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THE WANDERING SEARCH

James L. Bagby, Cdr., USN

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The aggressive search for a target believed to be remaining within a specified region is analytically modeled. The probability of detection is expressed, both for the random search path and for any deliberate, non-overlapping path. The results are valid for any distribution of target positions in the region, for any lateral range detection curve, and for any target presence probability, provided that the target is not actively observing and dodging the searcher's path. Both the mobile and the stationary

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20. Targets are considered, and targets having stochastic region-departure times are also considered. All detection expressions are closed-form and readily calculatable. Some points of strategy are made on selection of the search region and selection of the deliberate versus the random tactic.

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2. Among other results, this Research Contribution extends the applicability of the well-known Random Search Model so that it will no longer be viewed as being restricted to a uniform distribution for the target's position. It likewise extends the applicability of the Non-Overlapping Search Model.

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Much of the author's analysis in this paper was undoubtedly boosted by his having studied the two works referenced in the main body; one coordinated by the Operations Analysis Study Group of the U.S. Naval Academy recently and the other authored by Dr. Bernard O. Koopman in 1946.

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INTRODUCTION

A macro-model will be developed for the aggressive search for an item of interest believed to be positioned somewhere within a large region of specified area. Potential applications would include, for instance, one vessel searching for an enemy unit, a rescue search for a vessel or survivors in distress, and even a law enforcement unit's search for a discarded homicide weapon.

This model will permit the target's position to be "distribution free," in the sense that the probability-of-detection results will be valid for any target position distribution as well as for the possibility that the target is not even present in the region to be searched. The results will respond to any level of target mobility, provided that such motion is not guided by watchful and conscious evasion. An extension in the final section of this paper will permit the target to have an uncertain, sudden departure time. The model permits any type of Lateral Range Curve for prevailing search conditions, and if the area under this curve ("sweep width") can be estimated, then the estimated probability of detection for the search may be numerically calculated.

Each of two search tactics will be addressed separately, the purely random search and any deliberate, non-overlapping search. A comparison of these tactics will be made in the Summary section, where also some additional observations of strategy are made. It will be shown that the deliberate, non-overlapping tactic dominates the random tactic for any target position distribution, any level of target mobility, and any lateral range curve. Basic planning elements for a deliberate overlapping tactic will be suggested as well.

Considering the flexibility advertised in the second paragraph above (vis-a-vis target position distribution, target mobility, target departure, target existence, and lateral range curve), the reader may experience surprise when noting the arguments to be far more straightforward than clever. Moreover, the expressions for detection probability will be closed-form and readily calculatable.

In what perspective should the offerings of this paper be placed, considering the rich body of analytical work that has been published during the past 40 years; especially for the large-area search scenario? A complete answer to that question will not be attempted, but the following comments regarding this past work might be helpful to the less indoctrinated reader.

In 1946, a macroscopic model for the random search was presented by Dr. Bernard O. Koopman, who found it expedient and sufficient to assume a uniform, equally-likely target position distribution and a search system certain to detect the target within some "definite range," while sure to miss outside such range (reference 1). The product of that model is the well-known and often-used exponential detection equation, a simple function of relative search effort randomly applied. Unimpeded by the equally-likely target constraint on his random search model, Koopman focused the next chapter of "Search and Screening" upon a very interesting endeavor: the determination of optimal allocation of deliberate-search efforts against a target having any specific position distribution. During that successful pursuit, he allocated variable "search density," using his earlier exponential equation, to each point in the search region. The mathematical sophistication and abstractness of that result and supporting analysis was, however, at least one order of magnitude higher than for his macroscopic model.

Twenty years later, the first edition of reference (2) re-visited the macro, random search model and easily removed the "definite range" stipulation from the exponential detection equation. The equally-likely target constraint, however, remained in place. In the meantime, many other authors probed deeply into the large-area search, ultimately to suggest optimal allocations of deliberate-search efforts and often using the Bayesian technique to periodically exploit no-detection information. As pointed out by James Dobbie's excellent survey of such literature through 1967 (reference 3), papers addressing large-area search considered such things as false target environments, lost contacts, post-detection prosecutions, information-gain objectives, and even "two-sided" strategies for opposing parties (Game Theory). Most of the "one-sided" models considered stationary targets only. More recent work by Alfred Kaufman, however, accepts and uses distributions of conceivable target and searcher paths, assuming a known instantaneous probability density for detection (reference 4). Richardson and Belkin permit uncertain "sweep

widths" thus uncertain lateral range curves, which is most realistic (reference 5). John Pierce illuminates the relationship between the detection objective and the information-gain objective, for an exponential detection equation against a stationary target (reference 6). A recent textbook on Search Theory by Lawrence Stone is a very comprehensive, elaborate, and sophisticated mathematical treatment of large-area search against targets, stationary for the most part (reference 7).

Review of such splendid and sophisticated examples of past work compels this author to re-stress the macroscopic nature of "The Wandering Search." Discovery herein of the "distribution-free" quality of the exponential equation for a random search should enhance some of the more sophisticated, detailed works that have used this equation, by immediately extending their serviceability to non-uniform target distributions. Otherwise, this rather elementary paper should appeal to the undergraduate student of applied probability; and more importantly to the "undergraduate" searcher, whose academic degree, if even scientific, is found most often to be in a deterministic field as opposed to a stochastic one.

THE GENERAL MODEL

The wandering search, as illustrated by figure 1 below, consists of N identical track segments. Each such relatively small segment, of constant length, has a new course that is either randomly selected in every case or deliberately selected. For the former, purely random tactic, the complete search track of total length L will, upon reflection, appear to have "wandered" senselessly, as shown in figure 1. For the deliberate tactic, however, the search track would show planned "wandering" with a careful avoidance of segment overlap.

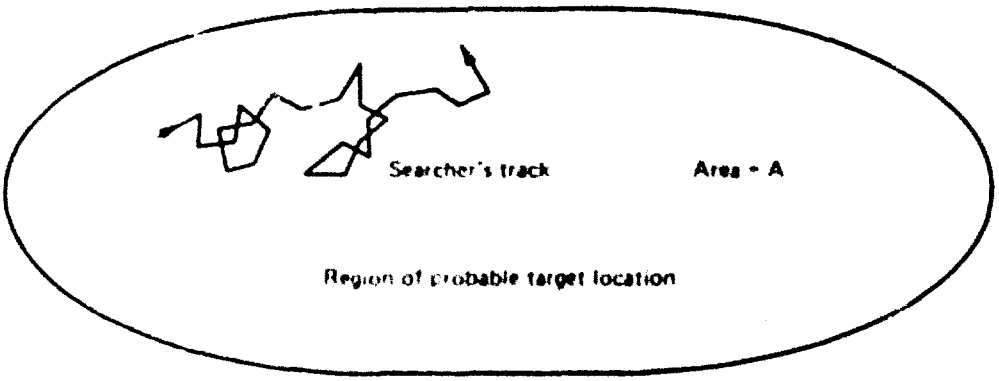

FIG 1 THE WANDERING SEARCH OF A LARGE REGION (RANDOM TACTIC)

The well-known Lateral Range Curve $\bar{P}(x)$, as depicted by figure 2 below, is merely the locus of conditional detection probabilities for various ranges-at-CPA (Closest Point of Approach) x between this searcher and this target under the prevailing environment conditions. P_m denotes the maximum possible detection range for all practical purposes. It turns out that the area under this curve has special significance, and it is given the name "Sweep Width" and the symbol W (reference 1).

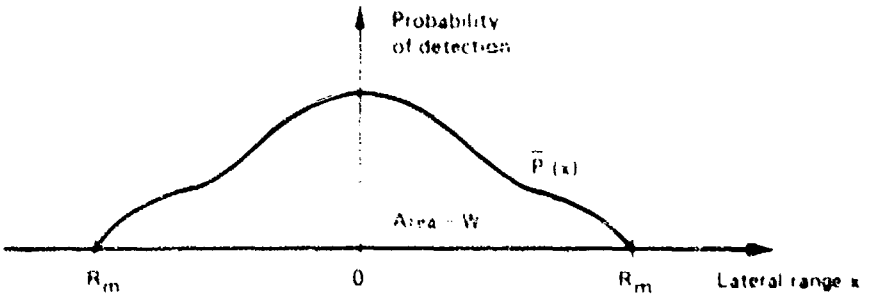

FIG 2 THE LATERAL RANGE CURVE (LRC)

The building block for this model is the typical search segment, and figure 3 below depicts the jth segment, j=1,2,...,N. During each such segment, detection may or may not occur. If the target is not located within the segment when that segment is actually executed, detection is clearly impossible, since its side boundaries are a distance R_m either side of track. On the other hand, if the target is so located, the probability of detection during this segment is approximately $W/(2R_m)$ (reference 2). It is noted that such location across the width of this segment (normally small, relative to the size of the whole region) is quite equally-likely.

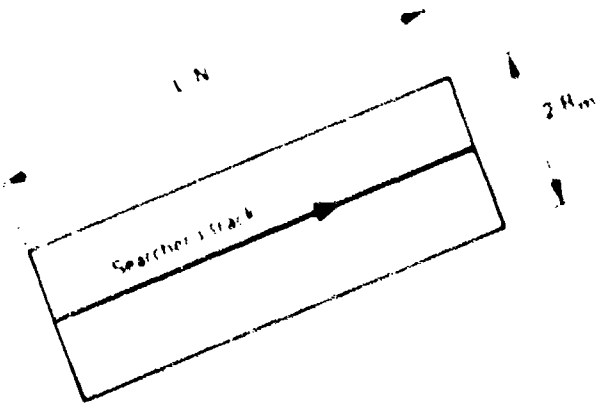

FIG 3 THE TYPICAL, jth SEARCH SEGMENT

The following two identities will be used for the analysis:

$$P[\text{detection}] = P[\bigcup_{j=1}^{N} \text{detection}_j] \tag{1}$$

and alternatively,

$$\begin{aligned} P[\text{detection}] &= 1-P[\underline{\text{no}}\ \text{detection}] \\ &= 1-P[\bigcap_{j=1}^{N} \text{no detection}_j] \\ &= 1-P[\text{no det}_1] \cdot P[\text{no det}_2 | \text{no det}_1] \\ &\quad \cdot P[\text{no det}_3 | \text{no det}_1 \cap \text{no det}_2] \ \ldots \\ &\quad \cdot P[\text{no det}_N | \text{no det}_1 \cap \text{no det}_2 \cap \ldots \\ &\quad \ldots \cap \text{no det}_{N-1}] \\ &= 1 - \prod_{j=1}^{N} P[\underline{\text{no}}\ \text{det}_j | \text{no det so far}], \end{aligned} \tag{2}$$

where det_j and $\underline{\text{no}}\ \text{det}_j$ are events tied exclusively to the jth search segment, $j=1,2,\ldots,N$.

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TACTIC 1: PURELY RANDOM TURNS

Suppose the jth search segment, $j=1,2,\ldots,N$, is equally-likely to lie anywhere within the search region, given merely that no detection has occured so far during the search. If the starting point or first segment is randomly placed (equally-likely), then the above supposition nicely approximates the random-new-course search tactic. Except for certain special conditions to be discussed in Appendix A, previous detection failure has little, if any, practical influence upon the marginal location distribution for the connected jth segment.[1] Knowing the actual location of the (j-1)th segment would indeed severely limit the possible locations of the connected jth segment; but we will design our analysis carefully, so that it will not be based upon knowing anything about the first (j-1) segments except that they each failed to detect the target, wherever they or the target may have been located.

At this point, the reader may wonder why we plan to condition each segment outcome upon "no detection so far." The reason the equation (2) identity on page 6 will be selected is that it will not force us to assume independence among the segments. The original "Random Search Model" (reference 1) used purely unconditional probabilities for its development, thereby imposing independence and leading the author to assume the search segments to be disconnected and unrelated.

Let us now develop an expression for eventual detection by this random tactic. Examination of equation (2) on page 6 reveals that we need to determine P[no detection on jth segment given no detection so far], for $j=1,2,\ldots,N$.

---

[1] By "connected" we mean that the jth search segment starts where the (j-1)th ended. The new course for the jth segment is purely random (i.e., uniform over 0 to 360°).

$P[\underline{no}\ \text{det}_j \mid \text{no det so far}]$

$$= 1 - P[\text{det}_j \mid \text{no det so far}] \qquad (3)$$

Now, detection during the jth segment is subevent of the necessary event that the target be present within the jth segment. Thus, the detection event is the intersection of the detection and target-presence events. Conditioning in every term the condition of no detection so far, we write

$P[\text{det}_j \mid \text{no det so far}]$

$$= P[\text{det}_j \cap \text{tgt in jth seg} \mid \text{no det so far}]$$

$$= P[\text{det}_j \mid \text{tgt in jth seg} \cap \text{no det so far}]$$

$$\cdot P[\text{tgt in jth seg} \mid \text{no det so far}]$$

$$= \frac{\hat{\alpha}}{\Sigma P_m} \cdot P[\text{tgt in jth segment} \mid \text{no det so far}] \qquad (4)$$

Now, let (X,Y) denote the stochastic, unknown coordinates of the target, at the time of the jth segment and let S_x, S_y be the range of possible values for X, Y respectively within the search region. Let $g(x,y)$ be the <u>appropriate</u>[1] PDF for target position <u>at the time of the jth segment</u>. For the time being, we will assume that the target indeed remains within the boundaries of the search region. This assumption will be relaxed later on page 11.

Using the continuous form of the Law of Total Probability, we can write

[1] "Appropriate", meaning the original Probability Density Function $g(x,y)$ for target location, adjusted perfectly for the no-detection-so-far knowledge at the time of the jth segment and for the target's motion, if any, during the first $(j-1)$ segments (using Bayesian techniques).

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$$P[\text{tgt in jth segment} \mid \text{no det so far}]$$

$$= \int_{S_y} \int_{S_x} P[\text{tgt in jth seg} \mid \text{tgt at posit } (x,y) \cap \text{no det so far}] \cdot g_j(x,y)\, dxdy$$

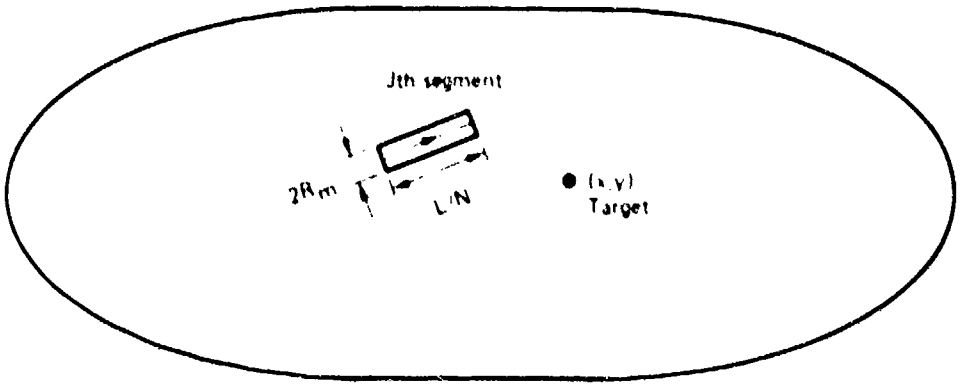

FIG 4 TARGET LOCATION AND PLACEMENT OF jth SEARCH SEGMENT (IN GENERAL)

Now consider the specific conditions of the first factor of the integrand above: the target is now at "time" j located at the specific position (x,y) and the search has failed so far. Recall from the Introduction that the scenario does not have the target actively observing and evading the searcher's path. Recall also our opening supposition that the jth segment is equally-likely to lie anywhere in the search region, given that no detection has occurred so far. Thus, the probability that it will contain any specific target point (x,y) equals the area of the segment divided by the area of the whole region, or $(2R_m \cdot L/N)/A$.[1]

[1] Argument as follows: Consider all those positions for the "initial starboard corner" of the segment which would cause the segment to contain the fixed (x,y). The quantity of such positions is represented by $2R_m \cdot L/N$, while the quantity of all possible "starboard corners" is represented by A. The author concedes that those relatively few target (x,y) positions adjacent to the region's boundary are somewhat less apt to be captured by this random segment (which must be entirely inside the region) than are the overwhelming majority of (x,y) away from the boundary.

Continuing,

$$P[\text{tgt in jth segment} \mid \text{no det so far}]$$

$$= \int_{S_y}\int_{S_x} \frac{2R_m \cdot L/N}{A} \cdot g_j(x,y)\,dxdy$$

$$= \frac{2R_m \cdot L/N}{A} \int_{S_y}\int_{S_x} g_j(x,y)\,dxdy$$

$$= \frac{2R_m \cdot L/N}{A} \tag{5}$$

The iterative process of backward substitutions (equation (5) into equation (4) and equation (4) into equation (3)) yields

$$P[\text{no det}_j \mid \text{no det so far}] = \left(1 - \frac{WL}{NA}\right) .$$

Equation (2) now becomes

$$P[\text{detection}] = 1 - \prod_{j=1}^{N} \left(1 - \frac{WL}{NA}\right)$$

$$= 1 - \left(1 - \frac{WL}{NA}\right)^N \tag{6}$$

Motivation to remove the variable N, if possible, leads to writing equation (6) as

$$P[\text{detection}] = 1-e^{\ln\left(1 - \frac{WL}{NA}\right)^N} = 1-e^{N\ln\left(1 - \frac{WL}{NA}\right)}$$

and noting that since $\frac{WL}{NA} = \frac{WL/N}{A}$ is very small

$$\ln\left(1 - \frac{WL}{NA}\right) \simeq - \frac{WL}{NA} \text{ (reference 2).}$$

Thus,

$$P[\text{detection}] \simeq 1-e^{-WL/A} , \tag{7}$$

where

W = "sweep width" (area under LRC)
L = Total Track Length of Search
A = Area of the Search Region

NOTE: WL/A is often called "Coverage Factor."

DISCUSSION

Equation (7) is valid for this random tactic, regardless of the actual probability distribution for target position, regardless of the targets movements during the search (provided, of course, that the target remains in the search region), and regardless of the Lateral Range Curve. The original "Random Search" model also achieved equation (7), but with the restrictive assumptions that the target position distribution was uniform (equally-likely) throughout the region, that the Lateral Range Curve was of the rare definite range[1] type, and that the search segments were disconnected and unrelated.

EXTENSION FOR DEBATABLE PRESENCE OF THE TARGET

This trivial extension is included for completeness and for some visibility supporting search strategy (see page 22). Suppose the target may or may not be in the search region through the period of the search.

Let a = P[tgt in the search region throughout the search] ≤ 1

Then,

$$P[\text{detection}] = P[\text{detection/tgt in search region}] \cdot P[\text{tgt in search region}]$$

$$\simeq (1-e^{-WL/A}) \cdot a \qquad (8)$$

REMARK

If the target is known to be somewhere in the search region (i.e., $a=1$) and if the searcher could search

[1] The "definite range" type applies if and only if there exists some definite range within which detection is absolutely assured and beyond which detection is impossible.

indefinitely (using, let's say, a constant speed of v), then detection is assured. That is, elapsed-time-until-detection T is a valid random variable having the following probability distribution (from equation 8):

$$F(t) = P[\text{det by time } t] = 1-e^{-WL/A}$$

$$= 1-e^{-Wvt/A} = 1-e^{-(\frac{Wv}{A})t},$$

the well-known <u>exponential</u> distribution having constant detection rate (or tendency) Wv/A and expected time until detection of $1/(Wv/A) = (A/Wv)$.

TACTIC 2: DELIBERATE TURNS

The searcher will select the course for each new segment carefully to avoid any segment overlap (researching previously searched area).[1] Suppose, for the time being, that the target is stationary (this supposition to be relaxed on page 16). The segment events $\{det_j\}_{j=1}^{N}$ are now mutually exclusive; that is, a detection cannot occur during more than one search segment. The equation (1) identity on page 6 will now be fruitful.

$$P[\text{detection}] = P[\bigcup_{j=1}^{N} det_j]$$

$$= \sum_{j=1}^{N} P[det_j], \text{ because of the mutually exclusive property.}$$

$$= \sum_{j=1}^{N} P[det_j | \text{tgt in } j\text{th segment}]$$

$$\cdot P[\text{tgt in } j\text{th seg}]$$

$$= \sum_{j=1}^{N} \frac{W}{2R_m} \cdot P[\text{tgt in } j\text{th seg}]$$

$$= \frac{W}{2R_m} \cdot \sum_{j=1}^{N} P[\text{tgt in } j\text{th seg}]$$

$$P[\text{detection}] = \frac{W}{2R_m} \cdot S \,, \qquad (9)$$

[1] Depending upon the segment's width and the rate-of-turn to each new course, a petite fraction of overlap may, in practice be unavoidable. The searcher may desire to choose a value for R_m where the Lateral Range Curve probability is, for instance, 0.10; thereby truncating the curve at that range.

where

$$\rho = \sum_{j=1}^{N} P[\text{tgt in } j\text{th segment}]$$

$$= P[\text{tgt in the } \underline{\text{subregion}} \text{ covered by the search}]$$

REMARK

1. There is a subtle upper limit on the total track length L, resulting from the searcher's avoidance of overlapping segments while still remaining within the search region:

$$2R_m L \leq A, \quad \text{or} \quad L \leq \frac{A}{2R_m}$$

2. The fact that the P[tgt in the jth segment] varies from segment to segment (for any non-uniform target position distribution) has not blocked us from achieving a concise expression for P[detection]. Note also that for this tactic, the derivation of the expression did not really require the segments to each be of the same length.

3. If the probability distribution for target position could be estimated, then ρ could be estimated for the <u>subregion</u> to be covered. Alternatively, ρ might be assessed subjectively using available intelligence regarding the target.

A SPECIAL CASE

Suppose intelligence reveals only that the target may (with probability 1) be somewhere within a search region of area A and if so, that she is stationary.

With no other information, it is natural to use the uniform, equally-likely distribution for the target's location (contingent upon her presence, of course). According to equation (9), P[detection] = $(W/2R_m)\cdot\rho$, where

ρ = P[tgt in <u>subregion</u> to be covered]. Because the subregion event for target location is a subset of the region event,

$$\beta = P[\text{tgt in subregion} \mid \text{tgt in region}] \cdot P[\text{tgt in region}]$$

$$= \frac{\text{area of subregion}}{\text{area of region}} \cdot \alpha = \frac{2R_m L}{A} \cdot \alpha.$$

Therefore,

$$P[\text{detection}] = \frac{W}{2R_m} \cdot \frac{2R_m L}{A} \cdot \alpha$$

$$= \frac{WL}{A} \cdot \alpha, \quad \text{"coverage factor itself times the likelihood of presence (for the special, equally-likely target position case).} \tag{10}$$

NOTE: Equation (10), like equation (9), requires $L \leq \frac{A}{2R_m}$ for the non-overlapping property to be maintained. Thus we have the following <u>practical maximum</u> probability:

$$P[\text{detection}]_{max} = \frac{W(A/2R_m)}{A} \cdot \alpha = \frac{W}{2R_m} \cdot \alpha,$$

the average height of the lateral range curve times the likelihood of target presence.

REMARK

If for the foregoing special case, the target's presence is <u>assured</u> ($\alpha = 1$), then

$$P[\text{detection}] = \frac{WL}{A} \tag{11}$$

which is the exact result of reference (1) for a non-overlapping search for a stationary target known to be somewhere equally-likely in a region of area A. The practical maximum is $\frac{W}{2R_m}$.

TARGET MOBILITY

Although the level of target movement had no effect upon P[detection] for the earlier random tactic, we have been forced to assume target immobility in the derivation of P[detection] for the deliberate tactic. Let us now examine the effect of target mobility for the deliberate tactic.

Suppose the other extreme to target immobility. That is, suppose hypothetically that the target is so mobile during the search that after the search has begun the target rapidly becomes equally-likely to be anywhere within the region (if still present at all). We have now lost our mutually exclusive property, even though the segments do not overlap.[1]

So, we must start with the equation (2) identity on page 6 as we did for the random tactic. In fact, the derivation starting with equation (3) on page 8 clearly applies to this situation as well. Examine equation (4) on page 8:

$$P[\text{det}_j \text{ no det so far}] = \frac{W}{2R_m}$$

$$\cdot\ P[\text{tgt in } j\text{th segment no det so far}]$$

We are now assuming such extreme target mobility that the target's location is equally-likely to be anywhere in the region.

Thus,

$$P[\text{tgt in } j\text{th segment no det so far}] = \frac{2R_m \cdot L/N}{A},$$

which is identical to equation (5) on page 10, although achieved via a different reason. So the subsequent work on page 10 for the random tactic applies here,

[1] If the target has any mobility, then there is some chance that she will be vulnerable to detection during some two (or more) search segments k and i. Thus $P[\text{det}_k \cap \text{det}_i] > 0$, making it invalid to assert that the events $\{\text{det}_j\}$ must be mutually exclusive.

yielding the following results for the deliberate tactic against a target with unreasonably high (i.e., unlimited) mobility:

$$P[\text{detection}] \simeq 1-e^{-WL/A}, \text{ if the target is indeed present} \quad (7)$$

$$P[\text{detection}] \simeq (1-e^{-WL/A}) \cdot \lambda, \text{ if the target's presence has probability } \lambda. \quad (8)$$

REMARK

A suggested method of handling moderate target mobility for this deliberate tactic will be presented in the summary on page 18.

SUMMARY AND SOME STRATEGY

THE RANDOM TACTIC

The wandering search that results from a tactic of purely random turns yields

$$P[\text{detection}] \simeq (1-e^{-WL/A}) \cdot \iota, \tag{8}$$

where

W = sweep width
L = total track length
A = area of search region
ι = P[tgt in search region]

for any lateral range curve, any level of target mobility, and any target position distribution.

THE DELIBERATE TACTIC

The systematic search that results from making deliberate turns to avoid re-searching any previously-searched area yields, for the stationary target case,

$$P[\text{detection}] = \frac{W}{2R_m} \cdot \rho, \tag{9}$$

where

W = sweep width
R_m = max detection range
ρ = P[tgt in subregion covered],

and where total track length $\leq \frac{A}{2R_m}$ (for execution feasibility);

This result applies for any lateral range curve and any target position distribution.

REMARKS

1. If, against this deliberate tactic, the unsuspecting target has some mobility within the search region,

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the probability of detection should lie somewhere between the (upper) value computed using equation (9) and a (lower) value computed using equation (8) (see pages 16 and 17 for unlimited target mobility). An ensuing argument under Strategy will show that for any given feasible track length L, equation (9) yields a larger numerical value than equation (8). At this time we should note that an alerted target who can keep track of the searcher and who does have enough quick mobility to remain well clear of the searcher will, with proper execution, enjoy a zero probability of detection.

2. If, as a special case, the target position distribution is assessed to be uniform, we get for the deliberate tactic

$$P[\text{detection}] = (WL/A) \cdot a, \quad \text{where} \tag{10}$$

$$a = P[\text{tgt in search region}]$$

STRATEGY

It seems that any sensibly-applied deliberate tactic would yield a detection probability greater than that for the random tactic, against targets not guided by conscious and watchful evasion. Toward that end, let us first show that indeed equation (9) will yield larger probability than equation (8), for any feasible investment in search time or track length L:

> Obviously, the deliberate searcher will apply his search within the region in such a way as to enhance δ, the P[tgt in subregion covered]. That is, he will place his "subregion covered" where the target is perceived to have elevated position probability. The worst case, in terms of yield for such ingenuity against this stationary target, exists when the target's position distribution is uniform, thereby precluding any elevated position probability for exploitation. Yet even for this worst case, P[detection] equals $(WL/A)\cdot a$ according to special case equation (10), which is itself greater than equation (8) for the random tactic: $(1-e^{-WL/A})\cdot a$. Thus, the worst case of equation (9) numerically exceeds equation (8).[1]

[1] It is, of course, a simple exercise in analytic geometry to show that $x>1-e^{-x}$, by analyzing $h(x) = x-(1-e^{-x})$.

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Figure 5 illustrates the relationship between equations (9): $(W/2R_m)\cdot\delta$ and equation (8): $(1-e^{-WL/A})\cdot a$. The family of equation (9) curves represents the exploitation of a family of non-uniform target position distributions, the most "bowed" curve being associated with the most peaked distribution for this stationary target. The linear "floor" for this family of curves is the worst case equation (10): $(WL/A)\cdot a$. For any given $(W/2R_m)\cdot\delta$ curve, δ is a function of subregion size thus of track length L. When L reaches its "saturated" maximum, the so-called subregion becomes the whole region, causing δ to equal a.

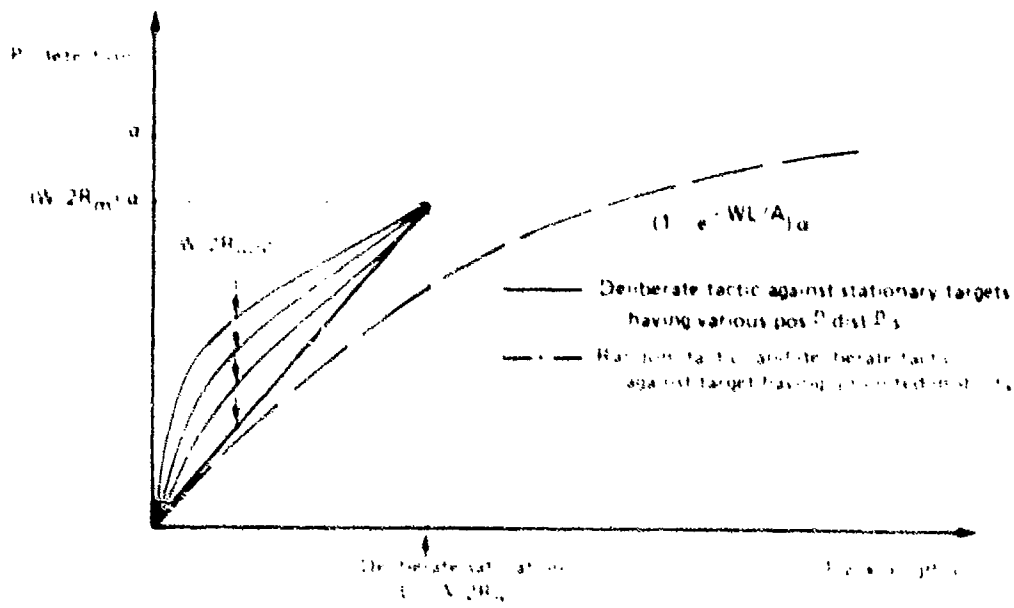

FIG 5 TACTIC COMPARISON

Now let us seriously consider <u>target mobility</u>, which we found to affect P[detection] when the deliberate tactic is used:

> The deliberate tactic against a stationary target yields detection probability $(W/2R_m)\cdot\delta \geq (WL/A)\cdot a$.
>
> The deliberate tactic against a target with <u>un</u>reasonable mobility yields the even lesser

$(1-e^{-WL/A})\cdot a$. The deliberate tactic against a target with reasonable mobility should yield a probability somewhere between these two extremes; and we must note that the lower extreme also happens to be the invariant yield for the random tactic.

Thus, dominance is indicated for the deliberate tactic over the random tactic for all feasible "unsaturated" track lengths regardless of (unsuspecting) target mobility and regardless of the applicable target position distribution.

Suppose the searcher has enough track length L to more than "saturate" the region using the preferred deliberate tactic. If detection does not occur after a complete ($L_1 = A/2R_m$) deliberate search, which tactic should be used for the remaining time or track length available? The answer lies in the realization that the searcher is simply faced with the same situation as he was originally, except that the target's presence probability, a, is now smaller due to a Bayesian adjustment for failure to detect. The deliberate, non-overlapping tactic would dominate the random tactic for this next phase as well, because both tactics are facing the identical adjusted value for a and the same, remaining track length available.

In some special cases, there may be a tactic even superior to the deliberate, non-overlapping search; namely a deliberate overlapping tactic that purposely re-searches segments still having peaked likelihood of target position (even after no success during the first execution of such segments). Of course, such a-posteriori position likelihoods would not remain high enough to justify this, unless the potential ($W/2R_m$) for detecting a target present in such a segment were quite low.[1] Although a detailed scheme for the optimal allocation of deliberate, overlapping effort is outside the intentions and scope of this paper, an outline of suggested analytical process is sketched as follows:

[1] A high $W/2R_m$ potential would probably force the target presence likelihood to become too depressed, after an unsuccessful execution of the segment, to justify a re-execution.

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1. Decide how often, during the search that it will be practical and desirable to revise the target's position distribution. Let ℓ denote the track length to be executed during each period between such re-assessments.

2. Place the first subregion (of area $2R_m\ell$) so as to maximize β, the probability of target's presence in the subregion. Search this subregion completely using the deliberate non-overlapping tactic, and if unsuccessful, execute step 3.

3. Make a rough, Bayesian adjustment to the target's a-priori position distribution, using the no-detection knowledge from this (most recent) failure. Place the next subregion so as to again maximize β, considering the new, a-posteriori target distribution. Continue, sequentially, the above re-assessments and re-executions of the "best" subregion of area $2R_m\ell$, until either the target is detected or the total available track length L is consumed.

A final point of strategy is noteworthy. The searcher may legitimately be able to enhance λ, thus P[detection] with a careful specification of the whole search region. Indeed, using available pre-search intelligence, such efforts are as commonplace as they are obvious. Merely increasing the size (A) of the region, however, will have mixed effects upon P[detection]. Although the factor λ might be increased by an expansion of A, the factors WL/A and $(1-e^{-WL/A})$ will be decreased, for any fixed investment in track length L. The net effect upon P[detection] would be assessed only through careful analysis centered upon the product of these factors.

STRATEGY SUMMARY

Against a target whose motion is not guided by conscious and watchful evasion, the deliberate, non-overlapping tactic is clearly superior to the random tactic; regardless of the assessed position distribution for the target, regardless of the applicable lateral range curve, and regardless of the target's level of mobility. The exponential, random search formula serves merely as an interesting quantity to see how poorly (vis-a-vis success probability) the searcher could do, if he were to be so random and blind in his prosecution.

Best use of the deliberate tactic demands one obvious and easy piece of planning and, if feasible, an enhancement which is more complicated. The obvious planning consideration is to "place" the subregion-to-be-covered (without overlap) where the assessed target position distribution is elevated. The enhancement involves a sequential planning process, whereby the target's position distribution is updated after convenient periods of failure. After each such Bayesian update, the next subregion to be covered is placed where the aposteriori distribution is elevated. In many cases, portions of the regions previously searched might end up being re-searched. Thus, while each subregion execution is a deliberate, non-overlapping search, the entire search might be called a deliberate, "overlapping" search.

UNKNOWN TARGET DEPARTURE TIME

Suppose the target may depart the search region before the searcher has the opportunity to execute all N search segments. What is the probability of detection for this situation?

Obviously, if we know exactly when the target will leave, we need only to recompute the number of search segments (N') executable before target departure time, and our previous work will readily apply (using N' vice N). The interesting case is the one characterized by uncertainty. Let's see what we can do.

Let the discrete random variable D denote the number of search segments executable before target departure time. For the time being we will condition our work on the presumption that the target is at least present in the search region when the search commences. Later, we will relax that assumption in the usual way, using π, the probability of original presence.

Let p(d) be the estimated[1] probability mass function for D, noting that the sample space of possible outcomes is 0,1,2,...,N, where N is, as before, the total number of segments the searcher desires to execute. Using the law of total probability,

$$P[\text{detection}] = \sum_{d=0}^{N} P[\text{detection/given } d] \cdot p(d) \qquad (12)$$

FOR THE RANDOM TACTIC

$$P[\text{detection}] \simeq \sum_{d=0}^{N} (1-e^{-WL/A}) \cdot p(d)$$

One might be tempted to bring $(1-e^{-WL/A})$ outside the summation, since d is not shown in that expression. Yet in fact, track length L is really a function of d.

[1] "Estimated" based upon a subjective evaluation of available target intelligence.

Although total planned track length is $(L/N)\cdot N = L$, total executable track length L' (with the target present) is really $(L/N)\cdot d = L(d/N)$. Therefore,

$$P[\text{detection}] \simeq \begin{cases} \sum_{d=0}^{N} [1-e^{-WL(d/N)/A}]\cdot p(d), & \text{if the target is originally present} \\ \alpha \cdot \sum_{d=0}^{N} [1-e^{-WL(d/N)/A}]\cdot p(d), & \text{if the target's original presence has probability } \alpha. \end{cases} \tag{13}$$

where

L = total planned track length
N = number of segments planned
d = number of segments executed before target departure time
$p(d)$ = estimated probability mass function for D.

REMARK

This also applies for the deliberate tactic against a target with unreasonably high mobility.

FOR THE DELIBERATE TACTIC

If the target is stationary until departure time D then departs suddenly (such as survivors who "disappear" after D rescue-search segments),

$$P[\text{detection}] = \sum_{d=0}^{N} \frac{W}{2R_m} \cdot \beta \cdot p(d),$$

where β = P[tgt in the subregion covered by the search]

Now β is a function of d, because β is a function of the the size of the subregion covered before the target departs $(2R_m[L/N]d)$. As a reminder of β's dependence upon d, we should write

$$P[\text{detection}] = \sum_{d=0}^{N} \frac{W}{2R_m} \cdot \beta(d) \cdot p(d)$$

$$= \frac{W}{2R_m} \cdot \sum_{d=0}^{N} \beta(d) \cdot p(d) \qquad (14)$$

REMARKS

1. As usual for the deliberate tactic, total track length $\leq \frac{A}{2R_m}$ for feasibility.

2. An alternative writing of equation (14) is

$$P[\text{detection}] = \frac{W}{2R_m} \cdot E[\beta]$$

where $E[\beta]$, the expected value of β, $= \sum_{d=0}^{N} \beta(d) \cdot p(d)$

3. If, as a special case, the target is equally-likely to be anywhere in the search region and is definitely present,

$\beta(d) = \frac{2R_m(L/N)d}{A}$. Thus from equation (14),

$$P[\text{detection}] = \frac{W}{2R_m} \cdot \sum_{d=0}^{N} \frac{2R_m(L/N)d}{A} \cdot p(d)$$

$$= \frac{WL}{NA} \cdot \sum_{d=0}^{N} d \cdot p(d)$$

$$= \frac{WL}{NA} \cdot E[D] = \frac{WL}{A} \cdot \frac{E[D]}{N}$$

$$= \frac{WL}{A} \cdot E[S], \qquad (15)$$

where $E[S]$ is the expected proportion of the planned search that the target will be present for.

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APPENDIX A

CONNECTED SEGMENTS FOR THE RANDOM SEARCH

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CONNECTED SEGMENTS FOR THE RANDOM SEARCH

The controlling supposition in the analysis for the Random Tactic was: the jth search segment is equally-likely to lie anywhere in the search region, given that no detection has occurred so far ($j=1,2,\ldots,N$), if merely the first segment is placed on an equally-likely basis. Let us discuss the practical implications of this supposition for the following special situation.

If the target has been relatively stationary at (x,y) <u>and</u> if the likelihood ($W/2R_m$) of detecting a target swept by any segment is large, then to know that all previous segments have failed will slightly disturb, in an a-posteriori sense, the marginal location distribution of the (j-1)th segment. Although the a-priori marginal distribution of the (j-1)th segment is made uniform by the randomness of the first segment, its a-posteriori distribution should have slightly reduced likelihood near the point (x,y), after detection failure under this special situation.

Now, if the search segments are <u>disconnected</u>, the disturbed a-posteriori distribution for the (j-1)th segment will <u>not</u> affect the uniform marginal location distribution for the disconnected jth segment.

If the actual search segments are <u>connected</u>, however, the jth segment's location distribution will also have some reduced likelihood near the point (x,y), causing $(2R_m \cdot L/N)/A$ to be only a close upper bound type approximation for the first factor of the integrand at the top of page 9.

$$P[\text{tgt in jth seg} \mid \text{tgt at posit } (x,y) \text{ no det so far}] < \frac{2R_m \cdot L/N}{A}$$

This, in turn, will make the equation (7) result an upper bound approximation for the probability of detection, for this special combination of target immobility, large $W/2R_m$, and connected segments.

In summary, then, if the target is mobile, or if $W/2R_m$ is not large, or if the segments are disconnected,

$$P[\text{detection}] \simeq 1-e^{-WL/A} \text{ for the random search.}$$

On the other hand, if the target is relatively stationary and $W/2R_m$ is close to unity, then

$$P[\text{detection}] \lesssim 1-e^{-WL/A} \text{ for the random search with } \underline{\text{connected}} \text{ segments}$$