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USAMC ltr, 7 Jan 1974
TECHNICAL REPORT M-68-1

DYNAMICS OF WHEELED VEHICLES

Report 1

A MATHEMATICAL MODEL FOR THE TRAVERSAL OF RIGID OBSTACLES BY A PNEUMATIC TIRE

APPENDIX B: DIGITAL IMPLEMENTATION OF SEGMENTED TIRE MODEL

by

N. R. Murphy, Jr.

August 1969

Sponsored by

U. S. Army Materiel Command

Conducted by

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

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Best Available Copy
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Foreword

The digital implementation of the segmented tire model is an extension of the study reported in the basic report and was developed at the U. S. Army Engineer Waterways Experiment Station (WES) as a part of the vehicle mobility research program under DA Project 1TO62103A046, "Trafficability and Mobility Research," Task 03, "Mobility Fundamentals and Model Studies," sponsored by the Directorate of Development and Engineering, U. S. Army Materiel Command.

The study described herein was accomplished by personnel of the Vehicle Dynamics Section (VDS), Mobility Research Branch, Mobility and Environmental Division, under the general supervision of Messrs. W. G. Shockley and S. J. Knight, and under the direct supervision of Dr. D. R. Freitag and Mr. A. J. Green. Mr. N. R. Murphy, Jr., of the VDS and Mr. J. F. Smith of the Mathematics and Analysis Section, Electronic Computer Branch, wrote the computer programs. Mr. Murphy prepared this appendix.

COL Levi A. Brown, CE, was Director of WES during the preparation of this appendix. Mr. F. R. Brown was Technical Director.
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<th>Section</th>
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### Conversion Factors, British to Metric Units of Measurement

British units of measurement used in this report can be converted to metric units as follows:

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<td>2.54</td>
<td>centimeters</td>
</tr>
<tr>
<td>pounds</td>
<td>4.444</td>
<td>newtons</td>
</tr>
<tr>
<td>pounds per square inch</td>
<td>0.689476</td>
<td>newtons per square centimeter</td>
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</table>
Summary

This appendix presents the procedures for digital implementation of the segmented tire model, developed in the basic report for an analog computer. Two procedures are required: (a) determination of the segment spring coefficient from measured load-deflection test results, and (b) computation of the resultant force vector transmitted to the axle. Digital programs for both procedures were written in Fortran IV for a GE-420 system, and are included.
APPENDIX B: DIGITAL IMPLEMENTATION OF SEGMENTED TIRE MODEL

1. For digital implementation of the segmented tire model, basically two procedures are required: (a) determination of the segment spring coefficient from measured load-deflection test results, and (b) computation of the resultant force vector transmitted to the axle. Therefore, two computer programs have been prepared.

**Program 1**

2. The first program (fig. B1) is used to compute the average deflection of each segment for a given vertical center-line deflection.* It is written in Fortran IV and programmed to run on a GE-420 system. The only input requirement is a card containing the tire radius in inches, the number of segments for 180 deg (current status of the program limits the maximum number of segments to 60), and the desired maximum vertical center-line deflection in inches multiplied by 10. This maximum deflection serves as a cutoff criterion for the computer and should be a number that does not exceed the tire section height. The actual maximum deflection is multiplied by 10 to ensure an integer value. This is a restriction imposed by the "DO-loop" that successively increments the center-line deflection.

3. A representative computer printout is shown in fig. B2 to illustrate the format of the output and to demonstrate, in conjunction with the following example, how the results are used to determine an appropriate segment spring coefficient:

---

* Appendix A to this report presents procedures for computing the necessary effective radial deflections.
a. Problem. A 9.00-14, 2-P8 tire with a $\frac{3}{16}$-in.* radius is inflated to 30-psi pressure; at this pressure it deflects 1 in. under a 100-lb load. Divide the lower half (180 deg) of the tire into 18 equal segments and determine the segment spring coefficient.

b. Solution. Prepare an input data card containing the tire radius in inches ($\frac{3}{16}$), the number of segments in 180 deg (18), and the maximum vertical center-line deflection in inches multiplied by 10 (say $3 \times 10 = 30$). Run using the program in ft. H1. Since the coordinate (1.0, 760) was chosen from the load-deflection curve, examine the computer printout (fig. B2) and find the values of segment deflections for a center-line deflection of 1 in. Two segments on each side of the vertical center line are seen to be influenced at this particular deflection. Compute the segment spring coefficient $K$ from the equation:

$$ K = \frac{F}{2 \sum_{i=1}^{2} \Delta_i \cos \phi_i} $$

$$ = \frac{760}{2(0.914 \cos 5^\circ + 0.384 \cos 15^\circ)} $$

$$ = \frac{760}{2.56} $$

$$ = 297 \text{ lb/in.} $$

This value is assigned as the spring coefficient for each segment and will change only if the number of segments or the inflation pressure is changed. Choosing a small deflection, such as 0.2 or 0.4 in., as a basis for determining the spring coefficient would most likely yield a $K$-value somewhat different from one with a larger value of deflection as a basis. Comparisons of measured load-deflection curves with those computed for $K$-values obtained from the segmented tire model at various deflections revealed that greater accuracy and consistency were obtained when $K$ was computed from a deflection of 1 in. This deflection is generally in the area of maximum curvature on the load-deflection curve.

---

A table of factors for converting British units of measurement to metric units is presented on page ix.

** See equation development in paragraph 21 of the main report.
Program 2

4. The second program (fig. B3) is used to compute the vertical and horizontal components of the resultant force vector transmitted to the axle. It, too, is written in Fortran IV for the GE-420 system. It is a complete, self-contained program that was assembled from a section of a more comprehensive one that represented a multidegree-of-freedom vehicle. Taken out of context, it contains none of the dynamics of the problem, i.e. the influence of the coupled differential equations describing the dynamics of the sprung mass of the system has been removed. Certain modifications were made regarding the input requirements, so the program demonstrates, in a static sense, how the components of the resultant wheel spring force are computed with the segmented tire concept. With other slight modifications, this program can be adapted as a subprogram to describe the tire compliance in other vehicle dynamics programs.

5. The solution is based on a purely geometric approach that treats each discrete axle movement interval in terms of space-oriented coordinates of the terrain and wheel center with respect to a fixed reference frame. The current program will handle up to 100 terrain profile points and up to $2k$ tire segments equally divided about a vertical axis through the axle of each wheel. Experience has indicated that twelve 10-deg segments, six on each side of the vertical, are sufficient to describe the tire compliance for most terrain conditions and obstacle configurations. The angle to each segment center line is measured from the vertical and is considered positive in a counterclockwise direction.

Computer calculations

6. The following sequence of calculations is used in this second program:

a. Dimension the appropriate space for segment angles, segment spring coefficients, terrain profile points, segment forces, etc.

b. Provide a "degree-to-radian" converter = $\pi/180$.

c. Read in: Coordinates of wheel center (X1,Y1), tire radius, number of tire segments, number of terrain points, number of positions to be calculated, and X and Y increments.
d. Check for end of data; if no more data, exit computer.
e. Read in and store all segment angles, spring coefficients, and terrain points.
f. Print headings and terrain profile table.
g. Designate a variable name for the number of tire segments; e.g. NM = NSEG.
h. Determine maximum horizontal projection of tire, i.e.
   \[ X_{RET} = X_1 - WRAD, \; X_{FET} = X_1 + WRAD. \]
i. Initialize ITR = 2.
j. Change all segment angles from degrees to radians.
k. Compute the coordinates of each (undeflected) tire segment
   (i) location with respect to the wheel axle:
      \[
      CX(i) = \sin(DSEG(i) \cdot WRAD) \\
      CY(i) = -\cos(DSEG(i) \cdot WRAD)
      \]
      (These then remain constant for a given wheel.)
l. Set \( VV(1) = 0 \) \( VV(2) = 0. \)
m. Now determine the coordinates of each (undeflected) segment
   with respect to the fixed reference system, e.g.
      \[
      RXY(1,i) = X_1 + CX(i) \\
      RXY(2,i) = Y_1 + CY(i)
      \]
n. At this time compute intersections of the tire segment
   centers with terrain. For each tire segment:
   (1) Set \( DMIN = 1.E10 \), i.e. some large number definitely
      greater than tire radius.
   (2) Compute slope of tire segment (i), e.g. \( SM = CY(i)/CX(i) \).
   (3) \( KTR = ITR \). This is a counter to account for terrain
      stations.
   (4) Set \( X2 = 0 \), i.e. first terrain point begins at \( X = 0 \).
   (5) Recall \( Y2 \) from proper storage location; now there is
      terrain coordinate \( X2,Y2 \).
   (6) Recall \( X3,Y3 \), i.e. the next forward terrain coordinate.
   (7) Obtain the slope of this line (terrain segment).
   (8) Compare it with the slope of \( i^{th} \) tire segment.
   (9) If slopes are not the same, the lines will intersect;
      use the point slope method to obtain coordinate of inter-
      section, i.e. \( XX_i,YY_i \) for \( i^{th} \) tire segment. (This is
coordinate of deflected tire segment with respect to fixed reference system.)

(10) Now determine whether this coordinate falls within the tire circumference.

(11) If not, then increment the terrain segment and recheck as before until a coordinate is determined that falls within the tire circumference.

(12) Compute the radial distance of this intersection from the wheel center and set this distance equal to the variable, DMIN. This represents a new "minimum" length.

(13) Check this length against last minimum value.

(14) If it is smaller, let the coordinates of this deflected tire segment be: \( RXY(1,i) = XX; RXY(2,i) = YY. \)

(15) If, however, it is larger, check to see whether forward edge of tire is ahead of the terrain station.

(16) If so, then advance the station by one. Go back to step (1).

(17) If not, go to next tire segment and repeat the procedures described above until all intersections of tire segment centers with terrain segments have been computed.

This operation should locate the positions (with respect to the fixed and moving reference) of each deflected tire segment.

9. Now compute the actual deflection of each tire segment beginning as follows:

\[
\begin{align*}
RXY(1,i) &= X1 - RXY(1,i) \\
RXY(2,i) &= Y1 - RXY(2,i)
\end{align*}
\]

These are vertical and horizontal components of new segment length.

p. Store these values in a temporary storage AA(1),AA(2).

q. Compute the radial distance of each segment using the Pythagorean Theorem, i.e. \( TMAG(i) = \text{square root of sum of squares of the respective components.} \)

r. Obtain the \( \sin \) and \( \cos \) of each segment angle as follows:

\[
\begin{align*}
RXY(1,i) &= AA(1)/TMAG(i) = \sin \text{ of the segment angle} \\
RXY(2,i) &= AA(2)/TMAG(i) = \cos \text{ of the segment angle}
\end{align*}
\]

s. Compute the radial force in each segment, i.e.
\[ TMAG(i) = (WRAD - TMAG(i)) \times SEGK(i) = K6 \]

where \( \delta = WRAD - TMAG(i) \)

t. Compute the horizontal and vertical components of the resultant force vector, e.g.

\[ VV(1) = ETMAG(i) \times RXV(1,i) \text{ horizontal component} \]
\[ VV(2) = ETMAG(i) \times RXV(2,i) \text{ vertical component} \]

u. Print results.
v. Increment tire center position and repeat procedures.

7. Computation of resultant tire spring force vector is summarized as follows:

a. The points where all tire segment centers (spring locations) intersect the undeflected tire circumference are located.

b. The points where tire segment centers intersect the terrain are located.

c. The radial reduction in length of each tire segment center intersecting the terrain is computed.

d. The force vector for each segment is computed using the displacement of the segment center and the segment spring coefficient.

e. The segment force vectors are summed to yield the vertical and horizontal components of the resultant spring force vector for the wheel.

Notation

8. Symbols used in the second program are defined below:

- \( CX, CY \): Coordinates (with respect to the wheel axle) of points where segment center lines intersect the undeflected tire circumference
- \( DSEG \): Segment angle in degrees; the angle measured from a vertical line through the wheel axle to center line of segment (positive value, counterclockwise)
- \( DTR \): Degree-to-radian conversion factor
- \( HGT \): Elevation of profile
- \( NPOS \): Number of wheel positions to be computed
- \( NSEG, NNN \): Number of tire segments
- \( NTP \): Number of terrain points to be used
RXY Absolute coordinates (with respect to fixed X-Y reference) of the points where segment center lines intersect the undeflected tire circumference

SEGK Segment spring coefficient (can vary with each segment)

ST Slope of a terrain segment

STA Horizontal distance of terrain profile

TMAG Radial force of segment

VVV(1) Horizontal component of force

VVV(2) Vertical component of force

WRAD Wheel radius

XX,YY Absolute coordinates of the intersection points of the terrain with tire segment

X1,Y1 Coordinates of wheel center

X2,Y2 Absolute coordinates of the rear terrain station

X3,Y3 Absolute coordinates of the adjacent terrain station in increasing X-direction

XFET,XRET Forward and rear extremes of tire (horizontal)

XINCR,YINCR Increment of axle movement of wheel in X- and Y-directions

Summary

9. The required input information for the second program consists of (a) the coordinates of the tire center, (b) the undeflected tire radius, (c) the number of segments to be used, (d) the number of terrain coordinates, (e) the number of axle positions to be calculated, (f) the X- and Y-increments (i.e. the movements of the axle), (g) each segment angle in degrees, (h) each segment spring constant (may be different for each segment if desired), and (i) the terrain profile coordinates. The tire-center coordinates are the only inputs that do not remain constant. When the program is used in an appropriate vehicle dynamics program that describes the motions of the sprung masses, these coordinates are determined at each time step by the differential equations that describe the dynamics of the vehicle system. The coordinates (X1,Y1) of the tire center are used to compute forward and rear extremes of the tire for each increment of movement. This is accomplished by subtracting or adding the tire radius (WRAD) to the coordinate X1. (See fig. B4.) The program can then be used to compute
the location of the points, relative to the axle, where each segment center intersects the tire circumference. These values never change and are stored in the computer under the identification CX(i) and CY(i).

10. Then for each increment of horizontal axle movement (XINCR), the program proceeds as follows:

   The points of the wheel at each segment's center line are located relative to the fixed (X-Y) reference frame and stored as coordinates under the identification RXY(1,i) and RXY(2,i). This gives the complete orientation of each undeflected segment "spring." The program then proceeds into a loop that calculates the intersections of the segment center lines and the terrain profile segments, which are constructed from straight-line connections of the terrain profile points. These intersections are determined by the point-slope method, which compares the slopes of each segment center line (proceeding in a counterclockwise sequence beginning with the rear segment) with the slopes of each terrain segment within the forward extreme of the tire. A check is made in each instance to determine if the intersections occur within the periphery of the undeflected tire. This indicates whether a particular segment is influenced by the terrain and the amount that each segment deflects in a radial direction. The radial deflection that is computed for each segment spring is multiplied by the appropriate spring constant to yield a segment force vector whose magnitude and orientation are known. These individual vectors are summed to yield the vertical and horizontal components of the resultant force vector acting at the axle. The axle is then advanced to the next position and the process is repeated.

Damping

11. No provision has been made to incorporate segment damping forces at this time. The influence of tire damping is currently computed as a gross vertical force from the relative motion between the axle and a point directly beneath the axle.
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<thead>
<tr>
<th>Line</th>
<th>Description</th>
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<tr>
<td>1</td>
<td>A PROGRAM TO COMPUTE AVG DEFORMATIONS OF PNEUMATIC TIRE SEGMENTS</td>
</tr>
<tr>
<td>2</td>
<td>TIME(S): AVERAGE</td>
</tr>
<tr>
<td>3</td>
<td>TAN(1) = SIN(1) / COS(1)</td>
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<td>4</td>
<td>READ I, R, H, IDEF</td>
</tr>
<tr>
<td>5</td>
<td>PRINT 159</td>
</tr>
<tr>
<td>6</td>
<td>PRNT 60</td>
</tr>
<tr>
<td>7</td>
<td>R = TIRE RADIUS IN IN., I = NO. OF SEGMENTS FOR 180 DEGREES</td>
</tr>
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<td>8</td>
<td>T = MAX. VERTICAL DEFLECTION AT VRP IN IN. TIMES 10</td>
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<tr>
<td>9</td>
<td>T = R / T</td>
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<tr>
<td>10</td>
<td>T = T * 100.0 / PI</td>
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<td>11</td>
<td>DEL = J/10</td>
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<td>X = SORT(1) - DELTA - DELTA - DELTA</td>
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<td>PHI = ATAN(X)</td>
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<td>15</td>
<td>ARC = ARC OF CONTACT, W = ARC OF ENTIRE SEGMENT</td>
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<td>16</td>
<td>PHI = PHI / T</td>
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<td>IF (I = 1) THEN 5, 6</td>
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<td>ARC = R * PHI</td>
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<td>19</td>
<td>M = NO. OF SEGMENTS INFLUENCED BY DELTA DEFLECTION</td>
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<td>AVGDEL(I) = R / SQRT((M * ARC)/W) - R / R^2 * ((R + DELTA) / R)</td>
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<td>21</td>
<td>AVGDEL(I) IS AVG DEFORMATION OF SEGMENT</td>
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<td>22</td>
<td>IF (I - 1) THEN 15, 16</td>
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<td>23</td>
<td>STOP</td>
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<tr>
<td>24</td>
<td>M = M - 1</td>
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<td>M = M - 1</td>
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<td>26</td>
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<tr>
<td>28</td>
<td>M = M - 1</td>
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<tr>
<td>29</td>
<td>ARC = R * PHI</td>
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<tr>
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Fig. B1. Program for computing average deflections of pneumatic tire segments
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*Note: This table represents the comparison of average deflections for different segments.*
| CL | NO | THETA | SEG1  | SEG2  | SEG3  | SEG4  | SEG5  | SEG6  | SEG7  | SEG8  | SEG9  | SEG10 | SEG11 | SEG12 | SEG13 | SEG14 |
|----|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.60| 1810.00 | 1.116 | 0.796 | 0.052 |
| 1.50| 1810.00 | 1.417 | 0.900 | 0.087 |
| 1.70| 1810.00 | 1.618 | 1.107 | 0.138 |
| 1.90| 1810.00 | 1.819 | 1.314 | 0.312 |
| 2.10| 1810.00 | 2.020 | 1.522 | 0.474 |
| 2.20| 1810.00 | 2.165 | 1.525 | 0.466 |
| 2.30| 1810.00 | 2.221 | 1.729 | 0.663 |
| 2.40| 1810.00 | 2.321 | 1.833 | 0.774 | 0.000 |
| 2.50| 1810.00 | 2.422 | 1.936 | 0.885 | 0.005 |
| 2.60| 1810.00 | 2.522 | 2.040 | 0.992 | 0.017 |
| 2.70| 1810.00 | 2.623 | 2.144 | 1.106 | 0.037 |
| 2.80| 1810.00 | 2.723 | 2.246 | 1.217 | 0.063 |
| 2.90| 1810.00 | 2.824 | 2.341 | 1.327 | 0.097 |
| 3.00| 1810.00 | 2.924 | 2.445 | 1.438 | 0.137 |

Fig. 42. Printout of computations of average deflections
06/30/69 INGRAM INGRAM TC1 04-G2-R0-010 7 FEBRUARY 1969 PAGE # 1 INGR SDL6

$ TITLE INGRAM TC1 04-G2-R0-010 7 FEBRUARY 1969
C THIS CODE WAS TAKEN FROM A VEHICLE DYNAMICS PROGRAM WHICH IS NOW
C BEING USED AT WES, BUT IS INTENDED ONLY TO DEMONSTRATE STATICALLY HOW THE
C PROGRAM PRESENTED HERE CONTAINS NONE OF THE DYNAMICS OF THE
C PROBLEM, BUT IS INTENDED ONLY TO DEMONSTRATE STATICALLY NOW THE
C RESULTANT WHEEL SPRING FORCE IS COMPUTED USING THE SEGMENTED WHEEL
C CONCEPT.
1 DIMENSION DSEG(24), SEK(24), STA(100), HGTr(100), TMAG(24), RX(2, 24)
2 DIMENSION VV(2), AA(2), CX(24), CY(24)
3 FORMAT ( F10.0 )
4 DTR = 3.141592653 / 180.
C THE ANGLE, DSEG1(1), TO EACH SEGMENT CENTER IS MEASURED FROM THE
C VERTICAL, POSITIVE COUNTERCLOCKWISE IN DEGREES.
C UNITS FOR OTHER VARIABLES ARE ARBITRARY, BUT SHOULD BE CONSISTENT.
5 100 READ 1, XI, YI, WRAD, NSEG, NTP, NPOS, XINC, YINC
6 CALL EOFST(50, ITEST)
7 GO TO(120,110), ITEST
8 110 CALL EXIT
9 120 READ 1, ( DSEG1(1), SEK1(1), 1 = 1, NSEG )
10 READ 1, ( STA(1), HGTi(1), 1 = 1, NTP )
11 PRINT 3, HWRAD, NSEG
12 3 FORMAT( 15Ho WHEEL RADIUS = F8.2, //
22Ho NUMBER OF SEGMENTS = I5//)
13 PRINT 4, 1, DSEG1(1), SEK1(1), 1 = 1, NSEG )
14 4 FORMAT ( 16Ho SEGMENT ANGLE TO SPRING /
1 36Ho NUMBER CENTERLINE CONSTANT //
2 ( 15, F15.2, F14.2 ) )
15 PRINT 5, ( STA(1), HGT1(1), 1 = 1, NTP )
16 5 FORMAT ( 16Ho TERRAIN PROFILE /
1 16Ho X /
2 ( 2F8.2 ) )
17 NNN = NSEG
18 XMET = XI - WF.D
19 XFET = XI + WRAD
20 1TR = 2
21 DO 1005 I = 1, NSEG
22 DSEG1(1) = DSEG1(1) + DTR
23 CX(I) = SIN(DSEG1(I)) * WRAD
24 CY(I) = COS(DSEG1(I)) * WRAD
25 1005 CONTINUE
26 PRINT 10
27 10 FORMAT( 50Ho LOCATION OF WHEEL CENTER RESULTANT FORCES /
1 7X, 1X, 1X, 1X, 1X, 1H, 1X, 1X, 1H, 1X, 1H )
28 DO 200 1 = 1, NPOS
29 VV(1) = 0.
30 VV(2) = 0.
31 DO 325 I = 1, NNN
32 RXY(1, I) = XI + CX(I)
33 RXY(2, I) = YI + CY(I)
34 325 CONTINUE
35 C ... INTERSECT SEGMENT CENTERS WITH TERRAIN
36 DO 335 I = 1, NNN
37 DMN = XI, E10
38 IF( DMN < CX(I) ) GO TO 39
39 C .... IMPOSSIBLE FOR WHEEL TO CONTACT TERRAIN
40 XXY = XXY + 0.0
41 335 CONTINUE
42 C ... PERPENDICULAR DISTANCE OF NODE FROM TERRAIN
43 DO 345 I = 1, NNN
44 DXY = DXY + 2.0
45 345 CONTINUE
46 C ... TOTAL IMPACT ENERGY
47 C 1/2 * WRT * WRAD
48 C 1/2 * VV * VV
49 C 1/2 * RXY * RXY
50 C 1/2 * CXY * CXY
51 C .. TOTAL IMPACT ENERGY = 1/2 * WRT * WRAD
52 C .. + 1/2 * VV * VV
53 C .. + 1/2 * RXY * RXY
54 C .. + 1/2 * CXY * CXY
55 C 1/2 * WRT * WRAD
56 C 1/2 * VV * VV
57 C 1/2 * RXY * RXY
58 C 1/2 * CXY * CXY
59 CONTINUE
60 CALL EOFST(60, ITEST)
61 GO TO(120,110), ITEST
62 CALL EXIT

33 HORT1=CY(1)*V1+CY(1) 769
34 5325 CONTINUE
35 C ..,INTERSECT SEGMENT CENTERS WITH TERRAIN 770
36 DO 5350 J=1,NNN 771
37 DMN=M/10 771
38 SM=CY(1)/CX(1).
39 KTR=1TR
40 X2=0.
41 5329 CONTINUE
42 X3=STA(KTR)-STA(ITR-1)
43 Y2=HGT(KTR)
44 SY=(Y2-Y3)/(X2-X3)
45 IF(SM=ST)5331,5330,5331
46 5330 XX=1,E35
47 GO TO 5333
48 5331 XX=(Y2-Y1)+SM*X1-ST*X2)/(SM-ST)
49 5333 YY=ST*(XX-X2)+Y2
50 KERR=KB7N(X2,XX,X3)
51 GO TO 5335,5335,5336,KERR
52 KERS=KB7N(Y2,YY,Y3)
53 GO TO 5335,5336,5337,KERS
54 KRS=KB7N(X1,XX,RY(2,1))
55 GO TO 5339,5339,5339,KERU
56 KERF=KB7N(Y1,YY,RX(2,1))
57 GO TO 5340,5339,5339,KERU
58 5336 IF(XFET=STA(KTR))5350,5336,5336
59 5338 KTR=KTR+1
60 X2=X3
61 Y2=Y3
62 GO TO 5329
63 5346 DMN=SQRT((XX-X1)**2*(YY-Y1)**2)
64 IF(DMN=DMIN)5341,5341,5336
65 5341 DMN=DMIN
66 RXY(1,1)=XX
67 RXY(2,1)=YY
68 GO TO 5355
69 5350 CONTINUE
70 5395 CONTINUE
71 DO 5356 J=1,NNN
72 RXY(1,1)=RXY(1,1)
73 RXY(2,1)=RXY(2,1)
74 5356 CONTINUE
75 C ..,NORMALIZE RXY AND LET TMAG BE MAGNITUDE
76 DO 5357 J=1,NNN
77 AA(1)=RXY(1,1)
78 AA(2)=RXY(2,1)
79 TMAG(1)=SQRT((AA(1)**2+AA(2)**2)**2)
80 RXY(1,1)=AA(1)/TMAG(1)
81 RXY(2,1)=AA(2)/TMAG(1)
82 5397 CONTINUE
83 C ..,COMPUTE FORCE FOR EACH SEGMENT
84 DO 5358 J=1,NNN
85 TMAG(1)=WRAD+TMAG(1)*SEGK(1)
86 5358 CONTINUE
87 C ..,COMPUTE COMPOSITE FORCE VECTOR
88 DO 5359 J=1,NNN
89 VVV(1)=VVV(1)+TMAG(1)*RXY(1,1)
90 VVV(2)=VVV(2)+TMAG(1)*RXY(2,1)
91 5359 CONTINUE
92 P5NT2,X1,Y1,VVV
93 200 CONTINUE
94 GO TO 100
95 END
DO 5356 T=1,NNN
RXY(1,1)=X2+RXY(1,1)
RXY(2,1)=Y1+RXY(2,1)
CONTINUE

C  ...NORMALIZE RXY AND LET TMAG BE MAGNITUDE
DO 5357 T=1,NNN
AA(1)=RXY(2,1)
AA(2)=RXY(1,1)
TMAG(I)=SORT((AA(1)*AA(1)*AA(2)*AA(2))
RXY(I,1)=AA(1)/TMAG(I)
RXY(I,2)=AA(2)/TMAG(I)
CONTINUE

C  ...COMPUTE FORCE FOR EACH SEGMENT
DO 5358 T=1,NNN
TMAG(I)=SUM(TMAG(I)+ SECK(I))
CONTINUE

C  ...COMPUTE COMPOSITE FORCE VECTOR
DO 5359 T=1,NNN
VVV(I,1)=VVV(1,1)+TMAG(I)*RXY(I,1)
VVV(I,2)=VVV(2,1)+TMAG(I)*RXY(I,2)
CONTINUE

PRINT 2,XY,XYY
2 FORMAT(2F10.2,10X,2F10.2)
Y1*Y1+YINC
Y2*Y2+YINC
CONTINUE
GO TO 100
END

FUNCTION KBWKN(A,P,B)
FI(A>P)35,40,20
C
20 IF(B>P)45,40,50
10 P BETWEEN A AND B
30 IF(B>P)50,40,40
C
VALID RETURN
40 KBWKN = 1
C
RETURN
C
0 RETURN
50 KBWKN = 2
9 END

Fig. 68. Program for computing vertical and horizontal components of the resultant force vector transmitted to the axle.
Fig. 3. Schemaic diagram depicting space coordinates of digital segmented fire program.
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**ABSTRACT**

This appendix presents the procedures for digital implementation of the segmented tire model, developed in the basic report for an analog computer. Two procedures are required: (a) determination of the segment spring coefficient from measured load-deflection test results, and (b) computation of the resultant force vector transmitted to the axle. Digital programs for both procedures were written in Fortran IV for a 36-600 system, and are included. ( )
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