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A CRITICAL EXAMINATION OF A NUMERICAL FRACTURE DYNAMIC CODE

by

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ABSTRACT

After upgrading the energy dissipation algorithm, numerical experiments were conducted to assess the reliability of the explicit dynamic finite element code, HCRACK. Two dynamic fracture specimens, i.e., the wedge-loaded rectangular DCB (RDCB) specimen and the wedge-loaded tapered DCB (TDCB) specimen, which were studied experimentally by Kalthoff et al, were then analyzed with this updated fracture dynamic code. Using the experimentally determined dynamic fracture toughness, $K_{ID}$, versus crack velocity, $\dot{a}$, relation, the RDCB specimen was analyzed first by the "propagation method" where good agreements between calculated and measured $K_{ID}$ versus $a$ relations were observed. The calculated $a$ versus time, $t$, relation was then used as input data in the "generation method" where the resultant $K_{ID}$ were virtually identical to those obtained in the propagation method. Error analyses of the generation method were also made first by using the experimentally determined $a$ versus $t$ relation and secondly by artificially perturbing this relation.

A TDCB specimen was then analyzed with both the propagation and generation methods by using the $K_{ID}$ versus $\dot{a}$ relation established for this specimen and the measured $a$ versus $t$ relation, respectively. The computed $K_{ID}$ obtained by both methods were in good agreement with the experimental results, showing that either approach can be used in analyzing fracture.

KEYWORDS

Dynamic fracture, dynamic finite element analysis, dynamic fracture toughness, crack arrest stress intensity factor.
INTRODUCTION

For the past three years, two of the authors have used a two-dimensional elasto-dynamic finite element code, which was based on HONDO [1], to compute the dynamic stress intensity factor for a crack propagating with a prescribed velocity [2-5] by applying to each node a nodal force sufficient to release the node. This numerical procedure was later modified to include a startup procedure for computing dynamic stress intensity factor, dynamic energy release rate, fracture energy, kinetic energy and strain energy at each increment of crack advance [6,7]. Also the impulse stress waves generated by the instantaneous application of a nodal force to model the release of a crack-tip node was reduced by varying the force over the time necessary for the crack tip to advance one nodal distance. Physically, this procedure models a more gradual transit of the crack-tip between two adjacent finite element nodes and is similar to that developed by Keegstra [8-10] with the exception that our restraining nodal force is completely eliminated when the crack-tip reaches its adjacent node. Other nodal force release mechanisms include those of Malluck and King [11] and Rydholm, Fredriksson and Nilsson [12] with different postulated rates of nodal force release. The dissipated energy during a crack extension based on any of the above three nodal force release mechanism is then computed from the nodal force versus nodal displacement relation during this incremental crack extension. In general this nodal force versus nodal displacement relation is non-linear and is governed by the dynamic state surrounding the propagating crack tip thus requiring the monitoring of nodal force or nodal displacement or both at every incremental time in the dynamic finite element analysis. Interestingly enough, recent studies showed that the differences in the mechanism of nodal force release [13,14] caused little changes in the resultant dynamic stress intensity factor. It is
thus of no surprise that good to excellent agreements were claimed by all [6,11,12] when these three crack tip energy dissipation procedures for computing the dynamic stress intensity factor was checked by analyzing the Broberg problem [15].

The above procedure of computing dynamic stress intensity factor for a crack whose velocity is prescribed to be equal to measured one in a dynamically fracturing specimen was termed "generation calculation" by Kanninen [16,17] who also expressed reservations on the accuracy of this numerical approach. The "propagation calculation" in contrast to the "generation calculation" is based on an assumed dynamic fracture toughness, $K_{ID}$, versus crack velocity, $\dot{a}$, relation which is then used to propagate a crack [16-23].* The assumed $K_{ID}$ versus $\dot{a}$ relation is considered correct when the calculated crack propagation history coincides with the experimental data, and the $K_{ID}$ at crack arrest, if any, is considered to be the crack arrest toughness, $K_{IA}$, sought by some in predicting fracture arrest of a dynamically propagating crack.

While one can debate the merits of propagation versus generation calculations, only one study which involved both propagation and generation calculation using the same numerical algorithm [23] has been published to date. Since the limited study in Reference [23] did not provide a comprehensive error assessment of the two procedures, this paper will report on our comparative studies using two Araldite B fracture specimens which were analyzed by Kalthoff et al by the method of caustics [24,25].

* Note that Keegstra in References [8,9,10] used the propagation calculation to compute $K_{ID}$ versus $\dot{a}$ relations in fracturing specimens.
DYNAMIC FINITE ELEMENT ANALYSIS

In the previous studies cited above [6,26], the dynamic fracture dynamic code HCRACK was shown to be an efficient and inexpensive method for simulating dynamic fracture problems. Numerical experiments proved that reasonable numerical accuracy can be obtained by using coarse meshes of conventional elements (see Figures 1 and 2) and a moderate number of time steps, e.g., about 150 steps for crack propagation and subsequent arrest in a RDCB specimen shown in Figure 1. Unlike the implicit dynamic finite element codes used by others, however, it was difficult to accurately prescribe the rate of nodal force release since the input nodal force would not generally be in equilibrium with the dynamic state of stress in the adjoining finite elements in this explicit finite element code. As a result, an in-depth study on the performance of our fracture dynamic code was conducted for different crack tip force release rates, different calculation procedures for the dynamic stress intensity factor, and different finite element breakdown. A brief description of some of these findings are presented in the following.

As mentioned above, the algorithm for artificially prescribing an input nodal force at the crack tip for each time step for prescribed decrease in the resultant residual nodal force in the dynamic code is not straightforward and often a complete release of the nodal force cannot be achieved in the prescribed time period. The basis of the numerical method is to define the force, \( F_n^1 \) which must be applied to a node at time step \( n \) such that the time variation of the stress follows the form shown on figure 3. In an implicit code, application of \( F_n \) would result in the same calculated force at the end of the time step. With the explicit code, however, the calculated force at the end of the increment, \( F_n^0 \), will rarely be equal to \( F_n^1 \). Accordingly, the force at the next time step \( F_{n+1}^1 \) must be adjusted
not only to compensate for the error, but also to yield the desired value at the end of the time step. The simplified method used in [26] was replaced by the following equation and typical results are as shown in figure 3.

\[
F_{n+1}^i = F_0^i - \sum_{j=1}^{n} \frac{F_j^i}{2^{n-j+1}} - F_n^i
\]

(1)

At the beginning of the crack propagation history when the first crack tip node is released (see Figure 3), excellent agreement between the prescribed (linear decrease in this case) and actually achieved nodal forces is noted while for the sixth node which was released later (see Figure 3), some deviations between both nodal forces are noted. Initial static equilibrium prior to crack propagation most likely contribute the excellent results in the former case.

Also noteworthy is the recent study by Malluck and King [13] who compared energy release rates for the two distinctly different functions of \( F/F_0 = \left[1 - b/\Delta \right]^{3/2} \) and \( F/F_0 = \left[1 - b/\Delta \right]^{1/2} \), where \( b \) is the distance between hypothetical crack tip location and the released crack tip node and \( \Delta \) is the inter-nodal distance and \( F \) and \( F_0 \) are the instantaneous and original crack tip nodal forces, respectively. Their results showed no significant differences in the calculated dynamic stress intensity factors for crack speeds lower than 25 percent of the shear wave velocity, i.e. \( c < 0.25c_2 \).

Our use of a linearly decreasing nodal force, \( F/F_0 = [1 - b/\Delta] \), with constant crack velocity between the two adjoining finite element nodes is thus justified.

The dynamic stress intensity factor was computed directly by the total strain energy released from an instantaneous balance of the total energy of the entire specimen [7] as
\[ G_I = 2(E_n - E_{n-1}) / (a_{n+1} - a_n) \]  

(2)

where \( E_n, E_{n+1} \) are the total strain energies for crack lengths of \( a_n \), \( a_{n+1} \), respectively when the crack extended from node \( n \) to node \( n+1 \). The dynamic stress intensity factor, \( K_I \), was then computed from \( G_I \) using Freund's relation [24]. Alternatively the value of \( G_I \) was computed by energy dissipated at the released node as

\[ G_I = (u_i \Delta F_i + \sum_{1-1}^m (u_i - u_{i-1}) \Delta F_i) / (a_{n+1} - a_n) \]  

(3)

where \( m \) is the number of time steps between nodes \( n \) and \( n+1 \), \( u_i \) and \( \Delta F_i \) are displacement and decrease of force at the released node \( n \), respectively.

Figure 4, shows the dynamic fracture toughness, \( K_{ID} \), associated with crack propagation and arrest in one of Kalthoff's RDCB specimens [24] computed by both equations (2) and (3) using the "propagation method." Although details of this analysis are described in the following section, the results are shown in this section as an indication that little difference can be noted in the \( K_{ID} \) obtained by the two algorithms.

As shown in Figure 4, the forced linear decrease in the crack tip nodal force improved the simulation of the smoothly propagating crack and eliminated the spurious oscillations in dynamic stress intensity factor observed previously [2-5]. It is uncertain, however, to what extent this smoothing procedure may hide the true oscillations of the dynamic stress intensity factor eventually induced by the reflected stress waves which emanated from the running crack.

SPECIMENS AND MATERIAL DATA

The two specimens analyzed by the dynamic finite element code are the wedge-loaded, RDCB and TDCB specimens which were investigated experimentally by Kalthoff et al [24,25]. Specimen geometries of these
Araldite B specimens and their finite element idealizations can be seen in Figures 1 and 2.

Although the rigid loading wedge between the two loading pins will prevent any inward displacement of the loading pins, these pins are free to leave the wedge and travel outwards. The resultant dynamic stress intensity factors in the presence of separating pins could vary significantly during crack propagation [26]. The smaller mass density and the two orders of magnitude larger compliance of the Araldite B specimens in comparison to the steel specimen studied in Reference [26] should have reduced the additional input energy due to any possible separation of the loading pins and thus constant loading pin displacement were prescribed at the pin holes.

Material constants of Araldite B used for this dynamic finite element analysis after [24] are modulus of elasticity \( E = 3.38 \text{ GPa} \), Poisson ratio of \( \nu = 0.33 \) and mass density, \( \rho = 1047 \text{ kg/m}^3 \). The experimentally determined dynamic fracture toughness \( K_{1D} \), versus crack velocity, \( \dot{a} \), relations used in the propagation calculations of RDCB and TDCB specimens are both plotted in Figure 5 [24,25] respectively. Crack length as a function of time used in the "generation calculations" of the RDCB specimen was taken from Figure 5 in Reference [24] but is not reproduced in this paper.

For the dynamic crack initiation in the RDCB specimen, the dynamic crack initiation stress intensity factor, \( K_{1Q} \), as reported in Reference [24], was used and the subsequent dynamic stress intensity factors were computed from the energy released at the node adjacent to the reference crack tip node except the set of \( K_{1D} \) data noted in Figure 4. Since an experimentally determined \( K_{1Q} \) was not reported in Reference [25], a statistically computed \( K_{1Q} \), which was back calculated from the median of
Kalthoff's measured oscillating $K_{ID}$ values [25] after crack arrest, was used in the analysis of the TDCB specimen.

RESULTS

RDCB Specimen

The first numerical analysis involved a propagation calculation for the RDCB specimen of Figure 1 using the $K_{ID}$ versus $\dot{\alpha}$ relation of Figure 5 and a $K_{IQ} = 2.32$ MNm$^{3/2}$. The resulting dynamic fracture toughness and crack tip motion of this propagation calculation are shown in Figures 6 and 7, respectively. The "propagation" crack tip motion from Figure 7 was then used as input data for the "generation" calculation. This result is not plotted in Figure 6 since the $K_{ID}$ versus $\dot{\alpha}$ relations obtained by both the propagation and generation calculation were indistinguishable.

As an additional numerical experimentation, however, the measured crack length, $a$, versus time, $t$, relation of Reference [24] was used as input to the "generation" calculation and the resultant $K_{ID}$ are also shown in Figures 6 and 7. Despite the lack of complete agreement between the two $K_{ID}$ curves obtained by propagation and generation calculation, the shapes of these two curves are very close. Although both $K_{ID}$ curves agree well with experimental data during the first half of dynamic crack propagation as shown in Figure 7, a distinct difference is noted by a second local maximum, which occurs in both propagation and generation calculations prior to crack arrest, but which does not occur in the experimental results. The similarity between the propagation and generation $K_{ID}$ curves is more apparent in Figure 8 where the second maxima in the two calculations occur at the same time. The higher $K_{ID}$ values in the generation calculation at lower measured crack velocities during much of the crack propagation will result in a general shift of two $K_{ID}$ versus $\dot{\alpha}$ relation in Figure 5.
Figure 6 also shows that the computed crack jump distance is 4% shorter of the measured one in the propagation calculation but by definition is equal to measured distance in the generation calculation. Although the propagation calculation is terminated when the computed dynamic stress intensity factor falls below the minimum $K_{ID}$ value in Figure 5, the generation calculation is continued up to the prescribed crack tip length and crack arrest time. Significantly lower dynamic stress intensity at the instant of crack arrest is noted.

The sensitivity of the dynamic stress intensity factor, which is calculated by the generation method, to the instantaneous crack velocity is further demonstrated in Figure 8. In order to assess the sensitivity of $K_{ID}$ obtained by the generation method to the input data, a numerical experiment was conducted by artificially perturbing the smooth experimental curve of the crack tip motion in Figure 8. The result was a severely perturbed $K_{ID}$ also shown in Figure 9, where discrete increases and decreases in crack velocities are followed by local minima and maxima of $K_{ID}$ respectively.

**TDCB Specimen**

Figure 9 shows the $K_{ID}$ as a function of $a$ computed by the propagation method, using the $K_{ID}$ versus $a$ relation of Figure 5 and by the generation method using experimentally determined $a$ versus $t$ relations for the TDCB specimen together with experimental data from Reference [25]. A second maximum, which resembles that found previously in the RDCB specimen, in $K_{ID}$ can be observed. The computed crack jump distance obtained by the propagation method is shorter than the experimental one by 12%. In the propagation calculation the computed $K_{ID}$ increased again to a value approaching experimental $K_{ID}$ after the initial crack arrest. Subsequently computed $K_{ID}$ oscillated around the few experimental points.
Figure 10 shows the $K_{ID}$ versus $t$ relations obtained by both propagation and generation calculations. Although the two calculated $K_{ID}$ are in excellent agreement with each other except for the initial phase of crack propagation in this TDCB specimen, the calculated $K_{ID}$ are lower than the measured $K_{ID}$ just prior to and after crack arrest. Previous experience with steel TDCB specimens [4,5,26] indicate that this small underestimate could be attributed to the possible separation of the loading pins from the loading wedge during crack propagation.

CONCLUSIONS

The results of the present and of the previous studies using HONDO II show that the dynamic stress intensity factor for a crack propagating in a finite two-dimensional body can be computed relatively inexpensively with an accuracy sufficient for many practical purposes. Very close agreements between the $K_{ID}$ obtained by the generation and by the propagation calculations should dispel the reservations [16,17] about this dynamic fracture algorithm.

When used in conjunction with measured crack position versus time data, the generation method with proper care can be used to accurately calculate the dynamic stress intensity factor during the fast crack propagation and crack arrest.

On the other hand the uncertainty in the $K_{ID}$ versus $a$ relations, particularly in the region of very low velocities together with limitation in the finite element modeling of dynamic crack propagation offers little chance for simulating the crack propagation and crack arrest event by the propagation method when the dynamic stress intensity factor oscillates in a narrow range about the crack arrest stress intensity factor as shown by some experimental results with the single edged notch specimens reported in [25].
DISCUSSION

It has been a common practice by all, including the authors, to verify their fracture dynamic code by analyzing the Broberg problem [15] for which the dynamic solution is available. Good agreements in these studies cannot be construed as verification of numerical solutions generated for cracks propagating in finite specimens composed of real materials. The discrepancies between the computed and the experimentally determined $K_{ID}$-values shown in Figures 6 and 9 could have arisen from the viscous damping in Araldite B which was not modeled in the elasto-dynamic analyses described in this paper. A study of the time-dependent energy balance during crack propagation and arrest suggests that the consistently appearing second maxima in the calculated $K_{ID}$-curves are real phenomena based on elastic analyses. It is interesting to note that the limited experimental $K_{ID}$ versus $a$ relation obtained for RDCB specimens machined from high strength steel [25] is in qualitative agreement with our elastic analysis of the RDCB specimen.

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FIGURE 2. WEDGE-LOADED TDCB SPECIMEN.
**FIGURE 3.** GRADUAL RELEASE OF THE NODAL FORCE AT THE CRACK TIP.
FIGURE 4. DYNAMIC FRACTURE TOUGHNESS IN WEDGE-LOADED RDCB SPECIMEN.
Figure 5. Crack velocity versus dynamic fracture toughness relations for Araldite B epoxy (Kalthoff et al.).
FIGURE 6. DYNAMIC FRACTURE TOUGHNESS IN WEDGE-LOADED RDCB SPECIMEN.
FIGURE 7. CRACK LENGTH, DYNAMIC FRACTURE TOUGHNESS AND CRACK VELOCITY VERSUS TIME IN WEDGE-LOADED RDCB SPECIMEN.
Figure 8. Crack length, dynamic fracture and crack velocity versus time in wedge-loaded RDCB specimen.
NUMERICAL RESULTS
- -O-- GENERATION CALCULATION FROM
MEASURED $a$ VS $t$
- - - - PROPAGATION CALCULATION FROM
MEASURED $K_{ID}$ VS $a$

EXPERIMENTAL RESULT
X KALTBOFF ET AL

FIGURE 9. DYNAMIC FRACTURE TOUGHNESS IN WEDGE-LOADED TAPERED DCB SPECIMEN.
NUMERICAL RESULTS

- GENERATION CALCULATION FROM MEASURED $a$ VS $t$
- PROPAGATION CALCULATION FROM MEASURED $K_{ID}$ VS $a$

EXPERIMENTAL RESULTS

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FIGURE 10. DYNAMIC FRACTURE TOUGHNESS AND CRACK LENGTH VERSUS TIME IN WEDGE-LOADED TAPERED DCB SPECIMEN.
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