SURVIVABILITY OF THE FIVE-INCH GUN LAUNCHED FINNED MOTOR CASE

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Survivability of the Five-Inch Gun Launched Finned Motor Case

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Survivability curves are presented for case thicknesses of 0.09 in., 0.10 in., 0.12 in., 0.15 in., and 0.19 in. and a maximum launch acceleration of 7,000 g's. The 0.19 in. case under 6,000 g's is also examined. Both bonded and unbonded propellants are considered. The results indicate that only the 0.19 in. case with unbonded propellant and subjected to 6,000 g's will survive the launch. The 0.19 in. case with bonded propellant under 6,000 g's is marginal as is the same case with unbonded propellant under 7,000 g's. All cases with a thickness less than 0.19 in. will not survive a launch of 7,000 g's.
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<td>gun-launched projectiles</td>
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ABSTRACT:

The structural integrity of the motor case of a five-inch gun launched projectile is examined as a function of case thickness and maximum launch acceleration. The analysis techniques developed in a previous study of the buckling failure of a three-inch projectile motor case are applied to the five-inch case. Survivability curves are presented for case thicknesses of 0.09 in., 0.10 in., 0.12 in., 0.15 in., and 0.19 in. and a maximum launch acceleration of 7,000 g's. The 0.19 in. case under 6,000 g's is also examined. Both bonded and unbonded propellants are considered. The results indicate that only the 0.19 in. case with unbonded propellant and subjected to 6,000 g's will survive the launch. The 0.19 in. case with bonded propellant under 6,000 g's is marginal as is the same case with unbonded propellant under 7,000 g's. All cases with a thickness less than 0.19 in. will not survive a launch of 7,000 g's.
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NOTATION

\( a_{\text{max}} \) maximum rigid body acceleration of the projectile

\( E \) elastic modulus in tension and compression

\( G \) modulus of rigidity

\( g \) acceleration due to gravity

\( h \) thickness of the motor case

\( L \) length of the motor case

\( P_b \) internal axial force at the base of the motor case with bonded propellant

\( P_o \) maximum external force acting on the base of the projectile

\( P_{ub} \) internal axial force at the base of the motor case with unbonded propellant

\( r \) radius of the motor case

\( t \) time

\( \alpha \) duration of the breech pressure

\( \nu \) Poisson’s ratio

\( \rho \) mass density of the motor case material

\( \sigma_b \) axial stress due to \( P_b \), i.e., the axial stress at the base of the motor case with the bonded propellant

\( \sigma_{cr} \) elastic buckling stress of a cylinder with a uniform axial stress

\( \sigma_{cr}/\eta \) if the buckling is elastic \( \eta = 1 \), if inelastic \( \eta < 1 \)

\( (\sigma_{cr})_{\text{inelastic}} \) inelastic buckling stress of a cylinder with a uniform axial stress

\( \sigma_{cy} \) 0.002 compressive yield stress

\( \sigma_{ub} \) axial stress due to \( P_{ub} \), i.e., the axial stress at the base of the motor case with the unbonded propellant

\( \omega \) density
INTRODUCTION

The Naval Weapons Center, at China Lake, California, is presently in the process of developing a series of gun launched guided projectiles. Recent experimental firings of a three-inch projectile showed the structural design of the motor case to be inadequate since the case buckled severely as the projectile passed through the gun barrel. The author was asked to perform an analysis of the motor case subjected to the launch environment to determine the cause of failure. That analysis and its results are presented in Reference 1. The essential conclusions were:

1. The dynamic effects of the launch are insignificant and the case responds to the breech pressure as if the pressure were applied in a static sense, i.e., the maximum internal axial load in the case can be computed on the basis of the mass distribution and the maximum rigid body acceleration of the projectile.

2. The loading is not impulsive, i.e., it is on the case for a sufficient length of time such that the shell will collapse if the applied load equals the static buckling load of the case.

3. Approximately 80% of the propellant inertial load is carried by shear along the case when the propellant is bonded.

4. The case will buckle inelastically due to the axial load caused by the launch.

5. If the obturator cracks, 10-20% of the breech pressure acting as a lateral pressure is sufficient by itself to cause elastic buckling.

6. The predicted buckling mode for axial load and for lateral pressure is the same and is essentially identical to the observed buckled shape of the launched projectile.
This report contains the results of a similar analysis performed on a five-inch motor case. A parametric study is made with the thickness of the case, the maximum launch acceleration and the bond of the propellant as the variables. The lateral pressure considered in Reference 1 is not considered here due to the improved obturator design. The results of the study are presented in the form of survivability curves with the thickness, acceleration, and bonding condition as the independent variables. The engineer can determine the survivability of a case by selecting a value for each of these three parameters and examining the curves.
DESCRIPTION OF THE FIVE-INCH PROJECTILE

The casing is 4130 steel alloy heat treated to 200 ksi. From MIL-HDBK-5A, Feb. 8, 1966:

Elastic modulus in tension and compression, \( E = 29 \times 10^6 \) psi
Modulus of rigidity, \( G = 11 \times 10^6 \) psi
Poisson's ratio, \( \nu = 0.32 \)
Density, \( \omega = \text{lbs/in.}^3 \)

0.002 Compressive Yield Stress, \( \sigma_{cy} = 198,000 \) psi

Total round weight: (from data sheet, L. R. 10-20-71)

\[
\begin{align*}
\text{Guidance Head} & : & 20.0 \text{ lbs} \\
\text{War Head} & : & 40.0 \text{ lbs} \\
\text{Motor Case*} & : & 17.9 \text{ lbs} \\
\text{Propellant} & : & 18.6 \text{ lbs} \\
\text{Fin Assembly} & : & 8.5 \text{ lbs aft} = 8.5 \text{ lbs} \\
& & 105 \text{ lbs}
\end{align*}
\]

Length, \( L = 20 \) in.
Radius, \( r = 2.5 \) in.
Thickness, \( h = 0.19 \) in., 0.15 in., 0.12, 0.10 in., 0.09 in.

*Based upon a thickness of 0.19 in.
DESCRIPTION OF THE EXTERNAL LOAD

The external load on the projectile is the breech pressure at the base of the projectile.* A typical time history of the breech pressure on the five-inch projectile (h = 0.15 in.) is shown in Figure 1. Thus, $a_{\text{max}}$, the predicted maximum rigid body acceleration of the projectile, is

$$a_{\text{max}} = \frac{\text{Maximum Force}}{\text{Total Mass}} = \frac{(34 \text{ ksi})(0.9424)(\pi)(2.5 \text{ in.})^2}{105 \text{ lbs/g}}$$

$$= 6,000 \text{ g}$$

The factor 0.9424 converts breech pressure to base pressure (data sheet for five-inch projectile, L.R., 10-20-71) and g is the acceleration due to gravity.

The maximum acceleration used in the following analysis is 6,000 g for the 0.19 in. thickness case and 7,000 g for all the thicknesses.

*The obturator design has been modified to prevent cracking; and, hence, there is no lateral pressure on the case.
DETERMINATION OF THE INTERNAL AXIAL FORCE

The results of the analysis presented in Appendix A of Reference 1 indicated that if the loading function on the case is of the form
\[ P_0 \sin \frac{\pi t}{\alpha}, \quad 0 \leq t \leq \alpha, \] and if the "natural frequency" of the case \( \sqrt{\frac{E}{\rho}} \frac{\pi}{L} \), is considerably higher than \( \frac{\pi}{\alpha} \), then the dynamic effects are negligible, i.e., the shell responds in a static sense to the changing load.

A half sine wave has been superimposed on the loading history in Figure 1. A comparison of the two curves reveals that the loading curve is approximately a half sine wave with a frequency \( (\pi/0.018) \) rad/sec.

For the five-inch case with \( L = 20 \) in.
\[
\sqrt{\frac{E}{\rho}} \frac{\pi}{L} = \sqrt{\frac{29 \times 10^6 \times 386}{0.283}} \frac{\pi}{20} = 9.95 \times 10^3 \text{ rad/sec}
\]
which is considerably higher than \( \pi/0.018 \). Hence, the dynamic effects will be neglected in this first order engineering analysis*.

The internal axial force in the motor case is due to the mass forward of the case, the mass of case, and some of the mass of the propellant. Assuming all of the propellant mass loading is carried by shear on the case, the internal axial force at the base of the case \( P_b \) is

\[
\text{Axial Force (bonded propellant)}
\]

<table>
<thead>
<tr>
<th>( a_{\text{max}} )</th>
<th>6,000 g</th>
<th>7,000 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>0.19 in</td>
<td>0.15 in</td>
</tr>
<tr>
<td>( P_b )</td>
<td>580 kips</td>
<td>675 kips</td>
</tr>
</tbody>
</table>

The stress due to \( P_b \) is
\[
\sigma_b = \frac{P_b}{2\pi rh}
\]

* The loading curve shown in Figure 1 is expressed in a Fourier sine series. The magnitude of the higher harmonics in that series will depend upon how close the loading function is to a single half sine wave. The higher harmonics have higher frequencies, and hence, the case will respond dynamically to these higher harmonics. A study of the significance of these higher harmonics will be made in the near future.
Hence

**TABLE 1**

<table>
<thead>
<tr>
<th>$a_{\text{max}}$</th>
<th>6000 g</th>
<th>7000 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.19 in.</td>
<td>0.19 in.</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>194 ksi</td>
<td>226 ksi</td>
</tr>
</tbody>
</table>

For the unbonded propellant, none of the propellant mass is carried by the case. Thus, $P_{ub}$, the force at the base with unbonded propellant is

**TABLE 2**

<table>
<thead>
<tr>
<th>$a_{\text{max}}$</th>
<th>6000 g</th>
<th>7000 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.19 in.</td>
<td>0.19 in.</td>
</tr>
<tr>
<td>$P_{ub}$</td>
<td>467 kips</td>
<td>545 kips</td>
</tr>
</tbody>
</table>

and the stress $\sigma_{ub}$ is

<table>
<thead>
<tr>
<th>$a_{\text{max}}$</th>
<th>6000 g</th>
<th>7000 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.19 in.</td>
<td>0.19 in.</td>
</tr>
<tr>
<td>$\sigma_{ub}$</td>
<td>157 ksi</td>
<td>183 ksi</td>
</tr>
</tbody>
</table>
DETERMINATION OF THE STATIC BUCKLING LOADS OF THE FIVE-INCH CASE

The results of Reference 1 and of the preceding analysis indicate that the motor case can be designed using standard procedures for estimating the static buckling strength of shells. The elastic buckling stress $\sigma_{cr}$ for the condition of uniform axial compression* was determined using the formula presented in Reference 2, page 528. The result is

<table>
<thead>
<tr>
<th>$h$</th>
<th>0.19 in.</th>
<th>0.15 in.</th>
<th>0.12 in.</th>
<th>0.10 in.</th>
<th>0.09 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{cr}$</td>
<td>530 ksi</td>
<td>417 ksi</td>
<td>334 ksi</td>
<td>278 ksi</td>
<td>250 ksi</td>
</tr>
</tbody>
</table>

Since $\sigma_{cr}$ is much larger than $\sigma_{cy}$, the buckling is inelastic, and the predicted elastic buckling loads are invalid. When estimating the inelastic buckling stress ($\sigma_{cr}^{inelastic}$) from the chart of Reference 2, page 704, we encountered some difficulty since $\sigma_{cr} = 530$ ksi was beyond the limit of the chart. Consequently, the maximum value of $\sigma_{cr}/h$ of 400 ksi was used for $\sigma_{cr} > 400$ ksi. Converting the inelastic buckling loads of page 704 to 200 ksi h.t. using the same procedure as described in Appendix D of Reference 1 gives

<table>
<thead>
<tr>
<th>$h$</th>
<th>0.19 in.</th>
<th>0.15 in.</th>
<th>0.12 in.</th>
<th>0.10 in.</th>
<th>0.09 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\sigma_{cr}^{inelastic}$)</td>
<td>182 ksi</td>
<td>182 ksi</td>
<td>178 ksi</td>
<td>175 ksi</td>
<td>174 ksi</td>
</tr>
</tbody>
</table>

* The axial load in the case is not uniform but varies along the length due to the distributed mass of the case and propellant. The assumption of uniform axial force is conservative and also allows the use of the charts of Reference 2. A varying load requires a computer program. This was considered in Reference 1, page 11, loading condition 1b. Further studies of this effect are planned in the near future.
CONCLUSION

The maximum value that \( \sigma_b \) (or \( \sigma_{ub} \)) can have is that given by \( (\sigma_{cr})_{\text{inelastic}} \) if the case is to survive the launch. If \( \sigma_b \) (or \( \sigma_{ub} \)) is greater than \( (\sigma_{cr})_{\text{inelastic}} \) then the case will buckle inelastically. Comparing the predicted axial stresses \( \sigma_b \) and \( \sigma_{ub} \) given in Tables 1 and 2 with the inelastic buckling stress given in Table 3 reveals that only the 6,000 g launch of the 0.19 in. case with unbonded propellant will probably survive. The 7,000 g launch of the 0.19 in. case with unbonded propellant is a border line case as is the same case under 6,000 g's with bonded propellant. All cases with thickness less than 0.19 in. will fail. This is shown graphically in Figure 2 where \( J_b, \sigma_{ub}, \sigma_{cr}, (\sigma_{cr})_{\text{inelastic}}, \) and \( \sigma_{cy} \) are plotted as a function of \( h \). The curves shown in Figure 2 are referred to here as the survivability curves. The survivability of a particular case to a specified acceleration is obtained by selecting the thickness along the abscissa and proceeding along the ordinate until either \( \sigma_b \) or \( \sigma_{ub} \), depending upon the assumed bonding condition, is reached. The region the point lies in determines its survivability.
FIGURE 2. SURVIVABILITY CURVES
REFERENCES
