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A FIRST-ORDER METHODOLOGY FOR CALCULATING PROBABILITY OF MISSION SUCCESS

Agbabian Associates
250 North Nash Street
El Segundo, California 90245

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A first-order methodology for calculating the probability that a system will successfully complete its mission was developed, illustrated, and validated. The methodology is directed to analysts, designers, managers, physicists, and planners who are unaccustomed to working with statistics and probability theory, but who are assumed to have had an introductory course in statistics and probability theory. The methodology, which makes use of closed-form solutions and is quite transparent, is designed to yield results quickly and

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20. ABSTRACT (Continued)

and without the use of a large computer. It is not intended to replace the formal, more accurate computerized methodologies that use Monte Carlo simulations or numerical partitioning. Within its specified applicable domain, the methodology yields quantitatively accurate results. The applicable domain is sufficiently large to encompass many problems of interest to the Defense Nuclear Agency community.

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SUMMARY

An analytical methodology is presented for calculating, to the first order, the probability that a system will successfully complete its mission. The methodology, which is quite transparent, is designed to yield results quickly and without the use of a large computer. It is not intended to replace the formal, more accurate computerized methodologies that use Monte Carlo simulation or numerical partitioning.

The probability of mission success is calculated by collapsing a network of probabilities. Each network probability represents the probability of no failure of the system in a failure mode that, either by itself or in concert with other failure modes, would abort the mission. The fact that there is a probability of failure of the system in a given failure mode reflects the uncertainty embodied in the system's capacity, or in the demand placed on the system, or both. Systematic and random uncertainties are differentiated. The systematic uncertainties are associated with the estimates of the capacity and demand means, which are treated as random variables. The systematic uncertainties are ultimately reflected as variability in the calculated probability of mission success.

The system failure modes that would abort the mission are grouped into sets such that within each set it can be reasonably assumed that there is perfect dependency; between sets, it can be reasonably assumed that there is statistical independency. Each such set represents one probability in the system's probability-of-mission-success network. Arbitrary correlation, reflecting the systematic uncertainty in the estimates of capacity and demand means, is admitted between any two probabilities in the network.

The density distributions for capacities, capacity means, demands, and demand means are assumed to be lognormal. The effect of this assumption and of other approximations inherent in the methodology are demonstrated by working illustrative problems with both the presented methodology and Monte Carlo simulation. The applicable domain of the methodology hereby established is sufficiently large to encompass many problems of interest to analysts, designers, managers, physicists, and planners in the Defense Nuclear Agency community.

The presented methodology (or for that matter any methodology of the same purpose) requires that systematic uncertainties be quantified using subjective reasoning. If a sponsor will not accept subjective estimates, then there can be no application of the methodology. Most sponsors, however, will entertain the idea of subjective estimates if the bases for these estimates are well documented.

PREFACE

The writers wish to thank A.H-S. Ang, Professor of Civil Engineering, University of Illinois, for his critical review of Appendixes A and B and for his constructive suggestions for improving the presentations. Finally, the valuable guidance and suggestions provided by CPT M. Moore, the Contracting Officer's representative, during the conduct of this project are gratefully acknowledged.

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SECTION 1

INTRODUCTION

1-1 PURPOSE.

The purpose of this document is to present a first-order methodology for calculating the probability that a system will successfully complete its mission, to illustrate the methodology, and to validate the methodology.

1-2 APPLICABILITY.

The methodology is directed to analysts, designers, managers, physicists, and planners who are unaccustomed to working with statistics and probability theory. The users, however, are assumed to have had an introductory course in statistics and probability theory. The methodology, which makes use of closed-form solutions and is quite transparent, is designed to yield results quickly and without the use of a large computer. It is not intended to replace the formal, more accurate computerized methodologies that use Monte Carlo simulation or numerical partitioning.

Within its specified applicable domain, the methodology yields quantitatively accurate results. Outside its domain, it yields qualitatively accurate results. The applicable domain is sufficiently large to encompass many problems of interest to the Defense Nuclear Agency community.

1-3 BACKGROUND.

Either an explicit or an implicit probability-of-mission-success criterion is imposed on the designing of a system, be it civil or military, small or large, simple or complex. Each mission of the system would have its own criterion. A typical explicit criterion would read:

> The lower one-sided Q-confidence limit for the probability that the system will successfully complete Mission X shall be at least PMS_o.

Implicit criteria, by contrast, make use of such terms as "factor of safety," "margin of safety," and "reserve capacity." Implicit criteria, which call for deterministic methodology, are not addressed here.

The probability of mission success is calculated by collapsing a series-parallel network of probabilities. Each network probability represents the probability of no failure of the system in a failure mode that, either by itself or in concert with other failure modes, would abort the mission. The fact that there is a probability of failure of the system in a given failure mode reflects the uncertainty embodied in the system's capacity, or in the demand placed upon the system, or both. In general, both random and systematic uncertainties are present. Random uncertainty cannot effectively be reduced by gathering more data or by conducting research and development. Systematic uncertainty, however, can be reduced by gathering more data or by conducting research and development, since it reflects parameter estimation and modeling errors; i.e., it reflects our ignorance. The presence of systematic uncertainty prevents us from calculating the probability of mission success with 100% confidence.

Refer to Reference 6 for additional background.

1-4 ORGANIZATION.

This report is organized into nine sections and two appendixes. Section 2 explains how to calculate the mean, the random coefficient of variation, and the systematic coefficient of variation of the system's capacity in a given failure mode and of the demand placed on the system. Section 3 explains how to calculate the probability that a capacity will exceed a single application of a demand. The methodology presented in Section 3 is extended in Section 4 to the problem of repeated applications of demand. Section 5 concludes that the methodology presented in Section 4 is, with a change of notation, also applicable to a set of failure modes. Section 6 explains how to use the methodology presented in Sections 2 through 5 to assess the probability of mission success of an existing system. Section 7 explains how the reverse concept of assessment, the allocation of the probability of mission success to an evolving system, is accomplished. Finally, closing remarks are made in Section 8. References are given in Section 9. Key derivations are given Appendixes A and B.

With the exception of Sections 5, 8, and 9, each section is organized into four subsections: introduction of the subject, methodology, example problems solved by the methodology, and validation of the methodology by re-solving the example problem with Monte Carlo simulation.

SECTION 2

CAPACITY AND DEMAND

2-1 INTRODUCTION.

A system responding in a given failure mode is characterized by its current capacity to resist failure and by the demand placed on the system. In general, both the capacity and the demand will be random variables. For our purposes, three descriptors are sufficient to describe the capacity or the demand: (1) its mean (expected, or average) value; (2) its random coefficient of variation* (COV); and (3) its systematic COV. Use the equations given below to calculate these three descriptors for the capacity. Change the notations used in these equations and use them to calculate the three descriptors for the demand.

2-2 METHODOLOGY.

First, either implicitly or explicitly state capacity C symbolically in terms of its (functionally) independent variables:

$$C = f(x_1, \ldots x_j, \ldots x_J) \tag{2-1}$$

The first-order approximation of the mean of C is simply†

$$\mu_C \doteq f\left(\mu_{x_1}, \ldots \mu_{x_j}, \ldots \mu_{x_J}\right) \tag{2-2}$$

where μ_{x_j} is the mean of x_j. Equation 2-2 is quite accurate, provided that the nonlinearity in $f(\cdot)$ with respect to x_j, near μ_{x_j}, is not severe and that the variability of x_j is not large. If required, the accuracy of Equation 2-2 can be increased by adding the second-order term:

$$\mu_C \doteq f\left(\mu_{x_1}, \ldots \mu_{x_j}, \ldots \mu_{x_J}\right) + 0.5 \sum_{j=1}^{J} \mu_{x_j}^2 \frac{\partial^2 f}{\partial x_j^2} \delta_{x_j}^2$$

$$+ 0.5 \sum_{\substack{i=1 \\ i \neq j}}^{J} \sum_{j=1}^{J} \mu_i \mu_j \frac{\partial^2 f}{\partial x_i \partial x_i} \rho_{ij} \delta_{x_i} \delta_{x_j} \tag{2-3}$$

---

*The standard deviation divided by the mean.

†See, for example, Reference 1 for the derivation of Equations 2-2, 2-3, and 2-4.

where $\partial^2 f/\partial x_j^2$ is to be evaluated at $\mu_{x_1}, \mu_{x_2}, \mu_{x_3}, \ldots$; δ_{x_j} is the COV of x_j; and ρ_{ij} is the correlation coefficient for x_i and x_j. The square of the COV of C is simply

$$\delta_C^2 \doteq \sum_{j=1}^{J}\left(\frac{\mu_{x_j}}{\mu_C}\right)^2\left(\frac{\partial f}{\partial x_j}\right)^2 \delta_{x_j}^2 + \sum_{\substack{i=1 \\ i\neq j}}^{J}\sum_{j=1}^{J} \frac{\mu_{x_i}\mu_{x_j}}{\mu_C^2}\,\frac{\partial f}{\partial x_i}\,\frac{\partial f}{\partial x_j}\,\rho_{ij}\delta_{x_i}\delta_{x_j} \tag{2-4}$$

Equation 2-4 is quite accurate for $\delta_{x_j}^2 \ll 1$. Notice that the estimates of μ_C and δ_C are dependent only on the means and COVs of the probability density distributions of the independent variables; distribution details enter into the higher-order terms, which are not used here.

Invariably, your values for the mean and COV of x_j will be only estimates. In general, the error in your estimates of δ_{x_j}, the actual COV of x_j, will not be of consequence relative to the error in your estimate of μ_{x_j}, the actual mean of x_j. Therefore, neglect the error in your estimate of δ_{x_j}, but a count for your error in estimating μ_{x_j} by assuming that $\bar{x}_j$, your best estimate of μ_{x_j}, is itself a random variable with actual mean μ_{x_j} and actual COV $\delta_{\bar{x}_j}$. Denote δ_{x_j} as the random COV of x_j, and $\Delta_{\bar{x}_j}$ as the systematic COV of $\bar{x}_j$. In a similar fashion, account for the random and systematic uncertainty in your functional form of C (Eq. 2-1).* Equations 2-2, 2-3, and 2-4 now become

$$\bar{C} \doteq f(\bar{x}_1, \ldots \bar{x}_j, \ldots \bar{x}_J) \tag{2-2a}$$

$$\bar{C} \doteq f(\bar{x}_1, \ldots \bar{x}_j, \ldots \bar{x}_J) + 0.5 \sum_{j=1}^{J} \bar{x}_j^2 \frac{\partial^2 f}{\partial x_j^2}\,\delta_{x_j}^2 + 0.5 \sum_{\substack{i=1 \\ i\neq j}}^{J}\sum_{j=1}^{J} \bar{x}_i\bar{x}_j \frac{\partial f}{\partial x_i \partial x_j}\,\rho_{ij}\delta_{x_i}\delta_{x_j} \tag{2-3a}$$

*This concept for accounting for your error in estimating δ_{x_i} and μ_{x_j} comes from Ang (see, for example, Ref.2). However, in what follows, we depart from Ang's method of application of this concept.

$$\delta_C^2 \doteq \delta_f^2 + \sum_{j=1}^{J}\left(\frac{\bar{x}_j}{\bar{C}}\right)^2\left(\frac{\partial f}{\partial x_j}\right)^2 \delta_{x_j}^2 + \sum_{\substack{i=1 \\ i\neq j}}^{J}\sum_{j=1}^{J} \frac{\bar{x}_i\bar{x}_j}{\bar{C}^2}\,\frac{\partial f}{\partial x_i}\,\frac{\partial f}{\partial x_j}\,\rho_{ij}\delta_{x_i}\delta_{x_j} \tag{2-4a}$$

where δ_f is your best estimate of the COV that reflects the random variability in the functional form of C for given values of x_j, and the remaining symbols, conventions, and restrictions are as previously defined. Use Equation 2-2a or 2-3a to calculate the mean of C, and Equation 2-4a to calculate the random COV of C.

Use the following approximate expression to calculate the square of the systematic COV of $\bar{C}$:

$$\Lambda_{\bar{C}}^2 \doteq \Lambda_{\bar{f}}^2 + \sum_{j=1}^{J}\left(\frac{\bar{x}_j}{\bar{C}}\right)^2\left(\frac{\partial f}{\partial \bar{x}_j}\right)^2 \Lambda_{\bar{x}_j}^2 + \sum_{\substack{i=1 \\ i\neq j}}^{J}\sum_{j=1}^{J} \frac{\bar{x}_i\bar{x}_j}{\bar{C}^2}\,\frac{\partial f}{\partial \bar{x}_i}\,\frac{\partial f}{\partial \bar{x}_j}\,\rho_{\bar{i}\bar{j}}\Lambda_{\bar{x}_i}\Lambda_{\bar{x}_j} \tag{2-5}$$

where $\Lambda_{\bar{x}_j}$ is the COV you associate with how well $\bar{x}_j$ represents μ_{x_j}; $\Lambda_{\bar{f}}$ is the COV you associate with the modeling error in your functional form of $\bar{C}$; $\rho_{\bar{i}\bar{j}}$ is the correlation coefficient for $\bar{x}_i$ and $\bar{x}_j$; and the remaining symbols, conventions, and restrictions are as previously defined. Notice the similarity between Equations 2-4a and 2-5.

Determine $\bar{x}_j$, δ_{x_j}, δ_f, and ρ_{ij} from appropriate statistical data. Choose $\Lambda_{\bar{x}_j}$ by quantifying your "degrees of belief" about how well $\bar{x}_j$ represents μ_{x_j}, the actual mean of x_j.* (Similarly for $\Lambda_{\bar{f}}$.) Use the information provided in Figure 2-1 to help quantify your degrees of belief. Use judgment to assign values for $\rho_{\bar{i}\bar{j}}$, the correlation coefficient for $\bar{x}_i$ and $\bar{x}_j$.

It should not be overlooked that a large, deterministic computer code can be used to calculate means and COVs with the above methodology. The code itself would be used to calculate the first-order estimate of the mean and the partial derivatives appearing in the equations for the COVs. The stringing together of these partials for the actual calculation of the COVs and the second-order estimate of the mean would be done outside the code.

---

*If experiments have been performed, use $\Lambda_{\bar{x}_j} \doteq \delta_{x_j}/\sqrt{n}$, where n is the number of experiments.

2-3 ILLUSTRATIVE USE.

Calculate the mean, the random COV, and the systematic COV of the capacity

$$C = x^2 \exp(0.50y)$$

where

$$\bar{x} = 1.00$$

$$\bar{y} = 0.50$$

$$\delta_x = 0.17$$

$$\delta_y = 0.23$$

$$\delta_f = 0.10$$

$$\Delta_{\bar{x}} = 0.03$$

$$\Delta_{\bar{y}} = 0.11$$

$$\Delta_{\bar{f}} = 0.15$$

$$\rho_{xy} = \rho_{yx} = 0.50$$

$$\rho_{\bar{x}\bar{y}} = \rho_{\bar{y}\bar{x}} = 0.00$$

First, calculate $\bar{C}$, δ_C, and $\Delta_{\bar{C}}$ using Equation 2-2a to estimate the mean of C.

<u>Equation 2-2a</u>.

$$\bar{C} \doteq \bar{x}^2 \exp(0.50\bar{y})$$

$$\doteq 1.284$$

<u>Equation 2-4a</u>.

$$\delta_C^2 \doteq \delta_f^2 + 4\delta_x^2 + 0.25\bar{y}^2\delta_y^2 + \bar{y}\rho_{xy}\delta_x\delta_y + \bar{y}\rho_{yx}\delta_x\delta_y$$

$$\doteq 0.385^2$$

Equation 2-5.

$$\Delta_{\bar{C}}^2 = \Delta_{\bar{f}}^2 + 4\Delta_{\bar{x}}^2 + 0.25\bar{y}^2\Delta_{\bar{y}}^2 + \bar{y}\rho_{\bar{x}\bar{y}}\Delta_{\bar{x}}\Delta_{\bar{y}} + \bar{y}\rho_{\bar{y}\bar{x}}\Delta_{\bar{x}}\Delta_{\bar{y}}$$

$$= 0.164^2$$

Now, calculate $\bar{C}$, δ_C, and $\Delta_{\bar{C}}$ using Equation 2-3a to estimate the mean of C.

Equation 2-3a.

$$\bar{C} \doteq \bar{x}^2 \exp(0.50\bar{y})\left[1 + \delta_x^2 + 0.125\bar{y}^2\delta_y^2 + 0.5\rho_{xy}\delta_x\delta_y + 0.5\rho_{yx}\delta_x\delta_y\right]$$

$$\doteq 1.284 \times 1.05$$

$$\doteq 1.348$$

Equation 2-4a.

$$\delta_C^2 = \frac{0.385^2}{1.05^2} = 0.367^2$$

Equation 2-5.

$$\Delta_{\bar{C}}^2 = \frac{0.164^2}{1.05^2} = 0.156^2$$

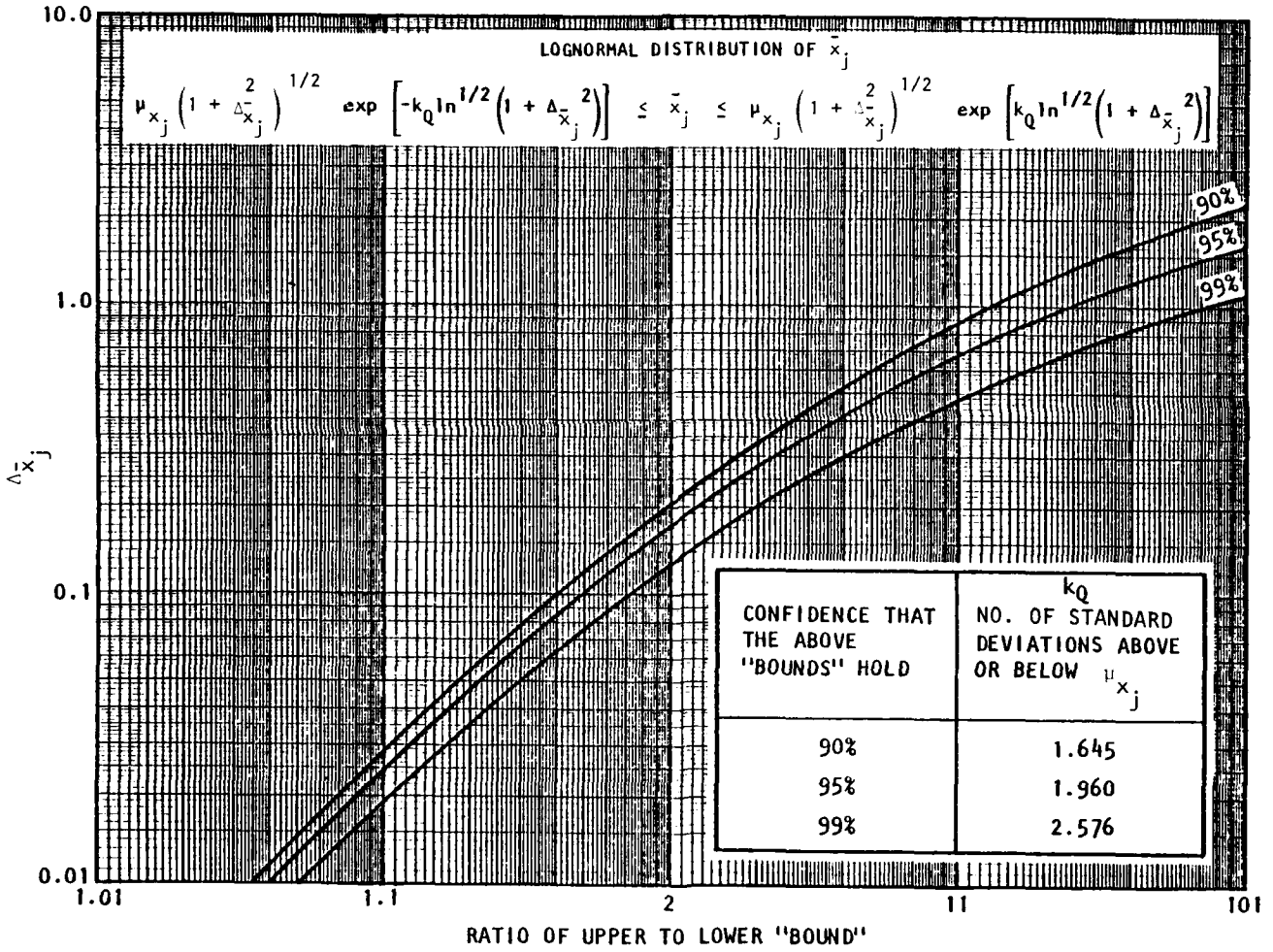
Since the second-order terms are only 5% of the first-order term, we can conclude that the estimates of $\bar{C}$, δ_C, and $\Delta_{\bar{C}}$ are, as will be seen below, relatively accurate and relatively insensitive to the assumed distributions of x, y, f, $\bar{x}$, $\bar{y}$, and $\bar{f}$.

2-4 VALIDATION

In order to validate the methodology presented in this document, we have used the Monte Carlo simulation technique to rework the illustrative problems. In this technique, pseudorandom variates are generated on a computer to form artificial samples. We used the Marsaglia-Bray method (Ref. 3) on a Data General Eclipse S/130 computer to generate pseudorandom normal and lognormal deviates. Cycling of the deviates was required for some of the problems because of inadequate word length on the computer.

In our simulations for the illustrative problem presented in Section 2-3, an outer loop of size 1000 controlled the generation of the systematic deviates. For each set of systematic deviates, an inner loop of 1000 sets of random deviates was collected. In our first simulation, all distribution models (x, y, f, $\bar{x}$, $\bar{y}$, $\bar{f}$) were assumed normal. The results were $\bar{C} = 1.339$, $\delta_C = 0.378$, and $\Lambda_{\bar{C}} = 0.152$. Our methodology, which makes no assumption as to distribution, yielded $\bar{C} = 1.348$, $\delta_C = 0.367$, and $\Lambda_{\bar{C}} = 0.156$. The agreement is seen to be quite good.

In our second simulation, all distribution models except f and $\bar{f}$ were changed to lognormal to test the sensitivity of the results to the assumed distribution models. The results were $\bar{C} = 1.344$, $\delta_C = 0.399$, and $\Lambda_{\bar{C}} = 0.153$. It is seen that the assumed distributions of x, y, f, $\bar{x}$, $\bar{y}$, and $\bar{f}$ do have an influence on $\bar{C}$, δ_C, and $\Lambda_{\bar{C}}$, but that it is small. In general, this influence will be small regardless of the distributions, provided the second-order term in Equation 2-3a is small compared to the first-order term.



CONFIDENCE THAT THE ABOVE "BOUNDS" HOLD	k_Q NO. OF STANDARD DEVIATIONS ABOVE OR BELOW μ_{x_j}
90%	1.645
95%	1.960
99%	2.576

Figure 2-1. Aid in assigning $\Delta_{\bar{x}_j}$.

SECTION 3

CAPACITY WILL EXCEED DEMAND

3-1 INTRODUCTION.

Consider a failure mode where the system's capacity is C under the demand D. The probability that C will exceed D can be calculated with 100% confidence if you know the distribution, the true value of the mean, the random COV, and the functional form of C and D, that is, if $\Delta_{\bar{C}} = \Delta_{\bar{D}} = \Delta_{\bar{f}_C} = \Delta_{\bar{f}_D} = 0$. For lognormal distribution of C and D, this probability is precisely

$$P(C > D) = \Phi\left\{\frac{\ln\left(\frac{\mu_C}{\mu_D}\sqrt{\frac{1+\delta_D^2}{1+\delta_C^2}}\right)}{\sqrt{\ln\left(1+\delta_C^2\right)\left(1+\delta_D^2\right)}}\right\} \tag{3-1}$$

where

μ_C = Mean of C

μ_D = Mean of D

δ_C = COV of C

δ_D = COV of D

$\Phi(\xi)$ = Cumulative probability of the standard normal variate ξ (see Table 3-1)

However, for the realistic case of nonzero Δ's, $P(C > D)$ itself becomes a random variable, in the Bayesian sense, with expected value $\bar{P}$ and systematic COV Δ_P. In what follows, we elect to write $Q(P > P_o)$ for the probability that $P > P_o$, and to read $Q(P > P_o)$ as "our confidence that P exceeds P_o."

This section shows how to calculate $\bar{P}$, Δ_P, and $Q(P > P_o)$ for lognormal distribution of C, D, $\bar{C}$, and $\bar{D}$ and for functionally and statistically independent C, D, $\bar{C}$, and $\bar{D}$. However, the methodology will yield reasonably accurate results for other unimodal distributions when $\bar{P}$, P_o, and Q are near 0.5, say, $0.01 < (\bar{P}, P_o, Q) < 0.99$.

3-2 METHODOLOGY.

Calculate the expected value of $P(C > D)$ using the following equation:*

$$\bar{P}(C > D) \doteq \Phi\left\{\frac{\ln\left[\frac{\bar{C}}{\bar{D}}\sqrt{\frac{\left(1+\delta_D^2\right)\left(1+\Delta_{\bar{C}}^2\right)}{\left(1+\delta_C^2\right)\left(1+\Delta_{\bar{D}}^2\right)}}\right]}{\sqrt{\left(1+T^2\right)\ln\left(1+\delta_C^2\right)\left(1+\delta_D^2\right)}}\right\} \tag{3-2}$$

where

$\bar{C}$ = Your best estimate of μ_C, the actual mean (average, or expected) value of C (see Eqs. 2-2a and 2-3a)

$\bar{D}$ = Your best estimate of μ_D, the actual mean (average, or expected) value of D

$\Delta_{\bar{C}}$ = COV that reflects how well $\bar{C}$ represents μ_C, the actual mean of C (see Eq. 2-5)

$\Delta_{\bar{D}}$ = COV that reflects how well $\bar{D}$ represents μ_D, the actual mean of D

$\Phi(\xi)$ = Cumulative probability of the standard normal variate ξ (see Table 3-1)

δ_C = COV that reflects your best estimate of the random nature of C (see Eq. 2-4a)

δ_D = COV that reflects your best estimate of the random nature of D

$$T = \sqrt{\frac{\ln\left(1+\Delta_{\bar{C}}^2\right)\left(1+\Delta_{\bar{D}}^2\right)}{\ln\left(1+\delta_C^2\right)\left(1+\delta_D^2\right)}} \tag{3-3}$$

Note that the ratio $\bar{C}/\bar{D}$ is, to the first order, the mean factor of safety. For small COVs (i.e., $\delta_C^2 << 1$, $\delta_D^2 << 1$, $\Delta_{\bar{C}}^2 << 1$, and $\Delta_{\bar{D}}^2 << 1$), Equation 3-2 reduces to

*The derivation of Equations 3-2 and 3-5 is given in Appendix A.

$$\bar{P}(C > D) \doteq \Phi\left\{\frac{\ln(\bar{C}/\bar{D})}{\sqrt{\left(1 + T^2\right)\left(\delta_C^2 + \delta_D^2\right)}}\right\} \tag{3-2a}$$

where

$$T = \sqrt{\frac{\Delta_{\bar{C}}^2 + \Delta_{\bar{D}}^2}{\delta_C^2 + \delta_D^2}} \tag{3-3a}$$

Equation 3-2a is displayed in Figure 3-1.

The median value of $P(C > D)$ is

$$\tilde{P}(C > 0) \doteq \Phi\left(k_{\bar{P}}\sqrt{1 + T^2}\right) \tag{3-4}$$

where $\tilde{P} = \Phi(k_{\bar{P}})$. *With the aid of* Equation 3-2, Equation 3-4 becomes

$$\tilde{P}(C > D) \doteq \Phi\left\{\frac{\ln\left[\frac{\bar{C}}{\bar{D}}\sqrt{\frac{\left(1 + \delta_D^2\right)\left(1 + \Delta_{\bar{C}}^2\right)}{\left(1 + \delta_C^2\right)\left(1 + \Delta_{\bar{D}}^2\right)}}\right]}{\sqrt{\ln\left(1 + \delta_C^2\right)\left(1 + \delta_D^2\right)}}\right\} \tag{3-5}$$

Use Figure 3-2, or the following approximate expressions, to determine the systematic COV of $P(C > D)$:*

$$\Delta_P^2 \doteq \frac{T^2}{2\pi\bar{P}^2(1 + T^2)}\exp\left(-k_{\bar{P}}^2 \cdot \frac{1 + T^2}{1 + \frac{3}{2}T^2}\right) \qquad \text{for } T < 0.7 \tag{3-6}$$

$$\Delta_P^2 \doteq \frac{\left(0.25 - \frac{1}{\pi}\cot^{-1}\sqrt{1 + 2T^2}\right)\left[4\bar{P}(1 - \bar{P})\right]^{1.327T^{-0.2155}}}{\bar{P}^2} \qquad \text{for } T > 0.7 \tag{3-7}$$

*Unfortunately, a closed-form expression for Δ_P could not be derived. The derivation of Equations 3-6 and 3-7 is given in Appendix B. It should be noted that Equation 3-7 is partially empirical.

Use the following equation to calculate your confidence that $P(C > D) > P_o$:*

$$Q(P > P_o) = \Phi\left(\frac{k_{\tilde{P}}\sqrt{1 + T^2} - k_{P_o}}{T}\right) \tag{3-8}$$

$$= \Phi\left(\frac{k_{\tilde{P}} - k_{P_o}}{T}\right) \tag{3-8a}$$

Equation 3-8a is plotted in Figure 3-3 for an illustrative value of $\tilde{P}$ and for various T. Note that Equation 3-8 plots as a straight line in normal-normal probability space.

The following alternative form of Equation 3-2 is better suited for calculating the required mean factor of safety of a failure mode:

$$\frac{\tilde{C}}{\tilde{D}} = \sqrt{\frac{\left(1 + \delta_C^2\right)\left(1 + \Delta_{\tilde{D}}^2\right)}{\left(1 + \delta_D^2\right)\left(1 + \Delta_{\tilde{C}}^2\right)}} \exp\left[k_{\tilde{P}}\sqrt{\left(1 + T^2\right) \ln\left(1 + \delta_C^2\right)\left(1 + \delta_D^2\right)}\right] \tag{3-9}$$

For small COVs, Equation 3-9 reduces to

$$\frac{\tilde{C}}{\tilde{D}} = \exp\left[k_{\tilde{P}}\sqrt{\left(1 + T^2\right)\left(\delta_C^2 + \delta_D^2\right)}\right] \tag{3-9a}$$

3-3 ILLUSTRATIVE USE.

Calculate the probability that capacity will exceed demand, where

$\tilde{C}$	= 1.28	δ_D	= 0.21
$\tilde{D}$	= 0.55	$\Delta_{\tilde{C}}$	= 0.16
δ_C	= 0.39	$\Delta_{\tilde{D}}$	= 0.24

*Substitution of the lower one-sided Q-confidence limit for μ_C/μ_D into Equation 3-1 yields, upon rearranging terms, Equation 3-8.

First calculate the parameter T (Eq. 3-3):

$$T = \sqrt{\frac{\ln(1 + 0.16^2)(1 + 0.24^2)}{\ln(1 + 0.39^2)(1 + 0.21^2)}} = 0.663$$

Next, calculate the expected probability of no failure (Eq. 3-2):

$$\bar{P}(C > D) = \Phi\left\{\frac{\ln \frac{1.28}{0.55}\sqrt{\frac{(1 + 0.21^2)(1 + 0.16^2)}{(1 + 0.39^2)(1 + 0.24^2)}}}{\sqrt{(1 + 0.66^2)\ln(1 + 0.39^2)(1 + 0.21^2)}}\right\}$$

$$= \Phi(1.51)$$

$$= 0.935 \text{ (Table 3-1)}$$

Next, for the purpose of later reference, calculate Δ_P^2 (Eq. 3-6):

$$\Delta_P^2 = \frac{0.663^2}{2\pi \times 0.935^2 \times (1 + 0.663^2)} \operatorname{Exp}\left[- 1.51^2 \left(\frac{1 + 0.663^2}{1 + 1.5 \times 0.663^2}\right)\right]$$

$$= 0.0877^2$$

Next, calculate three points on the Q vs. P_o curve using Equation 3-8:

$$Q = \Phi\left(\frac{1.51\sqrt{1 + 0.66^2} - k_{P_o}}{0.66}\right)$$

$$= \Phi\left(\frac{1.81 - k_{P_o}}{0.66}\right)$$

P_o	Q
0.5	0.997
0.965	0.5
0.999	0.26

Plotting these three points in normal-normal probability space yields the solid line shown in Figure 3-4. The open and filled circles are the data points that result from a reworking of this same problem by Monte Carlo simulation, as will be discussed in the next subsection.

3-4 VALIDATION.

In order to validate the methodology presented in Section 3-2, Monte Carlo simulation was used to rework the illustrative problem (see Sec. 2-4). An outer loop of 1000 and an inner loop of 855 were used.

In our first simulation, all distribution models (C, D, $\bar{C}$, $\bar{D}$) were assumed to be lognormal, like our methodology. The results were $\bar{P} = 0.938$ and $\Delta_P = 0.0883$. Our methodology, which yielded $\bar{P} = 0.935$ and $\Delta_P = 0.0877$, is in excellent agreement. Some of the percentiles for the Q function for this simulation are indicated in Figure 3-4 by the filled circles.

In our second simulation, all distribution models were assumed to be normal to test the sensitivity of the results to the assumed distribution models. The results were $\bar{P} = 0.900$ and $\Delta_P = 0.0932$. These results were indicated by the open circles plotted in Figure 3-4. For most applications, the disparity shown in Figure 3-4 for Q greater than about 0.1 would be acceptable.

Table 3-1. Tabulated cumulative probability of the normal distribution.

ξ	$\Phi(\xi)$
-3.090	0.001
-2.576	0.005
-2.326	0.010
-1.960	0.025
-1.645	0.050
-1.282	0.100
-1.036	0.150
-0.842	0.200
-0.674	0.250
-0.524	0.300
-0.385	0.350
-0.253	0.400
-0.126	0.450
0.000	0.500
0.126	0.550
0.253	0.600
0.385	0.650
0.524	0.700
0.674	0.750
0.842	0.800
1.036	0.850
1.282	0.900
1.645	0.950
1.960	0.975
2.326	0.990
2.576	0.995
3.090	0.999

*For a more extensive table, refer to a textbook or a handbook of statistics and probability.

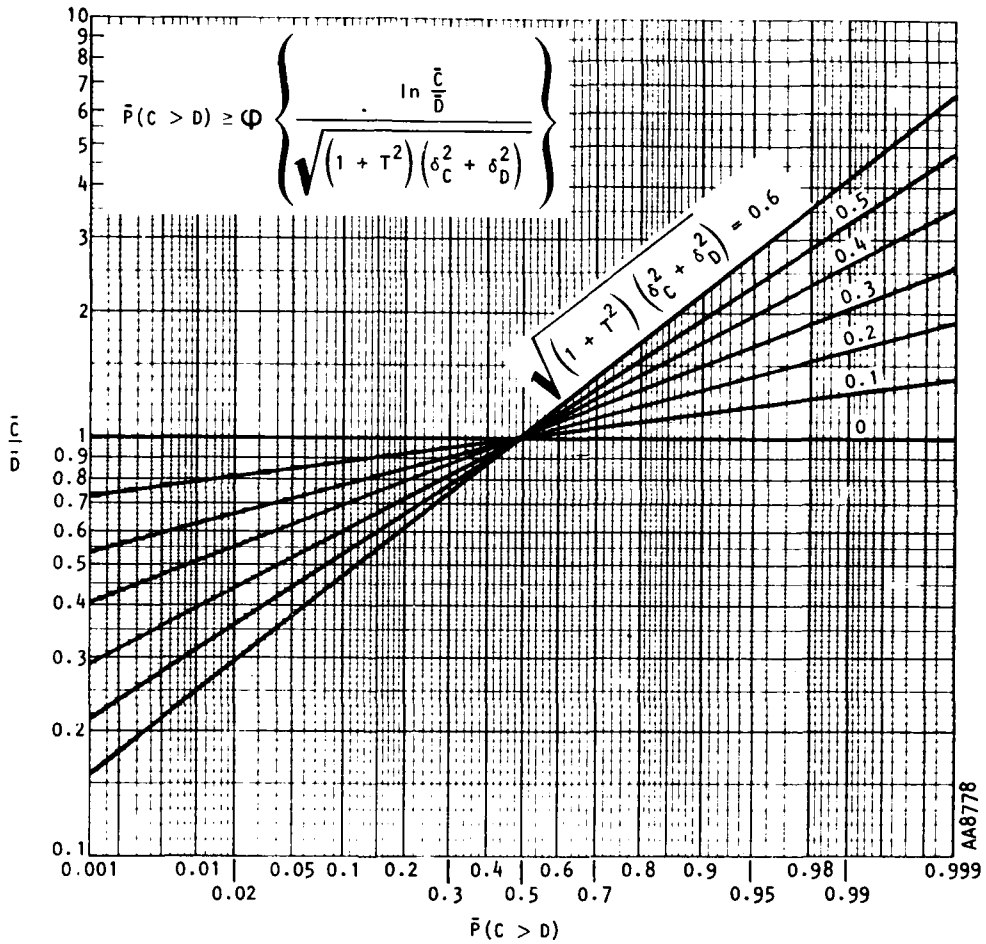


Figure 3-1. Expected probability that capacity will exceed demand — small COV'S.

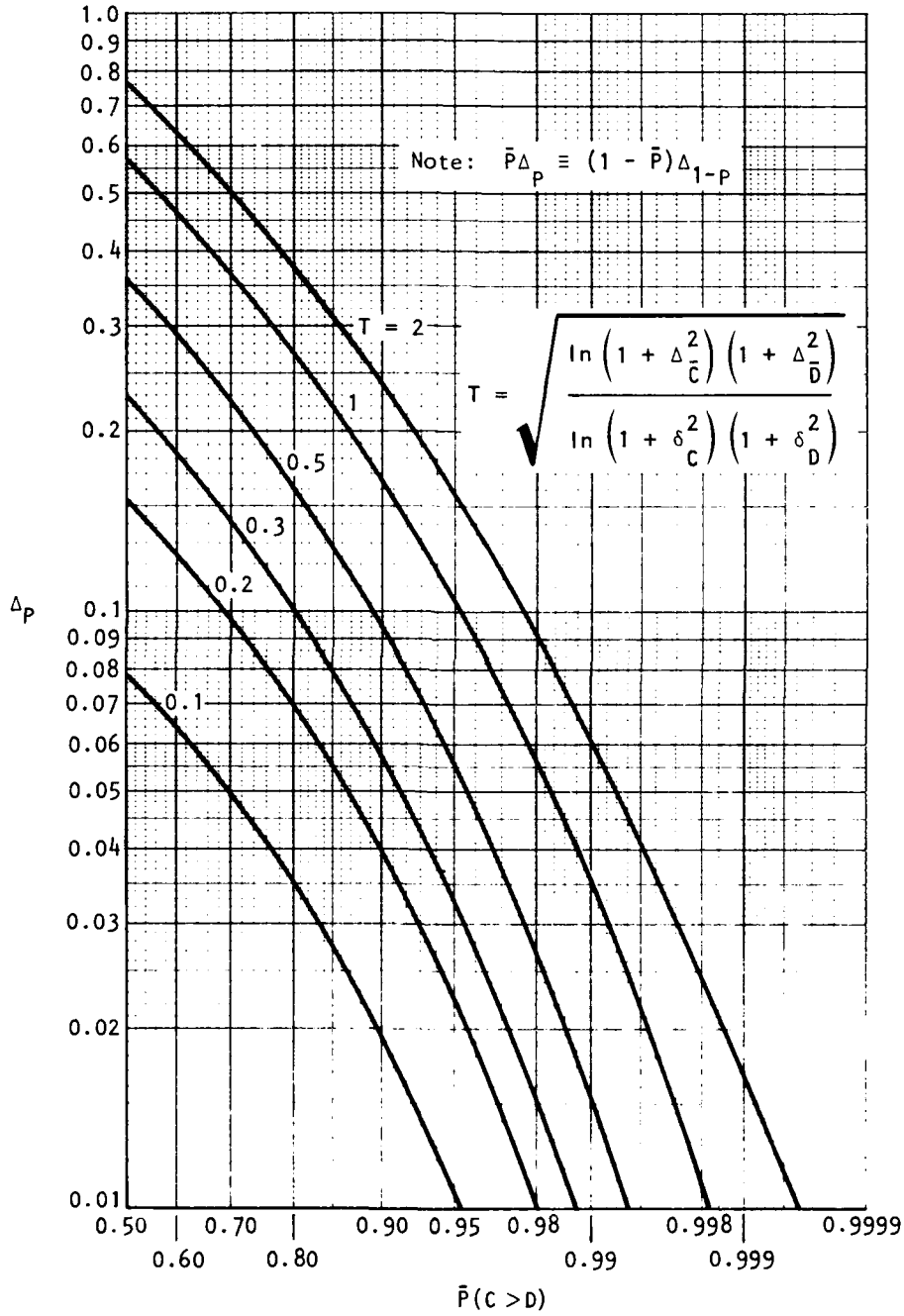


Figure 3-2. Systematic COV of P(C > D).

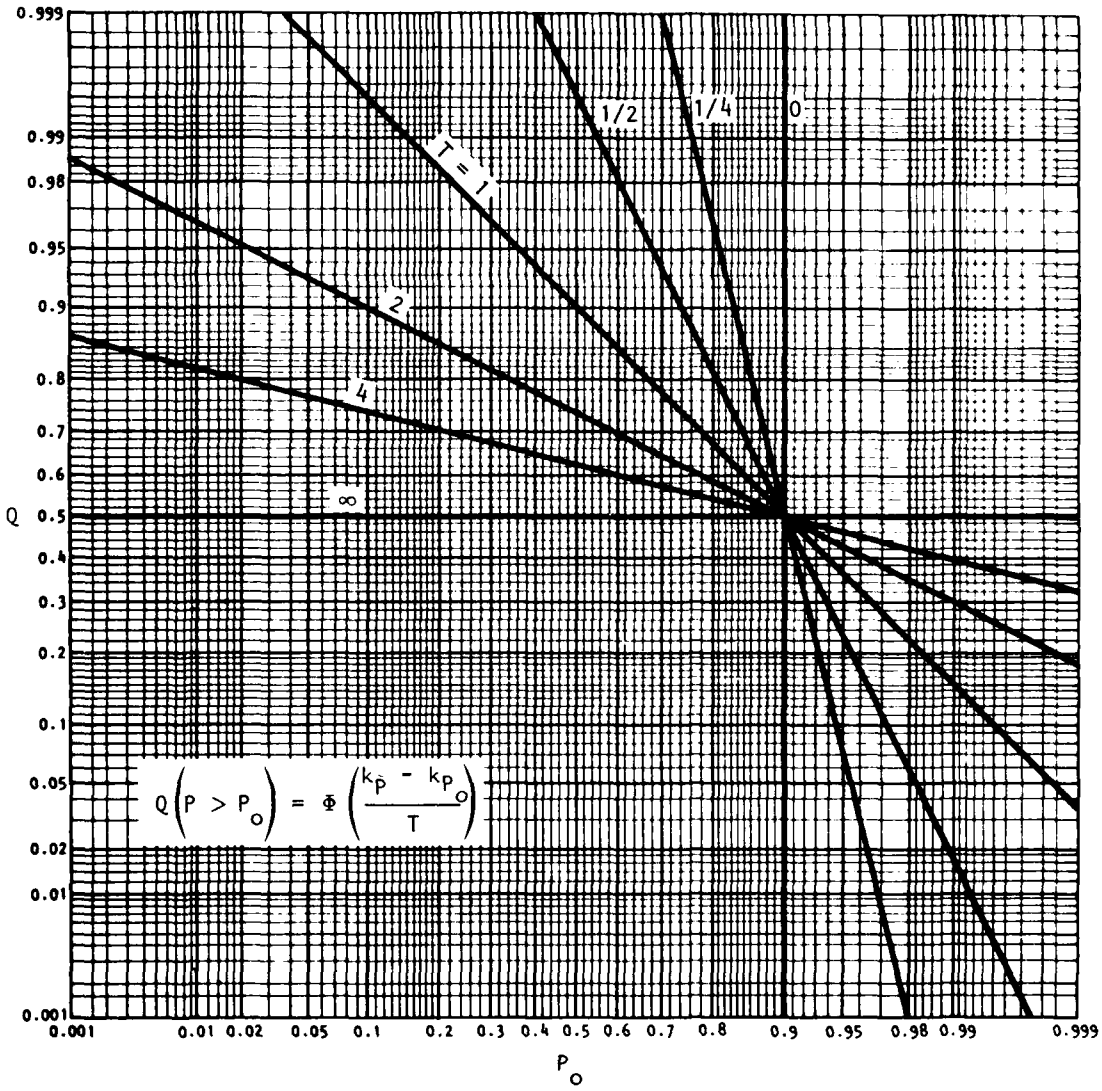


Figure 3-3. Plot of Equation 3-8a for $\tilde{P}$ = 0.9.

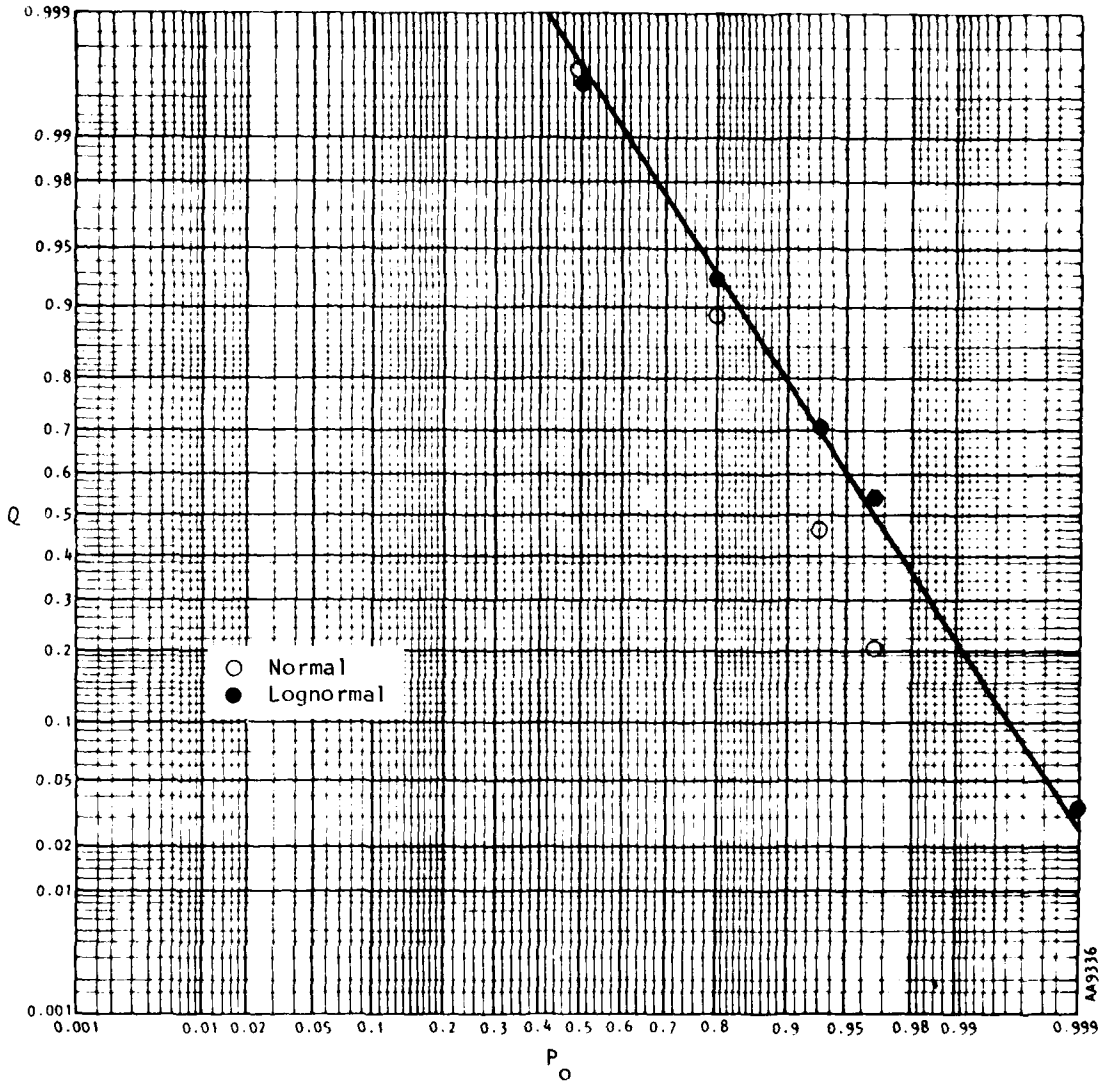


Figure 3-4. Illustrative problem.

4

SECTION 4

CAPACITY WILL EXCEED REPEATED DEMANDS

4-1 INTRODUCTION.

This section extends the results presented in Section 3 to the case of repeated application of demand. Capacity or demand or both may change randomly or deterministically from demand application to demand application. The set of demands may be deterministic, perfectly dependent, statistically independent, or partially dependent. All the demands of a deterministic set are known. None of the demands are known in an independent set. Given one demand, the other demands in a perfectly dependent set become deterministic. These same remarks apply to the set of capacities.

The probability that the system's capacity in a given failure mode will exceed repeated demands is shown in Table 4-1 for each of these states of dependency (after Ref. 4). For states where P is bounded and $\delta^2_{C_n}$ is either large (demands are, in effect, deterministic) or small (capacities are, in effect, deterministic) compared to $\delta^2_{D_n}$, the appropriate deterministic case is approximately correct. Figure 4-1 shows the error introduced when $\delta^2_{D_n}$ is neither large nor small compared to $\delta^2_{C_n}$. Notice that if the demand set has a common density distribution (i.e., a common mean and COV) and the capacity set has a common density distribution, the entries in Table 4-1 simplify to

$$P = P_{n'} = P_1 \tag{4-1}$$

$$P = \prod_{n=1}^{N} P_n = P_1^N \tag{4-2}$$

$$P_1^N \leq P \leq P_1 \tag{4-3}$$

Since P_n is a random variable, in the Bayesian sense (see Sec. 3), the resultant probability P is also a random variable. How to calculate P is shown below.

4-2 METHODOLOGY.

For those dependency states where P is equal to or can be approximated by $P_{n'}$, calculate $\bar{P}$ and Δ_P as follows:

$$\bar{P} = \bar{P}_{n'} \tag{4-4}$$

$$\Delta_P = \Delta_{P_{n'}} \tag{4-5}$$

where $\bar{P}_{n'}$ and $\Delta_{P_{n'}}$ are calculated using the information given in Section 3. For those states where P is equal to or can be approximated by $\prod_{n=1}^{N} P_n$, calculate $\bar{P}$ and Δ_P as follows:

$$\bar{P} = \prod_{n=1}^{N} \bar{P}_n \tag{4-6}$$

$$\Delta_P^2 \doteq \sum_{n=1}^{N} \Delta_{P_n}^2 + \sum_{\substack{n=1 \\ n \neq m}}^{N} \sum_{m=1}^{N} \rho_{\bar{P}_n \bar{P}_m} \Delta_{P_n} \Delta_{P_m} \tag{4-7}$$

where $\bar{P}_n$ and Δ_{P_n} are calculated using the information given in Section 3, and $\rho_{\bar{P}_n \bar{P}_m}$ is the correlation coefficient for $\bar{P}_n$ and $\bar{P}_m$. When neither Equation 4-4 nor Equation 4-6 is a reasonable approximation to the truth, interpolate between the two.

The correlation coefficient $\rho_{\bar{P}_n \bar{P}_m}$ is equal to zero if $\bar{C}_n$ and $\bar{C}_m$ are independent and $\bar{D}_n$ and $\bar{D}_m$ are independent. For other dependency states, calculate $\rho_{\bar{P}_n \bar{P}_m}$ as follows:

$$\rho_{\bar{P}_n \bar{P}_m} \doteq \frac{\rho_{\bar{C}_n \bar{C}_m} \sqrt{\ln\left(1 + \Delta_{\bar{C}_n}^2\right) \ln\left(1 + \Delta_{\bar{C}_m}^2\right)} + \rho_{\bar{D}_n \bar{D}_m} \sqrt{\ln\left(1 + \Delta_{\bar{D}_n}^2\right) \ln\left(1 + \Delta_{\bar{D}_m}^2\right)}}{\sqrt{\ln\left(1 + \Delta_{\bar{C}_n}^2\right)\left(1 + \Delta_{\bar{D}_n}^2\right) \ln\left(1 + \Delta_{\bar{C}_m}^2\right)\left(1 + \Delta_{\bar{D}_m}^2\right)}} \tag{4-8}$$

Calculate $Q(P > P_o)$ using Equation 3-8, where T is obtained by entering Figure 3-2 with $\bar{P}$ and Δ_P. Note that for $P \neq P_n$, $Q(P > P_o)$ is approximate and T has no physical meaning.

4-3 ILLUSTRATIVE USE.

Calculate the probability (P_o vs. Q) that the system's capacity in a given failure mode will exceed four applications of demand. Assume C_1, C_2, C_3, and C_4 are statistically independent, but have a common distribution function. Assume the same independence and commonality for the demands and the expected demands. However, assume that the expected capacities are perfectly dependent. Assume*

$$\bar{C} = 1.28 \qquad \delta_D = 0.21$$

$$\bar{D} = 0.55 \qquad \Delta_{\bar{C}} = 0.16$$

$$\delta_C = 0.39 \qquad \Delta_{\bar{D}} = 0.24$$

From Table 4-1, we know that

$$P = \prod_{n=1}^{n} P_n = P_1 P_2 P_3 P_4$$

From Section 3-3, we know that

$$\bar{P}_1 = \bar{P}_2 = \bar{P}_3 = \bar{P}_4 = 0.935$$

$$\Delta_{P_1} = \Delta_{P_2} = \Delta_{P_3} = \Delta_{P_4} = 0.0855$$

*Note: these are the same parameters used in Section 3-3.

Substituting into Equations 4-6 and 4-7 yields

$$\bar{P} = 0.935^4 = 0.764$$

$$\Delta_P^2 = 4 \times 0.0877^2 + 12 \times \rho_{\bar{P}_n \bar{P}_m} \times 0.0877^2$$

The correlation coefficient $\rho_{\bar{P}_n \bar{P}_m}$ is calculated using Equation 4-7 with $\rho_{\bar{D}_n \bar{D}_m} = 0$ and $\rho_{\bar{C}_n \bar{C}_m} = 1.0$. Equation 4-7 yields $\rho_{\bar{P}_n \bar{P}_m} = 0.311$ and Δ_P becomes 0.244, upon substitution for $\rho_{\bar{P}_n \bar{P}_m}$.

Entering Figure 3-2 with $\Delta_P = 0.244$ and $\bar{P} = 0.764$ yields $T = 0.7$. Entering Equation 3-8 with $T = 0.7$ and $\bar{P} = 0.764$ gives

$$Q = \Phi\left(\frac{0.72\sqrt{1 + 0.7^2} - k_{P_o}}{0.7}\right)$$

$$= \Phi\left(\frac{0.88 - k_{P_o}}{0.7}\right)$$

P_o	Q
0.5	0.90
0.81	0.5

The straight line shown in Figure 4-2 was constructed from the above data.

4-4 VALIDATION.

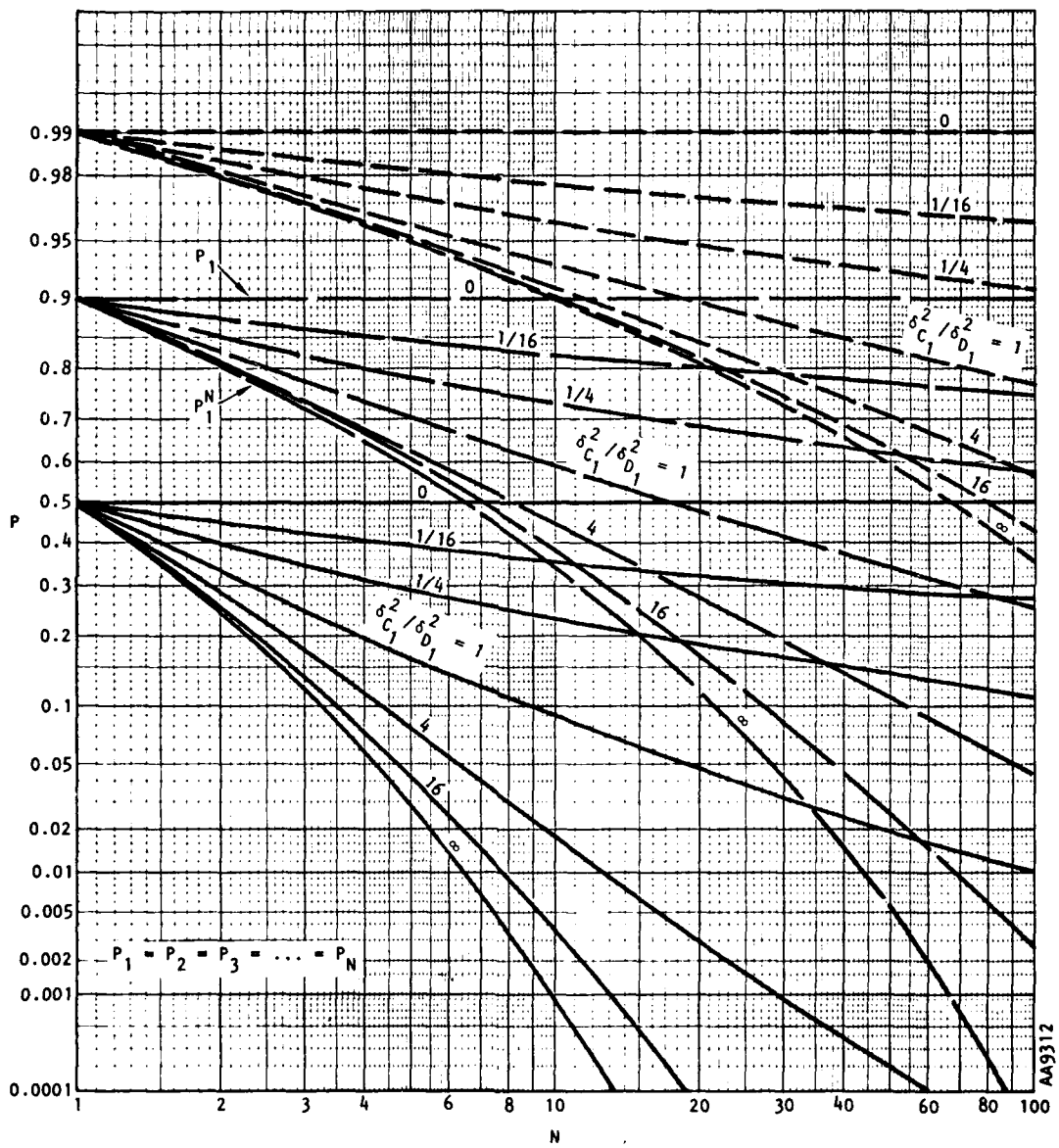
In order to validate the methodology presented in Section 4, Monte Carlo simulation was used to rework the illustrative problem. An outer loop of 139 and an inner loop of 855 was used. All distributions were assumed to be lognormal. The results were $\bar{P} = 0.783$ and $\Delta_P = 0.202$. Some of the percentiles for the Q function are shown in Figure 4-2 as the filled circles. The agreement between the two solutions is reasonably good.

Table 4-1. Probability that capacity will exceed N demands: One failure mode

Demands	Capacities			
	Deterministic	Perfectly Dependent	Statistically Independent	Unknown Dependency
Deterministic		$P = P_{n'}$	$P = \prod_{n=1}^{N} P_n$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$
Perfectly Dependent	$P = P_{n'}$	$P = P_{n'}$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$
Statistically Independent	$P = \prod_{n=1}^{N} P_n$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$	$P = \prod_{n=1}^{N} P_n$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$
Unknown Dependency	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$	$\prod_{n=1}^{N} P_n \le P \le P_{n'}$

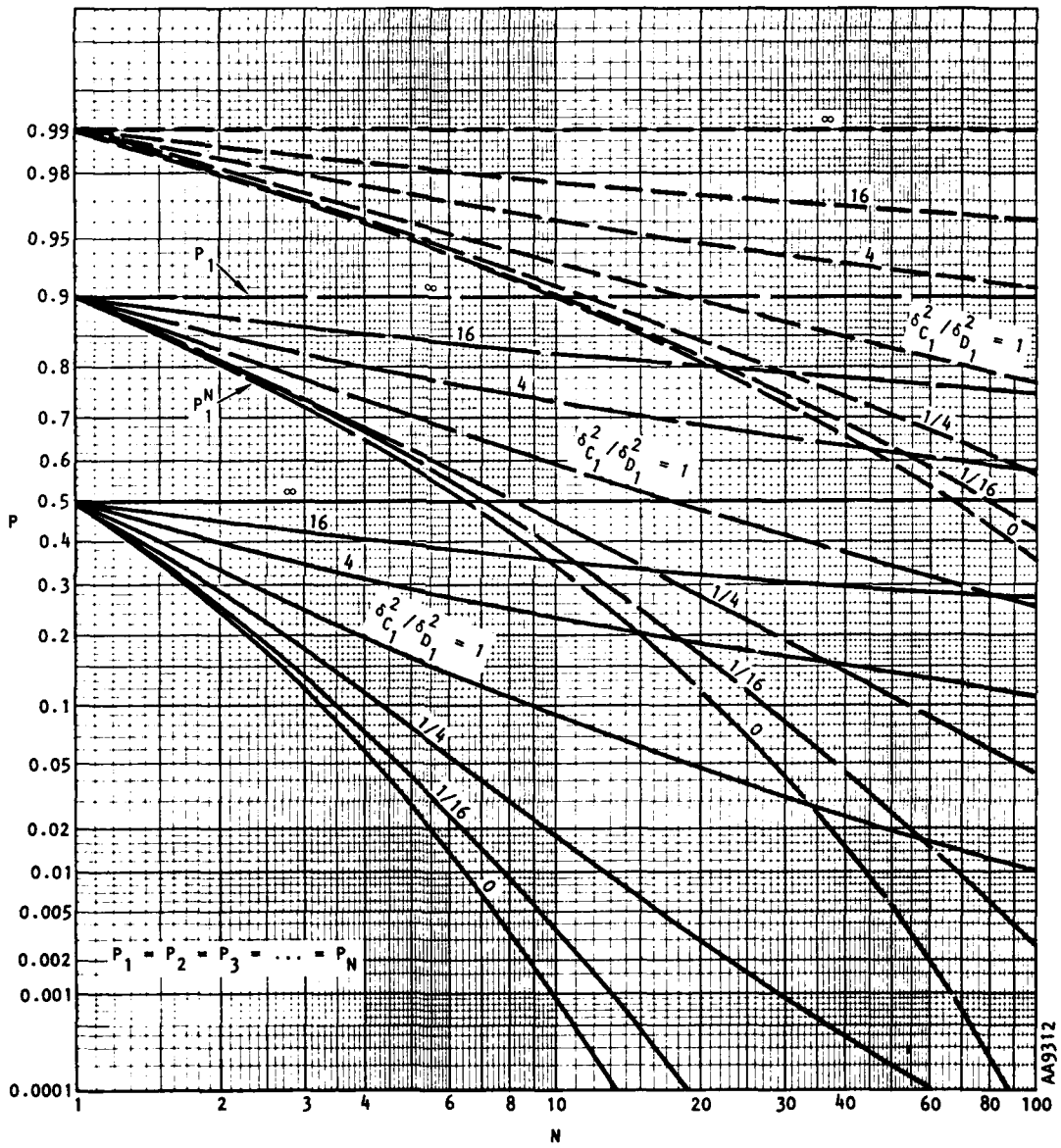
Note: 1. $P_n = P(C_n > D_n)$

2. $P_{n'}$ is the smallest probability in the set $P_1, P_2, P_3, \ldots, P_n, P_n, \ldots, P_N$



(a) Statistically independent capacities and perfectly dependent demands

Figure 4-1. Probability that capacity will exceed repeated demands: One failure mode.



(b) Perfectly dependent capacities and statistically independent demands

Figure 4-1. (Concluded).

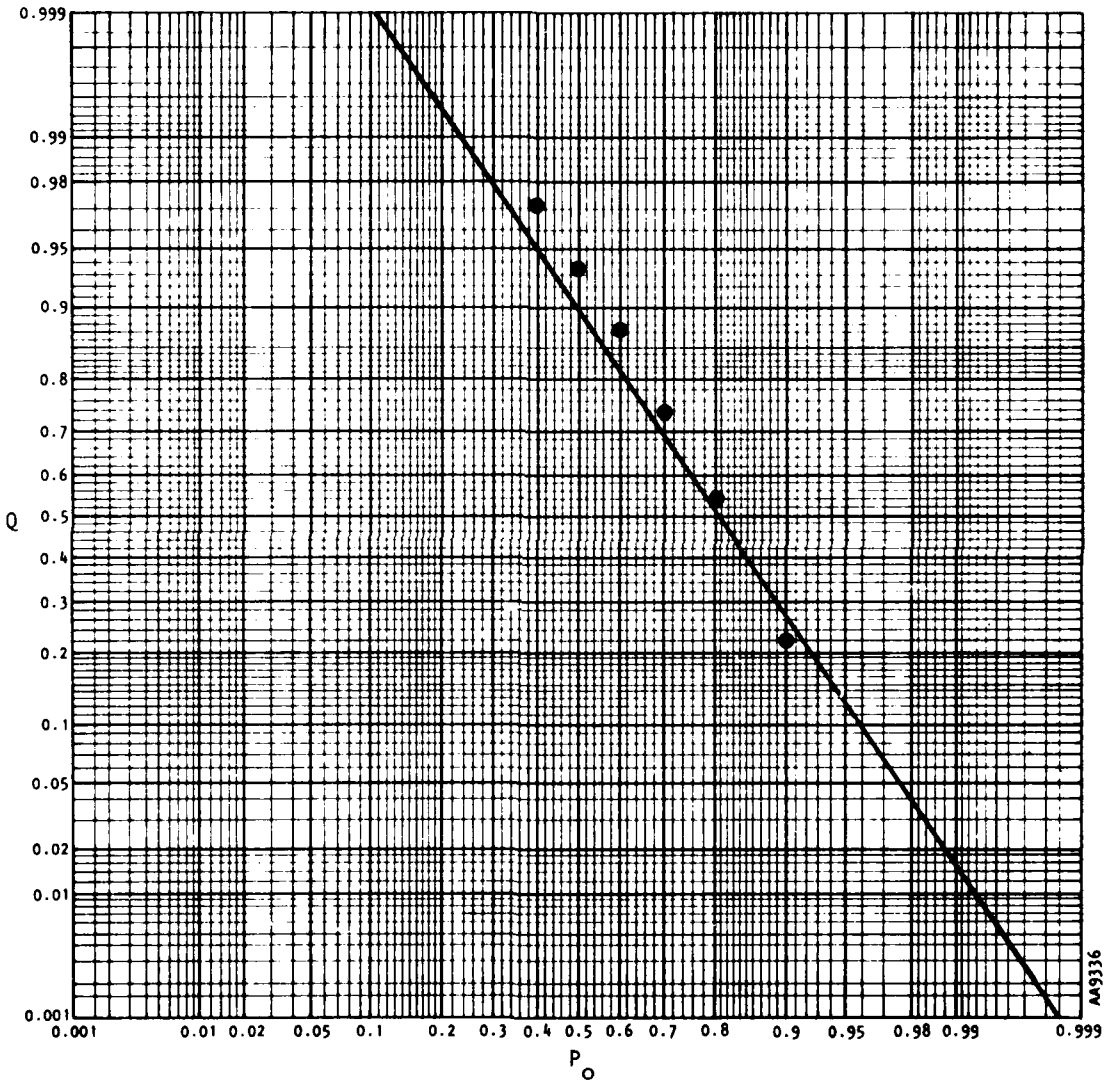


Figure 4-2. Illustrative problem.

SECTION 5

FAILURE-MODE SETS

This section extends the results presented in Section 3 to a set of failure modes. Only one application of demand per failure mode is considered. Capacity or demand or both may change randomly or deterministically from failure mode to failure mode. The set of capacities may be deterministic, perfectly dependent, statistically independent, or partially dependent. All the capacities of a deterministic set are known. None of the capacities of an independent set are known. Given one capacity, the other capacities in a perfectly dependent set become deterministic. The same remarks apply to the set of demands.

The probability that all the capacities in an M-failure-mode set will exceed their demands is shown in Table 5-1 for each of these states of dependency (after Ref. 4). Noting the similarity between Tables 4-1 and 5-1, we conclude that the methodology presented in Section 4 for a system responding in a given failure mode to N repeated demands is, with a simple change of notation, also applicable for an M-failure-mode set.

Table 5-1. Probability that capacity will exceed demand for each of M failure modes: One application of the demands

Mode Demands	Mode Capacities: Deterministic	Perfectly Dependent	Statistically Independent	Unknown Dependency
Deterministic		$P = P_{m'}$	$P = \prod_{m=1}^{M} P_m$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$
Perfectly Dependent	$P = P_{m'}$	$P = P_{m'}$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$
Statistically Independent	$P = \prod_{m=1}^{M} P_m$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$	$P = \prod_{m=1}^{M} P_m$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$
Unknown Dependency	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$	$\prod_{m=1}^{M} P_m \le P \le P_{m'}$

Note: 1. $P_m = P(C_m > D_m)$

2. $P_{m'}$ is the smallest probability in the set $P_1, P_2, P_3, \ldots, P_m, \ldots, P_M$

SECTION 6

ASSESSMENT

6-1 INTRODUCTION.

You will be faced either with assessing (synthesizing) the probability of mission success of an existing system or with allocating the probability of mission success to the failure modes of an evolving system. The former task, which is by far the easier of the two, is addressed here. The task of allocating the probability of mission success is addressed in Section 7.

6-2 METHODOLOGY.

Follow these eight steps to assess the probability of mission success of an existing or defined system:

1. Develop demand scenarios. Complete Steps 2 through 7 for each scenario.
2. Identify the system's failure modes.
3. Calculate the system's demand applied in each failure mode.
4. Calculate the system's capacity in each failure mode.
5. Calculate the probability of no failure of the system in each failure mode.
6. Construct the series-parallel network of no-failure probabilities.
7. Calculate PMS vs. Q.
8. Draw PMS vs. Q envelope.

What each of these eight steps entails is outlined below.

6-2.1 Step 1: Develop Demand Scenarios.

Exercise demand options (if any) to develop demand scenarios. Complete Steps 2 through 7 for each scenario.

6-2.2 Step 2: Identify Failure Modes.

Identify all the failure modes of the system that can abort the mission. This task will be made easier if the system failure modes are categorized and each category is addressed in turn.

6-2.3 Step 3: Calculate Demand.

Using the methodology presented in Section 2, calculate the mean, random COV, and systematic COV of the demand that is applied to the system in each failure mode identified in Step 2. If the system is subject to repeated demand application, calculate the demand descriptors for each application.

6-2.4 Step 4: Calculate Capacity.

Using the methodology presented in Section 2, calculate the mean, random COV, and systematic COV of the capacity of the system in each identified failure mode. If the system is subject to repeated demand application and the capacity changes from demand to demand, calculate the capacity descriptors for each demand.

6-2.5 Step 5: Calculate Probability of No Failure.

Using the methodology presented in Sections 3 and 4, calculate the expected probability of no failure of the system and its associated systematic COV in each identified failure mode. Account for repeated demand applications, as required.

6-2.6 Step 6: Construct System Network.

Construct the series-parallel network of the system failure mode probabilities. (Note that the expected values and systematic COVs of these probabilities were calculated in Step 5.) Do this by grouping the failure modes into sets such that within each set it can be reasonably assumed that there is perfect dependency; between sets, it can be reasonably assumed that there is statistical independence. Accomplish this step under the assumption that there is zero systematic uncertainty throughout the system.

6-2.2 Step 2: Identify Failure Modes.

Identify all the failure modes of the system that can abort the mission. This task will be made easier if the system failure modes are categorized and each category is addressed in turn.

6-2.3 Step 3: Calculate Demand.

Using the methodology presented in Section 2, calculate the mean, random COV, and systematic COV of the demand that is applied to the system in each failure mode identified in Step 2. If the system is subject to repeated demand application, calculate the demand descriptors for each application.

6-2.4 Step 4: Calculate Capacity.

Using the methodology presented in Section 2, calculate the mean, random COV, and systematic COV of the capacity of the system in each identified failure mode. If the system is subject to repeated demand application and the capacity changes from demand to demand, calculate the capacity descriptors for each demand.

6-2.5 Step 5: Calculate Probability of No Failure.

Using the methodology presented in Sections 3 and 4, calculate the expected probability of no failure of the system and its associated systematic COV in each identified failure mode. Account for repeated demand applications, as required.

6-2.6 Step 6: Construct System Network.

Construct the series-parallel network of the system failure mode probabilities. (Note that the expected values and systematic COVs of these probabilities were calculated in Step 5.) Do this by grouping the failure modes into sets such that within each set it can be reasonably assumed that there is perfect dependency; between sets, it can be reasonably assumed that there is statistical independence. Accomplish this step under the assumption that there is zero systematic uncertainty throughout the system.

6-2.7 Step 7: Calculate PMS.

The system's probability of mission success (PMS) is calculated in a manner similar to the calculation of capacity or demand (see Sec. 2). Accordingly, the actual PMS is represented by

$$PMS = g(P_1, \ldots, P_j, \ldots, P_J) \tag{6-1}$$

where P_j is the actual probability of no failure of the system in the jth set of failure mode probabilities identified in Step 6. The first-order approximation of the expected value of PMS is simply*

$$\overline{PMS} \doteq g(\bar{P}_1, \ldots, \bar{P}_j, \ldots, \bar{P}_J) \tag{6-2}$$

where $\bar{P}_j$ is the expected probability of P_j calculated in Step 5 and the function g is the mathematical equivalent of the series-parallel arrangement of $P_1, P_2, P_3, \ldots$ constructed in Step 5. Calculate $\overline{PMS}$ using the well-known rules for manipulating series-parallel arrangements of independent probability events.

Calculate the systematic COV of PMS (i.e., the COV of $\overline{PMS}$) using

$$\Delta^2_{PMS} \doteq \Delta^2_g + \sum_{j=1}^{J}\left(\frac{\bar{P}_j}{\overline{PMS}}\right)^2\left(\frac{\partial g}{\partial \bar{P}_j}\right)^2 \Delta^2_{P_j} + \sum_{\substack{i=1 \\ i \neq j}}^{J} \sum_{j=1}^{J} \frac{\bar{P}_i \bar{P}_j}{PMS^2} \frac{\partial g}{\partial \bar{P}_i} \frac{\partial g}{\partial \bar{P}_j} \rho_{\bar{P}_i \bar{P}_j} \Delta_{P_i} \Delta_{P_j} \tag{6-3}$$

where the partial derivatives are to be evaluated at $\bar{P}_1, \bar{P}_2, \bar{P}_3, \ldots$; Δ_{P_j} is the systematic COV of P_j calculated in Step 5; Δ_g is the systematic COV associated with the modeling error in your network; and $\rho_{\bar{P}_i \bar{P}_j}$ is the correlation coefficient for $\bar{P}_i$ and $\bar{P}_j$. Calculate $\rho_{\bar{P}_i \bar{P}_j}$ using Equation 4-8.

Finally, calculate your confidence that PMS exceeds PMS_o using

$$Q(PMS > PMS_o) \doteq \Phi\left(\frac{k_{\overline{PMS}}\sqrt{1 + T^2} - k_{PMS_o}}{T}\right) \tag{6-4}$$

*Add the second-order terms as required.

Obtain T by entering Figure 3-2 with $\overline{PMS}$ in place of P, and Δ_{PMS} in place of Δ_p.* Note that Equation 6-4 plots as a straight line in normal-normal probability space.

6-2.8 Step 8: Draw PMS Envelope.

Using the results of Step 7, plot the PMS_o vs. Q line for each demand scenario on lognormal-lognormal probability paper. The left-most envelope of these lines is the sought-after probability of mission success vs. confidence relationship for the particular mission under consideration (see Fig. 6-1).

6-3 ILLUSTRATIVE USE.

The seventh step of the above methodology is illustrated below for the system network shown in Figure 6-2 and for

$\bar{P}_1 = 0.99$

$\bar{P}_2 = \bar{P}_3 = 0.9$

$\bar{P}_4 = 0.95$

$\Delta_{P_1} = 0.04$

$\Delta_{P_2} = \Delta_{P_3} = 0.1$

$\Delta_{P_4} = 0.02$

$\Delta_{\bar{q}} = 0.02$

$\rho_{\bar{P}_2\bar{P}_3} = \rho_{\bar{P}_3\bar{P}_2} = 1.0$

All other $\rho \equiv 0.0$

* T has no physical meaning for systems comprising two or more independent failure modes.

Calculate PMS using Equation 6-1:

$$PMS = P_3P_4 + P_1P_2P_4 - P_1P_2P_3P_4$$

Calculate $\overline{PMS}$ using Equation 6-2:

$$\overline{PMS} = \bar{P}_3\bar{P}_4 + \bar{P}_1\bar{P}_2\bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_3\bar{P}_4$$

$$= 0.94$$

Calculate the partial derivatives in Equation 6-3:

$$\left.\frac{\partial PMS}{\partial P_1}\right|_o = \bar{P}_2\bar{P}_4 - \bar{P}_2\bar{P}_3\bar{P}_4 = 0.0855$$

$$\left.\frac{\partial PMS}{\partial P_2}\right|_o = \bar{P}_1\bar{P}_4 - \bar{P}_1\bar{P}_3\bar{P}_4 = 0.0941$$

$$\left.\frac{\partial PMS}{\partial P_3}\right|_o = \bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_4 = 0.1036$$

$$\left.\frac{\partial PMS}{\partial P_4}\right|_o = \bar{P}_3 + \bar{P}_1\bar{P}_2 - \bar{P}_1\bar{P}_2\bar{P}_3 = 0.9891$$

Substituting the above partials into Equation 6-3 yields

$$\Lambda^2_{PMS} = 0.02^2 + \left(\frac{0.99}{0.94}\right)^2 0.0855^2 \times 0.04^2 + \left(\frac{0.9}{0.94}\right)^2 0.0941^2 \times 0.1^2$$

$$+ \left(\frac{0.9}{0.94}\right)^2 0.1036^2 \times 0.1^2 + \left(\frac{0.95}{0.94}\right)^2 0.9891^2 \times 0.02^2$$

$$+ 2\,\frac{0.9 \times 0.9}{0.91 \times 0.94}\, 0.0941 \times 0.1036 \times 0.1 \times 0.1$$

$$= 0.034^2$$

Determine T by entering Figure 3-2 with $\overline{PMS} = 0.94$ and $\Delta_{PMS} = 0.034$:

$$T = 0.28$$

Finally, use Equation 6-4 to obtain the solid line plotted in Figure 6-2.

6-4 VALIDATION.

In order to validate the methodology presented in Section 6-2, the illustrative problem presented in Section 6-3 was reworked using binomial models as a basis for simulation. This simulation technique is explained below.

If $\bar{P}_i$ denotes the population mean and Δ_{P_i} denotes the coefficient of variation of an estimate (mean) from a sample of size N_i from the ith population, then

$$(1 - \bar{P}_i) / \bar{P}_i N_i = \Delta^2_{P_i}$$

Thus, from the problem input (Sec. 6-3), we infer

$$N_1 = 6.31 \doteq 6$$

$$N_2 = 11.11 \doteq 11$$

$$N_3 = 11.11 \doteq 11$$

$$N_4 = 131.58 \doteq 132$$

For each set of possible sample outcomes (S_1, S_2, S_3, S_4), where S_i denotes the number of successes in N_i trials, the system reliability estimate will be given by

$$PMS = P_3P_4 + P_1P_2P_4 - P_1P_2P_3P_4$$

where

$$P_i = S_i / N_i$$

The probability of this PMS value is given by

$$\prod_{i=1,2,4} \binom{N_i}{S_i} \bar{P}_i^{\,S_i} \, (1 - \bar{P}_i)^{N_i - S_i}$$

where

$$\binom{N_i}{S_i} = N_i! \,/\, S_i!(N_i - S_i)!$$

Note that the $i = 3$ term is ignored in the cell probability, since $\rho_{\bar{P}_2\bar{P}_3} = 1.0$ (i.e., $S_2 = S_3$).

This binomial model yields $\overline{PMS} = 0.932$ and $\Delta_{PMS} = 0.0348$. (Our methodology yielded $\overline{PMS} = 0.94$ and $\Delta_{PMS} = 0.034$.) Some of the percentiles for the Q function for this model are indicated in Figure 6-2 by the filled circles. Although the Q curve for the binomial model is not linear, the agreement with the presented methodology is reasonably good.

The general remarks about the applicable domain made in Sections 2-4 and 3-4 also apply here.

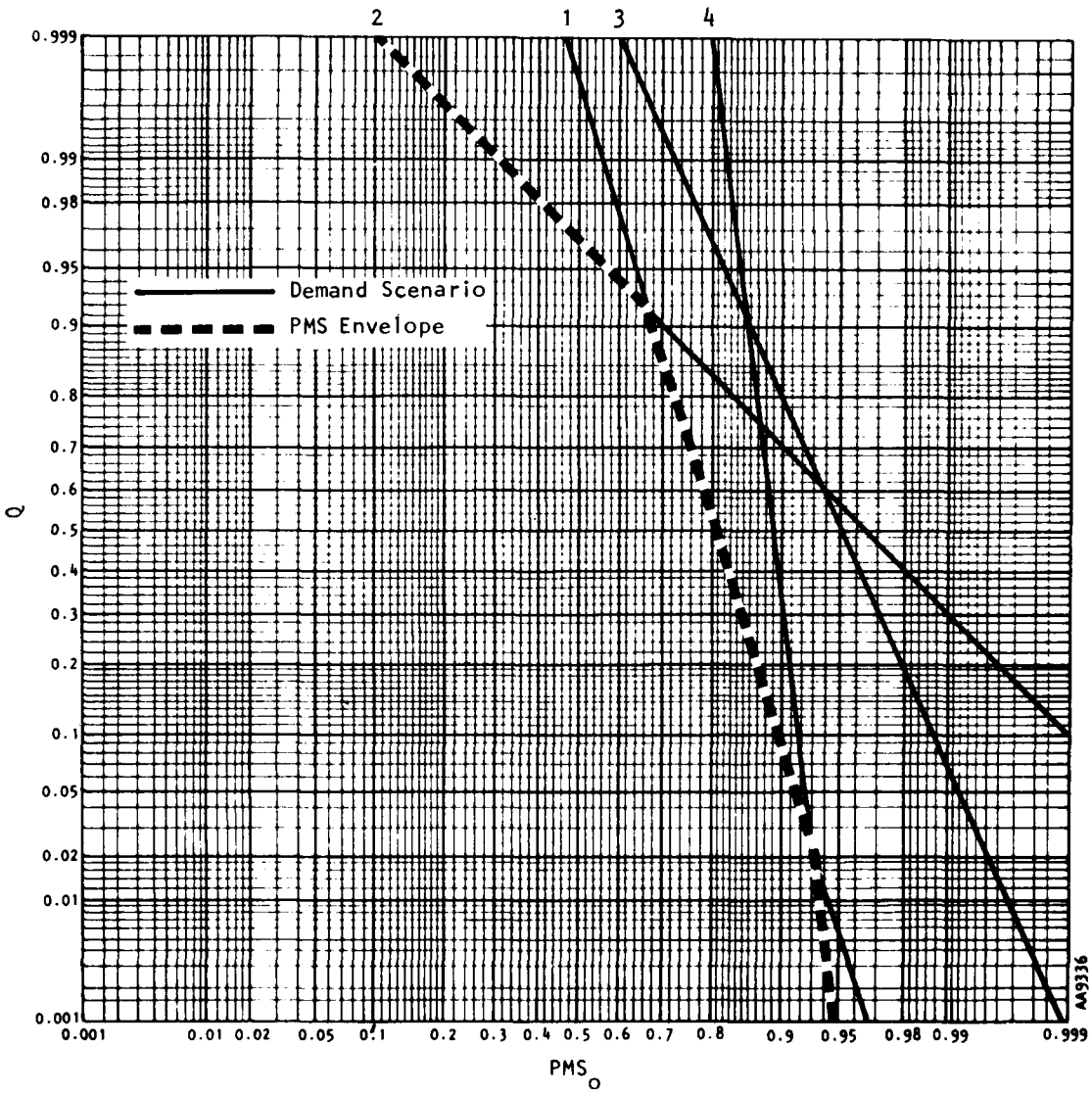


Figure 6-1. Probability-of-Mission-Success envelope.

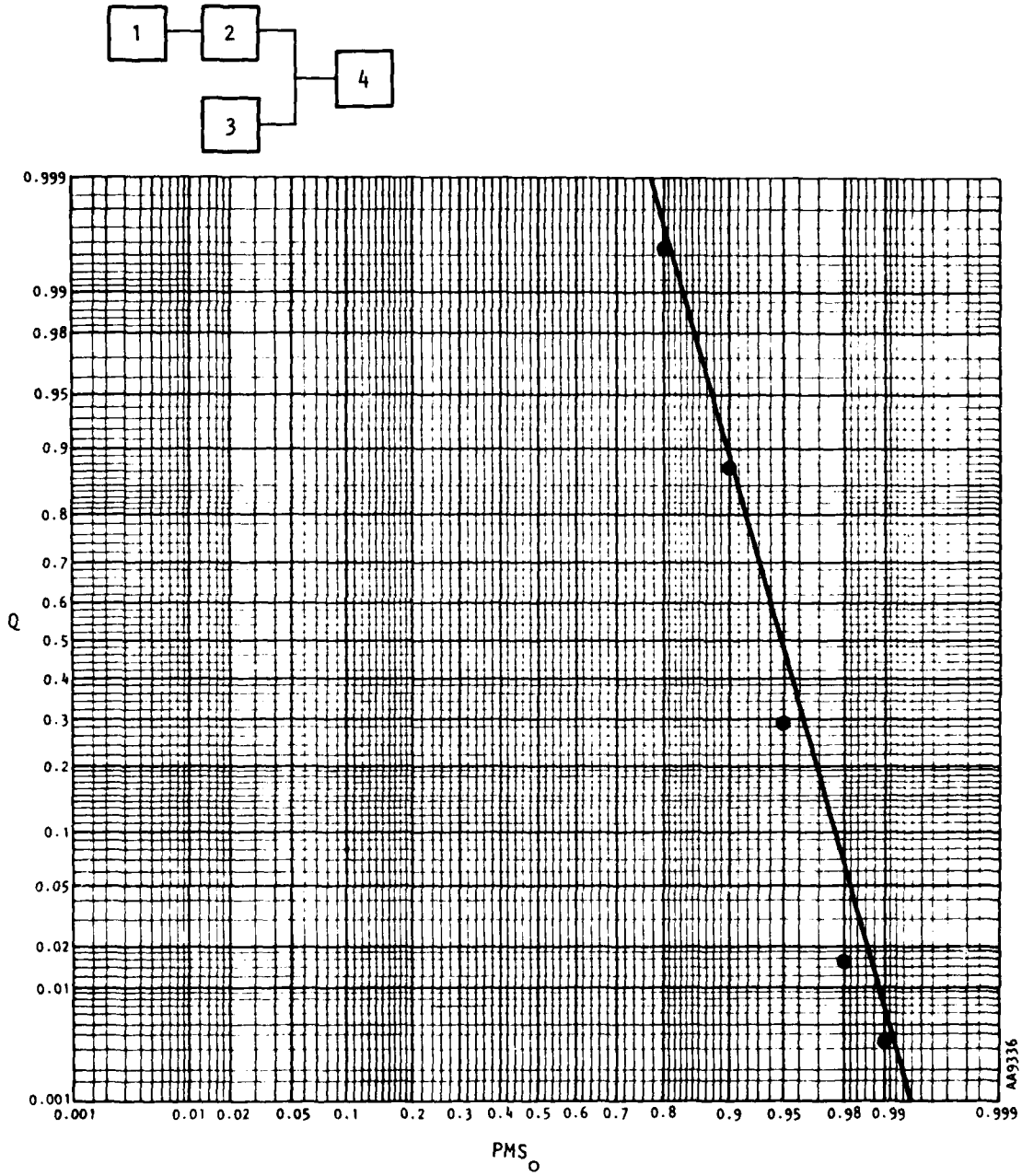


Figure 6-2. Illustrative problem.

SECTION 7

ALLOCATION

7-1 INTRODUCTION.

The reverse of assessment of the probability of mission success (PMS) of an existing system is the allocation of the PMS to the failure modes of an evolving system. Although, in concept, allocation is the reverse of assessment, the methodologies by which each is achieved are quite similar, as will be seen in this section.

7-2 METHODOLOGY.

Follow these 11 steps to allocate the probability of mission success (PMS):

1. Develop demand scenarios. Complete Steps 2 through 9 for each scenario.
2. Identify the system failure modes.
3. Calculate the system's demand applied in each failure mode.
4. Construct the series-parallel network of no-failure probabilities.
5. Select a trial value of $\overline{PMS}$.
6. Allocate $\overline{PMS}$ to each failure mode.
7. Calculate for each failure mode the mean capacity required to satisfy the allocated share of the $\overline{PMS}$.
8. Calculate Δ_{PMS}, the systematic uncertainty of $\overline{PMS}$.
9. Calculate $\overline{PMS}$. Repeat Steps 5 through 9 until the final and calculated $\overline{PMS}$'s are acceptably close.
10. Identify the critical mean capacity in each failure mode.
11. Write, for each failure mode, the deterministic design specification for the identified critical mean capacity.

Steps 1, 2, 3, and 4 are identical to the first four steps of assessment (see Sec. 6). Steps 8 and 10 have their counterparts in assessment and need not be discussed. Steps 5 6, 7, 9, and 11, however, are unique to allocation. Step 11 requires no explanation.

7-2.1 Step 5.

Select a trial value of the *expected* probability of mission success, $\overline{PMS}$. Solving Equation 6-4 for $\overline{PMS}$ and finding its maximum and minimum values with respect to T yield these bounds:

$$PMS_o \leq \overline{PMS} \leq \Phi\left(\sqrt{k^2_{PMS_o} + k^2_Q}\right) \tag{7-1}$$

where the probability that the system will successfully complete the mission is at least PMS_o, stated with Q confidence. Use Equation 7-1, with criteria PMS_o and Q, as an aid in selecting a trial value of $\overline{PMS}$.

7-2.2 Step 6.

The crux of Step 6 is to allocate the $\overline{PMS}$ in such a way that maximum cost effectiveness is achieved for the system. In general, this optimum state will be achieved by allocating the largest shares of the $\overline{PMS}$ to those failure modes that exhibit the smallest cost increments per share of the $\overline{PMS}$, and vice versa. See Reference 5 for an introduction to optimizing allocations.

7-2.3 Step 7.

Calculate for each failure mode the mean capacity required to satisfy the allocated share of the $\overline{PMS}$. Use Equation 3-9.

7-2.4 Step 9.

Calculate $\overline{PMS}$ using the following manipulated form of Equation 6-4:

$$\overline{PMS} = \Phi\left(\frac{Tk_Q + k_{PMS_o}}{\sqrt{1 + T^2}}\right) \tag{7-2}$$

where T is obtained by entering Figure 3-2 with the trial value of $\overline{PMS}$ and the Δ_{PMS} calculated in Step 3, and PMS_o and Q are given criteria values. Repeat Steps 5 through 9 until the trial and calculated $\overline{PMS}$'s are acceptably close.

7-3 ILLUSTRATIVE USE.

The following example illustrates Steps 5, 6, 7, and 9. Validation of this example is not necessary.

Step 1.

Only one demand scenario is assumed applicable for purposes of this illustrative example.

Step 2.

Only two failure modes are assumed significant.

Step 3.

Assume that we have determined the following demands for Failure Modes 1 and 2:

$\bar{D}_1$ = 28 units $\qquad$ $\bar{D}_2$ = 13 units

δ_{D_1} = 0.18 $\qquad$ δ_{D_2} = 0.20

$\Delta_{\bar{D}_1}$ = 0.15 $\qquad$ $\Delta_{\bar{D}_2}$ = 0.06

Step 4.

The two failure modes are assumed to be in series.

Step 5.

Assumed criteria

PMS_o = 0.5

Q = 0.9

Therefore, from Equation 7-1, we obtain

$$0.5 \leq \overline{PMS} \leq 0.9$$

Try $\overline{PMS} = 0.7$

Steps 6 and 7.

Assume, for purposes of this illustrative example, that

$$\text{Cost} \propto \bar{C}_1 + \bar{C}_2,$$

where $\bar{C}_1$ and $\bar{C}_2$ are the expected capacities of the system in Failure Modes 1 and 2, respectively. Moreover, assume that we have determined that

$$\delta_{C_1} = 0.09 \qquad \delta_{C_2} = 0.18$$

$$\Delta_{\bar{C}_1} = 0.15 \qquad \Delta_{\bar{C}_2} = 0.12$$

Equation 3-3a yields $T_1 = 1.0$ and $T_2 = 0.5$. Equation 3-9a yields

$$\bar{C}_1 = 28 \exp(0.29\, k_{\bar{P}_1})$$

$$\bar{C}_2 = 13 \exp(0.30\, k_{\bar{P}_2})$$

Thus,

$$\text{Cost} \propto 28 \exp(0.29\, k_{\bar{P}_1}) + 13 \exp(0.30\, k_{\bar{P}_2})$$

For $\overline{PMS} = \bar{P}_1 \bar{P}_2 = 0.7$, it can be verified that cost is approximately minimized for $\bar{P}_1 = 0.78$ and $\bar{P}_2 = 0.90$. Thus, $\bar{C}_1 = 35$ units and $\bar{C}_2 = 19$ units.

Step 8.

Entering Figure 3-2 with $T_1 = 1$ and $\bar{P}_1 = 0.78$ yields $\Delta_{P_1} = 0.27$. In a similar fashion we find $\Delta_{P_2} = 0.094$. From Equation 6-3,

$$\Delta^2_{\overline{PMS}} = \Delta^2_{P_1} + \Delta^2_{P_2} = 0.27^2 + 0.094^2 = 0.286^2$$

Step 9.

Entering Figure 3-2 with $\Delta_{PMS} = 0.286$ and $\overline{PMS} = 0.7$ yields $T = 0.7$. Finally, Equation 7-2 yields

$$\overline{PMS} = \Phi\left[\frac{0.7 \times 0 + 1.282}{\sqrt{1 + 0.7^2}}\right] = 0.85 \neq 0.7$$

Step 5a.

Try $\overline{PMS} = 0.85$.

Steps 6a and 7a.

For $\overline{PMS} = 0.85$, $\bar{P}_1 = 0.90$ and $\bar{P}_2 = 0.94$ approximately minimize cost. Thus, $\bar{C}_1 = 41$ units and $\bar{C}_2 = 21$ units.

Step 8a.

Entering Figure 3-2 with $T = 1$ and $\bar{P} = 0.90$ yields $\Delta_{P_1} = 0.155$. In a similar fashion we find $\Delta_{P_2} = 0.064$. From Equation 6-3,

$$\Delta^2_{PMS} = 0.155^2 + 0.064^2 = 0.168^2$$

Step 9a.

Entering Figure 3-2 with $\Delta_{PMS} = 0.168$ and $\overline{PMS} = 0.85$ yields $T = 0.7$. Equation 7-2 yields $\overline{PMS} = 0.85$ (trial and calculated $\overline{PMS}$ identical).

Step 10.

$\bar{C}_1 = 41$ units

$\bar{C}_2 = 21$ Units

SECTION 8

CLOSING REMARKS

The applicable domain of the methodology is summarized as follows. The calculation of $\bar{C}$, δ_C, and $\Delta_{\bar{C}}$ will be relatively accurate and insensitive to the distribution details of the independent variables of C if the second-order term in the expression for $\bar{C}$ (Eq. 2-3a) is small compared to the first-order term. (The same remark applies to $\bar{D}$, δ_D, and $\Delta_{\bar{D}}$.) The calculation of $P(C > D)$ will be relatively accurate, provided C, $\bar{C}$, D, and $\bar{D}$ are unimodal in distribution and P and Q are near 0.5, say $0.01 < (P, Q) < 0.99$. C, $\bar{C}$, D, and $\bar{D}$ will exhibit unimodal distributions if their independent variables exhibit unimodal distributions or if there are many independent variables. The calculation of $Q(PMS > PMS_o)$ will be relatively accurate, provided the second-order term of $\overline{PMS}$ is small and PMS and Q are near 0.5, say $0.01 < PMS, Q < 0.99$.

We have elected to write $Q(P > P_o)$ for the probability that $P > P_o$, and to read $Q(P > P_o)$ as "Our confidence that P exceeds P_o." The use of confidence in this context has as its precedent the statement of the one-sided confidence limit for the estimate of a model parameter in classical statistics. Thus, strictly speaking, $Q(P \geq P_o)$ should be read "The lower one-sided Q-confidence limit for P is P_o."*

Our methodology (or for that matter any methodology of the same purpose) requires that the systematic COVs be quantified using subjective reasoning.† If the sponsor of the application of the methodology will not accept subjective estimates, then there can be no application. Most sponsors, however, will entertain the idea of subjective estimates if the bases for these estimates are well documented.

Finally, it should be remarked that admitting even less general definitions of the terms "system" and "mission" than are normally held (or discarding the concepts of "system" and "mission" altogether) gives the presented methodology, or at least portions of the methodology, quite general applicability. For example, the methodology could be applied to cost estimating, to scheduling, and to RDT&E planning.

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*These remarks are also applicable for $Q(PMS > PMS_o)$.

†In the rare cases where specific experiments have been performed, systematic COVs can be calculated.

SECTION 9

REFERENCES

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APPENDIX A

DERIVATION OF EQUATIONS 3-2 AND 3-5

Suppose we know perfectly the "random" standard deviations σ_C and σ_D of the normal distributions of capacity and demand, respectively, but that the means μ_C and μ_D have an uncertainty represented by normal distributions with "systematic" standard deviations S_C and S_D in the respective unbiased estimates C and D. Let

$$\sigma^2 \equiv \sigma_C^2 + \sigma_D^2$$

$$S^2 \equiv S_C^2 + S_D^2$$

$$T \equiv S/\sigma$$

$$\mu \equiv \mu_C - \mu_D$$

$$m \equiv \bar{C} - \bar{D}$$

$$A \equiv m/\sigma$$

$$f(z) \equiv (2\pi)^{-0.5} \exp(-z^2/2)$$

$$\Phi(z) \equiv \int_{-\infty}^{z} f(t)\, dt$$

Then $\Phi(A)$ is known to be the median reliability, $\tilde{R}$. The problem is to find the mean reliability, $\bar{R}$.

THEOREM A-1: $\bar{R} = \Phi\left(A/\sqrt{1 + T^2}\right)$

Proof: The reliability is $\Phi(\mu/\sigma)$, where μ is normally distributed with mean m and standard deviation S. This is written as

$$\mu = N(m, S^2)$$

If we transform y to the variate

$$x = N(0, 1)$$

by the transformation

$$x = (y - m)/S$$

the reliability becomes

$$\Phi(Tx + A).$$

Then

$$\bar{R} = \int_{-\infty}^{\infty} \Phi(Tx + A)\ f(x)\ dx$$

$$\partial\bar{R}/\partial A = \int_{-\infty}^{\infty} f(Tx + A)\ f(x)\ dx$$

$$= (2\pi)^{-1} \int_{-\infty}^{\infty} \exp\left(-\{(Tx + A)^2 + x^2\}/2\right)\ dx.$$

Transform

$$w = x\sqrt{1 + T^2} + AT\Big/\sqrt{1 + T^2}$$

Then

$$\partial\bar{R}/\partial A = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp(-\{w^2 + A^2/(1 + T^2)\}/2)\ dw\Big/\sqrt{1 + T^2}$$

$$= \{2\pi(1 + T^2)\}^{-0.5} \exp\left(-A^2/2(1 + T^2)\right) \int_{-\infty}^{\infty} f(w)\ dw$$

$$= f\left(A\Big/\sqrt{1 + T^2}\right)\Big/\sqrt{1 + T^2}$$

$$\bar{R} = \Phi\left(A\Big/\sqrt{1 + T^2}\right) + K$$

Where K denotes an arbitrary constant of integration. Since $\bar{R} = 0.5$ when $A = 0$, we have

$$0.5 = \Phi(0) + K$$

$$K = 0. \qquad \text{QED.}$$

Suppose now that capacities and demands are lognormally distributed, not normally distributed.* Let

$$z = \log(\text{capacity}) = N(\mu, \sigma^2)$$

THEOREM A-2: The <u>nth</u> moment of capacity is $\exp(n\mu + n^2\sigma^2/2)$

Proof: Let $x = N(0, 1)$, and let $E[g]$ denote the expected value of g.

$$m_n = E[C^n] \equiv E[e^{zn}] = E[\exp((x\sigma + \mu)\, n)]$$

$$= (2\pi)^{-0.5} \int_{-\infty}^{\infty} \exp((x\sigma + \mu)\, n) \exp(-x^2/2)\, dx$$

Let

$$y = x - \sigma n$$

Then

$$m_n = (2\pi)^{-0.5} \int_{-\infty}^{\infty} \exp(-y^2/2 + \sigma^2 n^2/2 + \mu n)\, dy$$

$$= \exp(n\mu + n^2\sigma^2/2)$$

Corollary: $E[C] = \exp(\mu + \sigma^2/2)$.

---

*At this point we should note that the preferred procedure is to transform all observations by $z = \log C$ and work with the transformed sample, if this sample passes a goodness-of-fit test for normality. The remainder of this Appendix would then be unnecessary.

Corollary: If δ is the coefficient of variation of C, then

$$\delta^2 = m_2'/m_1^2 = \left|m_2 - m_1^2\right|/m_1^2$$

$$= \left|\exp(2\mu + 2\sigma^2) - \left\{\exp(\mu + \sigma^2/2)\right\}^2\right|/\left\{\exp(\mu + \sigma^2/2)\right\}^2 = \exp(\sigma^2) - 1$$

$$\sigma^2 = \log(1 + \delta^2)$$

Note that $\delta^2 \simeq \sigma^2 + \sigma^4/2 + \sigma^6/6$, small σ. Also $\delta^4 \simeq \sigma^4 + \sigma^6$, $\delta^6 \simeq \sigma^6$.

Corollary: The third central moment is

$$m_3' = m_3 - 3m_2m_1 + 3m_1^3 - m_1^3$$

$$= \exp(3\mu + 3\sigma^2/2)\left\{\exp(3\sigma^2) - 3\exp(\sigma^2) + 2\right\}$$

$$\simeq m_1^3\left\{3\sigma^4 + 4\sigma^6\right\}, \quad \text{for small } \sigma.$$

Corollary: The fourth central moment is

$$m_4' = m_4 - 4m_3m_1 + 6m_2m_1^2 - 4m_1^4 + m_1^4$$

$$\cong m_1^4\left\{3\sigma^4 + 19\sigma^6\right\}, \quad \text{for small } \sigma.$$

Let $\bar{C}_n$ denote the mean of a sample of size n.

Corollary: $E[\bar{C}_n] = \exp(\mu + \sigma^2/2)$

THEOREM A-3: The second central moment of $\bar{C}_n$ is $E^2[C]\ \delta^2/n$.

Proof:
$$E\left[(\bar{C}_n)^2\right] = E\left[\left(\sum_{i=1}^{n} e^{x_i}/n\right)^2\right]$$
$$= E\left[\sum_{i=1}^{n} e^{2x_i} + 2\sum_{i\neq j} e^{x_i x_j}\right]\Big/n^2$$
$$= \left(nm_2 + n(n-1)\ m_1^2\right)\Big/n^2$$
$$= \left(\exp(2\mu + 2\sigma^2) + (n-1)\ \exp(2\mu + \sigma^2)\right)\Big/n$$

$$E\left[(\bar{C}_n)^2\right] - E^2\left[\bar{C}_n\right] = \exp(2\mu + \sigma^2)\{\exp(\sigma^2) - 1\}\Big/n \cdot \text{QED}$$

Corollary: If Δ denotes the coefficient of variation of $\bar{C}_n$, then $\Delta^2 = \delta^2/n$.

THEOREM A-4:

$$E\left[\log(\bar{C}_n)\right] \approx \mu + \sigma^2(n-1)/2n - \sigma^4(n-1)/4n^2$$
$$- \sigma^6(n-1)(n-6)/12n^3, \quad \text{for small } \sigma.$$

Proof:

$$E\left[\log\left(\sum_{i=1}^{n} e^{z_i}/n\right)\right] = E\left[\log\left(\exp(\mu + \sigma^2/2) + \sum_{i=1}^{n}\left\{\exp(z_i) - \exp(\mu + \sigma^2/2)\right\}/n\right)\right]$$
$$= \mu + \sigma^2/2$$
$$+ E\left[\log\left(1 + \sum_{i=1}^{n}\left\{\exp(z_i) - \exp(\mu + \sigma^2/2)\right\}\Big/n\exp(\mu + \sigma^2/2)\right)\right]$$

Since

$$\log(1 + w) \simeq w - w^2/2 + w^3/3 - w^4/4 + w^5/5 - w^6/6$$

$$E[\log(\bar{C}_n)] \simeq \mu + \sigma^2/2 - n\delta^2/2n^2 + n(3\sigma^4 + 4\sigma^6)/3n^3$$

$$-n(3\sigma^4 + 19\sigma^6)/4n^4 - 3n(n - 1)\delta^4/4n^4$$

$$+ 10n(n - 1)(3\sigma^4\delta^2)/5n^5 - 15n(n - 1)(n - 2)\delta^6/6n^6$$

Ignoring terms of order σ^6/n^4 , σ^8, or less. Then

$$E[\log(\bar{C}_n)] \simeq \mu + \sigma^2(n - 1)/2n - \sigma^4(1/4n - 1/n^2 + 3/4n^3 + 3/4n^2 - 3/4n^3)$$

$$+ \sigma^6(-1/12n + 4/3n^2 - 19/4n^3 - 3/4n^2 + 3/4n^3$$

$$+ 6/n^3 - 5/2n^3) \cdot \text{QED}$$

THEOREM A-5: If

$$\hat{\mu}_C \equiv \log\left\{\bar{C}_n\sqrt{1 + \Delta^2}\Big/\sqrt{1 + \delta^2}\right\}$$

then

$$E[\hat{\mu}_C] \simeq \mu + \sigma^6(n - 1)/3n^3$$

<u>Proof</u>:

$$\frac{1}{2}\log\left\{(1 + \delta^2/n)/(1 + \delta^2)\right\} = \frac{1}{2n}\log\left\{1 + \delta^2/n\right\}^n - \frac{1}{2}\log(1 + \delta^2)$$

$$\simeq \frac{1}{2n}\log\left\{1 + \delta^2 + (n - 1)\,\delta^4/2n + (n - 1)(n - 2)\delta^6/6n^2\right\} - \sigma^2/2$$

$$\simeq \frac{1}{2n}\log\left\{[1 + \delta^2][1 + (n - 1)\delta^4/2n + (n - 1)(n - 2)\delta^6/6n^2 - (n - 1)\delta^6/2n]\right\}$$

$$- \sigma^2/2$$

$$\simeq \sigma^2/2n + \frac{1}{2n}\,[(n-1)(\sigma^4 + \sigma^6)/2n + \sigma^6(n^2 - 3n + 2 - 3n^2 + 3n)/6n^2] - \sigma^2/2$$

$$= -\sigma^2(n-1)/2n + \sigma^4(n-1)/4n^2 + \sigma^6[n^2 - 3n + 2]/12n^3$$

and the theorem follows from A-4. QED

From Theorem A-5, $\hat{\mu}_C$ is an asymptotically unbiased estimate of μ_C. We could then proceed in the same way with the demands. The result would be that $\hat{\mu}_C - \hat{\mu}_D$ would be an asymptotically unbiased estimate of $\mu_C - \mu_D$. The median reliability would then be

$$\tilde{R} \cong \Phi\left((\hat{\mu}_C - \hat{\mu}_D)\Big/\sqrt{\sigma_C^{\,2} + \sigma_D^{\,2}}\right)$$

which is Equation 3-5. Equation 3-2 follows from this and Theorem A-1.

APPENDIX B

DERIVATION OF EQUATIONS 3-6 AND 3-7

What is the variance of the reliability, given the same notation as in Appendix A? From that discussion, we have

$$R = \Phi(Tx + A)$$

where

$$x = N(0, 1).$$

The variance is

$$\sigma_R^2 = \int_{-\infty}^{\infty} \Phi^2(Tx + A)\ f(x)\ dx - \bar{R}^2$$

Let

$$\tilde{R} \equiv \Phi(A),$$

the median reliability. We have two approximations for σ_R^2, one for values $T < 0.3$ and the other for $T \geq 0.3$. These are tabulated for different values of $\tilde{R}$ and T and compared with the values from numerical integration (see Table B-1). The first approximation was uncovered as follows: Let

$$M_2(T) = \int_{-\infty}^{\infty} \Phi^2(Tx + A)\ f(x)\ dx.$$

Then

$$M_2'(T) = \int_{-\infty}^{\infty} 2x\Phi(Tx + A)\ f(Tx + A)\ f(x)\ dx$$

$$M_2''(T) = \int_{-\infty}^{\infty} 2x^2 f(x)\ dx \left[f^2(Tx + A) + \Phi(Tx + A)\ f'(Tx + A) \right]$$

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$$M_2'''(T) = \int_{-\infty}^{\infty} 2x^3 f(x)\ dx[3ff' + \Phi f'']$$

$$M_2''''(T) = \int_{-\infty}^{\infty} 2x^4 f(x)\ dx[3f'^2 + 4ff'' + \Phi f'''].$$

Since

$$f'(x) = -xf(x)$$

$$f''(x) = (x^2 - 1)\ f(x)$$

$$f'''(x) = (-x^3 + 3x)\ f(x)$$

we have

$$M_2(0) = \Phi^2(A)$$

$$M_2'(0) = 0$$

$$M_2''(0) = 2f(A)[f(A) - A\Phi(A)]$$

$$M_2'''(0) = 0$$

$$M_2''''(0) = 6f(A)[3A^2 f(A) + 4(A^2 - 1)\ f(A) + (-A^3 + 3A)\ \Phi(A)]$$

Expansion around $T = 0$ then gives

$$M_2(T) \cong \Phi^2(A) + T^2 f(A)\Big[f(A) - A\Phi(A)\Big] + T^4 f(A)\Big[f(A)\{7A^2 - 4\} + \Phi(A)[-A^3 + 3A]\Big]\Big/4$$

Similarly:

$$\bar{R}(T) = \int_{-\infty}^{\infty} \Phi(Tx + A)\ f(x)\ dx$$

$$\bar{R}'(T) = \int_{-\infty}^{\infty} xf(Tx + A)\ f(x)\ dx$$

$$\bar{R}''(T) = \int_{-\infty}^{\infty} x^2 f'(Tx + A)\ f(x)\ dx$$

$$\bar{R}'''(T) = \int_{-\infty}^{\infty} x^3 f''(Tx + A)\ f(x)\ dx$$

$$\bar{R}''''(T) = \int_{-\infty}^{\infty} x^4 f'''(Tx + A)\ f(x)\ dx$$

$$\bar{R}(0) = \Phi(A)$$

$$\bar{R}'(0) = 0$$

$$\bar{R}''(0) = -Af(A)$$

$$\bar{R}'''(0) = 0$$

$$\bar{R}''''(0) = 3Af(A)[3 - A^2]$$

$$\bar{R}(T) \simeq \Phi(A) - T^2 Af(A)/2 + T^4\ Af(A)[3 - A^2]/8$$

$$\bar{R}^2(T) \simeq \Phi^2(A) - T^2 Af(A)\Phi(A) + T^4 Af(A)[Af(A) + \Phi(A)\{3 - A^2\}]/4$$

$$\sigma_R^2 = M_2(T) - \bar{R}^2(T) \simeq T^2 f^2(A) + T^4 f^2(A)[7A^2 - 4 - A^2]/4$$

$$= f^2(A)\ T^2(1 + T^2(3A^2/2 - 1))$$

$$\simeq f^2\left(A \Big/ \sqrt{1 + 3T^2/2}\right) \Big/ (1 + T^2)$$

and our first approximation is

$$\sigma_R \simeq f\left(A \Big/ \sqrt{1 + 3T^2/2}\right) T \Big/ \sqrt{1 + T^2}, \text{ small } T.$$

For our second approximation, we notice that (see Table B-1)

$$\sigma_R^2 \simeq \sigma_R^2\Big|_{A=0} \left[4\bar{R}(1 - \bar{R})\right]^{T^{-0.2155}/0.7535}, \quad \text{large } T.$$

All we need, therefore, is $\sigma_R^2\Big|_{A=0}$

THEOREM B-1:

$$\sigma_R^2\Big|_{A=0} = 0.25 - (1/\pi)\cot^{-1}\sqrt{1 + 2T^2}$$

Proof:

$$\partial M_2(T)/\partial A = 2\int_{-\infty}^{\infty} \Phi(Tx + A)\ f(Tx + A)\ f(x)\ dx$$

$$= (1/\pi)\int_{-\infty}^{\infty} \Phi(Tx + A)\ \exp(-\{[Tx + A]^2 + x^2\}/2)\ dx$$

If we make the substitution

$$y = x\sqrt{1 + T^2} + AT \Big/ \sqrt{1 + T^2}$$

we have

$$\partial M_2(T)/\partial A = (1/\pi) \int_{-\infty}^{\infty} \Phi\left(Ty\Big/\sqrt{1 + T^2} + A/(1 + T^2)\right)$$

$$\exp\left(-\left\{y^2 + A^2/(1 + T^2)\right\}/2\right) dy\Big/\sqrt{1 + T^2}$$

$$= 2f(A') \int_{-\infty}^{\infty} \Phi(T'y + A'')\ f(y)\ dy\Big/\sqrt{1 + T^2}$$

where

$$A' = A\Big/\sqrt{1 + T^2}$$

$$T' = T\Big/\sqrt{1 + T^2}$$

$$A'' = A'\Big/\sqrt{1 + T^2}$$

Then the algebra of Theorem A-1 gives

$$\partial M_2(T)/\partial A = 2f(A')\Phi\left(A''\Big/\sqrt{1 + T'^2}\right)\Big/\sqrt{1 + T^2}$$

$$\partial M_2(T)/\partial A' = 2f(A')\Phi\left(A'\Big/\sqrt{1 + 2T^2}\right)$$

$$M_2(T) = 2 \int_{-\infty}^{A'} f(w)\Phi\left(w\Big/\sqrt{1 + 2T^2}\right) dw$$

Now let

$$r \equiv 1\Big/\sqrt{1 + 2T^2}$$

$$M_2(r) = 2 \int_{-\infty}^{A'} f(w)\Phi(rw)\ dw$$

$$\partial M_2(r)/\partial r = 2\int_{-\infty}^{A'} wf(w)\ f(rw)\ dw$$

$$= (1/\pi)\int_{-\infty}^{A'} \exp\left(-\left\{r^2 + 1\right\}w^2/2\right)\ wdw$$

$$= -\exp\left(-A'^2(r^2 + 1)/2\right)\Big/\pi(r^2 + 1)$$

$$= -\exp\left(-A^2r^2\right)\Big/\pi(r^2 + 1)^*$$

Now, if $A = 0$, then $\bar{R} = 0.5$ and

$$M_2(r) = 0.5 - \int_0^r dr/\pi(r^2 + 1) = 0.5 - (1/\pi)\tan^{-1}(r)$$

$$\sigma_R^2 = 0.25 - (1/\pi)\tan^{-1}(r) \cdot \text{QED}$$

---

*From this, we could get an approximation for extremely large T (small r):

$$M_2(r) \simeq 0.5 - r/\pi + r^3(A^2 + 1)/3\pi$$

$$\sigma_R^2 \simeq 0.5 - r/\pi + r^3(A^2 + 1)/3\pi - \bar{R}^2.$$

Table B-1. Exact and approximate values of σ_R

$\bar{R}$	T = 0.1		T = 0.2		T = 0.3		T = 0.5		T = 1.0		T = 2.0	
	Exact	Approx.[1]	Exact	Approx.[1]	Exact	Approx.[2]	Exact	Approx.[2]	Exact	Approx.[2]	Exact	Approx.[2]
0.5	0.039697	0.039696	0.078249	0.078239	0.114701	0.114701	0.179017	0.179017	0.288675	0.288675	0.384166	0.384166
0.6	0.038461	0.038455	0.075915	0.075862	0.111501	0.111064	0.174877	0.174568	0.284961	0.284790	0.382398	0.382376
0.7	0.034667	0.034644	0.068129	0.068550	0.101609	0.099952	0.161946	0.160762	0.273103	0.272418	0.376650	0.376562
0.8	0.028004	0.027955	0.056022	0.055658	0.083947	0.080629	0.138305	0.135889	0.250296	0.248756	0.365121	0.364907
0.9	0.017676	0.017605	0.036[illegible]	0.035523	0.055630	0.051146	0.098472	0.095106	0.207553	0.204900	0.341524	0.341091
0.95	0.010470	0.010401	0.021340	0.021307	0.034835	0.030824	0.066948	0.063883	0.167907	0.164803	0.316618	0.316008
0.99	0.002760	0.002724	0.006096	0.005800	0.010591	0.008878	0.025162	0.023958	0.098419	0.095883	0.261507	0.260700
0.999	0.000360	0.000351	0.000817	0.000793	0.001720	0.001437	0.005692	0.005694	0.044105	0.043099	0.196003	0.195283
0.9999	0.000044	0.000042	0.000115	0.000101	0.000263	0.000231	0.001237	0.001350	0.019526	0.019341	0.146138	0.145676
0.99999	0.000005	0.000005	0.000015	0.000012	0.000039	0.000037	0.000265	0.000321	0.008639	0.008718	0.108885	0.108681
0.999999	0.000001	0.000001	0.000002	0.000001	0.000006	0.000006	0.000056	0.000077	0.003833	0.003948	0.081212	0.081204
0.9999999	0.000000	0.000000	0.000000	0.000000	0.000001	0.000001	0.000012	0.000018	0.001707	0.001795	0.060676	0.060792

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