i/n$.

In Theorem 2.1 below, we shall show that $R_n \xrightarrow{a.s.} R_F$, and that R_n is asymptotically normally distributed. Thus R_n is an almost sure consistent estimator of R_F .

2.1 Sampling Properties of R_n

In what follows the following abbreviations will be used.

If $\{X_n\}$ is a sequence of random variables, and if $\{b_n\}$ is any other sequence, then the notation "$X_n = O_p(b_n)$" denotes the fact that $(X_n/b_n) \xrightarrow{P} 0$, where " $\xrightarrow{P}$ " denotes convergence in probability. The notation " $\xrightarrow{D} N(\mu,\sigma^2)$ " denotes convergence in distribution to a normally distributed random variable with mean μ and variance σ^2 .

Let $E(X \log X) = \alpha$, where α is finite, and let $E(X \log X)^2 < \infty$; also, let $\sigma^2(F) = (1/\mu^2)E(X \log X - \frac{\alpha X}{\mu} - X + \mu)^2$. *Then as* $n \to \infty$,

<u>Theorem 2.1</u>: $\sqrt{n}\,(R_n - R_F) \xrightarrow{D} N\left(0,\sigma^2(F)\right)$.

<u>Proof</u>: We shall first show that $R_n \xrightarrow{a.s.} R_F$. Write $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i \log X_i$; then, using the strong law of large numbers, we can write

$$R_n - \frac{\alpha}{\mu} + \log\mu = \frac{S_n}{\bar{X}} - \log\left(\frac{\bar{X}}{\mu}\right) - \frac{\alpha}{\mu}$$

$$= S_n\left(1+\frac{\bar{X}-\mu}{\mu}\right)^{-1}\mu^{-1} - \log\left(1+\frac{\bar{X}-\mu}{\mu}\right) - \frac{\alpha}{\mu}$$

$$= (S_n - \alpha + \alpha)\left(1 - \frac{\bar{X}-\mu}{\mu} + \ldots\right)\mu^{-1} - \left(\frac{\bar{X}-\mu}{\mu} + \ldots\right) - \frac{\alpha}{\mu}$$

$$= \left[(S_n-\alpha) + \alpha - \frac{\alpha}{\mu}(\bar{X}-\mu)\right]\mu^{-1} - \mu^{-1}(\bar{X}-\mu) - \frac{\alpha}{\mu} + O_p\left(\frac{1}{\sqrt{n}}\right)$$

$$= \mu^{-1} \frac{1}{n} \sum_{i}^{n} \left(X_i \log X_i - \frac{\alpha}{\mu} X_i - X_i + \mu \right) + O_p\left(\frac{1}{\sqrt{n}}\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} Z_i + O_p\left(\frac{1}{\sqrt{n}}\right) ,$$

where

$$Z_i = \mu^{-1}\left(X_i \log X_i - \frac{\alpha}{\mu} X_i - X_i + \mu\right) .$$

The result follows since $ES_n = \alpha$, and $X_i \log X_i$ are independent and identically distributed. To prove the statement of the theorem, we note that the Z_i's are independent and identically distributed with

$$EZ_i = 0 ,$$

and

$$EZ_i^2 = \sigma^2(F) ,$$

and invoke the central limit theorem. //

3. A Test for Exponentiality Based on R_n

Since R_n is scale-free, we can use the result of Theorem 2.1 to obtain a test for exponentiality by taking the underlying distribution, say G , to be a unit exponential; that is, $G(x) = 1-e^{-x}$. Using the fact that under G , $E(X \log X) = \Gamma^1(2) = .4227$, and that $E(X^2 \log X) = \Gamma^1(3) = 1.8455$, we can show that $\sigma^2(G) = E(X \log X - X E X \log X - X+1)^2 = .2898664$; $\Gamma^1(\cdot)$ is the digamma function. Thus, when $G(x) = 1-e^{-x}$,

$$\sqrt{n}(R_n - R_G) \xrightarrow{D} N(0,.2898664) ,$$

and so the above result can be used to test for exponentiality when n is large, using standard procedures for testing hypotheses.

The exact distribution of R_n when the underlying distribution is G is not known. Consequently, the quantiles of R_n , for $1 \leq n \leq$

30 , are obtained through a Monte Carlo simulation involving 10,000 random samples each of size n , generated from G using the subroutine GGAMR of the International Mathematical and Statistical Library program package. These quantiles are given in Table 3.1.

3.1 Power of the Test for Exponentiality Based on R_n

To study the power of a test for exponentiality based on R_n against several alternatives, and to compare the performance of this test versus other competing scale-free tests, we have to again resort to a Monte Carlo simulation. For convenience, we choose $n=20$ so that our test statistic becomes R_{20} , and we take 1000 replications of the test. The significance level is taken to be .05, and the alternatives considered are the Weibull, the uniform, the Pareto, the gamma, and the log-normal. Depending upon the nature of the alternative, either the one-sided or both the one-sided and the two-sided tests are studied. One-sided tests are used whenever the alternative F is star (anti-star) ordered with respect to the exponential distribution G , because then, $R_F \leq (\geq) R_G$ [Chandra and Singpurwalla (1980)]. Also, whenever $R_F \leq (\geq) R_G$, the test based on R_n is unbiased against the alternative F , in the sense of Lehman and Sheffé. The results of the power studies are summarized in column 2 of Table 3.2. In computing the entries in column 2, the quantiles of R_{20} given in Table 3.1 are used.

3.2 Comparison with Other Tests

Of the several goodness of fit tests for the exponential distribution which are available in the literature (Stephens, 1978), the one which appears to be the most recent and which enjoys the advantages of good power, good asymptotic relative efficiency, robustness to measurement error, etc., is the one based on the Gini statistic G_n (Gail and Gastwirth, 1978), where

TABLE 3.1

QUANTILES OF R_n FOR DIFFERENT VALUES OF n BASED ON A MONTE CARLO SIMULATION OF 10,000

n	.01	.025	.05	.10	.90	.95	.975	.99
2	0.00008	0.00034	0.00122	0.00532	0.49259	0.56993	0.61734	0.65771
3	0.00510	0.01217	0.02333	0.04388	0.53274	0.64686	0.74274	0.84830
4	0.01661	0.03065	0.04909	0.08060	0.56292	0.66990	0.75945	0.87696
5	0.03371	0.05483	0.08048	0.11372	0.57271	0.67130	0.76196	0.89225
6	0.05410	0.07883	0.10211	0.13992	0.58268	0.66895	0.75844	0.87641
7	0.07200	0.09540	0.12582	0.16153	0.57780	0.66236	0.74588	0.84759
8	0.08550	0.11248	0.14028	0.17900	0.57520	0.65668	0.74278	0.84773
9	0.09827	0.12611	0.15547	0.19029	0.57614	0.65756	0.72720	0.81783
10	0.10916	0.13937	0.16604	0.20092	0.57528	0.65064	0.72119	0.80938
11	0.12018	0.14841	0.17718	0.21146	0.57565	0.64365	0.71143	0.79680
12	0.13270	0.15778	0.18560	0.21990	0.57195	0.64383	0.70117	0.77535
13	0.13840	0.16789	0.19327	0.22929	0.56658	0.63012	0.69450	0.77214
14	0.14785	0.17491	0.20265	0.23555	0.56673	0.62765	0.68843	0.76281
15	0.15536	0.18469	0.21056	0.24169	0.56434	0.62193	0.67875	0.74745
16	0.16473	0.19231	0.21795	0.24782	0.55598	0.61718	0.67803	0.74812
17	0.17099	0.20053	0.22379	0.25433	0.55571	0.61133	0.67102	0.73812
18	0.17428	0.20158	0.22646	0.25749	0.55432	0.61367	0.66503	0.72869
19	0.18195	0.20812	0.23287	0.26366	0.55047	0.60271	0.65522	0.72314
20	0.18472	0.21430	0.23677	0.26518	0.54724	0.60260	0.64891	0.71128
21	0.19266	0.21802	0.24016	0.26705	0.54522	0.59751	0.65082	0.70937
22	0.19732	0.22258	0.24414	0.27134	0.54427	0.59567	0.64036	0.70218
23	0.19763	0.22546	0.24930	0.27593	0.54367	0.59740	0.64373	0.69910
24	0.20245	0.22842	0.24851	0.27745	0.53971	0.59058	0.63261	0.69686
25	0.20830	0.23180	0.25467	0.28102	0.54133	0.58599	0.62697	0.68312

TABLE 3.1--*continued*

n	.01	.025	.05	.10	.90	.95	.975	.99
26	0.21163	0.23450	0.25855	0.28513	0.53562	0.58352	0.62820	0.68123
27	0.21510	0.23802	0.26113	0.28680	0.53645	0.58311	0.62566	0.68014
28	0.22108	0.24216	0.26188	0.28854	0.53307	0.57787	0.61781	0.67050
29	0.22373	0.24651	0.26634	0.29224	0.53017	0.57604	0.61692	0.67211
30	0.22648	0.24758	0.26804	0.29380	0.53044	0.57313	0.61209	0.66050

TABLE 3.2

POWER COMPARISONS FOR A TEST FOR EXPONENTIALITY BASED ON THE REDUNDANCY AND THE GINI STATISTIC (based on 1,000 samples of size n=20, using a significance level of α=0.05)

Alternative	R_{20} 1-Sided Test	R_{20} 2-Sided Test	G_{20} 1-Sided Test	G_{20} 2-Sided Test
Weibull shape = 0.8	0.346	†	0.336	†
Weibull shape = 1.5	0.643	†	0.643	†
Uniform on (0,2)	0.826	†	0.829	†
Pareto DF $1 - 1/4x^2$	0.830	†	0.835	†
Gamma shape 2.0	0.631	†	0.622	†
Lognormal* σ=0.6, μ=1	0.873	†	0.873	†
Lognormal* σ=1.0, μ=1	0.144	0.154	0.117	0.126
Lognormal* σ=1.4, μ=1	0.629	†	0.620	†

$$*f(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-(\log x - \mu)^2/\sigma^2\right), \quad 0 < x < \infty .$$

†A two-sided test is inappropriate for this case.

$$G_n = \frac{\sum_{i=1}^{n-1} i(n-i)\left(X_{(i+1)} - X_{(i)}\right)}{(n-1)\sum_{i=1}^{n} X_{(i)}} ;$$

$X_{(1)} < \ldots < X_{(n)}$ are the order statistics of $X_1,\ldots,X_n$.

Thus it appears that it is appropriate for us to compare the test for exponentiality based on R_n with the one based on G_n with respect to both the power and the asymptotic relative efficiency. Another reason for choosing G_n as a basis for comparison is that Chandra and Singpurwalla (1980) have shown that a test for exponentiality based on G_n is identical to the one based on the "total time on test" of Barlow

and Doksum (1972). Since this latter test is known to be asymptotically minimax against a large class of alternatives defined by the Kolmogorov distance, so is the test based on G_n .

In column 3 of Table 3.2 we give the powers of the tests for exponentiality based on G_{20} against the several alternatives mentioned before. The entries in column 3 of Table 3.2 have been taken from Table 1 of Gail and Gastwirth (1978) for all the alternatives considered except the lognormal--this alternative was not considered by the above referenced authors. The powers of G_{20} against the lognormal alternatives were obtained by us with a Monte Carlo simulation of 1000 replications.

From a comparison of the entries in columns 2 and 3 of Table 3.2, we see that the two tests for exponentiality based on R_{20} or G_{20} perform equally well or poorly, depending on the alternative considered. For the lognormal alternative with $\sigma = 1$ and $\mu = 1$, the test based on R_{20} appears to have a slight edge over the one based on G_{20} . Both these tests have low power.

A reason why the tests for exponentiality based on the sample redundancy and the Gini index have low power for the lognormal alternative with $\mu = 1$ and $\sigma = 1$, is that this distribution is neither star nor anti-star ordered with respect to the exponential, whereas the other alternatives are. Furthermore, Chandra and Singpurwalla (1980) also show that when a distribution F is star (anti-star) ordered with respect to the exponential distribution G , then G_F , the Gini index of F , is less (greater) than G_G , the Gini index of G . Thus, it is not possible to claim unbiasedness of either R_{20} or G_{20} for the lognormal alternative in question, and our conclusion is that neither the test based on the Gini index nor the test based on the redundancy should be used under the circumstances described above.

$R_{F(\beta)} = \psi(\beta) - \log\beta$, where $\psi(x) = d \log \Gamma(x)/dx$. Hence $R^{(1)}_{F(1)} = \psi^{(1)}(2) - 1 = -0.3550668$, and from (4.2) we get the A.R.E. as 0.674.

For the Weibull alternatives $F(x,\beta) = \frac{1}{\lambda}(x/\lambda)^{\beta-1}\exp[-(x/\lambda)^{\beta}]$, the variance of $\sqrt{n}\hat{\beta}$ is $V = 0.60793$, again independent of λ , and $R_{F(\beta)} = \frac{1}{\beta}\psi\left(1+\frac{1}{\beta}\right) - \log\Gamma\left(1+\frac{1}{\beta}\right)$. Hence $R^{(1)}_{F(1)} = -\psi(2) - \psi^{(1)}(2) + \psi(2) = -0.6449332$, and from (4.2) we get the A.R.E. as 0.872.

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