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ASYMPTOTIC ANALYSIS OF von KARMAN FLOWS

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ABSTRACT

This paper is concerned with asymptotic expansions of solutions of von Karman's swirling flow problem. These expansions are used to prove the convergence of a class of approximative problems, which are set up by substituting for the infinite interval on which von Karman's problem is posed by a finite but large one and by imposing supplementary boundary conditions at the far end. The asymptotic expansions are crucial for the determination of the order of convergence. Exponential convergence is shown for well-posed approximative problems.

The given approach is applicable to general autonomous nonlinear boundary value problems on infinite intervals, for which the von Karman problem may be considered as a model problem.

AMS(MOS) Subject Classification: 26D05, 34C05, 34D05, 41A60

Key words: Navier-Stokes equation, singular points, asymptotic properties, asymptotic approximation

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SIGNIFICANCE AND EXPLANATION

This paper is concerned with the von Karman swirling flow problem which describes the velocity field of a fluid over an infinite rotating disk. It is assumed that the whole half space over the disk is filled with fluid. By restricting the class of admissible solutions von Karman (1921) was able to reduce the Navier-Stokes equations which describe this fluid dynamical configuration, and which are partial differential equations, to a system of ordinary differential equations which are posed on an infinite interval. This system of ordinary differential equations can well be considered as a model problem for boundary value problems on infinite intervals. The infinite interval is difficult for computation. It is a natural procedure to restrict the infinite interval to a finite but large one and to impose additional conditions at the far end which should describe the behavior of the solution far away from the disk. In this paper information about the asymptotic (which means far away from the disk) behavior is obtained and used for the construction of these far away boundary conditions. A theorem, which implies convergence of the 'infinite' solution to the 'infinite' solution as the length of the interval becomes infinite, is proved and exponential convergence is shown for suitable approximating problems. The extension of the derived statements to more general problems posed on infinite intervals is straightforward.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ASYMPTOTIC ANALYSIS OF von KARMAN FLOWS

Peter A. Markowich

1. Introduction

von Karman showed in 1921 that the Navier-Stokes equations for a stationary axisymmetric flow of a viscous incompressible fluid occupying the half space over an infinite rotating disk reduce to a system of two ordinary differential equations when using a cylindrical coordinate system (r,φ,z).

The disk is rotating in the plane $z = 0$ around the z-axis with angular velocity $\Omega_0 > 0$ and the angular velocity of the fluid in $z = \infty$ is $\gamma\Omega_0$. von Karman considered only the case $\gamma = 0$, but the generalization to $\gamma \in \mathbb{R}$ is straightforward.

With the normalization:

$$x = (\Omega_0/\nu)^{\frac{1}{2}} z \tag{1.1}$$

where ν is the specific viscosity of the fluid, the following similarity equations result from the Navier-Stokes equations:

$$\left.\begin{aligned} f'''(x) + 2f(x)f''(x) &= (f'(x))^2 - g^2(x) + \gamma^2 \quad (1.2)\\ g''(x) + 2f(x)g'(x) &= 2f'(x)g(x) \quad (1.3)\end{aligned}\right\} 0 \leq x < \infty$$

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The velocity field of the fluid in the (r,φ,z) - coordinate system is represented by $(\Omega_o r f'(x), \Omega_o r g(x), -2(\nu\Omega_o)^{1/2} f(x))$. Appropriate boundary conditions at the disk are:

$$(1.4) \qquad f(0) = a \in \mathbb{R}, \quad f'(0) = 0, \quad g(0) = 1.$$

A non zero value a represents blowing from the disk $(a < 0)$ or suction to the disk $(a > 0)$. The third boundary condition affecting the angular velocity implies that the fluid adhers to the disk. Boundary conditions at infinity are

$$(1.5) \qquad f'(\infty) = 0, \quad g(\infty) = \gamma.$$

For $\gamma = 0$ McLeod (1969a) has proven the existence of a solution of problem (1.2), (1.3), (1.4), (1.5) for all $a \in \mathbb{R}$. He obtained qualitative properties of this solution (f,g) which hold independently of the value of a:

$$(1.6.a) \qquad 0 \leq f' < 1, \quad (b) \lim_{x\to\infty} f(x) = f_\infty \in \mathbb{R}$$

$$(1.7) \qquad g > 0, \quad g' < 0$$

The solution (f,g) is non-oscillatory, i.e. neither f' nor g' oscillate around zero for $x \to \infty$. Moreover $f_\infty \in [-\infty,\infty)$ holds for <u>every</u> solution f (see McLeod 1970), and if $f_\infty \in \mathbb{R}^+$ then $f - f_\infty, f', f'', g, g'$ decay exponentially.

For $\gamma \neq 0$ McLeod (1969a) concluded that

$$(1.8) \qquad \text{signum } f'(x) \not\equiv \text{const}, \ \text{signum } g'(x) \not\equiv \text{const for } x \to \infty$$

for every solution (f,g). Furthermore he proved the existence of a solution for $\gamma > 0$ (1971) independently of the value of a, uniquenss for $\gamma = 1$ and $a \leq 0$ and a nonexistence theorem for $\gamma = -1$ and $a \leq 0$. Rogers and Lance (1960) heuristically derived asymptotic expansions for f and g in the case $\gamma \neq 0$ and McLeod (1969b) constructed a rigorous proof for these expansions which imply an exponentially dampened oscillatory decay of $f-f_\infty$ and $g-\gamma$. But his proof breaks down in the case $\gamma = 0$.

Lentini and Keller (1980) performed a numerical study of the swirling flow problem with $a = 0$ by reducing the infinite interval $[0,\infty]$ to a finite one $[0,X]$ with $X \gg 0$ and by imposing two additional asymptotic conditions, at $x = X$, which they derived from the requirement that unbounded solution components of the in $(f_\infty,0,0,\gamma,0)^T$ linearized problem vanish at infinity. Their results clearly indicate that infinitely many solutions (f_i,g_i) exist for $\gamma = 0$ which fulfill

$$(1.9) \qquad 0 < f_i(\infty) < f_{i-1}(\infty), \qquad i = 2,3,\ldots$$

There are two goals of this paper. The first is to derive an asymptotic expansion for f and g in the case $\gamma = 0$, $f_\infty > 0$, and to calculate the leading coefficients of the expansion

in the case $\gamma \neq 0$. The second goal is to use these expansions in order to prove convergence statements for reduced problems posed on finite intervals.

The organisation of the paper is the following: in paragraph 2 we reformulate the swirling flow problem to a first order boundary value problem on $[0,\infty]$ which have been treated by de Hoog and Weiss (1980) and Markowich (1980) and collect information on the solutions of these problems, in paragraph 3 we derive the asymptotic expansions for $\gamma = 0$, in paragraph 4 we investigate the case $\gamma \neq 0$ and in paragraph 5 asymptotic boundary conditions are treated.

The method employed to obtain the asymptotic expansions is linearization of (1.2), (1.3) around $(f_\infty,0,0,\gamma,0)$ (as used by McLeod (1969)) and a contraction mapping theorem which uses the closeness of the solutions of the linearized and non-linear problems.

2. Reformulation of the Swirling Flow Problem

We reduce (1.2), (1.3) to a first order system by substituting

$$(2.1)\qquad \begin{aligned} &y_1 = f,\ y_2 = f',\ y_3 = f'',\ y_4 = g,\ y_5 = g' \\ &y = (y_1, y_2, y_3, y_4, y_5)^T \end{aligned}$$

and get

$$(2.2)\qquad y' = F(y,\gamma) = \begin{pmatrix} y_2 \\ y_3 \\ -2y_1y_3 + y_2^2 - y_4^2 + \gamma^2 \\ y_5 \\ -2y_1y_5 + 2y_2y_4 \end{pmatrix}$$

The boundary conditions transform to

$$(2.3)\qquad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} y(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (By(0) = b)$$

$$(2.4)\qquad \lim_{x\to\infty} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} y(x) = \begin{pmatrix} 0 \\ \gamma \end{pmatrix} \quad (\lim_{x\to\infty} Cy(x) = c(\gamma)).$$

Because of (1.6.b) and (1.5):

$$(2.5)\qquad \lim_{x\to\infty} y(x) = y_\infty \in \mathbb{R}^5 \text{ and } y \in C([0,\infty])$$

holds and (2.4) can be changed to $C\,Y_\infty = c(\gamma)$. From (2.5) and (2.2) we deduce

$$(2.6) \qquad \lim_{x\to\infty} y'(x) = F(y_\infty,\gamma) = 0$$

This is a nonlinear equation which can be solved immediatly:

$$(2.7) \qquad y_\infty = (f_\infty,0,0,\gamma,0)^T \text{ with } f_\infty \in \mathbb{R}.$$

A one parameter solution manifold $y_\infty = y_\infty(f_\infty,\gamma)$ which is consistent with (2.4) is specified for fixed $\gamma \in \mathbb{R}$.

The next step is to calculate the matrix

$$(2.8) \qquad A(f_\infty,\gamma) := \frac{\partial F(y_\infty(f_\infty,\gamma),\gamma)}{\partial y} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2f_\infty & -2\gamma & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 2\gamma & 0 & 0 & -2f_\infty \end{bmatrix}$$

which has the eigenvalues v_i defined by the zeros of the equation

$$(2.9) \qquad v[v^2(v+2f_\infty)^2 + 4\gamma^2] = 0$$

These eigenvalues are:

$$(2.10) \qquad \begin{aligned} v_1(f_\infty,\gamma) &\equiv 0 \\ v_2(f_\infty,\gamma) &= -f_\infty + c_1(f_\infty,\gamma) + ic_2(f_\infty,\gamma) \\ v_3(f_\infty,\gamma) &= -f_\infty + c_1(f_\infty,\gamma) - ic_2(f_\infty,\gamma) = \bar{v}_2(f_\infty,\gamma) \\ v_4(f_\infty,\gamma) &= -f_\infty - c_1(f_\infty,\gamma) + ic_2(f_\infty,\gamma) \\ v_5(f_\infty,\gamma) &= -f_\infty - c_1(f_\infty,\gamma) - ic_2(f_\infty,\gamma) = \bar{v}_4(f_\infty,\gamma) \end{aligned}$$

where c_1, c_2 are defined by

$$(2.11)\qquad \begin{aligned} c_1(f_\infty,\gamma) &= \left(\frac{1}{2}\left(f_\infty^4 + 4\gamma^2\right)^{1/2} + \frac{f_\infty^2}{2}\right)^{1/2} \\ c_2(f_\infty,\gamma) &= \left(\frac{1}{2}\left(f_\infty^4 + 4\gamma^2\right)^{1/2} - \frac{f_\infty^2}{2}\right)^{1/2} \end{aligned}$$

If $\gamma = 0$ all eigenvalues are real and fulfill

$$(2.12)(a)\qquad \begin{aligned} &v_2 > 0,\ v_3 > 0,\ v_1 \equiv v_4 \equiv v_5 \equiv 0 \quad \text{if} \quad f_\infty < 0 \\ &v_1 \equiv v_2 \equiv v_3 \equiv 0,\ v_4 < 0,\ v_5 < 0 \quad \text{if} \quad f_\infty > 0 \\ &v_1 \equiv v_2 \equiv v_3 \equiv v_4 \equiv v_5 \equiv 0 \quad \text{if } f_\infty = 0 \end{aligned}$$

If $\gamma \neq 0$ the eigenvalues v_2, v_3, v_4, v_5 have a nonvanishing imaginery part for all $f_\infty \in \mathbb{R}$ and

$$(2.12)(b)\qquad \operatorname{Re} v_2 > 0,\ \operatorname{Re} v_3 > 0;\ \operatorname{Re} v_4 < 0,\ \operatorname{Re} v_5 < 0 \text{ for } f_\infty \in \mathbb{R}.$$

The decrease of the rank of $A(f_\infty,\gamma)$ in $\gamma = 0$ causes the different behavior of the velocity fields for $\gamma = 0$ and $\gamma \neq 0$ and it also requires different treatment. This will be pointed out in the paragraphs 3 and 4. Now we transform $A(f_\infty,\gamma)$ to its Jordan canonical form $J(f_\infty,\gamma)$:

$$(2.13)\qquad J(f_\infty,\gamma) = E(f_\infty,\gamma)^{-1} A(f_\infty,\gamma) E(f_\infty,\gamma)$$

and substitute

$$(2.13)\qquad E(f_\infty,\gamma) z = y - y_\infty(f_\infty,\gamma)$$

in (2.2). So we get the new problem

$$(2.14) \qquad z' = J(f_\infty,\gamma)z + h(z,f_\infty,\gamma)$$

$$(2.15) \qquad z(\infty) = 0$$

where $h(z,f_\infty,\gamma)$ is defined by

$$(2.16) \qquad h(z,f_\infty,\gamma) = E(f_\infty,\gamma)^{-1}F(E(f_\infty,\gamma)z + y_\infty(f_\infty,\gamma),\gamma) - J(f_\infty,\gamma)z$$

Regarding $h(z,f_\infty,\gamma)$ for fixed γ and f_∞ as a perturbation to equation (2.14) we come up with an 'inhomogenous' boundary value problem.

Using a suitable norm in $\mathbb{R}^5$ we derive

$$(2.17)(a) \qquad \|h(z,f_\infty,\gamma)\| \leq b_1(f_\infty,\gamma)\|z\|^2$$

$$(2.17)(b) \qquad \|h(z_1,f_\infty,\gamma) - h(z_2,f_\infty,\gamma)\| \leq b_2(f_\infty,\gamma)(\|z_1\|+\|z_2\|)\|z_1 - z_2\|$$

where $b_1(f_\infty,\gamma)$ and $b_2(f_\infty,\gamma)$ are independent of z resp. z_1 and z_2.

In order to solve problem (2.14) we have to find an appropriate particular solution and therefore let us briefly discuss the (general) problem

$$(2.18) \qquad u' = Au + f(x), \quad x \geq \delta \geq 0$$

where the Jordan matrix $A = \mathrm{diag}(A_o, A_+, A_-)$ is partitioned correspondingly to its eigenvalues will real part equal, larger and smaller zero.

An appropriate choice for a particular solution $up = (up_o, up_+, up_-)^T$, where $f = (f_o, f_+, f_-)^T$ is partitioned according to A, is the following:

$$\text{(a)}\quad up_o(x) = e^{A_o x} \int_{\infty}^{x} e^{-A_o s} f_o(s)\,ds$$

$$\text{(2.19)}\qquad \text{(b)}\quad up_+(x) = e^{A_+ x} \int_{\infty}^{x} e^{-A_+ s} f_+(s)\,ds$$

$$\text{(c)}\quad up_-(x) = e^{A_- x} \int_{\delta}^{x} e^{-A_- s} f_-(s)\,ds$$

if f_o fulfills $\|f_o(x)\| = O(x^{-r-\varepsilon})$ where $\varepsilon > 0$ and r is the dimension of the largest Jordan block in A_o. This choice was suggested by Lentini (1978) and de Hoog and Weiss (1980).

In operator form we can write

$$\text{(2.20)}\qquad up(x) = (Hf)(x) = (H_o f_o, H_+ f_+, H_- f_-)^T(x).$$

H is a linear operator on the space of all function f, for which $f \in C([\delta,\infty])$ and $\|f_o(x)\| = O(x^{-r-\varepsilon})$ holds.

If A_+, A_- are diagonalmatrices the following estimates are easily derived.

$$
(2.21)\qquad
\begin{aligned}
&(a)\quad \|(H_+f_+)(x)\| \leq \text{const}\cdot\|f_+\|_{[x,\infty]}\\
&(b)\quad \|(H_of_o)(x)\| \leq \text{const}\cdot x^{-\varepsilon}\max_{t\geq x}\|t^{r+\varepsilon}f_o(t)\|, \quad \varepsilon > 0\\
&(c)\quad \|(H_-f_-)(x)\| \leq \text{const}\cdot\|f_-\|_{[\delta,x]}\\
&(d)\quad \|(H_-f_-)(x)\| \leq \text{const}\cdot\|F_-\|_{[\delta,x]}x^{-\mu}\\
&\qquad \text{if } f_-(x) = F_-(x)x^{-\mu} \text{ with } \mu > 0
\end{aligned}
$$

where $\|\cdot\|_{[a,b]}$ denotes the max-norm on the interval $[a,b]$.

The constants depend on A but not on δ and f. If $\int_\delta^\infty e^{-A_-s}f(s)ds$ exists then $(H_-f_-)(x)$ can be split up into:

$$
(2.22)\qquad (H_-f_-)(x) = e^{A_-x}\int_\delta^\infty e^{-A_-x}\, f_-(s)ds + (\tilde{H}_-f_-)(x)
$$

where $\tilde{H}_-$ is defined by

$$
(2.23)\qquad (\tilde{H}_-f_-)(x) = e^{A_-x}\int_\infty^x e^{-A_-s}\, f_-(s)ds.
$$

$(\tilde{H}f) = (H_of_o, H_+f_+, \tilde{H}_-f_-)$ is also a particular solution of (2.18). If $f(x) = F(x)e^{-\eta_o x}$ with $\|F\|_{[\delta,\infty]} \leq$ const and $A_- = -\eta_1\cdot I$ with $\eta_o, \eta_1 > 0$, then the following estimates hold:

$$
(2.24)\qquad \|(\tilde{H}_-f_-)(x)\| \leq \text{const}\,\|F\|_{[x,\infty]}e^{-\eta_o x},\ \eta_o > \eta_1
$$

$$
(2.25)\qquad \|(H_-f_-)(x)\| \leq \text{const}\,\|F\|_{[\delta,x]}(e^{-\eta_o x} - e^{(\eta_1-\eta_o)\delta}e^{-\eta_1 x})
$$

$$
(2.26)\qquad \|(H_of_o)(x)\| \leq \text{const}\,\|F\|_{[x,\infty]}e^{-\eta_o x}
$$

3. Asymptotic Behaviour of Solutions in the case $\gamma = 0$.

After straightforward calculation we get

$$(3.1) \qquad J(f_\infty,0) = \begin{bmatrix} 0 & 1 & & & \\ & 0 & & & \\ & & 0 & & \\ & & & -2f_\infty & \\ & & & & -2f_\infty \end{bmatrix}$$

$$(3.2) \qquad E(f_\infty,0) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2f_\infty \\ 0 & 0 & 0 & 0 & 4f_\infty^2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2f_\infty & 0 \end{bmatrix}$$

$$(3.3) \qquad h(z,f_\infty,0) = \begin{pmatrix} \vdots \\ h_i(z,f_\infty,0) \\ \vdots \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{4f_\infty^2}(-8f_\infty^2 z_5 (z_1+z_5) + (z_2 - 2f_\infty z_5)^2 - (z_3+z_4)^2) \\ \frac{1}{2f_\infty}(-8f_\infty z_5 (z_1+z_5) + (z_2 - 2f_\infty z_5)^2 - (z_3+z_4)^2) \\ \frac{1}{2f_\infty}(4f_\infty z_4 (z_1+z_5) + 2(z_2 - 2f_\infty z_5)(z_3+z_4)) \\ -2 z_4 (z_1+z_5) - \frac{1}{f_\infty}(z_2 - 2f_\infty z_5)(z_3+z_4) \\ -2 z_5 (z_1+z_5) + \frac{1}{4f_\infty^2}((z_2 - 2f_\infty z_5)^2 - (z_3+z_4)^2) \end{pmatrix}$$

assuming $f_\infty > 0$. Here $z = (z_1,z_2,z_3,z_4,z_5)^T$ holds. The general solution of problem (2.14) for fixed f_∞ is :

$$(3.4) \qquad z = \exp(J(f_\infty,0)x)\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \xi_1\\ \xi_2\end{pmatrix} + (Hh(z,f_\infty,0))(x)$$

The proposition (2.15) holds iff $\nu_1 = \nu_2 = \nu_3 = 0$ because z decays exponentially and the estimates (2.17)(a) and (2.21)(b) hold.

For $f_\infty \in \mathbb{R}^+$; $\xi_1, \xi_2 \in \mathbb{R}$ fixed we define

$$(3.5) \qquad (\psi_0(z,f_\infty))(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ e^{-2f_\infty x} & 0 \\ 0 & e^{-2f_\infty x}\end{bmatrix}\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} + (Hh(z,f_\infty,0))(x)$$

as an operator on the Banachspace

$$(3.6) \qquad (A_{f_\infty} = \{u \mid u(x) = U(x)e^{-2f_\infty x},\ U \in C_b([\delta,\infty))\}, \|u\| = \|U\|_{[\delta,\infty]})$$

where $C_b([\delta,\infty))$ is the space of all functions which are continuous on $[\delta,\infty)$ and bounded on $[\delta,\infty]$. $\delta \geq 0$ will be chosen suitably.

$\psi_0(\cdot,f_\infty)$ maps A_{f_∞} into A_{f_∞} because of the estimates (2.17)(a), (2.21)(a), (b) and (2.25). Every solution z is a fixed point of $\psi_0(\cdot,f_\infty)$ and the existence and uniqueness of this fixed point is shown by the contraction mapping theorem, which is applicable for δ sufficiently large on any sphere with center $(0,0,0,e^{-2f_\infty x}\xi_1, e^{-2f_\infty x}\xi_2)^T$ because the estimates (2.17)(b),

(2.21)(a),(b) and (2.25) hold. As mentioned before all solutions z decay exponentially, however we have to prove that there is no fixed point of $\psi_o(\cdot,f_\infty)$ which decaysslower than $e^{-2f_\infty x}$.

This is done by regarding $\psi_o(\cdot,f_\infty)$ as an operator on the Banach-space

$$(3.7)\quad (B_\varepsilon = \{v \mid v(x) = V(x)x^{-2-\varepsilon}, V \in C_b([\delta,\infty)), \varepsilon > 0\}, \|v\| = \|V\|_{[\delta,\infty]}).$$

The contraction mapping theorem can be applied again for δ sufficiently large and because $A_{f_\infty} \subset B_\varepsilon$ holds for every $\varepsilon > 0, f_\infty > 0$ the established solutions $z \in A_{f_\infty}$ are unique in B_ε. The only difference in the proof of the contraction properties of $\psi_o(\cdot,f_\infty)$ in B_ε and A_{f_∞} is that the estimate (2.21)(d) has to be used instead of (2.25).

Assume now that $f_\infty < 0$. If $z \in B_\varepsilon$ then $Hh(z,f_\infty,0) \in B_\varepsilon$ and therefore $\nu_1 = \nu_2 = \nu_2 = \xi_1 = \xi_1 = 0$ has to be fulfilled and by the contraction mapping theorem $z = 0$ is the only fixed point. For $f_\infty = 0$ the same argument is valid in $B_{1+\varepsilon}$, because the largest Jordanblock of $J(0,0)$ is three dimensional.
We exclude $f_\infty \leq 0$ because we only look for solutions in B_ε for $f_\infty \neq 0$ and $B_{1+\varepsilon}$ for $f_\infty = 0$.

In order to investigate whether z might decay faster than $e^{-2f_\infty x}$ we substitute H_-, which is defined by (2.23), by $\tilde{H}_-$ and $\tilde{H} = (H_o, H_+, \tilde{H}_-)$ for H. Because of the estimate (2.24) with $\eta_o = 4f_\infty$ and $\eta_1 = 2f_\infty$ and because of (2.2b) we conclude that

$$(3.8) \qquad \|(\tilde{H}h(z,f_\infty,0))(x)\| = O(e^{-4f_\infty x}) \quad \text{for} \quad z \in A_{f_\infty}$$

holds. We have cut off the $O(e^{-2f_\infty x})$ term in H_-.

The (modified) operator $\tilde{\psi}_o(\cdot,f_\infty)$ is defined by

$$(3.9) \quad (\tilde{\psi}_o(z,f_\infty))(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ e^{-2f_\infty x} & 0 \\ 0 & e^{-2f_\infty x} \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + (\tilde{H}h(z,f_\infty,0))(x)$$

The contraction argument in A_{f_∞} applies again and because a fixed point z of $\tilde{\psi}_o(\cdot,f_\infty)$ establishes a solution of (2.14), (2.15) the equivalence of the solution manifolds defined by the fixed points of ψ_o and $\tilde{\psi}_o$ for $f_\infty \in \mathbb{R}^+$ and $(\xi_1,\xi_2) \in \mathbb{R}^2$. follows because $z \in A_{f_\infty}$ and because (2.22) holds.
Assume now that $(w_1,w_2) = (0,0)$. Then the unique fixed point of $\tilde{\psi}_o(\cdot,f_\infty)$ is $z = 0$ implying the trivial solution $f \equiv f_\infty$, $g \equiv 0$.

Moreover assume that $w_1 = 0$.

Then start the iteration $z^{n+1} = \tilde{\psi}_o(z^n,f_\infty)$ with some vector $z^o = (z_1^o,z_2^o,0,0,z_5^o)^T \in A_{f_\infty}$. (3.3) implies that $h(z^o,f_\infty,0) =$ $= (\tilde{z}_1,\tilde{z}_2,0,0,\tilde{z}_5)^T$.

Therefore the (unique) fixed point z^* of $\tilde{\psi}_o(\cdot,f_\infty)$ fulfills $z^* = \lim_{n\to\infty} z^n = (z_1^*,z_2^*,0,0,z_5^*)^T \in A_{f_\infty}$, which implies after premultiplication with $E(f_\infty,0)$ that $g \equiv 0$.

From the definitions of $\tilde{\psi}_o$ and h we derive

$$
\begin{aligned}
&\text{(a)}\ z_1 = O(e^{-4f_\infty x}) \\
&\text{(b)}\ z_2 = O(e^{-4f_\infty x}) \\
(3.10)\qquad &\text{(c)}\ z_3 = O(e^{-6f_\infty x}) \\
&\text{(d)}\ z_4 = e^{-2f_\infty x}\, w_1 + O(e^{-6f_\infty x}) \\
&\text{(e)}\ z_5 = e^{-2f_\infty x}\, w_2 + \left(\frac{1}{2f_\infty} w_1^2 + \frac{1}{8f_\infty^3} w_2^2\right) e^{-4f_\infty x} + O(e^{-6f_\infty x})
\end{aligned}
$$

(3.10)(c) and (d) hold because the terms of order $e^{-4f_\infty x}$ cancel in $h_3(z,f_\infty,0)$ and $h_4(z,f_\infty,0)$.

By applying the identity $y \equiv E(f_\infty,0)z + y_\infty(f_\infty,0)$, integrating $f'''(x) = 4f_\infty^2\, z_5(x)$ twice and by continuation from $[\delta,\infty]$ to $[0,\infty]$ we get

THEOREM I

Every solution of (1.2), (1.3), (1.5) with $f_\infty > 0$ and $\gamma = 0$ fulfills:

$$
\begin{aligned}
f(x) &= f_\infty + e^{-2f_\infty x}\, w_2 + \left(\frac{1}{8f_\infty} w_2^2 + \frac{1}{32f_\infty^3} w_1^2\right) e^{-4f_\infty x} + \\
&\quad + O(e^{-6f_\infty x}), \quad x \to \infty \\
g(x) &= e^{-2f_\infty x}\, w_1 + O(e^{-6f_\infty x}), \quad x \to \infty
\end{aligned}
$$

where $w_1, w_2 \in \mathbb{R}$. $w_1 = 0$ implies $g \equiv 0$, $w_1 = w_2 = 0$ implies $f \equiv f_\infty$, $g \equiv 0$. Therefore no nontrivial solution (f,g) oscillates for large x, the convergence of $f - f_\infty$ and g to 0 for $x \to \infty$ is monotonic.

The boundary value problem (2.2), (2.3), (2.4) is well-posed regarding the number of boundary conditions at $x = 0$ and $x = \infty$, because the three parameters (f_∞, w_1, w_2) have to be determined from the three boundary conditions at $x = 0$ given by (2.3).

4. Asymptotic Behaviour for $\gamma \neq 0$

In this case the eigenvalues of $A(f_\infty,\gamma)$ are distinct and we get

$$(4.1)\quad J(f_\infty,\gamma) = \mathrm{diag}(0, v_2(f_\infty,\gamma), v_3(f_\infty,\gamma), v_4(f_\infty,\gamma), v_5(f_\infty,\gamma))$$

$$(4.2)\qquad E(f_\infty,\gamma) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & v_2 & v_3 & v_4 & v_5 \\ 0 & v_2^2 & v_3^2 & v_4^2 & v_5^2 \\ 0 & \frac{v_2^2 v_5}{2\gamma} & \frac{v_3^2 v_4}{2\gamma} & \frac{v_4^2 v_3}{2\gamma} & \frac{v_5^2 v_2}{2\gamma} \\ 0 & \frac{v_2^3 v_5}{2\gamma} & \frac{v_3^3 v_4}{2\gamma} & \frac{v_4^3 v_3}{2\gamma} & \frac{v_5^3 v_2}{2\gamma} \end{bmatrix}$$

Pursuing as in paragraph 3 and using that $v_3 = \bar{v}_4$ we get

$$(4.3)\qquad z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ e^{v_4(f_\infty,\gamma)x} & 0 \\ 0 & e^{\bar{v}_4(f_\infty,\gamma)x} \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + (Hh(z,f_\infty,\gamma))(x).$$

Defining $\psi_\gamma(z,f_\infty)$ by the right hand side of (4.3) as an operator on the Banachspace:

$$(4.4)\ (A_{f_\infty,\gamma} = \{u \mid u(x) = U(x)e^{(\mathrm{Re}\,v_4(f_\infty,\gamma))x}, U \in C_b([\delta,\infty)\}, \|u\| = \|U\|_{[\delta,\infty]})$$

we conclude as we did in paragraph 3 that $\psi_\gamma(\cdot,f_\infty)$ is a contraction on any sphere with center $(0,0,0,e^{v_4 x}\xi_1,\ e^{\bar{v}_4 x}\xi_2)^T$, if δ is sufficiently large.

McLeod (1969b) proved that there is no solution (f,g) that decays slower to (f_∞,γ) than $e^{\mathrm{Re}\,v_3 x}$. We want to prove now that there is no solution z of (4.3), which decays faster than that. Therefore we again substitute H for $\tilde{H}$ and as in paragraph 3 we find that $w_1 = w_2 = 0$ yields $z \equiv 0$ as the unique fixed point of

$$(4.4)\quad (\tilde{\Psi}_\gamma(z,f_\infty))(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ e^{v_4 x} & 0 \\ 0 & e^{v_4 x} \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + (\tilde{H}h(z,f_\infty,\gamma))(x).$$

So we have established the existence of a unique (f_∞,w_1,w_2)-solution manifold for $f_\infty \in \mathbb{R}$, $(w_1,w_2) \in \mathbb{C}^2$.

The real solution manifold $y = E(f_\infty,\gamma)z + y_\infty(f_\infty,\gamma)$ is obtained by choosing $w_1 = \frac{1}{2}(\beta_1 - i\beta_2)$ and $w_2 = \bar{w}_1$. The three parameters are now $(f_\infty,\beta_1,\beta_2) \in \mathbb{R}^3$. In order to show this, we set $u = E(f_\infty,\gamma)z$ and get a fixed-point equation for u by

$$(4.5)\qquad u = E(f_\infty,\gamma)\tilde{\psi}_\gamma(E^{-1}(f_\infty,\gamma)u,f_\infty)$$

and therefore $u = \lim_{n\to\infty} u^n$ where

$$(4.6)\quad u^{n+1} = \exp(A(f_\infty,\gamma)x)E(f_\infty,\gamma)\begin{pmatrix} 0 \\ 0 \\ 0 \\ w_1 \\ \bar{w}_1 \end{pmatrix} + E(f_\infty,\gamma)(\tilde{H}h(E^{-1}(f_\infty,\gamma)u^n,f_\infty,\gamma))(x)$$

holds. The first term on the right hand side is real because $\bar{w}_1 = w_2$, the second is real if u_n is real because

$$(4.7) \qquad E(f_\infty,\gamma)\exp(J(f_\infty,\gamma)(x-s))E^{-1}(f_\infty,\gamma)=\exp(A(f_\infty,\gamma)(x-s))$$

holds. This yields inductively that u as well as $y = u + y_\infty$ are real, if the starting function u^0 is real. Therefore we get

THEOREM II

Every solution of (1.2), (1.3), (1.5) with $\gamma \neq 0$ fulfills for some $(\beta_1,\beta_2) \in \mathbb{R}^2$

$$f(x) = f_\infty + e^{-(f_\infty+c_1(f_\infty,\gamma))x}(\beta_1 \cos(c_2(f_\infty,\gamma)x) +$$

$$+ \beta_2 \sin(c_2(f_\infty,\gamma)x)) + O(e^{-2(f_\infty+c_1(f_\infty,\gamma))x}), \quad x \to \infty$$

$$g(x) = \gamma + \frac{1}{2\gamma} e^{-(f_\infty+c_1(f_\infty,\gamma))x}(\alpha_1 \cos(c_2(f_\infty,\gamma)x) +$$

$$+ \alpha_2 \sin(c_2(f_\infty,\gamma)x)) + O(e^{-2(f_\infty+c_2(f_\infty,\gamma))x}), \quad x \to \infty$$

where $\alpha_1 = \alpha_1(f_\infty,\gamma) = \mathrm{Re}(v_4^2(f_\infty,\gamma)v_3(f_\infty,\gamma)(\beta_1-i\beta_2))$

and

$\alpha_2 = \alpha_2(f_\infty,\gamma) = -\mathrm{Im}(v_4^2(f_\infty,\gamma)v_3(f_\infty,\gamma)(\beta_1 - i\beta_2))$.

Furthermore $\beta_1 = \beta_2 = 0$ yields $f \equiv f_\infty$ and $g \equiv \gamma$. All nontrivial solutions f and g oscillate for large x in the sense that there are sequences $\tilde{x}_i, \bar{x}_i \in \mathbb{R}$ with $\lim_{i\to\infty} \tilde{x}_i = \lim_{i\to\infty} x_i = +\infty$ which are the only zeros of $f - f_\infty$ resp. $g - \gamma$ and

$$\begin{Bmatrix} \tilde{x}_{i+1} - \tilde{x}_i \\ \bar{x}_{i+1} - \bar{x}_i \end{Bmatrix} \sim \frac{\pi}{\gamma}\sqrt{\frac{f_\infty^2}{2} + \frac{1}{2}\sqrt{f_\infty^4 + 4\gamma^2}}$$

holds for $i \to \infty$.

Again the boundary value problem on $[0,\infty]$ is well posed regarding the number of boundary conditions at $x = 0$ and $x = \infty$.

A convergent expansion for f and g of the form $\sum_{n=0}^{\infty} c_n(f_\infty, w_1, w_2)\exp(-n(f_\infty + c_1(f_\infty,\gamma))x)$ can be obtained for all $\gamma \in \mathbb{R}$ from the iteration $z^{n+1} = \tilde{\psi}_\gamma(z^n, f_\infty)$ by choosing $z^0 \equiv 0$ as starting function.

5. Asymptotic Boundary Conditions

Boundary value problems on inifinite intervals are often solved by restricting to a finite but large interval and by imposing supplementary boundary conditions on the right end (see Lentini (1978), de Hoog and Weiss (1980) and Markowich (1980)). The approximative problems for the swirling flow problem have the following form:

$$(5.1) \qquad u' = F(u,\gamma)$$

$$(5.2) \qquad Bu(0) = b \qquad 0 \leq x \leq X, \qquad X >> 0$$

$$(5.3) \qquad S(u(X),\gamma) = 0, \qquad S:\mathbb{R}^5 \to \mathbb{R}^2 \quad \text{for fixed } \gamma \in \mathbb{R}$$

The two additional asymptotic boundary conditions (5.3) shall express the desired behaviour of the exact solution y for large x so that they shall assure convergence of u to y in the following sense:

$$(5.4) \qquad \|u(\cdot,X)-y\|_{[0,X]} \to 0 \quad \text{as} \quad X \to \infty.$$

Moreover the order of convergence should be reasonably large in order to enable us to compute the solution u(x,X) of the two point boundary problem (5.1), (5.2), (5.3) sufficiently fast.

From Spijker's nonlinear stability-consistency concept (see Spijker (1972), paragraph 3.1) we conclude that it is sufficient for the convergence to require that 1) and 2) hold, where:

1) The problem

$$(5.5) \qquad v' = \frac{\partial F}{\partial y}(y(x),\gamma)v + f(x) \qquad 0 \leq x \leq X$$

$$(5.6) \qquad Bv = \beta$$

$$(5.7) \qquad \frac{\partial S}{\partial y}(y(X),\gamma)v(X) = \gamma(X)$$

is for all $f \in C([0,X])$, $\beta \in \mathbb{R}^3, \gamma(X) \in \mathbb{R}^2$ uniquely soluble and the solution v fulfills a stability estimate of the form:

$$(5.8) \qquad \|v\|_{[0,X]} \leq (\|\beta\| + X^{r-1}\|\gamma(X)\| + X^{2r-1}\|f\|_{[0,X]})$$

where r is the dimension of the largest Jordanblock belonging to zero eigenvalues of $A(f_\infty,\gamma)$, that means $r = 1$ for $\gamma \neq 0, f_\infty \in \mathbb{R}$ and $r = 2$ for $\gamma = 0$, $f_\infty > 0$.

2) the consistency assumption

$$(5.9) \qquad \|S(y(X),\gamma)\| = O(X^{-2(2r-1)-\varepsilon}), \qquad \varepsilon > 0$$

holds.

At least convergence of the order

$$(5.10) \qquad X^{2r-1}\|S(y(X),\gamma)\|$$

results and the approximative problems (5.1), (5.2), (5.3) are for all sufficiently large X soluble. This solution $u(x,X)$ is unique in a ball, whose center is the restriction of y to

$[0,X]$ and whose radius is smaller than const. $X^{-(2r-1)}$ (in the space $C^1([0,X])$ with $\|u\| = \|u\|_{[0,X]} + \|u'\|_{[0,X]}$) where const. is sufficiently small.

The basic assumption is to require the solution of (2.2), (2.3), (2.4) to be isolated, i.e. the linearized problem

$$(5.11) \qquad w' = \frac{\partial F}{\partial y}(y(x),\gamma)w, \qquad 0 \leq x \leq \infty, \qquad w \in C([0,\infty])$$

$$(5.12) \qquad Bw(0) = 0$$

$$(5.12) \qquad Cw(\infty) = 0$$

has the unique solution $w = 0$, so bifurcation and limit points are excluded from the convergence analysis. Writing (5.11) as

$$w' = \frac{\partial F}{\partial y}(y_\infty,\gamma)w + G(x)w$$

with $G(x) = O(\|y(x) - y_\infty(f_\infty,\gamma)\|$ we get

$$(5.13) \qquad w = E(f_\infty,\gamma)\exp(J(f_\infty,\gamma)x)\xi + E(f_\infty,\gamma)(HE^{-1}(f_\infty,\gamma)Gw)(x)$$

where $\xi = (\xi_1,\xi_2,\xi_3,\xi_4,\xi_5)$. By applying (5.12) we conclude that $\xi_2 = \xi_3 = 0$ for $\gamma = 0$ as well as $\gamma \neq 0$.

The general solution of (5.11), (5.12) is

$$(5.14) \qquad w(x) = E(f_\infty,\gamma)\phi(x)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

where $\phi(x)$ is a fundamental matrix of

$$u' = E^{-1}(f_\infty,\gamma)\frac{\partial F}{\partial y}(y(x),\gamma)E(f_\infty,\gamma)u$$

Therefore y is isolated, iff the 3×3 Matrix

$$(5.15) \qquad BE(f_\infty,\gamma)\phi(0)\begin{bmatrix} 1 & \\ 0 & 0 \\ 0 & \\ 0 & \\ 0 & I_2 \end{bmatrix}$$

is regular.

From this proposition and certain assumptions on $\frac{\partial S}{\partial y}(y(X),\gamma)$ Markowich (1980) concluded the unique solvability of (5.5), (5.6), (5.7) for all $(f,\beta,\gamma(x)) \in C([0,x]) \times \mathbb{R}^5$ and the estimate (5.8) similarly to de Hoog and Weiss (1980) who dealt with problems, where $F_y(y_\infty)$ has no eigenvalue with real part zero. These assumptions are:

$$(5.16) \qquad \left\| \left(\frac{\partial S}{\partial y}(y(X),\gamma)E(f_\infty,\gamma)\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right)^{-1} \right\| = O(1) \text{ for } X \to \infty$$

$$(5.17) \qquad \left\| \frac{\partial S}{\partial y}(y(X),\gamma) \right\| = O(1) \text{ for } X \to \infty$$

$$(5.18) \qquad \left\| \frac{\partial S}{\partial y}(y(X),\gamma)E(f_\infty,\gamma)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\| = O(X^{-r}) \text{ for } X \to \infty$$

Because y(X) decays exponentially it is sufficient to require $\frac{\partial S}{\partial y}(y_\infty(f_\infty,\gamma),\gamma)$ to fulfill (5.16), (5.17), (5.18) instead of $\frac{\partial S}{\partial y}(y(X),\gamma)$. Obviously (5.9) is fulfilled, if

$$(5.19) \qquad S(y_\infty(f_\infty,\gamma),\gamma) = 0$$

holds, and then the order of convergence is at least $X\cdot\exp(-(f_\infty + c_1(f_\infty,\gamma))X)$ for $\gamma \neq 0$ and $X^3\exp(-2f_\infty X)$ for $\gamma = 0$ because of the expansions for y. So (5.16), (5.17), (5.18) with $y_\infty(f_\infty,\gamma)$ substituted for $y(X)$ and (5.19) specify a class of admissible supplementary boundary conditions. A natural choice is

$$S(u(X),\gamma) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} u(X) - \begin{pmatrix} 0 \\ \gamma \end{pmatrix} = Cu(X) - c(\gamma)$$

Obviously (5.19) is fulfilled and it is checked by a straightforward calculation that (5.16), (5.17), (5.18) hold for $\gamma = 0$ as well as $\gamma \neq 0$. From the asymptotic expansion given in Theorems 1 and 2 we conclude that the order of convergence is minimal for this choice, i.e. $X\exp(-(f_\infty + c_1(f_\infty,\gamma))X)$ for $\gamma \neq 0$ and $X^3\exp(-2f_\infty X)$ for $\gamma = 0$, but even these linear inhomogenous supplementary boundary condition yield a reasonable - exponential - order of convergence.

Lentini and Keller (1980) used the following asymptotic boundary conditions

$$S_p(u(X),\gamma) = \begin{Bmatrix} (u_1(X)+c_1(u_1(X),\gamma))u_2(X)+u_3(X)-c_2(u_1(X),\gamma)(u_4(X)-\gamma) \\ c_2(u_1(X),\gamma)u_2(X)+(u_1(X)+c_1(u_1(X),\gamma))(u_4(X)-\gamma)+u_5(X) \end{Bmatrix} = 0$$

for $\gamma \neq 0$ and their limiting forms

$$S_p(u(X),0) = \begin{Bmatrix} 2u_1(X)u_2(X) + u_3(X) \\ 2u_1(X)u_4(X) + u_5(X) \end{Bmatrix} = 0$$

for $\gamma = 0$. These so called projection conditions are derived by setting the zero these solution components of the linear problem (2.14) (without the 'inhomogenous' term $h(z,f_\infty,\gamma)$) and (2.15) which do not automatically vanish at infinity. Checking the conditions (5.16), (5.17), (5.18) and (5.19) yields the stability and consistency of the projection conditions.

In order the determine the exact order of convergence which is given by (5.9) we substitute the asymptotic expansions for y from Theorem 1 in the case $\gamma = 0$ and from Theorem 2 for $\gamma \neq 0$ for u(X) into the projection conditions and get

$$(5.20) \qquad \|S_p(y(X),\gamma)\| = O(\exp(-2(f_\infty + c_1(f_\infty,\gamma))X)) \quad \text{for } \gamma \in \mathbb{R}$$

Therefore the projection conditions cancel the first two terms of the series expansion for y. The expression (5.20) also explains, why Lentini and Keller (1980), who assumed no blowing or suction at the disk had to increase the length of the interval X in order to achieve the same accuracy when computing higher order Karman swirling flows. Because of (1.9) the sequence $f_i(\infty) = f_\infty^i$ for $i = 1,2,\ldots,$ is decreasing, therefore the order of convergence decreases too unless X is increased.

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This paper is concerned with asymptotic expansions of solutions of von Karman's swirling flow problem. These expansions are used to prove the convergence of a class of approximative problems, which are set up by substituting the infinite interval on which von Karman's problem is posed, by a finite but large one and by imposing supplementary boundary conditions at the far end. The asymptotic expansions are crucial for the determination of the order of convergence. Exponential convergence is shown for well-posed approximative problems.

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20. (Abstract continued)

The given approach is applicable to general autonomous nonlinear boundary value problems on infinite intervals, for which the von Karman problem may be considered as a model problem.

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