TECHNICAL REPORT NO. 1 ON PHOTOELECTRON PROCESSES

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I. INTRODUCTION

Theoretical ionospheric models based on first principles require the electron and ion temperatures, especially in the region above the F2 peak where the altitude distribution of the electron density can be in part specified by a diffusing plasma with a characteristic scale height. Although basic processes involved in the heating of the ambient electrons by photoelectrons are known, the numerical techniques currently available use different approximations and in general do not give identical results from identical input data. Differences are greatest above 300 km where photoelectron transport and non-local heating become important.

Numerical methods which include most of the important processes have been developed by Nisbet and Swartz (Nisbet, 1968; Swartz, 1972), Banks and Nagy (1970a,b), Cicerone and Bowhill (1971), and Stolarski (1972). These approaches are outlined below.

In the Nisbet-Swartz method photoelectron diffusion equations with an additional loss term to remove escaping fast electrons are solved at discrete energy levels. This differential equation is given in Table I. The pitch angle distribution of the escape flux can be calculated, but only if the photoelectron number densities were previously computed.

Banks and Nagy use a two-stream approximation, based on the differential equations of up and down flux components given in Table I. As with the diffusion coefficient in the Nisbet-Swartz approach, the pitch angle distribution of the flux must be known a priori for the factor \( \langle \mu \rangle \). The method has the advantage of being able to assign a direction to the energy cascaded component.

Stolarski (1972) developed a very good "non-real world" analytical method for testing other photoelectron flux calculations. Unfortunately, this method
does not allow energy cascaded input terms, but does include pitch angle effects resulting in non-isotropic fluxes. His solutions for isotropic scattering and production are given in Table I.

In principle the Monte Carlo technique developed by Cicerone and Bowhill can include all effects and all processes of importance. In practice a huge number of samples is required to reduce the statistical fluctuations, and even very fast computers can only give approximate results in reasonable execution times.

II. AMBIENT ELECTRON HEATING

Of importance to calculations of the ambient electron heating rates is the photoelectron number densities at each energy and height, which can be obtained by each of the methods outlined above (See Table II). Figures 1 and 2 (from Cicerone et al., 1973), show comparisons of the densities as obtained by several of the methods. In general the agreement is quite good below 400 km. Hence, the heating of the ambient electrons and other photoelectron excitation processes will also agree below 400 km. For this "local" region, Swartz and Nisbet (1972) have developed a means of specifying the ambient electron heating. It should be stressed that these heating efficiencies are only for the local heating region, and cannot be extrapolated to higher altitudes.

III. PHOTOELECTRON FLUXES

Figure 3 shows a comparison of the photoelectron fluxes (also from Cicerone et al., 1973) and indicates flux differences on the order of a factor of two. At the start of this comparison no effort was made at standardizing any of the subroutines and some differences in the neutral model and cross sections were encountered. Even some major programming errors were found. The calculations for this comparison were made independently by each of the authors of the
various methods. The work at Cornell has been undertaken to resolve these discrepancies and to develop means of generating non-local heating efficiencies suitable for use in ionospheric modeling. Currently each of the programs has been put in standard test forms with interchangeable subroutines. Stolarski's method has been programmed in a general form to handle a large number of streams and some calculations have been performed for the 20-stream case. One result to date is that the diffusion calculation of Table III (Nisbet, 1968) is correct below the altitude where $\frac{dn_{PE}}{dZ} = 0$, while the Banks-Nagy fluxes appear to go numerically unstable in this region. Non-isotropic fluxes above this altitude appear to violate current assumptions in both the diffusion and 2-stream programs.

Since the current programs for actual ionospheric conditions do not provide for changes in the pitch angle distribution, a new generalized program has been written. Test runs have given the same results as Stolarski's analytic method. Basically, the approach is an extension of Nisbet's (1968) escape probability method as modified by Swartz (1972) but without the diffusion calculations. There is also a similarity with the Monte Carlo method in that iterations are performed for each elastic collision, but use of the distribution functions eliminates the statistical fluctuations of typical Monte Carlo results.

The number of elastic collisions occurring at a given altitude, $Z$, is

$$n_{PE}v_E \sum_j n_j(Z)\sigma_{ej}$$

where

- $n_{pe}$ = photoelectron number density
- $v_E$ = photoelectron velocity
- $n_j$ = neutral number density
- $\sigma_{ej}$ = elastic cross section for jth neutral.
From a given production profile \( q_o(Z) \), the number of photoelectrons coming from \( Z' \) and encountering their first elastic collision at altitude \( Z \) is

\[
q_1(Z) = \int_0^Z q_o(Z') P_c(Z,Z') dZ' + \int_Z^\infty q_o(Z') P_c(Z,Z') dZ'
\]

where \( P_c(Z,Z') \) is the probability of encountering an elastic collision after traveling the distance \( Z' \) to \( Z \), averaged over the scattering function.

From the \( q_1(Z) \) profile the process may be repeated, until \( q_n(Z) \ll q(Z) \).

Since the probability of encountering an elastic collision given a collision occurred is on the order of 0.7, ten iterations will be within 3\% of the true result, twenty iterations the error is less than 1/10 of one percent.

The summing sequence is simple enough that the required iterations can be performed in times comparable to numerically solving the second order differentiation equations of previous less general methods.

Finally, the photoelectron number density is given by

\[
n_{\text{PE}}(Z) = \frac{\sum_{j=1}^{N} q_j(Z)}{v_E \sum_j n_j(Z) \sigma_{ej}}
\]

Once the density profile is obtained, the net flux is given by

\[
\phi_{\text{net}}(Z) = \int_0^{90} g(\omega) \left\{ \int_0^Z \left[ q_o(Z') + v_E n_{\text{PE}}(Z') \sum_j n_j(Z') \sigma_{ej} \right] \exp - \sum_j \frac{(\sigma_e + \sigma_{a})}{\cos \alpha} \int_0^Z n_j(X) dX dZ' \right\} d\omega
\]

\[
- \int_Z^\infty \left[ q_o(Z') + v_E n_{\text{PE}}(Z') \sum_j n_j(Z') \sigma_{ej} \right] \exp - \sum_j \frac{(\sigma_e + \sigma_{a})}{\cos \alpha} \int_0^Z n_j(X) dX dZ' \right\} d\omega
\]

where \( \sigma_{ej} \) and \( \sigma_{aj} \) are the elastic and inelastic collision cross sections, respectively, \( g(\omega) \) is the scattering function and \( \alpha \) the pitch angle.

Several features of this generalized technique are still being tested, and more specific details will be included in future reports. Preliminary
Implications of current work suggest that the older photoelectron flux programs underestimated the escape fluxes by factors of two to four. Further comparisons are in progress.
The problem of calculating electron and ion temperatures for theoretical ionospheric models is complicated by photoelectron transport (i.e. "non-local" effects) in the upper F2 region. Since former computer programs which restrict the flux distribution function do not provide consistent fluxes, a new generalized method was developed, and is currently being tested (See Section III) with very gratifying results.

Temperatures for the "local" region can be generated from the photoelectron heating efficiency of Swartz and Nisbet (1972) (See Section II). Once the flux problems are solved, it is hoped that this simple efficiency concept can be extended to the "non-local" region.
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BASIC PHOTOELECTRON EQUATIONS

Modified
DIFFUSION

\[ \frac{d\phi}{dz} = q\left[1 - P_{es}\right] - \nu n[\sigma_e P_{es} + \sigma_a] \eta \]

Nisbet (1968)

Swartz and Nisbet (1972)

\[
\begin{align*}
\langle \mu \rangle \sin \frac{d\phi^+}{dz} &= \frac{q}{2} + q^+ - \phi^+ n[\sigma_e P_e + \sigma_a] + \phi^- n \sigma_e P_e \\
-\langle \mu \rangle \sin \frac{d\phi^-}{dz} &= \frac{q}{2} + q^- - \phi^- n[\sigma_e P_e + \sigma_a] + \phi^+ n \sigma_e P_e
\end{align*}
\]

Banks and Nagy (1970, 1971)

\[
\phi(\pm \mu) = \frac{I \sigma_1}{4\pi(\sigma_e + \sigma_a)} \sum_{a=1}^{N} \frac{L_{a \alpha} e^{-k_a \tau}}{1 + \mu k_a} + \frac{\Gamma_{e^{-\gamma \tau}}}{1 + \mu \gamma}
\]

Stolarski (1972)

2N-STREAM

\[
\phi(-\mu) = \frac{I \sigma_1}{4\pi(\sigma_e + \sigma_a)} \sum_{a=1}^{N} \frac{L_{a \alpha} e^{-k_a \tau} - e^{-\tau/\mu}}{1 - \mu k_a} + \frac{\Gamma_{e^{-\gamma \tau} - e^{-\tau/\mu}}}{1 - \mu \gamma}
\]

Table I.
CALCULATION OF PHOTOELECTRON NUMBER DENSITIES

LOCAL : \[ \eta_{PE} = \frac{q}{v \ n \ \sigma_a} \]

DIFFUSION : \[ \eta_{PE} = \frac{q}{v \ n \ \sigma_a} - \frac{v \cdot (\phi_d)}{v \ n \ \sigma_a} \]

Nisbet (1968) and
Swartz and Nisbet (1972)

DIFFUSION Modified by \[ \eta_{PE} = \frac{q[1-P_{es}]}{v \ n[\sigma_a + \sigma_e P_{es}]} \]

ESCAPE :

From TWO-STREAM : \[ \eta_{PE} = \frac{1}{v} \ [\phi^+ + \phi^-] \]

Banks and Nagy (1970, 1971)

From N-STREAM : \[ \eta_{PE} = \frac{1}{v} \sum_{j=1}^{N} a_j [\phi(u_j) + \phi(-u_j)] \]

Stolarski (1972)

Table II.
CALCULATION OF PHOTOELECTRON FLUXES

(Along $\bar{B}$)

**DIFFUSION:**

$$\dot{\phi}_d = -\frac{v \sin I}{3 \pi (\sigma_e + \sigma_a)} \cdot \frac{d \eta_{PE}}{dz}$$

Nisbet (1968)

**ESCAPE:**

$$\dot{\phi}_{es} = \int_{0}^{\sigma_e} \left[ q + \eta_{PE} (\sigma_e + \sigma_a) \right] \rho_{es} \, dz$$

Swartz and Nisbet (1972)

From

**TWO-STREAM:**

$$\phi = \langle u \rangle [\phi^+ - \phi^-]$$

Banks and Nagy (1970, 1971)

From

**N-STREAM:**

$$\phi = \sum_{j=1}^{N} a_j \nu_j [\phi(\nu_j) - \phi(-\nu_j)]$$

Stolarski (1972)

Table III.
\( \zeta = 0^\circ \)

\( T_\infty = 1114K \)

580km

- C-B
- N-B
- N-S