TO:
Approved for public release; distribution is unlimited. Document partially illegible.

FROM:
Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; JUN 1967. Other requests shall be referred to Naval Postgraduate School, Code 023, Monterey CA 93940. Document partially illegible. This document contains export-controlled technical data.

AUTHORIZED
usnps ltr, 21 jan 1972
SOME TWO-COMPONENT BETA DECAY MEASUREMENTS

by

Clark Tilton Ballard, Jr.
Captain, United States Army
B.S., United States Military Academy, 1963

and

James Robert Tichenor, III
Captain, United States Army
B.S., United States Military Academy, 1960

This document is subject to special export controls and each transmittal to foreign government or foreign nationals may be made only with prior approval of the U. S. Naval Postgraduate School.

(code 023) Monterey, Calif. 93940
Best Available Copy
SOME TWO-COMPONENT BETA DECAY MEASUREMENTS

by

Clark Tilton Ballard, Jr.
Captain, United States Army
B.S., United States Military Academy, 1963

and

James Robert Tichenor, III
Captain, United States Army
B.S., United States Military Academy, 1960

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL

June 1967

Signature of Authors:

Approved by

Thesis Advisor

Chairman, Department of Physics

Academic Dean
ABSTRACT

The shorter half-life of a two-component beta decay was determined. Natural silver was activated with thermal neutrons to produce isotopes Ag$^{108}$ and Ag$^{110}$. The shorter half-life decay (that of Ag$^{110}$) was isolated from that of Ag$^{108}$ by measuring the decay of only the higher energy betas associated with Ag$^{110}$. Because a large portion of the energy spectrum was excluded, pile-up corrections were necessary. The half-life was also determined using the more conventional two-component method. The single-component method yielded a half-life of $23.26 \pm 0.03$ seconds compared with $23.43 \pm 0.04$ seconds determined by the two-component method. (Only statistical uncertainties are shown above.) The techniques used for making pile-up corrections are described, and systematic errors are discussed. This investigation indicates that accurate half-life determinations can be obtained with the single-component method.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>7</td>
</tr>
<tr>
<td>II. Experimental Results</td>
<td>13</td>
</tr>
<tr>
<td>III. Experimental Details</td>
<td>16</td>
</tr>
<tr>
<td>IV. Analysis of Data</td>
<td>23</td>
</tr>
<tr>
<td>V. Equipment</td>
<td>31</td>
</tr>
<tr>
<td>VI. Determination of File-Up Corrections</td>
<td>45</td>
</tr>
<tr>
<td>VII. Conclusions and Recommendations</td>
<td>56</td>
</tr>
<tr>
<td><strong>Appendix</strong></td>
<td></td>
</tr>
<tr>
<td>1. SPECTRUM</td>
<td>60</td>
</tr>
<tr>
<td>2. ADDUP</td>
<td>62</td>
</tr>
</tbody>
</table>
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simplified Silver Decay Schemes</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Beta Detector Assembly</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>Rabbit (Side View)</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>Center Section of Pneumatic System</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>Pneumatic Tube (Top View)</td>
<td>38</td>
</tr>
<tr>
<td>6</td>
<td>Automatic Timing and Control Circuit</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>Cs(^{137}) Beta Spectrum Used in Pile-up Time Determination</td>
<td>44</td>
</tr>
<tr>
<td>8</td>
<td>Silver Beta Spectrum</td>
<td>52</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The authors wish to express their appreciation to their thesis advisor, Professor G. W. Rodeback, whose assistance in this undertaking was invaluable. Appreciation is also extended to Mr. R. C. Moeller, Mr. M. K. Andrews, Mr. H. L. McFarland, Professor W. W. Hawes, the authors' wives, and to the many others who assisted in the several facets of the project.
I. INTRODUCTION.

A conventional method for determining the half-life of one beta decay nuclide in the presence of another nuclide with an appreciable longer beta decay half-life, is to record the composite decay counts and to mathematically separate the two decay modes, thus yielding the half-lives of both modes. Some of the limitations on the statistical accuracy of the short half-life determined by the above method can be avoided if it is feasible to eliminate the longer half-life component from the decay counts. In this investigation the shorter half-life component had the higher of the two beta energies. By detecting only those betas above the maximum long-lived beta energy, a separation of the short-lived component was effected. However, even those high-energy betas initially detected comprise only a small fraction of the total composite beta spectrum.

In spite of the smaller counting rate (compared with detection of the entire composite spectrum) when detecting only the high-energy betas, the resulting statistical accuracy of the short half-life determined in this manner can be somewhat better than that of the short half-life determined in the conventional manner from the composite decay. However, there is no advantage in this single-component method when seeking an accurate half-life value, unless other possible errors are properly accounted for. Consequently,
this investigation is primarily concerned with properly accounting for the systematic errors associated with the above described single-component method of measuring the shorter half-life.

The short half-life was also determined by the more conventional method of detecting and analyzing the two-component decay. Agreement between the two methods thus constitutes a demonstration of the validity of the single-component technique.

A natural silver sample containing isotopes 107 and 109 was irradiated with thermal neutrons in the core of an AGN 201 Reactor and rapidly transferred by means of a pneumatic transfer system\(^1\) to the detector site. The beta particles emitted in the decay of the activated sample were detected using a Europium doped Calcium Fluoride scintillation crystal with photomultiplier tubes and associated electronics. The decay counts were stored in a multichannel scaler.

When activated by thermal neutrons, natural silver gives rise to two radioactive isotopes, (108 and 110), which decay with maximum beta particle energies of 1.65 MeV and 2.87 MeV respectively. (See Figure 1 for simplified decay schemes.) The currently published half-lives of these decays are \(2.42 \pm .02\) minutes for Ag\(^{108}\) and \(2.5 \pm .3\) seconds for Ag\(^{110}\)\(^2\).

The single-component method of measurement consisted
of counting only those pulses from the amplifier correspond-
ing to beta particle energies of 1.9 MeV or greater. This
was accomplished by pulse height discrimination using a
single-channel pulse-height analyzer. In this way, only beta
particles emitted by Ag\textsuperscript{110}, the short half-life isotope, were
selected for the multichannel scaler. The relatively high
discriminator setting on the pulse-height analyzer used in the
measurement above made it necessary to correct the data
for pile-up. Pile-up results when two beta particles are
detected by the scintillation counter within a time called
the pile-up time. The amplifier circuit is unable to resolve
the two incoming pulses in this time and its output is a
single pulse equal to the sum of the two incoming pulses.
The magnitude of this new pulse may now be of sufficient
amplitude to exceed the discriminator level and be erroneously
counted as a higher energy pulse. Three steps were
employed in order to determine the pile-up correction. First,
the effective pile-up time was calculated using measurements
on Cs\textsuperscript{137} sources of varying strengths. Second, pulse-
height spectrum measurements were made on silver from
which it was possible to deduce the composite beta spectrum
of the silver sample as a function of time. Third, the
time varying spectrum was used along with the pile-up time
to determine the number of pile-up counts which occurred
during a given time interval. These counts were then
subtracted from the raw decay counts before final analysis was performed.

The second more conventional method was used to measure the decay of Ag\textsuperscript{110} for the purpose of demonstrating the validity of the previous method. In this method the discriminator of the pulse-height analyzer was set at a minimum value so that beta particles of both isotopes were selected for counting. Pile-up corrections, although far less significant than in the first method, were also applied to this data.

Each set of corrected data was then least-squares fitted to an appropriately assumed mode of decay. The calculations were carried out by the FRANTIC computer program. Average values of the half-life of Ag\textsuperscript{110} determined by the two methods were consistent to within 0.7%. The disagreement is believed to be due to systematic errors to be discussed in Section IV.

In order to perform this experiment certain modifications of existing equipment were necessary. An air-tight, light-tight beta detection assembly built around the CaF\textsubscript{2}(Eu) crystal was installed permitting the uncovered lower face of the crystal to be placed directly over the activated sample and accommodate the pneumatically controlled sample holder (rabbit). A method was also devised to yield positive indication that the sample was in the same location in the
reactor core for each activation. The above modifications and others described in Section III resulted in efficient beta particle detection. Also decay counts were recorded with a minimum loss of time following activation and with a maximum reproducibility of activation for different data runs.
II. EXPERIMENTAL RESULTS.

The values of the half-lives and their statistical estimators of standard deviation determined by FRANTIC are given below. These estimators are purely statistical in nature and do not account for the much larger systematic errors discussed in Sections IV, V, and VI. The average values of half-life are weighted averages. The methods for determination of these averages and the final estimator of standard deviation for each average are explained in reference (3).

FRANTIC OUTPUT

One Component Runs

<table>
<thead>
<tr>
<th>Run</th>
<th>Half-Life Ag$^{110}$ (seconds)</th>
<th>Estimator of Standard Deviation (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.188</td>
<td>.064</td>
</tr>
<tr>
<td>2</td>
<td>23.187</td>
<td>.059</td>
</tr>
<tr>
<td>3</td>
<td>23.450</td>
<td>.103</td>
</tr>
<tr>
<td>4</td>
<td>23.243</td>
<td>.077</td>
</tr>
<tr>
<td>5</td>
<td>23.378</td>
<td>.112</td>
</tr>
<tr>
<td>6</td>
<td>23.430</td>
<td>.095</td>
</tr>
<tr>
<td>Average</td>
<td>23.26</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Two Components Runs

<table>
<thead>
<tr>
<th>Run</th>
<th>Half-Life Ag$^{110}$ (seconds)</th>
<th>Estimator of Standard Deviation (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>23.381</td>
<td>.109</td>
</tr>
<tr>
<td>8</td>
<td>23.479</td>
<td>.089</td>
</tr>
<tr>
<td>9</td>
<td>23.473</td>
<td>.085</td>
</tr>
<tr>
<td>10</td>
<td>23.487</td>
<td>.079</td>
</tr>
<tr>
<td>11</td>
<td>23.295</td>
<td>.077</td>
</tr>
<tr>
<td>12</td>
<td>23.462</td>
<td>.085</td>
</tr>
<tr>
<td>Average</td>
<td>23.43</td>
<td>.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Half-Life Ag$^{108}$ (seconds)</th>
<th>Estimator of Standard Deviation (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>141.452</td>
<td>.410</td>
</tr>
<tr>
<td>8</td>
<td>141.255</td>
<td>.323</td>
</tr>
<tr>
<td>9</td>
<td>141.361</td>
<td>.299</td>
</tr>
<tr>
<td>10</td>
<td>141.256</td>
<td>.271</td>
</tr>
<tr>
<td>11</td>
<td>141.317</td>
<td>.277</td>
</tr>
<tr>
<td>12</td>
<td>142.128</td>
<td>.310</td>
</tr>
<tr>
<td>Average</td>
<td>141.45</td>
<td>.13</td>
</tr>
</tbody>
</table>

Chi-square Test

The Chi-square test was applied to each of the above runs substituting the product of the degrees-of-freedom and the weighted-variance-of-fit as given by FRANTIC, for Chi-square. A confidence level of 0.05 was chosen, and run 7 was discarded from further consideration based on this criterion. The averages listed do not include this run.

References (3) and (4) are recommended for a more detailed discussion of "goodness-of-fit" as applied to this investigation.

Average Half-Lives

One Component  Ag$^{110}$  $T_{1/2} = 23.26 \pm .03$ sec.
Two Component $^{110}\text{Ag}$ $T_{1/2} = 23.43 \pm 0.04 \text{ sec.}$

Two Component $^{108}\text{Ag}$ $T_{1/2} = 141.45 \pm 0.13 \text{ sec.}$
III. EXPERIMENTAL DETAILS.

EQUIPMENT AND PROCEDURES

By means of a high speed pneumatic transfer system, a rabbit composed of balsa wood and polyethylene, onto which the silver sample was fastened, was transported from the counting area to the center of the reactor core, a region of high neutron flux. In this region the sample was activated with a slow neutron flux of $2.8 \times 10^7$ neutrons/cm$^2$-sec. for a period of two minutes with a reactor power level of 0.63 watts. (Two minutes is the time which will optimize the activity of the short-lived Ag$^{110}$ with respect to that of the longer-lived Ag$^{108}$.) The sample was then forced out of the reactor core by a positive pressure and seated under the detector assembly.

The detector assembly consisted of a 1/8 inch thick, 2 inch diameter, CaF$_2$(Eu) scintillation crystal optically coupled with an EMI/US 9536B photomultiplier tube. The output of the photomultiplier tube was fed to a preamplifier, followed by an amplifier and a single-channel analyzer. The output of the analyzer was then fed to a multichannel scaler.

In addition to the single-channel analyzer (described above) which selected the pulses for decay counting, there was a second single-channel analyzer paralleling the first. The discriminator of this second analyzer was set at a
minimum value which corresponded to 0.35 MeV energy. The recording of counts in the scaler which followed this analyzer was initiated at the same instant the rabbit was fired from the reactor. The total counts recorded by this scaler for a standard interval of time constituted a measure of the source strength for a particular irradiation of the silver sample. Thus, the relative source strengths of different irradiations of the silver sample were accurately known in terms of these “monitor” readings.

ENERGY CALIBRATION

Prior to each run the pulse height spectrum of a Cs\(^{137}\) source was recorded. The spectrum was recorded in 256 channels of the pulse-height analyzer. The high voltage power supply was subsequently adjusted, if necessary, so that the conversion electron peak (taken approximately as 0.64 MeV) appearing in the cesium spectrum, was centered in channel 45 of the pulse-height analyzer. A typical Cs\(^{137}\) spectrum is shown in Figure 7 in Section VI. A discussion of the spectrum is also included in Section VI.

A second check of the energy calibration at a higher energy was made by utilizing the end point of the 1.65 MeV beta spectrum of Ag\(^{108}\). A secondary silver sample was activated by slow neutrons and after a 4 minute wait, so that Ag\(^{110}\) had decayed to a negligible level, a pulse-height spectrum was recorded. A proper energy calibration required
that the beta spectrum end point of 1.65 MeV appear approximately at channel 120. These two pulse-height spectral measurements were made to ensure reproducibility of electronic settings for each decay run.

DECAY MEASUREMENTS

Two methods were used to measure the decay of the silver sample. In the first method a high discriminator setting on the single-channel pulse-height analyzer was used so that only those higher energy pulses greater than 1.9 MeV from the short-lived component, would be recorded. In the second method a much lower discriminator setting was used so that both the short and long half-life components were recorded. A total of twelve data runs were made, six for each method, with the same sample being used in all runs.

Single-Component

With this method, the integral discriminator dial of the single-channel pulse-height analyzer was set at a reading which corresponded to an energy of 1.9 MeV, which is well above the maximum long-lived beta energy of 1.65 MeV. The raw counts were recorded in 128 channels of a multichannel scaler set for two seconds per channel. Counting began at the instant the rabbit was fired from the reactor core. However, because the rabbit was not seated for the full period of the first two seconds of counting, the data in the first channel was not used in the analysis. The rabbit's
transit time is estimated to be a small fraction of a second. Monitor readings were recorded for these decay runs.

**Double-Component**

With this method the integral discriminator dial was set at a minimum which corresponded to 0.35 MeV energy. Both the 1.65 MeV and 2.87 MeV beta components were counted and recorded in 512 channels of the multichannel scaler with two seconds per channel. Monitor readings were also recorded for these decay runs.

**CORRECTION MEASUREMENTS**

**Pile-up Time Measurements**

Four Cs$^{137}$ sources of widely differing strengths were prepared for these measurements. These sources were mounted on aluminum planchets which were held in place beneath the beta detector by a specially designed source holder. The gain of the detector system was adjusted so that essentially the upper limit of the Cs$^{137}$ spectrum came at channel 130 out of a total of 256 channels of the pulse-height analyzer. (See Section VI for a discussion of the spectrum.) The pulse-height spectrum was determined using the weakest cesium source so that the dead time losses in the pulse-height analyzer would be minimized. The losses during this measurement were $(3.0 \pm 1.0)\%$. The pile-up effect for each of the sources was measured by a single-
channel analyzer whose discriminator setting corresponded
to channel number 140 on the pulse-height spectrum.
Monitor counts to determine relative source strengths were
taken for all four sources using a minimum discriminator
setting. See Section VI for a detailed discussion of the
use of the measurements taken in this section.

Pile-up Correction Measurements

In order to apply the pile-up corrections, the compo-
site beta energy spectrum of silver had to be known as a
function of time. To accomplish this, two measurements
of the silver beta energy spectrum were taken. For the
first measurement, the silver sample was irradiated for
2 minutes at 0.1 watts, and a pulse-height spectrum was
immediately recorded using 256 channels of the pulse-height
analyzer. The lower power (compared to the power for a
decay measurement) was used to reduce pulse-height analyzer
dead time losses. For the second measurement, the silver
sample was irradiated for 2 minutes at 0.63 watts, the
same as for a regular decay measurement, and a pulse-
height spectrum was recorded beginning 4 minutes after the
end of irradiation. This enabled the short-lived component
to decay for 10 half-lives, and, therefore, allowed us to
determine the spectrum of the longer-lived Ag$^{109}$. The
usual monitor readings at a minimum discriminator setting
were taken for one minute on both runs. A more detailed discussion of the use of these measurements is given in Section VI.

Dead Time Measurements

The two-source method was used to determine the dead time of the multichannel scaler (See Section IV for the equation used.) The gamma radiation from two approximately equal Cs\(^{137}\) sources was detected by a scintillation counter with a NaI(Tl) crystal. The amplifier output pulses were very nearly identical in shape and time duration to those from the CaF\(_2\)(Eu) beta detector used in measuring the silver decay. Therefore, it was assumed that the dead time of the multichannel scaler determined by the gamma pulses would differ little from a comparable determination using the beta detector.

Background

Background measurements were made prior to each decay run, using the same integral discriminator setting as for the decay run.

MISCELLANEOUS MEASUREMENTS

Activation of a Blank Rabbit

A rabbit, similar in every respect to the one containing the silver sample, but without any silver attached, was irradiated in a manner identical to that for the silver decay runs. The resulting activity from this blank rabbit
recorded for the discriminator settings used in the two types of regular runs was determined to be negligible compared to that of the rabbit with the silver sample.

Check on Timing Interval Calibration of Multichannel Scaler

To check the accuracy of the two second interval per channel of the multichannel scaler, a 10 KC signal from a crystal-controlled oscillator with a rated output specification of 100KC ± one cycle/second at 50°C was fed into the multichannel scaler and counted. Based on this measurement, the time interval was found to be in error by less than 0.005%.
IV. ANALYSIS OF DATA.

PRINCIPAL CALCULATIONS

Pile-Up

If two beta particles are detected by the scintillation counter within a time called the pile-up time, the amplifying circuit is unable to resolve the resultant pulses and therefore produces a single pulse whose height is equal to the sum of the two individual pulse heights. For the decay measurements utilizing an integral discriminator setting, two pulses normally not counted may combine into a single pulse which contributes an extra output pulse from the discriminator.

Pile-up is the predominant error occurring in the single-component decay measurement since the discriminator setting is such that initially, approximately 90% of all pulses in the silver spectrum are rejected. An appreciable portion of those pulses which are counted may arise from pile-up between pulses which normally would be below the discriminator setting. Descriptions of the techniques for determining the pile-up time and corrections to the decay data are given in detail in Section VI. Corrections for pile-up were applied to the compiled raw data points by the use of the computer program, ADDUP, described in Appendix 2.
Dead Time

The dead time loss most significantly affects the two-component decay measurement. The dead time for the equipment used in the decay measurements was determined by the two-source method as described in Section II. The following formula, which is accurate to within one per cent for counting rates typical in this measurement, was used to calculate the dead time:

\[ \tau_d = \frac{\hat{n}/2}{\hat{n}_1 \hat{n}_2 - \hat{n}_{12} \hat{n}_b} \]

\[ \hat{n} = \hat{n}_1 + \hat{n}_2 - \hat{n}_{12} - \hat{n}_b \]

\[ \hat{n}_1 = \text{counting rate of source 1} \]

\[ \hat{n}_2 = \text{counting rate of source 2} \]

\[ \hat{n}_{12} = \text{counting rate of sources 1 & 2} \]

\[ \hat{n}_b = \text{background counting rate} \]

Two sets of data were taken, and the resulting average dead time was determined to be:

\[ \tau_d = (2.5 \pm .2) \times 10^{-6} \text{ seconds} \]

where the uncertainty quoted is based only on the statistical uncertainties of the measurements. The calculation of dead time corrections to the decay data was carried out by the FRANTIC computer program. The dead time correction was applied to input data already corrected for pile-up. For th types of decay runs, the maximum error in a final
corrected decay-data point introduced by this procedure is estimated to be less than 0.05%.

Final Half-Life Calculation

The FRANTIC Program for Analysis of Exponential Growth and Decay Curves\(^4\) was used exclusively for the dead time correction of data and determination of half-lives. This program takes raw decay counts as input and fits the counting rates (corrected for dead time) by least squares analysis to an assumed mode of decay which is in general the sum of exponential decays. The output of the program includes the computed half-lives and their standard deviations, an analysis of the deviations of the data points, and a computed value of Chi-square. The results are listed in Section II.

Background

The background counting rate was measured prior to commencement of each run. This background rate was then normally applied as an input to the FRANTIC program. However, the assumption of the invariance of the background during the decay of the sample was tested for all of the two-component runs by allowing FRANTIC to compute the background as a component. In all cases the measured background was within the statistical uncertainty of the computed background. These results verified the original assumption that the decay consisted of a constant component and two
exponentially decaying components. A similar analysis of background invariance was not carried out for the one-component runs since the measured background counting rates were all considered to be negligible.

**ERROR ANALYSIS**

Possible sources of error include instabilities in electronic components (including the detector), impurities in the sample, Least Squares Analysis (FRANTIC), statistical errors in counts, statistical and systematic errors in pile-up and dead times, background and timing uncertainties, and the number of data points used in the analysis.

**Impurities in the Sample**

The silver used as a sample was 99.999% pure when procured. However, when being rolled into a strip for attachment to the rabbit, there is a possibility that impurities were picked up. Although the effect on beta-active impurities is believed to be negligible, gamma-active impurities may partially account for the disagreement in the results of the two methods and for their disagreement with the published values. The efficiency of the scintillation counter for detecting gammas is somewhat higher than the usual beta detector because of the thickness ($1/8''$) of the CaF$_2$(Eu) crystal. FRANTIC is incapable of resolving the effect of a small impurity such as this.
Least Squares Analysis (FRANTIC)

There is a possibility of an error inherent in the FRANTIC program although this seems unlikely. Statistical weighting of the data points was used in the FRANTIC Least Squares Analysis, which is normal practice. One might suspect that this would lead to a different result from unit weighting, for example. However, past experience indicates that the type of weighting used in the least squares analysis has little effect on the half-life results. (Private communication from Rodeback, G.W.)

Pile-Up Time and Dead Time

The statistical error associated with the measurement of the dead time (8%) was found to correspond to 0.1% error in the Ag\textsuperscript{110} half-life for the one-component runs and 0.2% error in this half-life for the two-component runs. Because the dead time was determined using a gamma detector (Section VI), there is a possibility of a systematic error in the dead time of the order of 10%.

The statistical and systematic errors associated with the measurements for pile-up time should be less than the statistical error for the dead time. A possible systematic error of 20% has been assumed for the pile-up time (See Section VI). This corresponds to about a 0.3% change in the Ag\textsuperscript{110} half-life as determined by the one-component runs.
Background and Timing Uncertainties

The effect of uncertainty in background was analyzed by computing half-lives with FRANTIC with input values of background rates 50% and 150% of the measured background rates. The effect on the half-life value was found to be negligible for both of the background values.

The uncertainty in the timing interval is known to be less than one part in 20,000 from the 10 KC calibration measurement. When this uncertainty was introduced into the FRANTIC input the final effect on the half-life value was negligible.

Number of Data Points

The sensitivity of the short half-life and its standard deviation as computed by FRANTIC, to the number of input data-points was tested. The ADDUP program was used to compile original raw-count-data into a specified number of input points. These input points always applied to the same overall time interval of measurement. The run with the smallest standard deviation for the shorter half-life was selected from the single and from the double-component decays, to be analyzed in this manner.

For the single-component decay 57, 36, 31, 28, 23 and 11 input data-points were used. A tendency toward a minimum standard deviation of the half-life was noticed in the vicinity of 30 points although the total variation in the
half-life value was less than 0.1%. In the double-component decay analysis 250, 125, 100, 50 and 25 points were used. There was no apparent trend in the standard deviation of the shorter half-life, and the observed variation in half-life value was even less than in the single-component runs.

It was concluded that the FRANTIC program is not sensitive to the number of good input data-points, the total of which cover the same total time interval. The timing interval of eight seconds per data-point (corresponding to 28 points for single-component runs and 125 points for double-component runs) that was chosen as the final data input to FRANTIC, resulted in half-life determinations whose standard deviations were not appreciably different from the optimum attainable (purely on the basis of statistical uncertainties).

Starting Time for Input Data

It was desirable to ascertain the effect of deleting some of the initial data-points on the various half-life determinations. For single-component runs the average value of the short half-life increased by .07 seconds when a starting time of $t=20$ seconds was used instead of $t=2$ seconds. For double-component runs the average value of the short half-life increased .04 seconds and the average value of the long half-life decreased .01 seconds when a starting time of $t=20$ seconds was used instead of $t=2$. 
It was concluded that the actual starting time used had little effect on the resulting half-life determinations. A starting time of $t=20$ seconds was used for the calculation of the half-lives reported in Section II.

Instabilities in Electronic Components

The photomultiplier tube (EMI/US-9536B) used in this investigation was especially chosen for its excellent stability with respect to large changes in count rate. Special measurements in the past with a comparable EMI tube and comparable electronic equipment have indicated negligible overall gain shift in the detector system during the period of time required for decay runs. For this investigation checks on the energy calibration following a series of decay runs indicated no appreciable changes from the initial energy calibration made prior to decay runs.
V. EQUIPMENT.

The equipment just external to the reactor shield, existing prior to the start of this investigation, was designed primarily for the detection of gammas from short-lived nuclides. Consequently, it was necessary to design and build a new beta detector assembly and a modified pneumatic transfer system in order to measure short-lived beta activity. Some new control features for the pneumatic system were also incorporated into the modifications. And an electronic, automatic control circuit for operation of the pneumatic transfer system was successfully bench tested.

BETA DETECTOR ASSEMBLY

The beta detector assembly is located outside the reactor at the end of the pneumatic transfer tube. This assembly (see Figure 2) was designed and constructed to provide a means for securing a CaF$_2$(Eu) scintillation crystal to the photomultiplier tube and so that the scintillation counter was placed directly above the termination of the pneumatic transfer tube. Reference (7) gives detailed specifications of the CaF$_2$(Eu) crystal. The aluminum shield and clamping devices shown in the figure are joined to each other, and the lower clamping device to the base (in Figure 2) by threaded joints. These clamping devices and shield serve three main purposes:
Figure 2. BETA DETECTOR ASSEMBLY
1. They clamp the photomultiplier tube, Lucite light pipe, and crystal together to provide good optical coupling.

2. They secure the entire assembly to the base. There is an "O" ring in the base around the opening in the pneumatic tube which bears on the crystal and provides a pressure seal for the system.

3. They act as light shields to ensure that the entire assembly is light-tight.

RABBIT

The previously existing rabbit specifications are explained by Littie (5). This existing design had to be modified for two reasons.

1. The activated silver sample was directly exposed to the CaF₂(Eu) crystal to eliminate unwanted absorption of emitted beta particles.

2. The rabbit had to incorporate a spike on the rear for mating with the center section of the pneumatic transfer system which will be discussed below.

The first requirement was met by rolling the silver sample into a band 3 mils thick and 3/32" wide. This band was then fastened (Duro Cement) flush with the rabbit surface so that it completely encircled the mid-portion.
See Figure 3 for a side view of the rabbit. The second requirement was met by affixing a polyethylene base to the rabbit with "Duco Cement." This base has a spike which is slightly larger in diameter than the inside diameter of the nylon grip described below. See Figure 3 for a side view of the spiked base on the rabbit.

CENTER SECTION

As previously mentioned some method had to be devised to ensure that the rabbit was held in the same position in the maximum flux region of the reactor core for each run. Further, a positive indication that the rabbit was in position was required. These requirements were realized in the present design of the center section of the pneumatic system. (See Figure 4)

That part of the pneumatic system which is inside the reactor and shield consists of two aluminum pipes and the center section. The assembly of the two pipes and center section is inserted into the gloryhole liner which is an aluminum pipe with an inside diameter of 15/16 inch. The front pipe of the pneumatic system which extends from the reactor shield to the center section has an outside diameter of 7/8 inch. The back pipe of the system which extends from the reactor shield to the center section has an outside diameter of 3/4 inch. This back pipe is insulated from the glory-hole liner by polyethylene rings. The nylon divider
Figure 4. CENTER SECTION OF PNEUMATIC SYSTEM
forms the bulk of the device, serves as a base, and insulates the front and back pipes from one another. It is threaded and fitted with "O" rings on both ends so as to form an air-tight, rigid, and insulated junction between the two aluminum pipes of the pneumatic system. The nylon grip consisting of 4 flexible fingers is screwed as a subassembly into the divider. The inside diameter of the grip receptical is slightly smaller than the outside diameter of the rabbit spike. Thin aluminum wires lead to the outer portions of the fingers of the nylon grip and lead through the divider to contact points which connect with the rear aluminum pipe.

When the rabbit is fired into the reactor core the polyethylene spike on the rear of the rabbit becomes securely lodged in the flexible grip. At the same time the electrical contacts in the grip are forced outward and contact is made with the front transfer pipe. This completes the circuit shown in the figure and a positive indication is given on the ohmmeter that the rabbit is located at the maximum flux position of the reactor core.

**SOURCE HOLDER**

An aluminum source holder was designed and constructed. This was used for reproducible positioning beneath the detector of various Cs$^{137}$ sources used in the experiment.
Figure 5. PNEUMATIC TUBE (Top View)
CONTROL OF PNEUMATIC SYSTEM

A complete cycle of the rabbit is best described with the aid of Figure 5. The rabbit is placed into the pneumatic tube manually at the counting position by temporarily removing a brass seat assembly at the forward end of the tube. The valves shown are manually operated by switches. The rear-vacuum valve which evacuates from the rear portion of the tube is opened. The return-air valve is then opened so that atmospheric pressure forces the rabbit into the center section of the system. After about one second, the return-air switch is released, closing the return-air valve. Next, the front-vacuum valve is opened (Note: rear-vacuum valve is still open) to vacuum, and the entire tube is now more completely evacuated. A few seconds prior to the end of the predetermined activation time the two vacuum valves are closed. Then at the end of the predetermined activation time the "fire" valve at the rear of the tube is opened, releasing an air pressure of 15 pounds per square inch against the rear of the rabbit. This forces the rabbit back into the counting position within a small fraction of a second. The "fire" switch which opens the "fire" valve also has other functions, as described below. After the rabbit is seated in the counting position, a bleeder valve may be momentarily opened reducing the pressure in the tube to atmospheric.
ADDITIONAL ELECTRICAL SYSTEM MODIFICATIONS

The number of functions of some of the switches used in the electrical control system for the solenoid valves described above were increased. The corresponding modifications resulted in an accurate and reproducible timing of activation of the sample, together with a precise initiation of two pulse counting circuits. An electric timer was actuated by the momentarily closed air-return solenoid switch through a holding relay. Thus, the timer was started simultaneously with the entrance of the rabbit into the reactor core. This timer was used to time sample activation. After the predetermined activation time, the "fire" switch was manually thrown for about one second. This switch opened a high-pressure solenoid valve at the rear of the pneumatic system. The high-pressure air then forced the rabbit out of the core to the detector assembly. Two other functions were also carried out by the "fire" switch:

1. The monitor scaler and associated timer were actuated by the "fire" switch, by means of another holding relay. This caused monitor counting to begin simultaneously with removal of the rabbit from the core.

2. The momentary closing of the "fire" switch was also used to initiate the operation of the multi-channel scaler by means of a special pulse-forming
circuit. Thus, the multichannel scaler was started at the instant the rabbit was removed from the reactor core.

AUTOMATIC TIMING AND CONTROL CIRCUIT

The control circuit shown in Figure 6 was designed to automatically remove the rabbit from the reactor after a predetermined activation time. The circuit was designed and a prototype was constructed, and bench tested. This control system was considered feasible but was not incorporated into the pneumatic transfer system.

The control circuit was designed around Fairchild integrated circuits including: the 90029 buffer, the 91429 two-input gate, the 92329 flip-flop, and the 92729 inverter. These circuit elements are shown on the circuit diagram as 900, 914, 923, and 927 respectively.

The input to the control circuit can be any pulse with relatively sharp leading and trailing edges. For this application a 10 KC signal was fed into a 4 decade scaler. An output pulse rate of one pulse per second was fed from the scaler, into the control circuit. The input is only effective under the clear condition of the counter (upper row of 923's) which would exist only if the rabbit is in the grip of the center section (Figure 4).

The desired sample activation time is determined in seconds, and this time, minus one second, (called the set
count) is set in binary numbers on the five flip-flop circuits in the upper half of the diagram. (The time base could be increased from one second in powers of 2 by adding flip-flops between the input and the first flip-flop of the diagram. The present activation time capability of 32 seconds could be extended by adding flip-flops and gates to the left of the top line of 923's.) On the set count the vacuum circuit flip-flop experiences a falling voltage and cuts transistor Q1 off which in turn closes the vacuum relays and solenoids, sealing the system under a vacuum. On the set count plus one the flip-flop of the fire circuit experiences a falling voltage and, hence, forces transistor Q2 into saturation opening the fire relay and solenoid. The rabbit is then forced by compressed air cut of the nylon grip, at which time all elements are preset returning Q1 to saturation and Q2 to cutoff. The circuit is now prepared for another cycle.\(^8\).
Figure 7. \( ^{137} \text{Cs} \) BETA SPECTRUM USED IN PILE-UP TIME DETERMINATION

\[ \bar{N} \text{ (Counts/Sec/Channel)} \]

versus

ENERGY (Channel No.)
VI. DETERMINATION OF PILE-UP CORRECTIONS.

DETERMINATION OF PILE-UP TIME

The effective pile-up time of the circuitry employed in recording the beta decay of silver was determined with the use of the beta activity of \(^{137}\text{Cs}\) sources. The strengths of the four sources used were such that the counting rates ranged from approximately 400 c/sec to 40,000 c/sec. The discussion which follows describes the measurements which were made and the analysis that was used.

Consider Figure 7, which is the observed pulse-height spectrum (using the multi-channel analyzer) obtained from the weakest of the above-mentioned sources. This spectrum is also assumed to represent the shape which would be obtained for any of the \(^{137}\text{Cs}\) sources used.

In what follows, channel numbers will sometimes be used to specify the energy or energy ranges of pulses in various regions of the spectrum. The spectrum peak, somewhat below channel 130, originates from the K and L conversion electrons, as well as from the 0.657 MeV gamma pulses from the \(^{137}\text{Ba}\) decay, the latter being detected by the relatively thick (for betas) \(\text{CaF}_2(\text{Eu})\) scintillation crystal.

Although the spectrum has almost negligible amplitude above channel 130, for sufficiently strong \(^{137}\text{Cs}\) sources
an integral discriminator setting corresponding to channel 140 will yield a measurable counting rate. This counting rate is assumed to consist of three components: pile-up pulses produced principally by pulses between channels 10 and 130, background pulses of greater amplitude than channel 140, and beta particles from the high-energy portion of the relatively weak intensity 1.17 MeV beta transition of Cs$^{137}$.

For a given Cs$^{137}$ source let $\bar{n}$ represent the total counting rate in the interval of the pulse height spectrum from channels 10-130; and let $\bar{m}$ be the experimentally determined counting rate with an integral discriminator setting corresponding to channel 140. The three contributions to $\bar{m}$ are now represented by:

\[ \bar{m} = \bar{m}^p + \bar{m}^b + \bar{m}^c \]

where $\bar{m}^p$ is the pile-up time which is to be determined, $B$ is the ratio of the portion of the 1.17 MeV beta spectrum counting rate lying above channel 140 to $\bar{n}$, and $C$ is the observed background counting rate above channel 140. $B$ can also be determined from the analysis. The first term on the right side of the equation thus represents the pile-up counting rate above channel 140.

*Five percent of the total Cs$^{137}$ beta decays are contributed by this high-energy beta.*
In brief, all the quantities except $T_p$ and $B$ in equation (1) are measured or independently calculated ($F$ is a factor that is calculated with the use of the spectrum). Therefore, by taking measurements using two or more sources that give appreciable pile-up counts, these two quantities, $T_p$ and $B$, can be calculated using equation (1).

**DETERMINATION OF $F$**

Assuming that the given Cs$^{137}$ spectrum (Figure 1) applies for a given source strength, the pile-up counting rate for an integral discriminator setting corresponding to channel 140 can be calculated in terms of $T_p$. If the amplitude of the spectrum is assumed to be negligible from channels 130 to 140, then only pulses within the interval of channels 10 to 130 contribute to this pile-up rate.

In the following derivation the pulse energy (i.e., the abscissa of Figure 1) is treated as a continuous variable. The ordinate variable will be called $N (E')$, the counting rate per unit energy interval of the spectrum (lying between channels 10 and 130), the pulses of which are capable of yielding pile-up pulses. Thus $N (E') dE'$ represents the counting rate of those pulses lying between $E'$ and $E' + dE'$. $N (E'') dE''$ represents the counting rate of those pulses lying between $E''$ and $E'' + dE''$.

$T_p$ is defined to be the time interval during which it is possible for two different pulses to combine into a single...
pulse whose height is the sum of the two individual pulse
heights. Now, consider a small finite time interval $\Delta t \gg \tau_p$;$\text{\(N(E')dE'\Delta t\)}$ is the number of pulses from $dE'$ in time $\Delta t$, and $\tau_pN(E')dE'\Delta t$ is the total time available for pile-up during $\Delta t$.

Thus $\tau_pN(E')dE'\Delta t \int [N(E'')dE'']$ is the total number of pile-up counts between the two energy intervals during the time $\Delta t$ and, $\tau_pN(E')dE'\int N(E'')dE''$ is the average pile-up count rate over the interval $\Delta t$.

Thus for a given interval, from $E'$ to $E'+dE'$ the total pile-up rate is seen to be: $\tau_pN(E')dE' \int_{E'='10}^{130} N(E'') dE''$, and since $E'$ may take values from 10 to 130, the overall pile-up count rate is: $\tau_p \int_{10}^{130} N(E') \int_{E'='10}^{130} N(E'') dE'' dE'$. Or the overall pile-up counting rate is $\tau_pS$ where $S$ is the above double integral. Thus, the pile-up counting rate can be expressed as $\tau_p \left( \frac{S}{\hat{n}} \right) \hat{n}^2$ where $\hat{n} = \int_{10}^{130} N(E')dE'$.

But a consideration of $N(E')$ for various source strengths indicates that the corresponding values of $S$ are proportional to the source strength squared. Consequently, if the shape of the spectrum is assumed, as a first approximation, to be independent of the source strength, $\frac{S}{\hat{n}^2}$ should be constant. Accordingly the quantity $F$ is defined to be $\frac{S}{\hat{n}^2}$ where $\hat{n}_1$ is the counting rate of the weakest source over the interval 10 to 130, and $S$ is the corresponding double
integral. File-up effects for the weakest source are assumed to be negligible.

To determine $F$, the double integral $S$ was evaluated graphically with the use of a planimeter. The value for $n_1$, as obtained from the measured spectrum, was correlated with a simultaneous scaler measurement taken from 24 to 130.

**DETERMINATION OF $\gamma_p$**

The determination of $n$ in equation (1) for the other three sources was based upon a direct comparison of their activity with that of the weakest source. This was done simply by using an integral discriminator setting corresponding to channel 24 and by measuring the counting rates for each of the four sources. The above counting rates were then corrected for the dead time of the scaler that was used.

The discriminator setting corresponding to channel 24 was sufficiently high that the above counting rates were in error due to pile-up that is contributed by the portion of the spectrum below channel 24. An analysis of this effect (similar to that used to determine the principle pile-up term in equation (1)) resulted in converting equation (1) into a quadratic equation in $\gamma_p$ where $n$ was replaced by the corresponding counting rate uncorrected for pile-up. Observed data for the two strongest sources then led to
a quadratic equation in $\tau_p$ with $B$ eliminated. An accurate approximate solution of this equation yielded a value of $\tau_p = 0.28 \mu$ sec. The effect of pile-up in determining the true source strengths was to increase the "uncorrected" pile-up time by 4%.

The consistency of the entire set of data was checked by using equation (1) with the above value of pile-up time, to predict $m$ for the two weakest sources. The agreement with the measured values of $m$ was excellent; therefore, the determination of pile-up time should be reliable. The principle uncertainty undoubtedly rests with the assumption that the factor $F$ is constant for all source strengths. The uncertainty in $\tau_p$ due to this assumption is arbitrarily assumed to be 20%.

**DETERMINATION OF THE COMPOSITE SILVER PULSE-HEIGHT SPECTRUM AS A FUNCTION OF TIME**

In order to calculate the pile-up corrections for each of the two types of decay runs as a function of time, it was necessary to deduce the composite silver pulse-height spectrum as a function of time. This was done by measuring the pulse-height spectrum in two different ways. The first spectrum was obtained by activating the silver sample in the identical manner used in regular decay runs. However,

$B$ was found to be $3.87 \times 10^{-4}$. 

50
counts were not recorded on the pulse-height analyzer until after a four minute delay from the termination of activation. This delay constituted ten of the short half-lives; therefore, most of the $^{110}\text{Ag}$ activity had died out.* Therefore, the resulting spectrum was principally that of the longer-lived (2.4 min.) $^{108}\text{Ag}$. The counts recorded on the pulse-height analyzer were corrected for analyzer dead time. The initial counting rate (due to $^{108}\text{Ag}$) for a given channel (corresponding to the end of the activation period) was computed using the 2.4 minute half-life and the corrected counts for the given channel.

The second spectrum was recorded immediately following activation. However, in order to decrease the effect of analyzer dead time, the neutron flux was 16% of the flux used in the regular decay runs. The channel counts for this run were corrected for dead time and normalized to the first pulse-height spectrum run described above. The normalization was accomplished by use of the monitor counts which were recorded in each case.

The normalized counts in a particular channel now represented counts from the composite spectrum. The counts for this same channel obtained from the long half-life

* Actually, approximately $\frac{1}{2}$ of the activity 4 minutes after activation was due to $^{110}\text{Ag}$.  

51
Figure 8. Counting Rate vs. Channel No. for Silver Beta Spectrum.

3. "..."
spectrum were now subtracted from the above normalized counts; the result represented the total counts arising from the short-lived component ($\text{Ag}^{110}$). Using the published value of the short half-life and the total counts arising from $\text{Ag}^{110}$, the initial counting rate (due to $\text{Ag}^{110}$) for this channel was then calculated. The two initial counting rates, calculated as indicated above, were then used to determine the overall counting rate for the $i$th channel versus time by using the following relation:

$$\hat{n}_i = \hat{n}_{os_i} e^{-\lambda_s t} + \hat{n}_{ol_i} e^{-\lambda_l t}$$

where $\hat{n}_{os_i}$ and $\hat{n}_{ol_i}$ are the initial counting rates for the short and long-lived components calculated as described above. $\lambda_s$ and $\lambda_l$ are the decay constants for the short and long half-lives. $\hat{n}_i$ is the predicted counting rate for the $i$th channel at the time, $t$.

The calculations of the two initial channel counting rates, as well as the channel counting rates versus time were made with the SPECTRUM computer program. (See Appendix 1 for SPECTRUM.) These spectra were plotted (See Figure 8) and used as described in the following section in the calculation of pile-up corrections.
CALCULATION OF PILE-UP CORRECTIONS

By using the formula expressing counting rate of any channel versus time for the silver pulse-height spectrum, the spectrum can be constructed for any instant of time following the end of silver activation. (See Figure 7 showing the silver spectrum at three different times.) If $E$ represents the integral discriminator setting used for recording decay counts, then in a manner similar to that used in the previous section (Determination of Pile-Up Time), the pile-up count rate recorded at this instant of time should be:

$$\int_{E}^{E'} N(E) \left[ \int_{E}^{E'} N(E') \, dE' \right] \, dE' = \mathcal{I}_p S$$

where $N(E')$ represents the counts/sec-energy interval of the silver pulse-height spectrum at the given time, at the energy $E'$.

For the single-component decay runs, $E$ corresponded to channel 130, and for the double-component runs $E$ corresponded to channel 24. By plotting the composite silver spectrum for the times $t = 0, 40, 80, 120, 160, 200$, and $240$ seconds, the quantity $S$ was computed. (Integration was performed with a planimeter) for each type of run (i.e., for both values of $E$) at each of the
above times after activation. With the value of $T_p$ as determined previously, the pile-up count rate was calculated for each of the above times for both types of decay runs.

For a given type of run, a plot of the pile-up count rate versus time was made using the above results. Values of pile-up counts/interval corresponding to the mid-points of the timing intervals used in the decay measurements were taken from the resulting curve. The corrections were then applied to the raw data according to the expression:

$$\frac{\text{Corrected counts/interval}}{\text{Raw counts/interval}} = \frac{\text{Monitor Counts}}{(\text{Monitor Standard})^2} \left( \frac{\text{Monitor Standard}}{\text{Monitor Counts}} \right) (\text{Pile-up counts/interval})$$

where Monitor Counts are those counts taken during a particular measurement as described in Section III, and the Monitor Standard refers to the monitor counts taken during the first silver beta spectrum measurements (obtained by activating the silver in a manner identical to that of the regular decay runs).

The two-second raw data-points were compiled in groups of four giving an eight second timing interval per input point to FRANTIC. The pile-up correction described above was then applied to the compiled data. Compilation and corrections were performed by the ADDUP computer program (see Appendix 2). ADDUP also prepared other input cards necessary for the FRANTIC program.
VII. CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

The most likely sources of systematic error in the investigation are the dead time, pile-up time, sample impurity, and spurious pulses in the detection system from electrical pick-up. An increase in the pile-up time of the order of 20% would bring the average values of the $^{110}\text{Ag}$ half-life determined by the two methods, to within 0.3% of perfect agreement. An increase in the dead time would also bring the values into closer agreement. It, therefore, seems reasonable that much of the disagreement between the two methods could easily be accounted for by systematic errors in the pile-up time and the dead time.

Some of the disagreement between the calculated half-life values and the published values of 24.5 seconds and 2.4 minutes (2) could be explained by the presence of impurities in the sample. If the disagreement was caused by impurities (which are most likely gamma-active), then it would appear that there was at least one impurity whose half-life is less than 24 seconds and another whose half-life lies between 24 seconds and 2.4 minutes. If the disagreement was caused by spurious pulses, the rate of these spurious pulses must be dependent on the counting rate entering the detection system.

Although systematic errors of the types mentioned above may account for the disagreement with published
values of the half-lives, the agreement between the two methods for finding the shorter half-life, indicates the validity of the single-component method. This is further supported by the fact that neither the measured short or long half-lives (determined by either the single or the double-component methods) were altered appreciably by changing the start of the first input data point to FRANTIC from 2 to 20 seconds after the end of activation (see Section IV). It is therefore concluded that this method of half-life determination, as demonstrated in this investigation, is valid and is inherently capable of yielding high-accuracy results.

RECOMMENDATIONS

The following recommendations are presented as possible areas for improvement so that this investigation might be carried on to more fruitful results:

1. A new detector base should be designed which will allow removal of the rabbit without turning off the high voltage supply to the photomultiplier tube. The detector system is extremely sensitive to the photomultiplier high voltage, therefore, this possible source of error should be eliminated before further investigations are made.

2. The automatic timing and control circuit should be built and put into operation.
3. A thorough check of the electronic equipment should be made and if necessary the isolation of this equipment from outside sources of interference should be effected.

4. An even more detailed calculation of the pile-up time should be carried out. This investigation would use $^{137}\text{Cs}$ sources, as was done here, but would take the counting-rate dependence of the spectral shape into account. This more detailed calculation would reduce the systematic error in the pile-up time.

5. A more accurate determination of dead time should be conducted using two beta sources. This would require careful design of sources and source holder.

6. An independent examination of the sample purity could be conducted using spectroscopic analysis. If this is not feasible, a direct investigation should be made to determine possible beta and gamma contaminating activities in the sample.
BIBLIOGRAPHY


PROGRAM SPECTRUM
0DIMENSION R1(500),R2(500),R3(500),R4(500),R5(500),R6(500),CL(500)
1CT(500),L(500),M(500),CH(500),TAU(500),RG(500),DT(500),DL(500)
T0 = 0.0
T1 = 4.0
T2 = 8.0
T3 = 12.0
T4 = 16.0
T5 = 20.0
T6 = 24.0
READ 10*(M(I),I=1,86)
10 FORMAT (1U18)
READ 1*(CL(I),I=1,86)
READ 1*(DT(I),I=1,86)
1 FORMAT (10F8.0)
100 READ 5*X*Y
5 FORMAT (2F8.3)
DO 2 I=1,86
L(I) = I
CT(I) = X*CT(I)
CL(I) = Y*CL(I)
0R1(I) = ((0.050100*CL(I))*EXP(-0.00481*T0)) + ((0.16240*CT(I)) -
1(0.1026*CL(I))*EXP(-0.00481*T0))
0R1(I) = ((0.050100*CL(I))*EXP(-0.00481*T1)) + ((0.16240*CT(I)) -
1(0.1026*CL(I))*EXP(-0.00481*T0))
0R2(I) = ((0.050100*CL(I))*EXP(-0.00481*T2)) + ((0.16240*CT(I)) -
1(0.1026*CL(I))*EXP(-0.00481*T2))
0R3(I) = ((0.050100*CL(I))*EXP(-0.00481*T3)) + ((0.16240*CT(I)) -
1(0.1026*CL(I))*EXP(-0.00481*T3))
0R4(I) = ((0.050100*CL(I))*EXP(-0.00481*T4)) + ((0.16240*CT(I)) -
1(0.1026*CL(I))*EXP(-0.00481*T4))
0R5(I) = ((0.050100*CL(I))*EXP(-0.00481*T5)) + ((0.16240*CT(I)) -
1 \{0.1025^2 \cdot \gamma \} \exp (-0.0289 \cdot T5)
20R611 = \{0.050100 \cdot CL \cdot I \} \exp (-0.00401 \cdot T6) + \{0.16240 \cdot CT(I) -
1 \{0.1026 \cdot CL(I) \} \exp (-0.0289 \cdot T6)
PRINT 11 \cdot X \cdot Y
11 FORMAT (1H1+20x+3HX =F8.3+5X+3HY =F8.3)
PRINT 3
30FORMAT (1H0+93HPT CH RATE(0) RATE(40) RATE(80) RATE(1)
120) RATE(160) RATE(200) RATE(240) //
0PRINT 4+((L(I)+M(I))*R0(I)+R1(I)+R2(I)+R3(I)+R4(I)+R5(I)+R6(I)+M(I)
1+I=1,06)
40FORMAT (1X+12+1X+13+2X+E10+4+2X+E1 +4+2X+E10+4+2X+E10+4+2X+E10+4+2
1X+E10+4+2X+E10+4+2X+I3)
GO TO 100
END
END


APPENDIX A

PROGRAM ADDOP
DIMENSION DATA(1000),X(1000),Y(1000)
READ 1
1 FORMAT (216)
READ 1(X(1)),Y(1)
1 FORMAT (L106+1)
10C READ 2,NPTS
2 FORMAT (18)
READ 3,C10
3 FORMAT (F6.6,16)
PRINT 4,N10
4 FORMAT (1H12HN0 OF GRP=1,J=6,X=12NPTS=1 IN GRP=137)
PUNCH 21,10
21 FORMAT (52HAG HALF-LIFE DETERMINATION DATA IDENTIFICATION CG, 14,
124X)
IF (NPTS=200) 40,42,42
40 PUNCH 41,1D+M
41 FORMAT (3HAG 14,3X=14,6H=3 26DX)
42 PUNCH 11,1D+M
11 FORMAT (3HAG 14,3X=14,3CH 3+3 136X)
PUNCH 14
14 FORMAT (7GH=000000+0G+25CCCO-06+25
100+1000LC+01CX)
.MT=2
IF (MT=10) 190,20,20
10 PUNCH 9,1D+MT
9G TO 50
9 FORMAT (13,13H=000000+0G+11,10HG
2G TO 50
2G FORMAT (13,13H=000000+0G+12,9HGG
50 READ 7,(DATA(J),J=1,NPTS)
7 FORMAT (F6.6,8FY+0)
D = C/1203757.0
DO 6 I=1*M
Y(I) = 1.18*X(I)
SUB = 0.0
L=N*I
K = L - N + 1
DO 5 J=K,L
5 SUB = SUB + DATA(J)
6 SUM(I) = SUB - (((Y(I))**(D**2))
PRINT 8*(SUM(I)*I=1*M)
PUNCH 8*(SUM(I)*I=1*M)
8 FORMAT (10F8.0)
GO TO 100
END
END
FINIS
-EXECUTE.
The shorter half-life of a two-component beta decay was determined. Natural silver was activated with thermal neutrons to produce isotopes $^{109}$Ag and $^{110}$Ag. The shorter half-life decay (that of $^{110}$Ag) was isolated from that of $^{109}$Ag by measuring the decay of only the higher energy betas associated with $^{110}$Ag. Because a large portion of the energy spectrum was excluded, pile-up corrections were necessary. The half-life was also determined using the more conventional two-component method. The single-component method yielded a half-life of $23.26 \pm 0.03$ seconds compared with $23.43 \pm 0.04$ seconds determined by the two-component method. (Only statistical uncertainties are shown above.) The techniques used for making pile-up corrections are described, and systematic errors are discussed. This investigation indicates that accurate half-life determinations can be obtained with the single-component method.
Silver

Half-life of silver 110

Beta Decay

Two-component Decay