DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE OF VOICE A-ETC(U)

FEB 80

J P LEHOCZKY, J P AVER

UNCLASSIFIED
DIFFUSION APPROXIMATIONS FOR
THE COOPERATIVE SERVICE
OF VOICE AND DATA MESSAGES

by
J. P. Lehoczky
and
D. P. Gaver
February 1980

Approved for Public Release; Distribution Unlimited.

Prepared for:
Naval Postgraduate School
Monterey, California 93940

80 5 30 083
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

Rear Admiral J. J. Ekelund
Superintendent

J. R. Borsting
Provost

This report was prepared by:

J. P. Léhoczky, Professor
Carnegie-Mellon University

D. P. Gaver, Professor
Department of Operations Research

Reviewed by: Released by:

Michael G. Sovereign, Chairman
Department of Operations Research

William M. Tolles
Dean of Research
**Diffusion Approximations for the Cooperative Service of Voice and Data Messages.**

J. P. Lehoczky and D. P. Gaver

Naval Postgraduate School
Monterey, CA 93940

**Research and Development:**

A probability model is presented for a set of communication channels that share the service of data and voice transmissions. A diffusion-theoretic approximation is derived, utilizing new results of Burman (1979). It is shown that the data queue (which is of low priority relative to voice) is approximated by a Wiener process.

Queues Semigroup Theory
Communications Probability Modeling
Data Transmission
Voice Transmission

Approved for Public Release; Distribution Unlimited.
DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE

OF VOICE AND DATA MESSAGES

by

J. P. Lehoczky
Carnegie-Mellon University
Pittsburgh, PA

and

D. P. Gaver
Naval Postgraduate School
Monterey, CA

INTRODUCTION

In this paper we study the behavior of a queueing system which arises in the study of certain communication networks. Specifically we study a queueing phenomenon which arises with the SENET network, as described by Coviello and Vena (1975) or Barbacci and Oakley (1976). This network allows for both voice and data messages to be transmitted over the same channels by using a special type of integrated circuit and packet-switched multiplexor structure. The two classes of traffic have substantially different performance requirements. Voice messages tend to possess great redundancy, and hence not to be sensitive to channel error rates, while data is very sensitive to channel error, having essentially no redundancy. Voice messages on the other hand have critical timing requirements and cannot be queued, while data is
relatively insensitive to timing and can be queued. Additionally, voice messages tend to be very long relative to data messages which can be broken up into small packets. These special requirements have led to the following queueing network. A node of the network consists of $c + v$ channels or servers. The voice messages are assigned to $v$ channels and do not queue. Thus the voice messages operate as a loss system. Data messages may use $c$ channels exclusively and any unused voice channels; however, voice preempts data using voice channels. Data messages are queued if necessary. Typical performance measures that one may wish to calculate include the loss rate of voice traffic and the mean data queue length.

We make standard probabilistic assumptions. Specifically, we assume voice traffic arrives according to a Poisson($\lambda$) process and each voice message has an independent exponential($\mu$) service time. Data messages are assumed to have independent exponential($\eta$) service times and arrive according to a Poisson($\delta$) process. With these assumptions voice is an $M/M/v/v$ loss system, and data is an $M/M/S$ system where $S = c + v - V(t)$ with $V(t) =$ number of voice messages in service. The stochastic process $((X(t), V(t)), t \geq 0)$ is Markov with state space $\mathbb{Z}^+ \times \{0, 1, \ldots, v\}$ where $X(t) =$ data system size at time $t$. One can easily write the Kolmogorov forward equations appropriate for this system; however, these equations do not yield a closed form solution. To describe this system
one must either numerically solve the forward equations or introduce approximations.

This system has been studied previously by a number of researchers including Halfin and Segal (1972), Halfin (1972), Fischer and Harris (1976), Bhat and Fischer (1976), Fischer (1977), Chang (1977), and Gaver and Lehoczky (1979a,b). The last two papers introduce a "fluid flow" and a diffusion approximation and derive explicit formulas for data queue behavior. These papers focus on the important case in which
\[ \rho_d = \delta/n > c. \]
In such a situation the data messages must have access to voice channels for the system to be stable. Furthermore, it was assumed that \( n/\mu \) was large, say 10^4. Under these circumstances the data flow could be treated deterministically. Suppose we define
\[ \rho_v = \lambda/\mu, \]
and
\[ q = (\rho_v^v/v!)/\sum_{j=0}^{\infty} \rho_v^j/j!, \]
the Erlang B blocking probability. The total traffic intensity on the \( c + v \) channels is given by
\[ \rho_d + \rho_v(1-q), \]
or we could define
\[ \rho = (\rho_d + \rho_v(1-q))/(c+v). \]
A heavy traffic approximation can be derived for this case \( \rho \rightarrow 1 \). Such an approximation was derived in Gaver and Lehoczky (1979b) assuming \( n/\mu \) was large; a Wiener process with reflecting boundary was found appropriate. In this paper we derive a heavy traffic approximation for the system without the fluid flow assumption that \( n/\mu \) is large.

The methodology is drawn heavily from the approach of Burman (1979). In this approach one characterizes a Markov process
by its infinitesimal generator. One next suitably normalizes
the process so that the generator converges to a limiting
infinitesimal generator (in this case to that of a reflected
Brownian motion). This convergence allows the conclusion that
the finite dimensional distributions of the normalized Markov
process converge. The diffusion approximation consists of
treating the actual process through its limiting behavior.
The details are somewhat complicated by the presence of a
boundary.
2.

Let \( \{(X(t), V(t)), t \geq 0\} \) be a bivariate Markov process with state space \( S = \mathbb{Z}^+ \times \{0, 1, \ldots, v\} \). Here \( \{V(t), t \geq 0\} \) is marginally an M/M/\( v \)/\( v \) loss system with arrival rate \( \lambda \) and service rate \( \mu \). Conditional on \( V(t) \), \( \{X(t), t \geq 0\} \) is an M/M/(\( c + v - V(t) \)) queueing system with arrival rate \( \delta \) and service rate \( \eta \). We say that the \( V \) process subordinates the \( X \) process. We let

\[
Q = \begin{pmatrix}
\rho_V & -\rho_V \\
1 & -(1+\rho_V) & \rho_V \\
2 & -(2+\rho_V) & \rho_V \\
& \ddots & \ddots & \ddots \\
& & \rho_V \\
v-1 & -(v-1+\rho_V) & \rho_V \\
v & -v
\end{pmatrix},
\]

the infinitesimal generator of the \( V \) process.

The generator of the \( (X,V) \) process is given by

\[
Af(x,k) = \begin{cases} 
Qf(x,k) + \delta(f(x+1,k) - f(x,k)) \\
+ \eta(c+v-k)(f(x-1,k) - f(x,k)) \\
\text{if } x \geq c+v-k \\
Qf(x,k) + \delta(f(x+1,k) - f(x,k)) \\
+ \eta f(x-1,k) - f(x,k)) \\
\text{if } x < c+v-k
\end{cases}
\]

(2.2)
for \( f: S \to \mathbb{R} \) continuous where

\[
Qf(x,k) = \rho_v f(x,k+1) - (k+\rho_v) f(x,k) + kf(x,k-1)
\]  
(2.3)

\( v \geq k \geq 0 \)

and \( f(x,-1) = f(x,v+1) = 0 \). Clearly \( Qf(x) = 0 \), that is \( Q \) annihilates functions of \( x \) alone. We next normalize the \((X,V)\) process by defining \( X_n(t) = X(nt) / \sqrt{n} \) and \( V_n(t) = V(nt) \). One can calculate the generator of the Markov process \( \{(X_n(t), V_n(t)), t \geq 0\} \) having state space \( S_n = \{0, 1/\sqrt{n}, 2/\sqrt{n}, \ldots \} \times \{0,1,\ldots,v\} \) to be

\[
A_n f(x,k) = \begin{cases} 
    nQf(x,k) + \delta n(f(x + 1/\sqrt{n},k) - f(x,k)) \\
    \quad + \eta_n (f(x - 1/\sqrt{n},k) - f(x,k)) \\
    \quad \text{if} \quad x > \frac{c + v - k}{\sqrt{n}}
\end{cases}
\]

\( \text{if} \quad x = 0, 1/\sqrt{n}, \ldots, (c+v-k) / \sqrt{n} \).

We assume \( f(x,k) \) has three bounded derivatives in \( x \) for each fixed \( k \). With this assumption one can expand terms in (2.4) in a Taylor series and rewrite as
$$A_n f(x,k) = \begin{cases} 
Q f(x,k) + n^{1/2} f_x(x,k) (\delta - n(c+v-k)) \\
+ \frac{1}{2} f_{xx}(x,k) (\delta + n(c+v-k)) \\
+ O(n^{-1/2}) \\
\text{if } x \geq (c+v-k)/\sqrt{n} \\
Q f(x,k) + n^{1/2} f_x(x,k) (\delta - n\sqrt{n}x) \\
+ \frac{1}{2} f_{xx}(x,k) (\delta + n\sqrt{n}x) + O(n^{-1/2}) \\
\text{if } x = 0, 1/\sqrt{n}, \ldots, (c+v-k)/\sqrt{n} 
\end{cases}$$

(2.5)

with $f_x(x,k) = \frac{\partial}{\partial x} f(x,k)$ and $f_{xx}(x,k) = \frac{\partial^2}{\partial x^2} f(x,k)$.

We ultimately wish to prove that the finite dimensional distributions of $\{X_n(t), t \geq 0\}$ converge to those of a Wiener process with reflecting barrier at the origin. This can be restated in terms of semi-groups. We let $\{T^n_t, t \geq 0\}$ be the semi-group of operators associated with $\{(X_n(t), V_n(t)), t \geq 0\}$ and $\{T^\infty_t, t \geq 0\}$ be that associated with a Wiener process having reflecting barrier at 0. Let $g$ be a continuous function $g: \mathbb{R}^r \to \mathbb{R}^r$. Knowledge of the semi-group is equivalent to knowledge of the transition functions by taking a sequence of $g$'s which approximate indicator functions. We wish to prove $|T^n_t g(x,k) - T^\infty_t g(x)| + O$ as $n \to \infty$ for all $(x,k)$. Here $T^n_t g(x,k) = E(g(X_n(t)|X_n(0) = x, V_n(0) = k)$. The presence of the variable $k$ prevents this.
from being done directly. The method we use is to construct a convenient sequence of functions \( \langle g_n \rangle_{n=1}^\infty \) which converge in some sense to \( g \). We write

\[\|T^n_t g - T^\infty_t g\|_n \leq \|(T^n_t g) - T^\infty_t g\|_n + \|T^n_t g - T^n_t g_n\|_n + \|T^n_t g_n - (T^n_t g)_n\|_n \] (2.6)

where \( \| \cdot \|_n \) refers to the sup norm over \( S_n \). Both \( T^n_t \) and \( T^\infty_t \) are contraction semigroups.

\( \langle (T^n_t g) \rangle_{n=1}^\infty \) is the sequence of functions constructed from \( T^n_t g \). Our goal is to show that each of the three terms on the right side of (2.6) converges to 0. The first and second terms can be handled similarly. For any function \( g \), we must guarantee that the constructed \( \langle g_n \rangle_{n=1}^\infty \) sequence satisfies \( \|g_n - g\|_n + 0 \). It will follow that \( \|(T^n_t g) - T^\infty_t g\|_n + 0 \).

Moreover, since \( \{T^n_t, t \geq 0\} \) is a contraction semi-group

\[\|T^n_t g - T^n_t g_n\|_n \leq \|g - g_n\|_n \] which also converges to 0. The sequence \( \langle g_n \rangle_{n=1}^\infty \) will be chosen in such a way that the third term converges to 0.

We focus on a convergence determining class of functions \( g \), those which are bounded and have three bounded derivatives. For such a function \( g(x) \) we define

\[g_n(x,k) = g(x) + \frac{1}{\sqrt{n}} u(x,k) + \frac{1}{n} v(x,k) \] (2.7)
where \( u \) and \( v \) have two bounded derivatives in \( x \) for each fixed \( k \). The functions \( u \) and \( v \) will be determined explicitly later and are chosen to control the third term in (2.6). Clearly when \( g \) is defined by (2.7), \( \| g - g_n \|_n = O(n^{-1/2}) \) and therefore converges to 0 as required.

One can apply the generator \( A_n \) to \( g_n \) to derive

\[
A_n g_n(x, k) = \begin{cases} 
\frac{nQg(x)}{2} + n^{1/2} \left[ Qu(x, k) + g'(x) \delta \eta (c+v-k) \right] \\
+ [Qv(x, k) + u_x(x, k) \delta \eta (c+v-k)] \\
\quad + \frac{1}{2} g''(x) \delta \eta (c+v-k)] + O(n^{-1/2}) \\
\text{if } x \geq (c+v-k)/\sqrt{n}
\end{cases}
\]

\[
A_n g_n(x, k) = \begin{cases} 
\frac{nQg}{2} + n^{1/2} \left[ Qu(x, k) + g'(x) \delta \eta \right] \\
+ [Qv(x, k) + u_x(x, k) \delta \eta n^{1/2}x] \\
\quad + \frac{1}{2} g''(x) \delta \eta n^{1/2}x + O(n^{-1/2}) \\
\text{if } x = 0, 1/\sqrt{n}, \ldots, (c+v-k)/\sqrt{n}
\end{cases}
\]

where \( u_x(x, k) = \frac{\partial}{\partial x} u(x, k) \). Recall that \( Q \) annihilates functions of \( x \) alone, thus \( nQg(x) = 0 \). We want to have \( A_n g_n(x, k) \) converge to a finite limit and to have that limit be independent of \( k \). For this to occur, the \( n^{1/2} \) term must be controlled and the functions \( u \) and \( v \) must be chosen in such a way as to eliminate the variable \( k \).
The $n^{1/2}$ coefficient in (2.8) can be rewritten by adding and subtracting

$$
\sum_{k=0}^{\ell} \pi_k g'(x) (\delta - \eta(c+v-k)) = -\eta(c+v)(1-\rho)g'(x).
$$

We next pick $u(x,k)$ to be a solution of

$$
\begin{align*}
0u(x,k) &= -(g'(x) \eta(\rho_d-(c+v-k)) + g'(x) \eta(c+v)(1-\rho)) \\
&= -g'(x) \eta(k-\rho_v(1-q))
\end{align*}
$$

(2.9)

When $u(x,k)$ is any solution of (2.9), the coefficient of the $n^{1/2}$ term in (2.8) becomes

$$
\begin{align*}
-g'(x) \eta(c+v)(1-\rho) & \quad \text{if } x \geq \frac{c+v-k}{\sqrt{n}} \\
g'(x) \eta((c+v)\rho - n^{1/2}x - k) & \quad \text{if } 0 \leq x < \frac{c+v-k}{\sqrt{n}}
\end{align*}
$$

Equation (2.9) can be solved explicitly. Define

$$
a_k = -g'(x) \eta(k-\rho_v(1-q))/\nu, \text{ so that (2.9) can be written as}
$$

$$
\begin{align*}
-\rho_v(u(x,k)-u(x,k-1))-(k-1)(u(x,k-1)-u(x,k-2)) &= a_{k-1}, \quad k = 1, \ldots, v \\
-\nu(u(x,v)-u(x,v-1)) &= a_v
\end{align*}
$$

(2.10)
Equation (2.10) has a solution since \( \sum_{k=0}^{V} \pi_k a_k = 0 \), where \( \pi_k \) is the stationary distribution associated with \( Q \), or \( \pi_k = (\rho_k^k/k!)/(\sum_{i=0}^{V} \rho_i^i/i!) \). The solution is given by

\[
    u(x,k) - u(x,k-1) = \frac{\sum_{i=0}^{k-1} \pi_i a_i}{\rho_x \pi_{k-1}} = \frac{-g'(x)\eta T_{k-1}}{\mu p_v n^{T_{k-1}}}
\]

where

\[
    T_k = \sum_{i=0}^{k} \pi_i (1-p_v(1-q)) \quad \text{and} \quad T_v = 0.
\]

Clearly

\[
    u(x,k) = u(x,0) - \frac{g'(x)}{\mu p_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}}, \quad 1 \leq k \leq v \quad (2.11)
\]

where \( u(x,0) \) is arbitrary. We let \( u(x,0) = \frac{1}{2} g'(x) \) so

\[
    u(x,k) = g'(x) \left( \frac{1}{2} \frac{1}{\mu p_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} \right), \quad 0 \leq k \leq v \quad (2.12)
\]

For the choice of \( u \) specified by (2.12) we next wish to insure that the limiting generator is independent of the variable \( k \). The function \( v \) is chosen to eliminate the dependence on \( k \). The \( O(1) \) term of (2.8) is given, for \( x > (c+v-k)/\sqrt{n} \), by
\[ Qv(x,k) + g''(x) \left[ \frac{1}{2} - \frac{n}{\mu \rho_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} (\delta - \eta(c+v-k)) \right] \]

\[ + \frac{1}{2} g''(x) \delta + \eta(c+v-k) \]

\[ = Qv(x,k) + H(x,k). \]

Let \( \bar{H}(x) = \sum_{k=0}^{V} \pi_k H(x,k) \) and consider \( Qv(x,k) + (H(x,k) - \bar{H}(x)) + \bar{H}(x) \).

We now let \( v(x,k) \) be any solution of

\[ Qv(x,k) = -(H(x,k) - \bar{H}(x)). \]  \[ (2.13) \]

Equation (2.13) has a one-parameter family of solutions, since \( \sum_{k=0}^{V} \pi_k (H(x,k) - \bar{H}(x)) = 0 \). When \( v(x,k) \) is chosen to be any solution of (2.13), the \( O(1) \) term of (2.3), for \( x > (c+v-k)/\sqrt{n} \), will become \( \bar{H}(x) \) and will therefore be independent of \( k \). It remains to calculate \( \bar{H}(x) \).

\[ \bar{H}(x) = g''(x) \left[ \sum_{k=0}^{V} \pi_k \left( \left( \frac{1}{2} - \frac{n}{\mu \rho_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} \right) (\delta - \eta(c+v-k)) \right) \right] \]

\[ + \frac{1}{2} (\delta + \eta(c+v-k)) \]

\[ = g''(x) \left[ \delta - \frac{n}{\mu \rho_v} \sum_{k=0}^{V} \pi_k (\delta - \eta(c+v-k)) \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} \right] \]  \[ (2.14) \]

\[ = g''(x) \left[ \delta - \frac{n^2}{\mu \rho_v} \sum_{k=0}^{V} \pi_k (k - \rho_v(1-q)) \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} \right. \]

\[ + \frac{n(c+v)(1-q)}{\mu \rho_v} \sum_{k=0}^{V} \pi_k \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}} \]
The second term can be rewritten by interchanging the order of summation. The third term is $O(1-p)$. We find

$$\tilde{H}(x) = g''(x) \left[ \delta - \frac{n^2 v-1}{\mu \rho_v} \sum_{i=0}^{V-1} \frac{T_i}{\pi_i} \sum_{k=i+1}^{\pi_k} (k-\rho_v(1-q)) + O(1-p) \right]$$

$$= g''(x) \left[ \delta - \frac{n^2 v-1}{\mu \rho_v} \sum_{i=0}^{V-1} \frac{T_i}{\pi_i} (T_v-T_i) + O(1-p) \right]$$

with $T_v = 0$ or

$$\tilde{H}(x) = g''(x) \eta \left[ \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{V-1} \frac{T_i^2}{\pi_i} + O(1-p) \right]$$

(2.15)

For the functions $u$ and $v$ specified by (2.12) and (2.13), equation (2.8) can be rewritten as

$$\Lambda_n g_n(x,k) = \begin{cases} 
-n^{1/2}(1-p)(\alpha+v) \eta g'(x) \\
+ \eta \left[ \rho_d + \frac{1}{\mu \rho_v} \sum_{i=0}^{V-1} \frac{T_i^2}{\pi_i} + O(1-p) \right] g''(x) + O(n^{-1/2}) \\
\text{for } x \geq (c+v-k)/\sqrt{n}
\end{cases}$$

(2.16)

$$\Lambda_n g_n(x,k) = \begin{cases} 
\eta^{1/2} \eta [(c+v) \rho - n^{1/2} x - k] g'(x) \\
+ \eta g''(x) \left[ \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{V-1} \frac{T_i^2}{\pi_i} + O(1-p) \\
-(c+v-k-n^{1/2} x) \frac{n}{\mu \rho_v} \sum_{i=0}^{k} \frac{T_i}{\pi_i} \right] + O(n^{-1/2}) \\
\text{for } x \leq (c+v-k)/\sqrt{n}
\end{cases}$$
We now introduce the "heavy traffic approximation."

In order for the generator to converge to a limiting generator we must have $1 - \rho = O(n^{-1/2})$. Specifically, we assume $\rho = \rho_n = 1 - (\theta/\sqrt{n})$ for some $\theta > 0$. In this case, $n^{1/2}(1 - \rho) = \theta$, and (2.16) becomes

\[
\begin{align*}
A_n g_n(x, k) = & \begin{cases} 
-\theta n(c+v)g'(x) + n \left[ \rho_d + \frac{n}{\mu v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right] g''(x) + O(n^{-1/2}) \\
& \text{for } x \geq (c+v-k)/\sqrt{n}
\end{cases}
\end{align*}
\]

We now define a limiting generator $A_\infty$ with domain consisting of all functions $g$ having three bounded derivatives and $g'(0) = 0$. Let

\[
A_\infty g(x) = -\theta n(c+v)g'(x) + n \left[ \rho_d + \frac{n}{\mu v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right] g''(x), \quad x > 0
\]
$A_\infty$ is the generator of a Markov process which corresponds to a Wiener process with drift $-\theta n(c+v)$, scale

$$2n \left[ \rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=1}^{v-1} \frac{T_i^2}{\pi_i} \right],$$

and a reflecting barrier at 0. The $O(n^{-1/2})$ terms involve the first three derivatives of $g$ which are bounded. It is clear from a direct comparison of (2.16) and (2.18) that

$$|A_n g_n(x,k) - A_\infty g| + 0 \text{ as } n \to \infty \text{ for all } x > 0 \text{ and } k \text{ arbitrary. In addition, } g'(0) = 0 \text{ is necessary for the generator to converge at } x = 0. \text{ Unfortunately even assuming } g'(0) = 0,$

$$|A_n g(0,k) - A_\infty g(0)| + (c+v-k) \frac{2}{\mu \rho_v} \sum_{i=0}^{k} \frac{T_i^2}{\pi_i} g''(0) \text{ as } n \to \infty$$

rather than to 0. One needs a special argument to handle this lack of convergence at the boundary.

We set out to prove the third term in (2.6) converges to 0. Standard semi-group results (see Burman, 1979, p. 33) give

$$(T^\infty_T g)_n - T^n_T g_n = \int_0^t T^n_{t-S} ((A_n w)_n - A_n w_n) dS \quad (2.19)$$

where $w = w(t,x) = T^\infty_T g(x)$. Recall that $w_n = w + (1/\sqrt{n})u + (1/n)v$ with $u$ and $v$ defined by (2.12) and (2.13) with $g$ replaced by $w$. It follows that
\[ \|T_t^n g - (T_t^g)_n\|_n \]
\[ = \left\| \int_0^t T_{t-S}((A_\infty w)_n - A_\infty w + A_\infty w - A_n w_n) \, dS \right\|_n \]
\[ \leq \int_0^t \|T_{t-S}((A_\infty w)_n - A_\infty w)\|_n \, dS + \int_0^t \|T_{t-S}(A_\infty w - A_n w_n)\|_n \, dS \]
\[ \leq \int_0^t \|A_\infty w - A_\infty w\|_n \, dS + \int_0^t \|T_{t-S}(A_\infty w - A_n w_n)\|_n \, dS \]

The first term is clearly \( O(n^{-1/2}) \). It remains to show that the second is \( O(n^{-1/2}) \) as well. We have shown \( |A_\infty w - A_n w_n| = O(n^{-1/2}) \) except at the boundary where it is \( O(1) \). We split the integral into two parts, for one of which the process is away from the boundary, and for the other, near the boundary. The integral away from the boundary has an integrand which is \( O(n^{-1/2}) \). The integral near the boundary is also \( O(n^{-1/2}) \) since under a heavy traffic assumption the process is rarely near the boundary. The details are merely summarized here; they are based on the ideas of Burman (1979).

Let \( I_{on} \) be the indicator function of \[ \left[ 0, \frac{c+v-k}{\sqrt{n}} \right) \]
and \( I_{ln} \) be the indicator of
\[
\left[ \frac{c+\nu-k}{\sqrt{n}}, -\right)
\]

We have

\[
\int_0^t T^n_{t-S} (A_w - A_n w_n) dS |_{n} \leq \int_0^t T^n_{t-S} (A_w - A_n w_n) I_{\ln dS} |_{n} + \int_0^t T^n_{t-S} (A_w - A_n w_n) I_{0n dS} |_{n}
\]

\[
\leq \int_0^t (A_w - A_n w_n) I_{\ln dS} |_{n} + \| A_w - A_n w_n \| \int_0^t T^n_{t-S} I_{0n dS} |_{n}.
\]

The first term is \( O(n^{-1/2}) \), since \( |A_w - A_n w_n| = O(n^{-1/2}) \) off the boundary. The factor \( \| A_w - A_n w_n \| = O(1) \), thus it remains to show that

\[
\int_0^t T^n_{t-S} I_{0n dS} |_{n} = O(n^{-1/2}).
\]

This gives the total time in \([0, t]\) spent near the boundary.

We bound

\[
\int_0^t T^n_{t-S} I_{0n dS} |_{n}
\]

by first introducing a function \( h(x) \) not in the domain of \( A_n \). We let \( h(x) \) have bounded support, be infinitely differentiable and be given by \( h(x) = x \) for \( x \) near 0. One can construct \( h_n(x) \) using (2.7) and apply \( A_n \) to \( h_n \) to find

17
\[ A_n h_n = \begin{cases} 0(1) & \text{if } x \geq \frac{c+v-k}{\sqrt{n}} \\ n^{1/2}((c+v)_n - n^{1/2}x - k) + O(1) & \text{if } x < \frac{c+v-k}{\sqrt{n}} \end{cases} \] (2.20)

One has

\[ T^n_{s_n} - h_n = \int_0^t T^n_{s_n} h_n dS \]

\[ = \int_0^t T^n_{s_n} h_n I_0 dS + \int_0^t T^n_{s_n} h_n I_0 dS. \]

It follows that

\[ \| \int_0^t T^n_{s_n} h_n I_0 dS \|_n \leq \| T^n_{s_n} - h_n \|_n + \| \int_0^t T^n_{s_n} h_n I_0 dS \|_n \]

\[ \leq 2\| h_n \|_n + O(1). \]

We have shown \( \| \int_0^t T^n_{s_n} h_n I_0 dA \|_n \) to be bounded in \( n \). An application of (2.2) shows

\[ \| \int_0^t T^n_{s_n} h_n I_0 dA \|_n = n^{1/2} \| (c+v)_n - n^{1/2}x - k + O(1) \| \int_0^t T^n_{s_n} I_0 dS \|_n \]

is bounded in \( n \). It follows that \( \| \int_0^t T^n_{s_n} I_0 dS \|_n = O(n^{-1/2}). \)
This finally concludes the argument which shows
\[ \|T^n_{t} g_n - (T^n_{t} g)_{n}\|_n = O(n^{-1/2}), \text{ hence by (2.6) } \|T^n_{t} g - T^n_{t} g\|_n = O(n^{-1/2}). \]

We have thus shown that the finite-dimensional distributions of the \((X_n(t), V_n(t))\) process converge to those of a Wiener process with drift \(-\theta n(c+v)\) scale

\[ \eta \left( \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_{i}^2}{\pi_i} \right), \]

and reflection at 0. The diffusion approximation treats \(X_n(t)\) as though it were such a Wiener process. For instance, the limiting Wiener process has a stationary exponential distribution with parameter

\[ \frac{\theta (c+v)}{\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} (T_{i}^2/\pi_i)}. \]

This is a distribution for \(X(nt)/\sqrt{n}\) and suggests \(X(t)\) will have a steady state distribution given approximately by an exponential with parameter

\[ (c+v)(1-\rho) \left( \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_{i}^2}{\pi_i} \right). \]

The steady state mean data queue length would then be
\[
E(X(t)) = \frac{\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}}{(c + v)(1-\rho)}.
\] (2.21)

It is interesting to consider the special case \( c = 0, \ v = 1 \) where the two types of traffic use the same channel.

Under heavy traffic, \( \rho = \rho_d + \rho_v / (1 + \rho_v) \), so \( \rho_d \approx (1 + \rho_v)^{-1} \).

The mean data queue length derived from the diffusion approximation (2.21) will be

\[
\left( \rho_d + \frac{n}{\mu} \frac{\rho_v}{(1 + \rho_v)^{\frac{3}{2}}} \right) / (1-\rho) \approx \frac{\rho_d}{1-\rho} \left( 1 + \frac{n}{\mu} \frac{\rho_v}{(1 + \rho_v)^{\frac{3}{2}}} \right).
\]

The latter is the exact expression derived by Fisher (1978) for this case. The expression (2.21) represents a generalization of the results of Gaver and Lehoczky (1979b). In this paper, a diffusion approximation is given based on the fluid flow assumption for the data. For this case the result is the same except that the scale is given by

\[
\frac{n^2}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}
\]

rather than

\[
n \left( \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right).
\]

The results derived in this paper therefore definitely generalize...
the results of Gaver and Lehoczky (1979b) since the variability in the data queue is now included. When $n/\mu$ is large, the second term dominates, and the fluid flow approximation is satisfactory.

The Wiener process approximation for the $X(t)$ process provides a method for studying the dynamics of that process. For instance, suppose the data queue were at level $x$ at time $t$ where $x$ is large. One might wish to study the time that elapses until the queue becomes empty. This is essentially the duration of the busy period under heavy traffic and corresponds to a first-passage time for a Wiener process. Let us denote it by $T_x$. Straightforward martingale arguments provide for its transform

$$E(e^{-ST_x}) = \exp \left[ (\frac{x}{\mu} - \langle m \rangle)^2 \right]$$

where

$$m = \theta(c+v) \eta \approx n^{1/2}(1-\rho)(c+v) \eta$$

$$\frac{\sigma^2}{2} = \eta \left( \rho_d + \frac{n}{\langle \mu \rho_v \rangle} \sum_{i=0}^{n-1} \frac{T_i^2}{\pi_1} \right)$$

It is also easy to find the mean first-passage time

$$E(T_x) = \frac{x}{m}$$
One might also be interested in the area beneath the sample path until emptiness occurs, since this area represents the total time waited by all data customers involved in the busy period. If $A_x$ represents this area, simple backward equation arguments give

$$E(A_x) = \frac{x^2}{2m} + \frac{\sigma^2}{2m^2} x$$  \hspace{1cm} (2.24)

where $m$ and $\sigma^2$ are given in (2.22).

**Acknowledgment.** This research was supported in part by a contract from the Office of Naval Research.
BIBLIOGRAPHY


23
### INITIAL DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>Number of Copies</th>
<th>Defense Technical Information Center</th>
<th>Cameron Station</th>
<th>Alexandria, VA  22314</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Library Code</td>
<td></td>
<td>Code 0142</td>
</tr>
<tr>
<td></td>
<td>Naval Postgraduate School</td>
<td></td>
<td>Monterey, CA  93940</td>
</tr>
<tr>
<td></td>
<td>Library Code 55</td>
<td></td>
<td>Naval Postgraduate School</td>
</tr>
<tr>
<td></td>
<td>Monterey, Ca.  93940</td>
<td></td>
<td>Dean of Research</td>
</tr>
<tr>
<td></td>
<td>Code 012A</td>
<td></td>
<td>Naval Postgraduate School</td>
</tr>
<tr>
<td></td>
<td>Monterey, Ca.  93940</td>
<td></td>
<td>Attn: A. Andrus, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D. Gaver, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D. Barr, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P. A. Jacobs, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P. A. W. Lewis, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P. Milch, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R. Richards, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M. G. Sovereign, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R. J. Stampfel, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R. R. Read, Code 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>J. Wozencraft, Code 74</td>
</tr>
<tr>
<td></td>
<td>Mr. Peter Badgley</td>
<td></td>
<td>ONR Headquarters, Code 102B</td>
</tr>
<tr>
<td></td>
<td>ONR Headquarters, Code 102B</td>
<td></td>
<td>800 N. Quincy Street</td>
</tr>
<tr>
<td></td>
<td>Arlington, VA  22217</td>
<td></td>
<td>Dr. James S. Bailey, Director</td>
</tr>
<tr>
<td></td>
<td>Dr. James S. Bailey, Director</td>
<td></td>
<td>Geography Programs,</td>
</tr>
<tr>
<td></td>
<td>Geography Programs,</td>
<td></td>
<td>Department of the Navy</td>
</tr>
<tr>
<td></td>
<td>Department of the Navy</td>
<td></td>
<td>ONR</td>
</tr>
<tr>
<td></td>
<td>ONR</td>
<td></td>
<td>Arlington, VA  93940</td>
</tr>
<tr>
<td></td>
<td>Prof. J. Lehoczky</td>
<td></td>
<td>Dept. of Statistics</td>
</tr>
<tr>
<td></td>
<td>Prof. J. Lehoczky</td>
<td></td>
<td>Carnegie Mellon University</td>
</tr>
<tr>
<td></td>
<td>Carnegie Mellon University</td>
<td></td>
<td>Pittsburgh, PA.  15213</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DISTRIBUTION LIST</td>
<td>No. of Copies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>---------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATISTICS AND PROBABILITY PROGRAM</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OFFICE OF NAVAL RESEARCH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CODE 436</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARLINGTON VA</td>
<td>22217</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OFFICE OF NAVAL RESEARCH</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NEW YORK AREA OFFICE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>215 BROADWAY - 5TH FLOOR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: CH. ROGER GRAFTON</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NEW YORK, NY</td>
<td>10033</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTOR</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OFFICE OF NAVAL RESEARCH BRANCH OFF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>536 SOUTH CLARK STREET</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: DEPUTY AND CHIEF SCIENTIST</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHICAGO, IL</td>
<td>60605</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LIBRARY</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAVAL OCEAN SYSTEMS CENTER</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAN DIEGO CA</td>
<td>92152</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAVY LIBRARY</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NATIONAL SPACE TECHNOLOGY LAB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: NAVY LIBRARIAN</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BAY ST. LOUIS MS</td>
<td>39522</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAVAL ELECTRONIC SYSTEMS COMMAND</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAVAL LABORATORY CENTER NO. 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARLINGTON VA</td>
<td>20360</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTOR NAVAL RESEARCH LABORATORY</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: LIBRARY (JRNL)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CODE 2026</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, D.C.</td>
<td>20375</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TECHNICAL INFORMATION DIVISION</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAVAL RESEARCH LABORATORY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, D.C.</td>
<td>20375</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Department</td>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------</td>
<td>----------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>PROF. C. R. BAKER</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>UNIVERSITY OF NORTH CAROLINA</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CHAPEL HILL, NORTH CAROLINA</td>
<td></td>
</tr>
<tr>
<td>PROF. R. E. DECHTER</td>
<td>DEPARTMENT OF OPERATIONS RESEARCH</td>
<td>CORNELL UNIVERSITY</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ITHACA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NEW YORK 14850</td>
<td></td>
</tr>
<tr>
<td>PROF. A. J. PERSHAC</td>
<td>SCHOOL OF ENGINEERING</td>
<td>UNIVERSITY OF CALIFORNIA</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IRVINE, CALIFORNIA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>92664</td>
<td></td>
</tr>
<tr>
<td>P. J. BICKEL</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>UNIVERSITY OF CALIFORNIA</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BERKELEY, CALIFORNIA</td>
<td>54720</td>
</tr>
<tr>
<td>PROF. F. W. BLOCK</td>
<td>DEPARTMENT OF MATHEMATICS</td>
<td>UNIVERSITY OF PITTSBURGH</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PITTSBURGH, PA</td>
<td>15260</td>
</tr>
<tr>
<td>PROF. JOSEPH BLUM</td>
<td>DEPT. OF MATHEMATICS, STATISTICS AND COMPUTER SCIENCE</td>
<td>THE AMERICAN UNIVERSITY</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WASHINGTON, DC</td>
<td>20016</td>
</tr>
<tr>
<td>PROF. R. A. BRADLEY</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>FLORIDA STATE UNIVERSITY</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TALLAHASSEE, FLORIDA 32306</td>
<td></td>
</tr>
<tr>
<td>PROF. R. E. B.A.RLOW</td>
<td>OPERATIONS RESEARCH CENTER</td>
<td>COLLEGE OF ENGINEERING</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UNIVERSITY OF CALIFORNIA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BERKELEY, CALIFORNIA 94720</td>
<td></td>
</tr>
<tr>
<td>MR. C. A. FENNETT</td>
<td>NAVAL COASTAL SYSTEMS LABORATORY</td>
<td>COC: P7466</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PANAMA CITY, FLORIDA 22401</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The document contains a table listing individuals and institutions along with the number of copies they have requested.
<table>
<thead>
<tr>
<th>DISTRIBUTION LIST</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF. L. N. PHAT</td>
<td>1</td>
</tr>
<tr>
<td>COMPUTER SCIENCE / OPERATIONS RESEARCH CENTER</td>
<td></td>
</tr>
<tr>
<td>SOUTHERN METHODIST UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>DALLAS, TEXAS 75275</td>
<td></td>
</tr>
<tr>
<td>PROF. W. A. ELISCHKE</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF QUANTITATIVE BUSINESS ANALYSIS</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY OF SOUTHERN CALIFORNIA</td>
<td></td>
</tr>
<tr>
<td>LOS ANGELES, CALIFORNIA</td>
<td>90007</td>
</tr>
<tr>
<td>DR. DERRILL J. BERDELON</td>
<td>1</td>
</tr>
<tr>
<td>NAVAL UNDERWATER SYSTEMS CENTER</td>
<td></td>
</tr>
<tr>
<td>CODE 21</td>
<td></td>
</tr>
<tr>
<td>NEWPORT</td>
<td>02840</td>
</tr>
<tr>
<td>J. E. BOYER JR</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF STATISTICS</td>
<td></td>
</tr>
<tr>
<td>SOUTHERN METHODIST UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>DALLAS, TX</td>
<td>75275</td>
</tr>
<tr>
<td>DR. J. CHANDRA</td>
<td>1</td>
</tr>
<tr>
<td>U. S. ARMY RESEARCH</td>
<td></td>
</tr>
<tr>
<td>P. O. BOX 12211</td>
<td></td>
</tr>
<tr>
<td>RESEARCH TRIANGLE PARK</td>
<td></td>
</tr>
<tr>
<td>NORTH CAROLINA</td>
<td>27716</td>
</tr>
<tr>
<td>PROF. E. CHERNOFF</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF MATHEMATICS</td>
<td></td>
</tr>
<tr>
<td>MASS INSTITUTE OF TECHNOLOGY</td>
<td></td>
</tr>
<tr>
<td>CAMBRIDGE, MASSACHUSETTS 02139</td>
<td></td>
</tr>
<tr>
<td>PROF. C. GERMAN</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF CIVIL ENGINEERING AND ENGINEERING MECHANICS</td>
<td></td>
</tr>
<tr>
<td>COLUMBIA UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>NEW YORK</td>
<td>10027</td>
</tr>
<tr>
<td>PROF. R. L. DISNEY</td>
<td>1</td>
</tr>
<tr>
<td>VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>DEPT. OF INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH</td>
<td></td>
</tr>
<tr>
<td>BLACKSBURG, VA</td>
<td>24061</td>
</tr>
</tbody>
</table>
DISTRIBUTION LIST

1

MR. GENE P. GLEISSNER
AFFLIEG MATHMATICS LABORATCGY
NAVY TAYLOR NAVAL SHIP RESEARCH
AND DEVELOPMENT CENTER
BETHESDA
MD 20004

1

PROF. S. S. GLPTA
DEPARTMENT OF STATISTICS
PURCUE UNIVERSITY
LAFAYETTE
INODA 47907

1

PROF. C. L. HANSON
DEPT. OF MATHEMATICAL SCIENCES
STATE UNIVERSITY OF NEW YORK,
BINGHAMTN
BINGHAMTON
NY 13901

1

Prof. M. J. Hinich
Dept. of Economics
Virginia Polytechnica Institute
and State University
Blacksburg, VA 24061

1

Dr. D. Depriest,
ONR, Code 102B
800 N. Quincy Street
Arlington, VA 22217

1

Prof. G. E. Whitehouse
Dept. of Industrial Engineering
Lehigh University
Bethlehem, PA 18015

1

Prof. M. Zia-Hassan
Dept. of Ind. & Sys. Eng.
Illinois Institute of Technology
Chicago, IL 60616

1

Prof. S. Zacks
Statistics Dept.
Virginia Polytechnic Inst.
Blacksburg, VA 24061

1

Head, Math. Sci Section
National Science Foundation
1800 G Street, N.W.
Washington, D.C. 20550
No. of Copies

1

Dr. H. Sittrop
Physics Lab., TNO
P.O. Box 96964
2509 JG, The Hague
The Netherlands

1

DR. R. ELASHOFF
BIONETHEMATICS
UNIV. OF CALIF.
LOS ANGELES
CALIFORNIA 90024

1

PROF. GEORGE S. FISHMAN
UNIV. OF NORTH CAROLINA
CUR. IN CR AND SYS. ANALYSIS
PHILLIPS ANNEX
CHAPEL HILL, NORTH CAROLINA 20742

1

DR. R. GNANAGESIKAN
EELL TELEPHONE Lab.
NEW DELHI, INDIA 07733

1

DR. A. J. GOLDSMITH
CHIEF, CR
DIV. 2C5.CZ, ADMIN. A428
U.S. DEPT. OF COMMERCE
WASHINGTON, D.C. 20234

1

DR. R. HIGGINS
53 BONN I, POSTFACH 585
NASSESTRASSE 2
WEST GERMANY

1

DR. P. T. HOLMES
DEPT. OF MATH.
CLEMSON UNIV.
CLEMSON
SOUTH CAROLINA 29631

1

Dr. J. A. Hocke
Bell Telephone Labs
Whippany, New Jersey 07733

1

Dr. Robert Hooke
Box 1982
Pinehurst, No. Carolina 28374
Dr. D. L. Iglehart
DEPT. CF C.F.
STANFORD UNIV.
STANFORD
CALIFORNIA

Dr. O. Trizna, Mail Code 5323
Naval Research Lab
Washington, D.C. 20375

Dr. E. J. Wegman,
ONR, Code 436
Arlington, VA 22217

Dr. H. Kgeyashi
IBM
WHITE PLAINS
NEW YORK

Dr. A. Lemoine
1020 GUINCA ST.
PALO ALTO
CALIFORNIA

Dr. J. MacQueen
UNIV. OF CALIF.
LOS ANGELES
CALIFORNIA

Prof Kneale Marshall
Scientific Advisor to DCNO (MPT)
Code Op-DIT, Room 2705
Arlington Annex
Washington, D.C. 20370

Dr. M. Mazumcar
PATH. DEPT.
ESTINGHOUSE RESEARCH LABS
FORDHAM U.C.F.
PITTSBURGH
PENNSYLVANIA

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY PUBLISHED TO DDS
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF. W. P. FIPSCH</td>
<td>INSTITUTE OF MATHEMATICAL SCIENCES</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>NEW YORK UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NEW YORK 10453</td>
<td></td>
</tr>
<tr>
<td>PROF. J. E. KACANE</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>CARNEGIE-MELLON</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PITTSBURGH, PA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PENNSYLVANIA 15213</td>
<td></td>
</tr>
<tr>
<td>DR. RICHARD LAVI</td>
<td>EFFECTOR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>OFFICE OF NAVAL RESEARCH BRANCH OFF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1030 EAST GREEN STREET</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PASADENA, CA 91101</td>
<td></td>
</tr>
<tr>
<td>DR. A. R. LAUER</td>
<td>EFFECTOR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>OFFICE OF NAVAL RESEARCH BRANCH OFF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1030 EAST GREEN STREET</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PASADENA, CA 91101</td>
<td></td>
</tr>
<tr>
<td>PROF. P. LEADBETTER</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>UNIVERSITY OF NORTH CAROLINA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CHAPEL HILL, NC 27514</td>
<td></td>
</tr>
<tr>
<td>CR. J. S. LEE</td>
<td>J. S. LEE ASSOCIATES, INC.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2001 JEFFERSON DAVIS HIGHWAY</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SUITE 802, ARLINGTON</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VA 22202</td>
<td></td>
</tr>
<tr>
<td>PROF. R. S. LEVENDRETH</td>
<td>DEPARTMENT OF STATISTICS</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BLACKSBURG, VA 24061</td>
<td></td>
</tr>
<tr>
<td>PROF. R. S. LEVENDRETH</td>
<td>DEPT. OF INDUSTRIAL AND SYSTEMS ENGINEERING</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>UNIVERSITY OF FLORIDA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GAINESVILLE, FL 32411</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** This page is the best quality practicable from copy furnished to OCR.
DISTRIBUTION LIST

No. of copies

1

G. LIEPERMAN
STANFORD UNIVERSITY
DEPARTMENT OF OPERATIONS RESEARCH
STANFORD CALIFORNIA 94305

1

JAMES R. MAAR
NATIONAL SECURITY AGENCY
FORT MEADE, MARYLAND 20755

1

R. H. MACSEK
DEPARTMENT OF STATISTICS
UNIVERSITY OF MISSOURI
COLUMBIA MO 65201

1

N. R. MAAM
SCIENCE CENTER
ROCKWELL INTERNATIONAL CORPORATION
P.O. BOX 1085
THOUSAND CAVES
CALIFORNIA 91362

2

W. E. PARLIS
PROGRAM IN LOGISTICS
THE GEORGE WASHINGTON UNIVERSITY
707 22ND STREET, N.W.
WASHINGTON, D.C. 20057

1

E. PASRY
DEPT. APPLIED PHYSICS AND
INFORMATION SCIENCE
UNIVERSITY OF CALIFORNIA
LA JOLLA
CALIFORNIA 92038

1

B. E. MCGONALD
SCIENTIFIC DIRECTOR
SCIENTIFIC LIAISON GROUP
OFFICE OF NAVAL RESEARCH
AMERICAN EMBASSY - TOKYO
AFC SAN FRANCISCO 96503

32
Dr. Leon F. McGinnis  
School of Ind. And Sys. Eng.  
Georgia Inst. of Tech.  
Atlanta, GA 30332

Dr. D. R. McNeil  
DEPT. OF STATISTICS  
PRINCETON UNIV.  
PRINCETON  
NEW JERSEY  
08540

Dr. F. Mosteller  
STAT. DEPT.  
HARVARD UNIV.  
CAMBRIDGE  
MASSACHUSETTS  
02139

Dr. H. Reiser  
IBM  
THOMAS J. WATSON FES. CTR.  
YORKTOWN HEIGHTS  
NEW YORK  
10598

Dr. J. Riepen  
DEPT. OF MATHEMATICS  
ROCKEFELLER UNIV.  
NEW YORK  
NEW YORK  
10021

Dr. Linus Schrage  
UNIV. OF CHICAGO  
GRAD. SCHOOL OF BLS.  
5836 GREEN ACRE AVE.  
CHICAGO, ILLINlS  
60637

Dr. Paul Schweitzer  
University of Rochester  
Rochester, N.Y. 14627

Dr. V. Srinivasan  
Graduate School of Business  
Stanford University  
Stanford, CA. 94305

Dr. Roy Welsch  
M.I.T. Sloan School  
Cambridge, MA 02139
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>DISTRIBUTION LIST</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CR. JAMET P. MYHRE</td>
</tr>
<tr>
<td></td>
<td>THE INSTITUTE OF DECISION SCIENCE</td>
</tr>
<tr>
<td></td>
<td>FOR BUSINESS AND PUBLIC POLICY</td>
</tr>
<tr>
<td></td>
<td>CLAREMONT PENTS COLLEGE</td>
</tr>
<tr>
<td></td>
<td>CLAREMONT</td>
</tr>
<tr>
<td></td>
<td>CA 91711</td>
</tr>
<tr>
<td>1</td>
<td>MR. F. NISSELICA</td>
</tr>
<tr>
<td></td>
<td>BUREAU OF THE CENSUS</td>
</tr>
<tr>
<td></td>
<td>ROCH 2025</td>
</tr>
<tr>
<td></td>
<td>FEDERAL BUILDING 3</td>
</tr>
<tr>
<td></td>
<td>WASHINGTON</td>
</tr>
<tr>
<td></td>
<td>D. C. 20033</td>
</tr>
<tr>
<td>1</td>
<td>MISS E. S. CRLEANS</td>
</tr>
<tr>
<td></td>
<td>NAVAL SEA SYSTEMS COMMAND</td>
</tr>
<tr>
<td></td>
<td>(SEA 03)</td>
</tr>
<tr>
<td></td>
<td>RM 1050</td>
</tr>
<tr>
<td></td>
<td>ARLINGTON VIRGINIA 20360</td>
</tr>
<tr>
<td>1</td>
<td>PROF. C. E OHER</td>
</tr>
<tr>
<td></td>
<td>DEPARTMENT OF STATISTICS</td>
</tr>
<tr>
<td></td>
<td>SOUTHERN METHODIST UNIVERSITY</td>
</tr>
<tr>
<td></td>
<td>DALLAS</td>
</tr>
<tr>
<td></td>
<td>TEXAS 75222</td>
</tr>
<tr>
<td>1</td>
<td>Prof. E. Parzen</td>
</tr>
<tr>
<td></td>
<td>Statistical Science Division</td>
</tr>
<tr>
<td></td>
<td>Texas A &amp; M University</td>
</tr>
<tr>
<td></td>
<td>College Station TX 77843</td>
</tr>
<tr>
<td>2</td>
<td>DR. A. PETRASOVITS</td>
</tr>
<tr>
<td></td>
<td>ECECH 2076, ECEC AND GRLG ELDG.</td>
</tr>
<tr>
<td></td>
<td>TUNNEY'S PASTUNE</td>
</tr>
<tr>
<td></td>
<td>OTTOA, CANTARIC KIA-CL2,</td>
</tr>
<tr>
<td></td>
<td>CANADA</td>
</tr>
<tr>
<td>1</td>
<td>PROF. S. L. PHEONIX</td>
</tr>
<tr>
<td></td>
<td>SIEGY SCHOOL OF MECHANICAL AND</td>
</tr>
<tr>
<td></td>
<td>AEROSPACE ENGINEERING</td>
</tr>
<tr>
<td></td>
<td>CORNELL UNIVERSITY</td>
</tr>
<tr>
<td></td>
<td>ITHACA</td>
</tr>
<tr>
<td></td>
<td>NY 14850</td>
</tr>
<tr>
<td>1</td>
<td>DR. A. L. POWELL</td>
</tr>
<tr>
<td></td>
<td>DIRECTOR</td>
</tr>
<tr>
<td></td>
<td>OFFICE OF NAVAL RESEARCH BRANCH OFF</td>
</tr>
<tr>
<td></td>
<td>455 SUMNER STREET</td>
</tr>
<tr>
<td></td>
<td>BOSTON</td>
</tr>
<tr>
<td></td>
<td>MA 02210</td>
</tr>
<tr>
<td>1</td>
<td>MR. F. R. FRICK</td>
</tr>
<tr>
<td></td>
<td>CODE 222, OPERATIONSL TEST AND ONRS</td>
</tr>
<tr>
<td></td>
<td>EVALUATION FORCE (OPTEVFOR)</td>
</tr>
<tr>
<td></td>
<td>MCHELLEN</td>
</tr>
<tr>
<td></td>
<td>VIRGINIA 20300</td>
</tr>
</tbody>
</table>

This page is best quality practicable from copy furnished to IERG.
<table>
<thead>
<tr>
<th>DISTRIBUTION LIST</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF. M. L. PURI</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF MATHEMATICS</td>
<td>47401</td>
</tr>
<tr>
<td>P.O. BOX 4</td>
<td></td>
</tr>
<tr>
<td>INDIANA UNIVERSITY FOUNDATION</td>
<td></td>
</tr>
<tr>
<td>ELKHART, IN</td>
<td></td>
</tr>
<tr>
<td>PROF. H. ROEBSINS</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF MATHEMATICS</td>
<td></td>
</tr>
<tr>
<td>COLUMBIA UNIVERSITY</td>
<td>NEW YORK,</td>
</tr>
<tr>
<td>NEW YORK 10027</td>
<td></td>
</tr>
<tr>
<td>PROF. H. ROSENBATT</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF MATHEMATICS</td>
<td>UNIVERSITY OF CALIFORNIA 92093</td>
</tr>
<tr>
<td>SAN DIEGO LA JOLLA</td>
<td>CALIFORNIA</td>
</tr>
<tr>
<td>PROF. S. M. RCSS</td>
<td>1</td>
</tr>
<tr>
<td>COLLEGE OF ENGINEERING</td>
<td>UNIVERSITY OF CALIFORNIA</td>
</tr>
<tr>
<td>BERKELEY CA</td>
<td>94720</td>
</tr>
<tr>
<td>PROF. J. RUBIN</td>
<td>1</td>
</tr>
<tr>
<td>SCHOOL OF ENGINEERING AND APPLIED SCIENCE</td>
<td>UNIVERSITY OF CALIFORNIA 90024</td>
</tr>
<tr>
<td>LOS ANGELES, CALIFORNIA</td>
<td></td>
</tr>
<tr>
<td>PROF. J. R. SAVAGE</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td>YALE UNIVERSITY</td>
</tr>
<tr>
<td>NEW HAVEN, CONNECTICUT</td>
<td></td>
</tr>
<tr>
<td>LOR 6520</td>
<td></td>
</tr>
<tr>
<td>PROF. L. L. SCHAEF FR</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF ELECTRICAL ENGINEERING</td>
<td>COLORADO STATE UNIVERSITY</td>
</tr>
<tr>
<td>FT. COLLINS, COLORADO</td>
<td></td>
</tr>
<tr>
<td>E0521</td>
<td></td>
</tr>
<tr>
<td>PROF. R. SERFLING</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td>FLORIDA STATE UNIVERSITY</td>
</tr>
<tr>
<td>TALLAHASSEE, FLORIDA 32306</td>
<td></td>
</tr>
<tr>
<td>PROF. H. R. SCHLACNY</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td>SOUTHERN METHODIST UNIVERSITY</td>
</tr>
<tr>
<td>DALLAS, TEXAS</td>
<td></td>
</tr>
<tr>
<td>75222</td>
<td></td>
</tr>
</tbody>
</table>

35
<table>
<thead>
<tr>
<th>DISTRIBUTION LIST</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF. C. C. SIEGMUND</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF STATISTICS</td>
<td></td>
</tr>
<tr>
<td>STANFORD UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>STANFORD CA</td>
<td>64305</td>
</tr>
<tr>
<td>PROF. M. L. SIEGMAN</td>
<td>1</td>
</tr>
<tr>
<td>DEPT. OF ELECTRICAL ENGINEERING</td>
<td></td>
</tr>
<tr>
<td>POLYTECHNIC INSTITUTE OF NEW YORK</td>
<td></td>
</tr>
<tr>
<td>BROOKLYN NEW YORK</td>
<td>11201</td>
</tr>
<tr>
<td>DR. A. L. SLAFKOSKY</td>
<td>1</td>
</tr>
<tr>
<td>SCIENTIFIC ADVISER</td>
<td></td>
</tr>
<tr>
<td>COMMANDANT OF THE MARINE CORPS</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON D. C.</td>
<td>20380</td>
</tr>
<tr>
<td>DR. C. E. SMITH</td>
<td>1</td>
</tr>
<tr>
<td>DESCARTES INC.</td>
<td></td>
</tr>
<tr>
<td>P.O. BOX 618</td>
<td></td>
</tr>
<tr>
<td>STATE COLLEGE</td>
<td></td>
</tr>
<tr>
<td>PENNSYLVANIA</td>
<td>16801</td>
</tr>
<tr>
<td>PROF. W. L. SMITH</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY OF NORTH CAROLINA</td>
<td></td>
</tr>
<tr>
<td>CHAPEL HILL</td>
<td></td>
</tr>
<tr>
<td>NORTH CAROLINA 27514</td>
<td></td>
</tr>
<tr>
<td>Dr. H. J. Solomon</td>
<td>1</td>
</tr>
<tr>
<td>ONR</td>
<td></td>
</tr>
<tr>
<td>223/231 Old Marylebone Rd</td>
<td></td>
</tr>
<tr>
<td>London NW1 5TH ENGLAND</td>
<td></td>
</tr>
<tr>
<td>MR. GLENN F. STAHL</td>
<td>1</td>
</tr>
<tr>
<td>NATIONAL SECURITY AGENCY</td>
<td></td>
</tr>
<tr>
<td>FORT MEACE</td>
<td></td>
</tr>
<tr>
<td>PARYLAND 20755</td>
<td></td>
</tr>
<tr>
<td>Mr. J. Gallagher</td>
<td>1</td>
</tr>
<tr>
<td>Naval Underwater Systems Center</td>
<td></td>
</tr>
<tr>
<td>New London CT</td>
<td></td>
</tr>
<tr>
<td>Dr. E. C. Monahan</td>
<td>1</td>
</tr>
<tr>
<td>Dept. of Oceanography</td>
<td></td>
</tr>
<tr>
<td>University College</td>
<td></td>
</tr>
<tr>
<td>Galway Ireland</td>
<td></td>
</tr>
</tbody>
</table>

36
<table>
<thead>
<tr>
<th>DISTRIBUTION LIST</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR. R. M. Stark</td>
<td>1</td>
</tr>
<tr>
<td>Statistics and Computer Sci.</td>
<td></td>
</tr>
<tr>
<td>Univ. of Delaware</td>
<td></td>
</tr>
<tr>
<td>Newark</td>
<td>19711</td>
</tr>
<tr>
<td>Prof. John W. Tukey</td>
<td>1</td>
</tr>
<tr>
<td>Fine Hall</td>
<td></td>
</tr>
<tr>
<td>Princeton Univ.</td>
<td></td>
</tr>
<tr>
<td>Princeton</td>
<td>08540</td>
</tr>
<tr>
<td>Dr. Thomas C. Varley</td>
<td>1</td>
</tr>
<tr>
<td>Office of Naval Research</td>
<td></td>
</tr>
<tr>
<td>Code 436</td>
<td></td>
</tr>
<tr>
<td>Arlington</td>
<td>22217</td>
</tr>
<tr>
<td>Prof. G. Watson</td>
<td>1</td>
</tr>
<tr>
<td>Fine Hall</td>
<td></td>
</tr>
<tr>
<td>Princeton Univ.</td>
<td></td>
</tr>
<tr>
<td>Princeton</td>
<td>08540</td>
</tr>
<tr>
<td>Mr. Cavin A. Shick</td>
<td>1</td>
</tr>
<tr>
<td>Advanced Projects Group</td>
<td></td>
</tr>
<tr>
<td>Code 6123</td>
<td></td>
</tr>
<tr>
<td>Naval Research Lab.</td>
<td></td>
</tr>
<tr>
<td>Washington</td>
<td>20375</td>
</tr>
<tr>
<td>Mr. Wendell G. Sykes</td>
<td>1</td>
</tr>
<tr>
<td>Arthur C. Little, Inc.</td>
<td></td>
</tr>
<tr>
<td>Livermore Park</td>
<td></td>
</tr>
<tr>
<td>Cambridge</td>
<td>02140</td>
</tr>
<tr>
<td>Prof. J. R. Thompson</td>
<td>1</td>
</tr>
<tr>
<td>Department of Mathematical Science</td>
<td></td>
</tr>
<tr>
<td>Rice University</td>
<td></td>
</tr>
<tr>
<td>Houston, Texas</td>
<td>77001</td>
</tr>
<tr>
<td>Prof. W. A. Thompsen</td>
<td>1</td>
</tr>
<tr>
<td>Department of Statistics</td>
<td></td>
</tr>
<tr>
<td>University of Missouri</td>
<td></td>
</tr>
<tr>
<td>Columbia, Missouri</td>
<td>65201</td>
</tr>
</tbody>
</table>
## DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>Name</th>
<th>Department</th>
<th>Copies</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRCF. F. A. TILLMAN</td>
<td>DEPT. OF INDUSTRIAL ENGINEERING</td>
<td>1</td>
<td>KANSAS STATE UNIVERSITY, MANHATTAN, KS 66506</td>
</tr>
<tr>
<td>PROF. A. F. VEINOTT</td>
<td>DEPARTMENT OF OPERATIONS RESEARCH</td>
<td>1</td>
<td>STANFORD UNIVERSITY, STANFORD, CALIFORNIA 94305</td>
</tr>
<tr>
<td>DANIEL H. WAGNER</td>
<td>STATION SQUARE ONE</td>
<td>1</td>
<td>FACULTY, PENNSYLVANIA, 19301</td>
</tr>
<tr>
<td>PROF. GRACE WAMBA</td>
<td>DEPT. OF STATISTICS</td>
<td>1</td>
<td>UNIVERSITY OF WISCONSIN, MADISON, WI 53706</td>
</tr>
<tr>
<td>PROF. K. T. WALLENIUS</td>
<td>DEPARTMENT OF MATHEMATICAL SCIENCES</td>
<td>1</td>
<td>CLEMSON UNIVERSITY, CLEMSON, SOUTH CAROLINA 29631</td>
</tr>
<tr>
<td>PROF. BERNARD WIDOSH</td>
<td>STANFORD ELECTRONICS LAB</td>
<td>1</td>
<td>STANFORD UNIVERSITY, STANFORD, CA 94305</td>
</tr>
</tbody>
</table>

---

38
<table>
<thead>
<tr>
<th>DISTRIBUTION LIST</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFFICE OF NAVAL RESEARCH</td>
<td>1</td>
</tr>
<tr>
<td>SAN FRANCISCO AREA OFFICE</td>
<td>760 MARKET STREET</td>
</tr>
<tr>
<td>SAN FRANCISCO CALIFORNIA 94102</td>
<td></td>
</tr>
<tr>
<td>TECHNICAL LIBRARY</td>
<td>1</td>
</tr>
<tr>
<td>NAVAL CREWNADE STATION</td>
<td></td>
</tr>
<tr>
<td>INDIAN HEAD MARYLAND 2064C</td>
<td></td>
</tr>
<tr>
<td>NAVAL SHIP ENGINEERING CENTER</td>
<td>1</td>
</tr>
<tr>
<td>PHILADELPHIA</td>
<td></td>
</tr>
<tr>
<td>DIVISION TECHNICAL LIBRARY</td>
<td>19112</td>
</tr>
<tr>
<td>PHILADELPHIA PENNSYLVANIA</td>
<td></td>
</tr>
<tr>
<td>BUREAU OF NAVAL PERSONNEL</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF THE NAVY</td>
<td></td>
</tr>
<tr>
<td>TECHNICAL LIBRARY</td>
<td>20370</td>
</tr>
<tr>
<td>WASHINGTON D.C.</td>
<td></td>
</tr>
<tr>
<td>PROF. H. ADEEL-NAEED</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF MATHEMATICS</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY OF NORTH CAROLINA</td>
<td>28223</td>
</tr>
<tr>
<td>CHARLOTTE NC</td>
<td></td>
</tr>
<tr>
<td>PROF. T. W. ANDERSON</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td></td>
</tr>
<tr>
<td>STANFORD UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>STANFORD, CALIFORNIA 94305</td>
<td></td>
</tr>
<tr>
<td>PROF. F. J. INSOMBE</td>
<td>1</td>
</tr>
<tr>
<td>DEPARTMENT OF STATISTICS</td>
<td></td>
</tr>
<tr>
<td>YALE UNIVERSITY</td>
<td></td>
</tr>
<tr>
<td>NEW HAVEN</td>
<td>6520</td>
</tr>
<tr>
<td>CONNECTICUT</td>
<td></td>
</tr>
<tr>
<td>PROF. L. A. ARCIA</td>
<td>1</td>
</tr>
<tr>
<td>INSTITUTE OF INDUSTRIAL ADMINISTRATION</td>
<td></td>
</tr>
<tr>
<td>UNION COLLEGE</td>
<td></td>
</tr>
<tr>
<td>SCHENECTADY</td>
<td>1430</td>
</tr>
<tr>
<td>NEW YORK</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td></td>
</tr>
</tbody>
</table>