Optical Pulse Squaring Effects in a Dispersive Delay Line

R. H. Lehmerg, J. Reintjes, and R. C. Eckardt

Laser Plasma Branch
Plasma Physics Division

May 1977

Work performed at the Naval Research Laboratory under the auspices of the U.S. Energy Research and Development Administration.
Using a pair of diffraction gratings, we have produced temporal squaring effects on 30 picosecond pulses that were negatively self phase modulated in cesium vapor. The results are in good agreement with the theory, which is discussed in detail. Further enhancement of the squaring by a saturable absorber is also discussed.
OPTICAL PULSE SQUARING EFFECTS IN A DISPERSIVE DELAY LINE

INTRODUCTION

A well known pulse compression technique consists of propagating a long phase-modulated (chirped) pulse through a dispersive delay line whose delay is greatest for those frequencies that appear earliest.\textsuperscript{1-12} At optical frequencies, the phase modulation can be self-induced on the pulse by a Kerr medium whose nonlinear change in refractive index $\delta n(t)$ responds to the instantaneous intensity; i.e., $\delta n(t) \sim n_2 I(t)^{5-10}$. For the normal case where the nonlinear refractive index $n_2$ is positive, the center portion of the pulse can be temporally compressed in a negatively dispersive delay line such as a pair of diffraction gratings.\textsuperscript{5-9}

In this paper, we show that if the sign of either $n_2$ or dispersion is reversed, this technique will no longer shorten the pulse, but can instead produce a nearly square time dependence. Using the negative $n_2$ of cesium vapor around 1.06 $\mu$m \textsuperscript{13,14}, we have observed the squaring effect of a grating pair on 35 psec pulses in good agreement with our theoretical prediction.

Such squared-off pulses could have several applications. For example, a square pulse contains more energy than any other shape of equal peak intensity and total duration. This makes it an attractive alternative to the bell-shaped pulses that are currently encountered in high power short pulse laser amplifiers operating near damage threshold levels. Another application would be the experimental study of various coherent nonlinear phenomena, such as stimulated Raman scattering, in which well-known analytic solutions exist for the case of a square driving pulse.\textsuperscript{15} Additional applications can be found in the realm of signal processing, such as fast optical gating and

Note: Manuscript submitted May 13, 1977.
ultrashort ramp pulse generation. Although short square pulses could also be generated by other techniques, such as clipping due to multiphoton absorption in semiconductors, the present technique has the advantage that it requires minimal insertion loss and allows faster risetimes.

**THEORY**

An optical pulse propagating through a medium of instantaneous nonlinear refraction \( \delta n(t) = n_2 \langle E^2 \rangle \) develops an intensity dependent phase shift:

\[
B(t) = \frac{\delta n_2}{n_0 \lambda c} \int_0^L I(z,t)dz,
\]

where \( I(z,t) \) is the intensity and \( L \) is the path length in the nonlinear medium. The complex field envelope \( \xi(t) \sim [I(t)]^{1/2} \exp[IB(t)] \) at the output characterized by an instantaneous carrier frequency shift \( \omega(t) = -\dot{B}(t) \sim -\dot{I}(t) \), as shown in Fig. 1 for the case where \( I(t) \) has a Gaussian time dependence of FWHM width \( \Delta t \).

If this pulse is now propagated through a delay line with dispersion \( \tau' = d\tau/d\omega \) in the group time delay \( \tau(\omega) \), the transmitted pulse \( I_T(t) \) will be temporally compressed or expanded, depending upon the relative signs of \( \tau' \) and the chirp \( d\omega/\omega \). In ordinary situations where \( n_2 > 0 \), the chirp is positive around the center portion of \( I(t) \), but the negative in the leading and trailing edges (solid line of Fig. 1). Since a negatively dispersive delay line \( (\tau' < 0) \) will cause the greatest delay to the low frequencies, it will tend to compress the center \( I(t) \), but expand the outer portions; hence, \( I_T(t) \) will have a sharp central spike, with either a long tail or secondary peak on each side.

If \( n_2 < 0 \), then \( B(t) \) and, therefore, \( \omega(t) \) reverse sign, as indicated by the dashed curve of Fig. 1. A negatively dispersive delay
line would then expand the center portion of $I(t)$ while compressing the leading and trailing edges, thereby tending to produce a square or flat-top time dependence $I_T(t)$. Identical results would be obtained if $n_2$ and $\tau'$ were both positive.

The time dependence of the transmitted pulse is given by the expression

$$I_T(t) \sim \int_{-\infty}^{\infty} dt' [I(t')]^{\frac{3}{2}} \exp \left[i\varphi(t, t')\right]^2$$

(2a)

where

$$\varphi(t, t') = B(t') - (t - t')^2/2\tau'.$$

(2b)

Figure 2 shows numerical solutions for Gaussian input pulses with different values of the peak phase shift $B_0$, and a fixed value of $\tau'$. As $|B_0|$ increases, $I_T(t)$ broadens and becomes more flat-topped until a nearly optimum squaring condition is attained (curve c). Larger values of $|B_0|$ or $|\tau'|$ result in a central dip (curve d), which continues to deepen and broaden as these quantities are increased.

If $|B_0| \gg 1$, Eq. (2) can be accurately approximated by the stationary phase result.18

$$I_T(t) \sim 2\pi I(t_0')/|\varphi''(t, t_0')|,$$

(3a)

where

$$\varphi''(t, t_0') = \varphi(t, t')/3t'^2|_{t' = t_0'},$$

(3b)

and $t_0'$ is defined by the condition

$$\varphi'(t, t_0') = \varphi(t, t')/3t'|_{t' = t_0'} = 0.$$
For a Gaussian input pulse with a FWHM of $\Delta t$, this gives

$$I(t) \sim |2\pi K(1 - 2\pi s^2) + \exp (\pi s^2)|^{-1}, \quad (4a)$$

where $\pi = 4\pi n^2$, $K = \tau' B_o / \Delta t^2$, and $s$ satisfies the relation

$$s[1 + 2\pi K \exp (-\pi s^2)] = t / \Delta t. \quad (4b)$$

Expression (4) remains quite accurate even for modest values of $|B_o|$, e.g., it agrees to within $\pm 1\%$ with curves b-d of Fig. 2.

It is apparent from (4) that the output shape depends only upon the parameter $K$. The condition for optimum squaring can be written as $\delta^2 I'(t) / \delta t^2 = 0$ as $t \to 0$; however, since $s \to 0$ as $t \to 0$, this is equivalent to expanding the exponential of (4a) and equating the total coefficient of $s^2$ to zero. The optimum value of $K$ is then

$$\left(\tau' / B_o / \Delta t^2\right)_{op} = 1/(4 \eta) \approx 0.090, \quad (5)$$

which is close to the value $K = 1/10$ used in curve c of Fig. 2.

One can obtain a more intuitive understanding of these results by noting that the maximum carrier frequency variation in Fig. 1 is $\Delta \omega_c = 2.36 \, |B_o| / \Delta t \approx |B_o| \Delta \omega_o$, where $\Delta \omega_o$ is the time-limited bandwidth. If $|B_o| \gg 1$, then $\Delta \omega_o$, and each segment of $I(t)$ experiences a well-defined group time delay $\omega(t) \tau'$ in the dispersive line. One would therefore, expect the transmitted pulseshape to depend only upon $\Delta \omega_c \tau'$ in relation to $\Delta t$; i.e., upon $\Delta \omega_c \tau' / \Delta t = 2.36 \, K$. For optimum squaring, $\Delta \omega_c \tau' = .26 \, \Delta t$.

**EXPERIMENT**

In order to test the pulse squaring effect, we performed the experiment indicated in Fig. 3. The incident radiation consisted of single pulses from a mode-locked Nd:YAG laser system. They contained nominally 40 mJ of energy with a 35 psec pulse width (FWHM), as
determined by a 5 psec resolution streak camera. The spatial profile, as determined by a silicon diode array was an Airy pattern with a 4.0 mm 1/e diameter, which was truncated at its first minimum by aperture A₁. After a 2:1:1 expansion by a Galilean telescope to minimize the spatial effects of self defocusing, the pulses were propagated through a 1 m-long, two-temperature cesium vapor cell, which has been described elsewhere. The peak on-axis intensity at the entrance was approximately 1 GW/cm², and the insertion loss (due to Cs dimers) was less than 20%.

Due to the negative $n_2$ of Cs vapor around 1.06 $\mu$m, the transmitted pulse is self phase modulated in the negative sense described earlier. The on-axis portion of the transmitted beam is expanded and propagated through a grating pair $G₁ G₂$, which is characterized by the negative dispersion:

$$\tau' = \frac{L\lambda}{2\pi\Delta\lambda^2} \left( \frac{\sin \theta_1 + \sin \theta_2}{\cos \theta_2} \right)^2$$

Here $L$ is the grating spacing, and $\theta_1$ and $\theta_2$ are, respectively, the angles of incidence and reflection at $G₁$. For our experiment, $\theta_1 = \theta_2 = 40^\circ$, and $L = 4$ m; hence, $\tau'/\Delta\lambda^2 = -0.17$.

The on-axis portion of the pulse was recorded both before and after the grating pair by a 5 psec resolution streak camera. A variable optical delay line before $G₁$ allows the input and output pulse separation at the streak camera to be adjusted so that they may be recorded on the same film. Densitometer traces of a typical shot are shown in Fig. 4. The pulse incident on the gratings has essentially the same shape and width as it had before the cesium cell; however, at the output of the gratings it has broadened to approximately 50 psec, with a noticeable flattening around the top.

No attempt was made in this experiment to optimize the squaring effect. Comparison of Fig. 4b with Fig. 2 suggests that the peak phase
shift in this shot was $B_0 = -1.1 \pi$, giving $B_0\tau' = 26 \Delta t^2$ rather than Eq. (5). This value of $B_0$ is consistent with the values measured in a related experiment\textsuperscript{14} where the cesium cell was used to compensate self phase modulation due to the laser plus two $CS_2$ cells. In that experiment, the phase modulation due to the cesium vapor at density $N = 7.6 \times 10^{17}$ cm\textsuperscript{-3} was typically $-1.5 \pi$ to $-2 \pi$, while that contributed by the laser was $+0.6 \pi$ to $+0.8 \pi$, leaving a contribution from the Cs cell of $-0.9 \pi$ to $-1.2 \pi$.

\textbf{DISCUSSION}

Although the dispersive technique described here can produce flat-topped picosecond pulses, it alone cannot produce risetimes significantly shorter than those indicated in Fig. 2; i.e., according to Eq. (5), one cannot shorten the risetime of the optimized pulse simply by increasing the chirp bandwidth. It can be shortened, however, by use of a fast saturable absorber\textsuperscript{17} at the output of the gratings. An example of this is illustrated in Fig. 5, which shows calculated pulse shapes before and after a saturable absorber. One quantitative index of the squaring effect is the ratio $t(90,90)/t(10,90)$, where $t(90,90)$ is the interval between the $90^\circ$ intensity levels, and $t(10,90)$ is the risetime from the $10^\circ$ to $90^\circ$ levels. In this example, $t(90,90)/t(10,90) = 2.7$ before the saturable absorber (Fig. 5a), and 4.4 after it (Fig. 5b). The transmission around the center of the pulse is 40\%. Although $t(10,90)$ could be reduced further by using lower values of the saturation intensity and small signal transmission, and effective lower limit of $t(10,90)$ would be determined by the relaxation time of the saturable absorber.
REFERENCES

Fig. 1 — Optical pulse of Gaussian time dependent intensity $I(t)$ showing the carrier frequency displacement $\omega_\gamma(t) \sim -n I(t)$ resulting from self phase modulation in a medium of nonlinear refractive index $n_2 \geq 0$.
Fig. 2 — Calculated squaring effects from a delay line of dispersion $\tau'$ upon a self phase modulated pulse of peak nonlinear phase shift $B_0$ for the case where $B_0 \tau' > 0$. (a) Incident Gaussian pulse $I(t)$, (b-d) transmitted pulse $I_T(t)$ for $|\tau'| = 0.017 \Delta t^2$, and (b) $|B_0| = 1.1 \pi$, (c) $1.9 \pi$, (d) $2.8 \pi$. All pulses have been normalized at $t = 0$.

Fig. 3 — Schematic illustration of the Cs pulse squaring experiment
Fig. 4 — Observed (—) and calculated (---) squaring effect on negatively self phase modulated pulses due to a grating pair of dispersion $r' = -2.07 \times 10^{-23} \text{ sec}^2$. (a) Incident pulse, compared to a Gaussian dependence, (b) transmitted pulse, compared to Fig. 2b ($B_o = -1.1 \pi$).

Fig. 5 — Additional squaring effect from a fast saturable absorber after the dispersive delay line. (a) Optimized input pulse $I_T(t)$ (Fig. 2c), (b) output from a saturable absorber of small signal transmission $2.62 \times 10^{-5}$ and saturation intensity $I_{SAT} = .0625 I_T(0)$. Both pulses have been normalized to the same intensity at $t = 0$. 

10