MECHANICAL DESIGN REQUIREMENTS FOR CLOSED COMBUSTION BOXES

by

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FOREWORD

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A theoretical analysis is made of the mechanical effect of a detonation occurring in a closed bomb during routine propellant testing in the pressure range for which convective burning is expected to develop. A hitherto unreported phenomenon was found, consisting of an amplification of the peak reflected detonation impulse by products of combustion and leading to a maximum impulse as a result of a detonation occurring during the first third of burning. Graphical results of the analysis are presented for a standard combustion bomb using an experimental propellant. Limitations imposed by instrumentation, especially pressure gauges, appear to be more severe than those imposed by mechanical construction over the range of experimental parameters of interest.

ABSTRACT

A theoretical analysis is made of the mechanical effect of a detonation occurring in a closed bomb during routine propellant testing in the pressure range for which convective burning is expected to develop. A hitherto unreported phenomenon was found, consisting of an amplification of the peak reflected detonation impulse by products of combustion and leading to a maximum impulse as a result of a detonation occurring during the first third of burning. Graphical results of the analysis are presented for a standard combustion bomb using an experimental propellant. Limitations imposed by instrumentation, especially pressure gauges, appear to be more severe than those imposed by mechanical construction over the range of experimental parameters of interest.
NOMENCLATURE

$\alpha$  Acoustical velocity in gas
$C$  Acoustical velocity in bomb material
$C_{vp}, C_{vo}$  Isochoric specific heat (propellant and fill gas)
$C_i$  Constants in detonation equations
  $C_1 = 3 \times 454/(4nY)^{1/3} \approx 30.48$
  $C_2 = T_r u_o / T_o u_r P_r$
  $C_3 = R_T u_r / u_o P_r$
  $C_4 = (T_v - T_o) / T_o$
  $C_5 = \mu_o / \rho p u_o$
  $C_6 = \mu C_{vo} / R_T C_{vp}$
$\Delta e_p$  Specific energy of reaction of test propellant
$f$  Weight fraction of test charge burned
$I$  Reflected impulse per unit area in interaction between detonation wave and bomb wall
$k$  Constant in Eq. (2) for resonant period of bomb
$m$  Mass
$p$  Pressure
$R$  Radius
$R_u$  Universal gas constant = 8.316 MPa - cm$^3$/mol·°K
$s_{1,2,3}$  Tensile stresses (defined in text)
$s_g$  Shear stress
$T$  Temperature
$t$  Time
$U$  Velocity
$V$  Volume
$Y$  Yield factor to give TNT equivalent
$z$  Scaled distance
Symbols

\( \gamma \)  
Ratio of specific heats

\( \delta \)  
Loading density of propellant into bomb \( \delta = \frac{m_p}{V_b} \)

\( \mu \)  
Molecular weight

\( \rho \)  
Density

\( \sigma_{s,y} \)  
Yield stress, shear

\( \sigma_{t,y} \)  
Yield stress, tension and compression

\( \tau \)  
Wall thickness

Subscripts

\( a \)  
Ambient, just prior to detonation

\( b \)  
Bomb

\( c \)  
Combustion conditions (maximum pressure)

\( cr \)  
Critical conditions

\( d \)  
Detonation conditions

\( e \)  
Effective

\( f \)  
Expressed in feet

\( i \)  
Inside

\( lb \)  
Expressed in pounds

\( m \)  
Mean value

\( \text{max} \)  
Maximum value

\( o \)  
Outside (radius); with no combustion (impulse); otherwise denotes fill gas conditions

\( P \)  
Resonance time

\( p \)  
Propellant

\( r \)  
Reference conditions

\( s \)  
Scaled value

\( v \)  
Isochoric

\( l \)  
Corresponding to maximum \( I_d \)
INTRODUCTION

The closed combustion bomb is regarded as a valuable tool for studying deflagration properties of solid propellants used in guns and rockets. It is desired to extend the pressure range of such studies to provide data on anomalous combustion resulting either from changes in the basic mechanism governing the conductive regression rate, or from development of burning in physical grain defects — the so-called convective burning mode. Tests would be carried out over a range of initial pressures and loading densities in order to achieve control over pressure range and pressurization rate. There is no assurance that anomalous burning, considered as the first stage of transition from deflagration to detonation (DDT), can be limited to deflagration under test conditions. Therefore, it is necessary to consider the design criteria for containing a detonation if it should occur during a routine deflagration study. This report presents principles and an illustrative example related to determination of requirements for containing peak deflagration pressures and impulsive detonation loadings for closed bombs over a range of design and operating parameters. The study is limited to spherical bombs, for which the theory is better established than for other shapes. Results are based on typical data and are intended to indicate trends rather than establish a final design, which would require additional calculations relevant to actual construction materials and propellants.

The purpose of this study is to present conservative design principles in order to avoid destroying or damaging material facilities. Under no circumstances should caution be lessened when human safety is involved; remote operations are always advised when high explosives are concerned.

DESIGN PRINCIPLES

Typical schematic pressure-time histories for detonation and deflagration in a closed bomb are shown in Figure 1. The pressure at time A is the initial pre-pressure provided for the test. Normal deflagration proceeds along the line ABDE, the pressure rising in a nearly spatially uniform manner from $p_A$ to $p_E$. The portion of the curve BCD indicates
the effect of a detonation occurring at time $t_B$ after an initial period of normal burning which may even approach zero. Because of the propagative nature of the detonation, the pressure-time history shown as BCD would vary, depending on the radial distance from the detonating source. It is convenient to consider three regions surrounding the detonation; detonations at standard atmospheric conditions these are: (1) the region of direct explosion effects, extending from the center of the explosion to a radial distance of 1.6 charge radii, and characterized by peak pressures varying from several hundred kilobars down to several hundred bars; (2) the region of composite blast (involving products and ambient gas), extending outward to a distance of approximately 16 charge radii and peak pressure of 175 psia; and (3) the region of air shock beyond 16 charge radii in which the shock pressure eventually decays with increasing distance to a simple sound wave. It should be noted that in an unconstrained environment, the line AE would be horizontal.

A tractable analysis is attainable only for simple geometries. The simplest is the sphere with a centrally situated spherical charge so that the expanding shock reaches all portions of the wall simultaneously without the shock reinforcement afforded by angular configurations. Departures from this ideal configuration have not been treated comprehensively by any quantitative analysis and would result in more severe...
requirements for containment; it is estimated that at an additional safety factor of 1.5 would be needed for a cylindrical bomb with an L/D ratio of unity.

In order to avoid the extremely high explosion pressures of region one, loading densities are limited to values below 0.4 g/cm$^3$ and associated static pressure rises of approximately 90 kpsi. The final pressure $p_E$ is readily calculated from thermochemical and gas law considerations; standard unfired pressure vessel design principles suffice for estimating static pressure mechanical requirements. Dynamic requirements depend on determination and analysis of the curve ABCDE appropriate to the bomb radius. The two following sections of this report will consider the dynamic and long range static aspects of detonation in a closed bomb. Dynamic or detonation effects will be discussed first because the static pressure conditions, existing after decay of the detonation transient, can be obtained as a limiting case of one of the equations required to determine detonation conditions. Except for different heat loss effects and possible non-equilibrium chemistry accompanying detonation, the final static pressure is independent of the occurrence of detonation.

DETONATION EFFECTS

While internal pressure alone dictates the design of a closed bomb to contain static loads, the situation is entirely different for the dynamic loading presented to the bomb wall by a detonation shock. It has been found that the effect of a shock wave depends primarily upon the impulse per unit area (integral of the pressure-time curve) and its time of delivery in the interaction between shock and wall. When a critical impulse is delivered to the bomb wall within a critical time, it may be expected that the bomb will suffer unacceptable damage. Four quantities must therefore be determined in order to estimate the response of the bomb to an internal detonation: (1) actual impulse, (2) actual action time, (3) critical impulse, and (4) critical action time. Items (1) and (2) represent detonation parameters which depend on the size and energy of the detonating charge, the distance of the bomb wall from the charge, and the nature of the intervening medium just prior to detonation. The critical parameters, items (3) and (4), depend upon material properties, especially under conditions of dynamic loading, and upon bomb geometry.

Detonation Parameters

Items (1) and (2), mentioned above, are the detonation parameters which are needed to determine the ability of a closed bomb to withstand an internal detonation. A third quantity, the peak reflected pressure, will also be calculated as an indication of the requirement of instrumentation to withstand short duration pulses of extremely high magnitudes compared to the static pressure capabilities.
The bases of calculation of detonation parameters are model experiments and scaling laws derived from dimensional analysis. Figure 2, adapted from Cohen, shows typical relationships between scaled parameters pertinent to the problem at hand. A discussion of the rationale of scaling laws is contained in reports by Kinney and Baker, while the use of alternate sets of parameters (leading to similar results) is covered by Rinehart and Pearson. It is important to note that, usually, scaled parameters are not dimensionless groups, but, as in Figure 2, have dimensions determined by the units in which the model experiment was conducted. While it would be possible to convert the number scales on the coordinate axes to agree with the SI, c.g.s., or some other standard system rather than the English, it is more convenient to convert the quantities with which the figure is entered from c.g.s. to English.

The scaled parameters of Figure 2 contain four scale factors (see the Appendix for details): acoustic velocity ratio $a_a/a_r$, density ratio $\rho_a/\rho_r$, pressure ratio $p_a/p_r$, and charge weight $m_p$. The last is also a ratio, but since the reference weight in Figure 2 is 1 lb of TNT, the ratio is numerically equal to the actual weight in pounds.

It is convenient for computational purposes to express the scaled parameters in terms of design parameters of the bomb, namely, radius, loading density, and fraction of propellant mass burned, as well as other constants characterizing the initial fill gas and propellant combustion products. An inert fill gas, such as nitrogen, will be assumed, thereby eliminating additional chemical heating from interactions of fill gas and propellant products. Details of this transformation are shown in the Appendix. The ambient conditions, denoted by subscript $a$, refer to the conditions of the bomb contents at the instant of detonation. They are a function not only of the fill gas parameters, but of the fraction of the propellant charge burned at the time of

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FIGURE 2. Scaled Reflected Impulse, Impulse Duration, and Reflected Pressure as Functions of Scaled Distance. (See the Appendix for definitions of scaled quantities.)
detonation. This latter effect distinguishes the current study from available calculations neglecting detonation augmentation resulting from partial combustion.

A computer program, incorporating Figure 2 and the expressions developed in the Appendix, was used to calculate $I_d$, $P_d$, and $t_d$ as functions of $f(0-1)$, $\delta(0.005-0.3g/cm^3)$, and $p_o(0.1-30$ MPa). Propellant gas properties were obtained by running the NWC propellant evaluation program (PEP) for ALTU-16, an experimental propellant considered for the actual tests; the fill gas was assumed to be nitrogen; and the reference conditions applicable to Figure 2 are air at one atmosphere and 70°F (294 K). A bomb ID of 5 inches was used, corresponding to a commercially available bomb. A summary of the constants used is provided in Table 1. As mentioned before, impulse is one of two important detonation parameters; a noteworthy result of the calculations is the existence of a maximum impulse in the $I_d$ vs $f$ relationship as a result of competing effects of charge weight and ambient gas conditions. This effect prevails over the range of parameters investigated and is shown, for illustrative purposes, in Figure 3 for specified values of loading density ($\delta$) and fill gas pressure ($p_o$). The maximum $I_d$ and corresponding value of $f$ vary with $\delta$ and $p_o$; maximum $I_d$ is shown as a function of $p_o$ and $\delta$ in Figure 4 over the range of values investigated. Also indicated (for $\delta$ = 0.005 and 0.3) are typical impulse values calculated by assuming no combustion, i.e., the entire charge detonates at the initial fill gas pressure. It is seen that the trends as a function of fill gas pressure are opposite for the two methods of calculation. If impulse from a detonation under initial conditions (no combustion, $f=0$) is compared with the maximum impulse ($f$ ranging from 0-0.3 depending on parameters), an impulse amplification factor ($I_d/I_{d0}$) is observed which ranges up to nine. Figure 5 is a plot of this factor as a function of $\delta$ and $p_o$ over the range of parameters studied. Since the instant of DDT is not yet predictable, one should base the detonation capabilities of the bomb on maximum impulse.

The second important detonation parameter is the time duration of the reflected impulse, shown in Figure 6 as a function of loading density and fill gas pressure. The relevance of $t_d$ will become clear in a later section.

Although peak detonation pressure is not included among the design requirements for the bomb, it is important in assessing requirements and capabilities for instrumentation, especially pressure transducers. Like impulse, detonation pressure has a maximum when considered as a function of $f$; however, maximum impulse usually results from a detonation which occurs earlier ($f=0-0.3$) in burning than does maximum detonation pressure ($f=0.6-0.7$). Figure 7 presents maximum calculated detonation pressures vs. initial fill pressure and propellant loading density over the range of parameters of this study.
TABLE 1. Physical Properties and Conditions Used in Calculations.

<table>
<thead>
<tr>
<th>Reference conditions (air)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, $P_r$</td>
<td>0.10136 MPa</td>
</tr>
<tr>
<td>Temperature, $T_r$</td>
<td>294 K (70°F)</td>
</tr>
<tr>
<td>Molecular weight, $\mu_r$</td>
<td>29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, $p_0$</td>
<td>Variable</td>
</tr>
<tr>
<td>Temperature, $T_o$</td>
<td>240 K (70°F)</td>
</tr>
<tr>
<td>Molecular weight ($N_2$), $\mu_o$</td>
<td>28</td>
</tr>
<tr>
<td>Specific heat, $C_{vo}$</td>
<td>0.1786 cal/g K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Propellant gas (ALTU-16)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochoric flame temperature, $T_{vp}$</td>
<td>4295 K</td>
</tr>
<tr>
<td>Molecular weight, $\mu_{vp}$</td>
<td>19.73</td>
</tr>
<tr>
<td>Specific heat, $C_{vp}$</td>
<td>0.503 cal/g K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solid propellant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho_p$</td>
<td>1.75 g/cm³</td>
</tr>
<tr>
<td>Yield factor</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The ambient pressure at all burnt (f=1) is the pressure which would be generated in the absence of detonation and would also be the pressure after detonation transients decayed, assuming similar heat losses and products of reaction in the two cases. We consider this pressure now because it is the natural limit of no detonation. Figure 8 shows the static, long term, combustion pressure as a function of $\delta$ and $p_0$. The effect of $p_0$ is mainly due to dissimilarities in properties between the fill gas and propellant combustion products. The combustion pressure is used to determine static design as will be presented in a subsequent section.

This concludes the discussion on calculation of detonation parameters. The next section considers the determination of critical values of impulse and time, necessary to the estimation of the capabilities of the bomb under detonation loading.
FIGURE 3. Relation of Reflected Impulse in a Closed Bomb to Fraction of Propellant Burned, Loading Density, and Fill Pressure. (ALTU-16 propellant, 5-inch ID spherical bomb.)
FIGURE 4. Variation in Reflected Impulse With Loading Density and Fill Pressure for Closed Bomb. (ALTU-16 propellant, 5-inch spherical bomb.)
FIGURE 5. Impulse Amplification Factor for Detonation in a Closed Bomb (ALTU-16 Propellant).

FIGURE 6. Impulse Duration Corresponding to Maximum Impulse for Detonation in Closed Bomb. (ALTU-16 propellant, 5-inch ID spherical bomb.)
FIGURE 7. Maximum Detonation Pressure in a Closed Bomb (ALTU-16 Propellant).
Critical Parameters

In order for a detonation to cause failure, the reflected impulse and its time duration must bear a certain relationship to critical values of impulse and time which depend upon material and structural details. The criterion is: impulse must exceed the critical value and must be delivered in a time not exceeding the critical time. The principles of calculation of the critical values are presented in the following sections.

Critical Impulse. The critical impulse is related to the critical particle velocity of the resisting structure, which is actually that velocity to which one portion of a structure must be accelerated relative to another in order that failure may occur. This velocity (\(U_{cr}\)) for steel is around 200 ft/s and is related to critical impulse by the expression

\[\text{Critical Impulse} \propto U_{cr}\]

\[ I_{cr} = \rho_b \tau U_{cr} \]  

where \( \rho_b \) is the density of the metal in slugs/ft\(^3\) and \( \tau \) is the thickness of the resisting medium in feet. For steel, a good value for \( I_{cr} \) is 1750 psi-ms/in., which means that an impulse of 1750 psi-ms, applied to a 1-inch-thick steel plate, will cause failure, if the time duration of the pulse does not exceed a critical time.

**Critical Time.** The concept of critical impulse within a critical time is discussed by Kinney and Sewell. Both the results of detailed analysis and physical reasoning suggest a critical time equal to one-quarter of the natural resonance period of the structure. The physical argument is that maximum displacement occurs if the impulse is delivered during the interval of increasing displacement, i.e., the first quarter period. For a thick spherical shell, loaded symmetrically, the fundamental frequency is given by Seide. The corresponding quarter period is given by

\[ \frac{t_p}{4} = \frac{kR_m}{C} \]  

where \( R_m \) is the mean radius, \( C \) is the velocity of sound in the structural material, and \( k \) is a constant depending on Poisson's ratio and thickness. Considering the accuracy of the other available data, it is sufficient to use \( k = 0.75 \). The calculated quarter periods will then be in error by no more than 5% for steel with a Poisson ratio of 0.33 regardless of wall thickness.

Knowing the critical impulse and time, we can now determine the effective impulse. If impulse action time, \( t_d \), is less than or equal to a quarter period, all the impulse is delivered within the required critical time and the effective impulse is equal to the actual impulse. If the action time is greater than a quarter period, only a portion of the actual impulse is delivered within the required critical time. The calculation of effective impulse is then determined by assuming the pressure time curve to decay linearly (adequate accuracy compared to other assumptions) so that

\[ \frac{t_c}{t_d} = \frac{t_{cr}}{t_{dl}} \left( 2 - \frac{t_{cr}}{t_{dl}} \right) ; \frac{t_{cr}}{t_{dl}} < 1 \]  

\[ t_c = t_{dl} \frac{t_{cr}}{t_{dl}} > 1 \]  

Having now completed the discussion of detonation principles, we turn next to the relatively simpler matter of requirements imposed by static pressure in a spherical pressure vessel.

STATIC PRESSURE CAPACITY OF THICK WALLED SPHERES

The pressure capability of a thick walled sphere is presented in numerous references based on various criteria which are still subject to controversy. The thick walled vessel is complicated by the non-uniform distribution of stresses across the wall and associated combined stresses. We have chosen the following formulas as a basis for design:

\[ s_{\text{max}_1} = s_{\text{max}_2} = p \frac{R_o^3 + 2R_i^3}{2(R_o^3 - R_i^3)} \]  
\[ s_{\text{max}_3} = p \]  
\[ s_{\text{max}} = p \frac{3R_o^3}{4(R_o^3 - R_i^3)} \]  

The quantities \( s_1, s_2, \) and \( s_3 \) are the meridional wall stress, hoop wall stress, and radial wall stress, while \( s_8 \) is the shear stress. All four stresses are maximum at the inner radius, \( R_i \). Important parameters are the working strength in tension (assumed equal to compression) and shear (usually taken to be 0.6 of tensile). Based on the foregoing, shear stress is limiting for \( R_i/R_o \) less than 0.91, while tensile stress is limiting for \( R_i/R_o \) greater than 0.91. Since \( R_i/R_o \) is less than 0.91 for most thick wall vessels, the static pressure design will be based on Eq. (7) with \( s_{\text{max}} \) equated to the working shear stress for the material in question.

This completes the section on design principles for containing impulsive and static loads resulting from a detonation occurring inside a spherical bomb. We now proceed to the application section where we will examine the capabilities of a given bomb which would be suitable as a vehicle for studying development of convective burning at moderate pressures.

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APPLICATION

The purpose of this section is to illustrate the principles just developed by a practical example in which the operating limits are to be estimated for an available high pressure bomb, whose specifications are:

- Inside diameter - 5 inches
- Outside diameter - 10 inches
- Material - 316 stainless steel
- Working Pressure - 16,000 psi

It is desired to know the allowable limits of propellant loading density to keep within design specifications for static pressure as well as impulsive loading. Also, it would be interesting to know, as a check-point, if the specified working pressure is over-conservative. The tensile yield stress for 316 stainless steel is found to be 50,000 psi.⁸ Using \( \sigma_y / \sigma_{ty} = 0.6 \) and a safety factor of 2, we calculate a working shear strength of 15,000 psi. From Eq. (7), the working pressure is calculated to be 17,500 psi so that the 16,000 psi specified working pressure is reasonable.

The critical impulse, at 1750 psi-ms/in. of wall thickness is

\[
I_{cr} = 2.5 \times 1750 = 4375 \text{ psi-ms}
\]  
(8)

The critical time, given by Eq. (2), using 15,000 ft/s as the acoustic velocity in steel, is

\[
t_{cr} = \frac{kR}{C} = \frac{0.75(3.75/12)}{15000} = 15.6 \mu s
\]  
(9)

Operational limits are determined from the working pressure, critical impulse, and critical time just computed. The principles involved are:

1. Working pressure is the limiting consideration for combinations of \( \delta \) and \( p_0 \) which produce a final static pressure greater than 16,000 psi (110 MPa) as determined from Figure 8.

2. Impulse is limiting for combinations of \( \delta \) and \( p_0 \) which produce a maximum impulse as determined from Figure 4 which leads to an effective impulse of 4375 psi-ms. In this example, Figures 4 and 6 show that whenever impulse exceeds 4375 psi-ms, the

action time is less than the critical time of 15.6 \mu s, so that impulses are always 100\% effective when they are limiting.

Limiting combinations of \( \delta \) and \( P_0 \) are shown for static and dynamic criteria in Figure 9. The region below the line ABC is available for conducting experiments. It is seen that static considerations are limiting at high fill pressures, while dynamic considerations are limiting at low fill pressures. Lines of constant maximum static and detonation pressures are also indicated in the region which is open to experiment.

Two additional curves are shown, labeled "attainable static limit" and \( R_0 = 5.5 \) inches. The first is the limiting static performance capability of a 5-inch ID sphere, regardless of the wall thicknesss, and corresponds to the limiting form taken by Eq. (7) when \( R_0 \) becomes very large. The second curve shows the improvement in dynamic performance afforded by increasing the wall thickness from 2.5 to 3.0 inches.

Figure 9 is a summary of results as well as a typical design chart giving relations between experimentally chosen parameters and performance. The limit line ABC not only delineates the region of available combinations of \( p_0 \) and \( \theta \) for safe operation, but separates propellant dependent quantities from bomb dependent quantities. The bomb design does not influence the lines of constant maximum reflected detonation pressure \( (p_d) \) and constant maximum combustion pressure \( (p_c) \) which depend solely on fill gas and propellant gas properties. Likewise the result of changing the bomb design would only change the limit of the region of safe operation, i.e., the line ABC. The dynamic limit line AB could theoretically be increased indefinitely by thickening the wall; improvement in the static limit is constrained by Eq. (7) unless construction materials of higher shear strengths are used. However, the limiting feature in the currently available 5-inch ID design is more likely the high detonation pressure peaks, even though they have a duration of only a few microseconds. It will probably be necessary to interpose a low pass filter, cutting off below 100 kHz in order to protect any sensitive pressure transducer. If this can be accomplished, then a loading density of 0.02 g/cm\(^3\) (20-g charge) will provide a maximum test pressure ranging between 35 and 100 MPa (5075 and 14,500 psia) depending on initial pressurization. Although the propellant data leading to Figure 9 are based on a particular propellant, it would be simple to construct analogous graphs for other propellants.
Appendix

TRANSFORMATION OF SCALED DETONATION PARAMETERS TO BOMB PARAMETERS

The correlations of detonation parameters upon which Figure 2 is based were originally determined for ambient conditions corresponding to air at sea level pressure and 70°F (294 K). In order to apply this information to the current study, two transformations were required. First, a scaling to convert to different ambient conditions (determined by partial combustion of the propellant charge) was applied in accordance with the discussions in Ref. 2. Second, the resulting scaling laws were expressed in terms of bomb parameters and properties of the propellant and fill gases. The result of the first transformation is shown, without discussion (see Ref. 2), in the following equations.

\[ z = \frac{R_f}{m_{1b,e}^{1/3}} \left( \frac{\rho_a}{\rho_r} \right)^{1/3} \]  
\[ (A-1) \]

\[ p_s = \frac{p_d}{(p_a/p_r)} \text{ psia} \]  
\[ (A-2) \]

\[ l_s = \frac{l_d}{m_{1b,e}^{1/3}} \left( \frac{a_a}{a_r} \right) \left( \frac{\rho_a}{\rho_r} \right)^{1/3} \text{ psi-ms} \]  
\[ (A-3) \]

\[ t_s = \frac{t_d}{m_{1b,e}^{1/3}} \left( \frac{a_a}{a_r} \right) \left( \frac{\rho_a}{\rho_r} \right)^{1/3} \text{ ms} \]  
\[ (A-4) \]

Subscripts on the symbols for weight and radius emphasize the dimensional nature of the scaled parameters. In addition, the subscript e, on charge weight, implies that, for the propellant considered, the equivalent weight of the reference explosive TNT should be used. This point will be covered later.

Calculation of the actual detonation parameters (subscript d) from the scaled values (subscript s) require the calculation of ratios of ambient to reference conditions for pressure, density, and acoustic velocity in terms of experimental parameters. In addition, changes in the system of units from English to metric are applied to scaled distance and pressure.

The first factor \( R_f/m_{1b,e}^{1/3} \) in the scaled distance, is converted to metric by using
\[
R_f = R_i / 30.48 \quad (A-5)
\]
\[
m_{lb,e} = 4/3 \pi R_i^3 \delta (1 - \xi) Y / 454 \quad (A-6)
\]

where \(Y\) is the yield factor for converting actual propellant weight to TNT equivalent. We have as a result

\[
R_f / m_{lb,e}^{1/3} = C_1 / \delta^{1/3} (1 - \xi)^{1/3} \quad (A-7)
\]

The density ratio, \(\rho_a / \rho_r\), which occurs repeatedly as a scale factor, is expressed in terms of experimental parameters, thus:

\[
\rho_a = \frac{m_o + \delta V_b}{V_b - (1 - \xi) \delta V_b / \rho_p} \quad (A-8)
\]
\[
m_o = \frac{\mu_o p_o V_b}{RT_o} (1 - \delta / \rho_p) \quad (A-9)
\]

and

\[
\rho_r = \frac{\mu_r p_r}{R T_r} \quad (A-10)
\]

Combining Eq. (A-8) through (A-10) provides the desired expression

\[
\frac{\rho_a}{\rho_r} = \frac{C_2 p_o (1 - \delta / \rho_p) + C_3 \delta}{1 - (1 - \xi) \delta / \rho_p} \quad (A-11)
\]

The perfect gas law is used throughout the analysis as an adequate approximation of the equation of state for the pressures involved in view of the other assumptions which are made.

The pressure ratio is most easily determined indirectly in terms of temperature and molecular weight ratios. For temperature ratio, \(T_a / T_r\), the ambient temperature is deduced from the energy equation, using the conservative assumption of no heat loss.

\[
f m_o \Delta e_p + (T_a - T_o) (f m_p C_{vp} + m_o C_{vo}) = 0 \quad (A-12)
\]

Equation (A-12) evaluated for \(m_o = 0\) (no fill gas), becomes

\[
\Delta e_p + (T_v - T_o) C_{vp} = 0 \quad (A-13)
\]
Assuming constant $\Delta e_p$, using Eq. (A-9), and noting that, by definition of $\delta$

$$m_p = \delta V_b$$  (A-14)

we obtain

$$\frac{T_a}{T_r} = \frac{T_o}{T_r} + C_4 \frac{f \delta}{(1 - C_5 p_o) \delta + C_6 p_o}$$  (A-15)

To derive an equation for $\mu_r/\mu_a$, it is required only to manipulate the expression for mass average molecular weight ($\mu_a$ in this instance).

During burning, propellant gas mixes with fill gas so that

$$\frac{m_o + f \delta V_b}{\mu_a} = \frac{m_o + f \delta V_b}{\mu_o}$$  (A-16)

By multiplying Eq. (A-16) by $\mu_r$ and substituting Eq. (A-9) for $m_o$, we obtain

$$\frac{\mu_r}{\mu_a} = \frac{\mu_r p_o / \mu_o + \mu_r g / \mu_o}{\rho_o + g}$$  (A-17)

where

$$g = \frac{R T_o (1 - f) \delta}{\mu_o (1 - \delta / \rho_p)}$$  (A-18)

The ratios $p_a / p_r$ and $a_a / a_r$ are now given in terms of the ratios already presented

$$p_a / p_r = (p_a / p_r)(T_a / T_r)(\mu_r / \mu_a)$$  (A-19)

$$a_a / a_r = (T_a / T_r)^{1/2}(\mu_r / \mu_a)^{1/2}(\gamma_a / \gamma_r)^{1/2}$$  (A-20)

In this report, the change in $\gamma$ during burning is neglected, an approximation which errs on the safe side in estimating the bomb capability since $\gamma$ actually decreases during burning. Assuming a constant $\gamma_a$ (equal to $\gamma_r$), therefore, results in a reflected impulse which is on the high side.

In summary, the equations which are actually used in the calculations are

$$p_d = (p_a / 145)(p_a / p_r)$$  (A-21)

$$e_d = R t_s / 2 (T_a / T_r)^{1/2}(\mu_r / \mu_a)^{1/2}$$  (A-22)
\[ I_d = \frac{R(\rho_a/\rho_r)(T_a/T_r)^{1/2}(\mu_r/\mu_a)^{1/2}I_s/Z}{Z = C_1(\rho_a/\rho_r)^{1/3}/6^{1/3}(1 - f)^{1/3}} \]  

(A-23)

(A-24)

The quantity, 145, appearing in Eq. (A-21) converts from psi to MPa. In the computer program, values of \( p_s \), \( t_s \), and \( I_g \) are obtained from Figure 2 by an interpolation scheme based on fitting the curves to a series of straight line segments (18 segments for each correlating curve).
REFERENCES


