CONDENSED STORAGE OF DIFFUSION EQUATION SOLUTIONS
FOR ATMOSPHERIC DENSITY MODEL COMPUTATIONS.

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It is shown that all neutral thermospheric component densities in diffusive equilibrium can be derived from a single function of height. This is used to permit considerable savings in storage of Jacchia 1977 model densities compared to storing the solution for each component, which requires use of external random access files. The single solution can be built into the computer code, making it more transportable to different computers.
subroutine has been developed implementing this method, including the latest results for atomic nitrogen and the non-diffusive equilibrium solution for atomic hydrogen.
The author wishes to acknowledge the assistance of the following individuals:

Dr. Kenneth S. W. Champion, of LKB, Mr. Edward C. Robinson, of SUWA, Dr. Jeffrey M. Forbes, of Boston College, and Jack W. Slowey, of Smithsonian Astrophysical Observatory, for their valuable advice.
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1. INTRODUCTION

Due to the increasing complexity of recent Jacchia density models\textsuperscript{1,2} it becomes necessary to store in tabular form the solution of the diffusion equation for each constituent, even if only the total mass density is to be computed. This is because the models call for corrections to the diffusion process for such effects as local time phase variations, seasonal-latitudinal variations and geomagnetic activity, all of which differ for the different constituents. The Jacchia 1964 model\textsuperscript{3}, on the other hand, required only temperature corrections, and hence only the total mass density needed to be stored as a function of height and exospheric temperature. As a result, one must either set aside a large block of storage (the current SAO version of the Jacchia 1977 model\textsuperscript{2} would require in excess of 17,000 words), or resort to random access disk storage. The first approach is undesirable in almost all cases, particularly with small computers. The second poses problems in implementation on different computers.

A third approach is presented here which takes advantage of the fact that, with only two exceptions, all terms in the diffusion equation are analytically integrable. The exceptions are the scale height term, which differs for different constituents merely by a mass factor, and the vertical flux term for hydrogen, which is important only for low altitudes. It therefore is necessary to store only the integral of the common mass-independent factor of the former term, and the solution for hydrogen whenever that constituent is of special interest.
2. ANALYSIS

The diffusion equation for the $i$th constituent, in the Jacchia 1977 (J77) density model is 2.4

$$\frac{d n_i}{n_i} + \frac{dT}{T} (1 + \alpha_i) + \frac{dh}{H_i} + \frac{\dot{z}_i}{D} \frac{dh}{n_i} = 0$$

where

$n_i$ = $i$th constituent number density
$T$ = temperature
$\alpha_i$ = $i$th constituent thermal diffusion coefficient
$h$ = height
$H_i = R_i T / M_i g$
$R = 8.31432 \times 10^3 \text{ J} (\text{kg} \cdot \text{mol})^{-1}/\text{K}$
$M_i$ = $i$th constituent molecular mass
$g = 9.80665 \ (1 + h/R_e)^{-2} \text{ m/sec}^2$
$R_e = 6.35766 \times 10^6 \text{ m}$
$\dot{z}_i$ = $i$th constituent vertical flux
$D$ = diffusion coefficient = $2 \times 10^{20} \sqrt{T/N}$
$N$ = total number density

The $\alpha_i$ are assumed values of -0.38 and -0.25 for helium and hydrogen and 0 for all others. The $\dot{z}_i$ are 0 for all but hydrogen. Neglecting the vertical flux term leads to 4

$$n_i (h, T) = n_i (h_0, T_\infty) \left[ \frac{T(h_0, T_\infty)}{T(h, T_\infty)} \right]^{1+\alpha_i} \exp \left[ M_i F(h_0, h, T_\infty) \right]$$

where

$$F(h_0, h, T_\infty) = \int_{h_0}^{h} g(z) / R * T(z, T_\infty) \, dz$$

$T_\infty$ = exospheric temperature
2.1 Tabulation

In practice it is therefore necessary to tabulate only $F$ and the density for one of the constituents at $h_0$ as a function of $T$, if $h_0$ is chosen to be the homopause, 100km. If $N_2$ is chosen, then the others, except hydrogen, are given by

$$\log n_i(h_0,T) = \log n_{28}(h_0,T) + Q_i$$

where the subscript $r$ indicates species and the $Q_i$ are constants:

$$Q_1 = \log \left( \frac{q_1}{q_{28}} \right) \quad (14,32)$$

$$Q_{16} = -\log q_{28} - \log \left( \frac{M'}{M''} \right)$$

$$+ \log [2(1-M'/M'')]$$

$$Q_{32} = -\log q_{28} - \log \left( \frac{M'}{M''} \right)$$

$$+ \log \left[ \frac{M'}{M''} (1 + q_{32}) - 1 \right]$$

Where

$q_i = \text{sea level concentration of } i^{\text{th}} \text{ constituent}$

$M' = \text{mean molecular mass at 100km (Eq. 5, ref. 2)}$

$M'' = \text{mean molecular mass at sea level}$

If one is interested in including the escape flux term for hydrogen, special tables would still be necessary; however the storage requirement would still be considerably less than if separate tables are used for all constituents. Furthermore, since the escape flux term is important only below 500km, it would be necessary to store results only for that region. If one excludes the escape flux term, the $H$ density is computed using:

$h_0 = 500\text{km}; \text{ eq. 17 of ref. 2}$
\[ \log n_i (500, T_{\infty}) = 5.94 + 28.9 T_{\infty}^{1/4}, \]

and

\[ F (500, h, T_{\infty}) = F (100, h, T_{\infty}) - F (100, 500, T_{\infty}). \]

2.1.1 Homogeneous Layer (90km \leq h \leq 100km)

For the homogeneous layer the diffusion equations are replaced by a single barometric equation for the mass density, from which component densities may be derived, as indicated in reference 2. Hence only the mass density need be stored. Alternatively one may store the density for \( N_2 \) and derive the others from it, as for the homopause boundary. The equations are the same except that \( \bar{M}' \) would be the mean molecular weight at the height of interest, given by Eq. 5. of ref. 2.
3. RESULTS

A subroutine has been constructed to implement the above procedures. Atomic nitrogen has been included, based on the most recent SAO subroutine version, which incorporates the AE OSS mass spectrometer data.\textsuperscript{5,6} For this we have taken the fractional sea-level composition given by Table 1, with the resulting mean-molecular weight $\overline{M}_0 = \overline{M}_0' = 28.9586$.

The tables for the diffusion function $F$ are built-in along with the $N_2$ densities for the homogeneous layer and the atomic hydrogen solution including escape flux. Together these occupy less storage than required for a single height segment (4000 words) when the solutions are tabulated for each component on a random access file. Hence no external files are required. Run times on the CDC 6600 are comparable to the random access version, with essentially identical results.

A FORTRAN deck of this subroutine is available on request.
Table 1. Sea-Level Composition

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Fraction by Volume</th>
<th>Molecular Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular Nitrogen (N&lt;sub&gt;2&lt;/sub&gt;)</td>
<td>0.78103</td>
<td>28.0134</td>
</tr>
<tr>
<td>Oxygen (O&lt;sub&gt;2&lt;/sub&gt;)</td>
<td>0.20953</td>
<td>31.9988</td>
</tr>
<tr>
<td>Argon (Ar)</td>
<td>0.009342</td>
<td>39.948</td>
</tr>
<tr>
<td>Helium (He)</td>
<td>0.000005242</td>
<td>4.0026</td>
</tr>
<tr>
<td>Atomic Nitrogen (N)</td>
<td>0.00007502</td>
<td>14.0067</td>
</tr>
</tbody>
</table>
4. REFERENCES


