Interferometric Measurement of Small Refraction Angles

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    Radial and axial refraction angles are separately measured for rays passing through an azimuthally symmetric medium. The fringe slopes from complementary interferograms are used in a derived expression. Shearing interferometry is used in an example.
INTERFEROMETRIC MEASUREMENT OF SMALL REFRACTION ANGLES

By utilizing the fringe slopes in complementary interferograms (discussed later), one can measure small angles of refraction for rays passing through a medium with transverse refractive index gradients. This technique is useful as a plasma diagnostic. The refraction angles for a plasma yield general information about perpendicular gradients and mass distribution. Small refraction angles through a low density region may represent significant mass due to the large volume involved. For the special case of a cylindrical plasma, it has been shown that the electron density can be determined from refraction measurements. The technique described in this letter allows the separate measurements of both the radial and axial refraction angles, and thus extends the technique of Ref. (1) to the more general case of a plasma with azimuthal symmetry.

The emphasis in this technique is on using fringe slopes to obtain refraction angles. Small fringe slopes can be measured accurately and unambiguously whereas relatively more error is involved in interpolating or extrapolating small fringe spacing.

Consider a medium which is symmetric about the z-axis so that the refractive index \( \tilde{n} \) is a function of \( z \) and the radial distance \( r \) from the z-axis. A laser beam passing through the medium perpendicular to the z-axis is combined with a reference beam at a small wavefront angle \( \phi = \lambda \theta_0 / d \) where \( \lambda \) is the background fringe spacing and \( \lambda \) is wavelength. The wavefront intersection is taken, for simplicity, to be perpendicular to the z-axis. The

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resulting interferogram has fringes which are projected into the image of a plane which passes through the z-axis and is perpendicular to the laser beam. In this plane, each fringe is an equiphase curve which is a function of z and the projected radial distance \( \varphi_\perp \) (ray impact parameter) of the ray from the z-axis. The phase for a ray along a path \( s = \sqrt{r^2 - \varphi_\perp^2} \) can then be expressed as

\[
\delta(r,z) = \pm \frac{2 \pi z}{d} + \frac{2 \pi}{\lambda} \int_{-\infty}^{\infty} n(z, \varphi_\perp, s) ds
\]  

(1)

where, in the first term, two signs are allowed for the wavefront angle. The two interferograms described by Eq. (1) are complementary in that one can be obtained from the other by an inversion of the wavefront angle. This is done physically by interchanging the reference and probing beams or by reflecting both beams from a mirror.

As a concrete example of the technique, consider the interferograms formed by passing a probing laser beam through a medium (laser produced plasma) small compared to the cross-section of the beam and then letting the beam impinge at 45 degrees incidence onto a wedged shearing plate. In our experiment, this was a 5 cm diameter by 1 cm thick plate of fused quartz with a 6 minute wedge. The front and back reflections interfere and the wedge and beam collimation produce a background fringe pattern. Due to the 1 cm plate thickness, the phase-disturbed region of the beam for one surface reflection falls on the undisturbed region of the beam from the other surface reflection which serves as a reference beam. The shearing plate thus acts as a folding-wave-front shearing interferometer and allows the simultaneous recording of the complementary interferograms. Such interferograms are shown in Fig. 1 with a 200 micron reference marker given near the bottom. The axis of symmetry (z-axis) is horizontal along the bottom
of the Figure. The five fringes coming in from the top from a large radial distance are seen to curve away from the plasma in the left-hand interferogram while they curve into the plasma in the complementary interferogram shown on the right. This behavior is now examined more quantitatively.

The fringes define a set of curves in the $p_z$-plane ($p_z = p_z(t)$, $z = z(t)$) each having a constant phase. The phase change along a fringe $\frac{\partial \delta}{\partial p_z} dp_z + \frac{\partial \delta}{\partial z} dz$, thus vanishes. This shows that the fringe slope $z' = dz/dr$ can be expressed as $-(\frac{\partial \delta}{\partial r})/(\frac{\partial \delta}{\partial z})$. Using Eq. (1) then gives

$$z' = -\frac{\theta_r + \theta_w}{\theta_z - \theta_w}$$

(2)

where $\theta_r = \int (\frac{\partial n}{\partial p_z}) ds$ and $\theta_z = \int (\frac{\partial n}{\partial x}) ds$ are the refraction angles for gradients along the $p_z$ and $z$ axes, respectively, and $\theta_w$ is the magnitude of the wavefront angle. When the two complementary interferograms are simultaneously recorded, then, at each $p_z$, $z$ position, Eqs. (2) give the slope $z_1'$ (+$\theta_w$) as well as the slope $z_2'$ ($-\theta_w$). These two equations give,

$$\theta_r = \frac{2z_1' z_2'}{z_1' - z_2'} \theta_w; \quad \theta_z = -\frac{z_1' + z_2'}{z_1' - z_2'} \theta_w$$

(3)

The method is illustrated at a location (near the middle in Fig. 1) where the fringe slopes in the complementary interferograms are considerably different. The fringe slope at the indicated point on the left-hand interferogram is $z_1' = -1.28$ while it is $z_2' = +1.57$ at the corresponding location in the right-hand interferogram. From the background fringe spacing $d$ (175 microns) and probe wavelength (0.527 microns) the wavefront angle $\theta_w$ is $\lambda/d = 3.0$ mrad. Using these results in Eqs. (3) gives the refraction angles as $\theta_r = 4.2$ mrad and $\theta_z = 0.31$ mrad. The density gradient at this location is seen to be primarily in the radial direction.
Fig. 1 — Complementary interferograms of a laser-produced plasma taken simultaneously with a folding-wave-front shearing interferometer. The axis of symmetry is at the bottom in both cases and a 200 micron marker is given. Note how, in the normal interferogram on the left, the fringes bend away from the center while, in the inverted interferogram on the right, they bend toward the center.
One should avoid applying Eqs. (3) at locations where $z_1' \leq z_2'$. This occurs in regions (near the lower-right corners in Fig. 1) where the index gradients are so large that the fringe slopes are nearly equal, $(\theta_w \ll \theta_r, \theta_z)$ in the complementary interferograms. One can, even at these locations, determine $\theta_r/\theta_z$ directly from Eq. (2) by ignoring $\theta_w$. One could apply Eqs. (3) to larger index gradient regions by using larger wavefront angles. However, it is better in the region of steep index gradients to determine $\theta_r, \theta_z$ directly from fringe spacing. From Eq. (1), one finds $\partial \delta/\partial \theta = 2 \pi \theta_r/\lambda$ and $\partial \delta/\partial z = 2 \pi (\theta_z + \theta_w)/\lambda$. Thus, $\theta_r$ is approximately $\lambda/\Delta \theta$, where $\Delta \theta$ is the radial (vertical in Fig. 1) distance between successive bright (or dark) fringes at the given location. As a check, note that $\Delta \theta$ is about 120 microns at the location indicated. This gives $\theta_r$ (4.2 mrad) in agreement with the value determined by Eqs. (3).

A rough, worst case (no imaging) interferometric limitation on the size of the refraction angle $\theta$ for a path $D$ in a medium is obtained by requiring that the difference in the actual and projected path $(D \cos \theta)$ be small compared to a probe wavelength $\lambda$. Thus, (for small $\theta$) $\theta \approx \sqrt{2 \lambda / D}$. For the example given in Fig. 1, the target was 600 microns in diameter. Thus, taking $\lambda \approx 5$ microns and $D \approx 10^3$ microns shows $\theta$ should be less than about 30 mrad. More exact numerical studies show that, with proper imaging, accurate interferometric results can be obtained even with much larger refraction angles.

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References


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