INDUCED ENHANCEMENT OF THE PLASMA LINE IN THE BACKSCATTER SPECTRUM BY IONOSPHERIC HEATING

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Prepared for:
Office of Naval Research
Advanced Research Projects Agency

June 1972
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by

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Approved for public release; distribution unlimited.

SUIPR Report No. 484
and
Technical Report No. 5

Prepared for
Office of Naval Research

Sponsored by
Defense Advanced Research Projects Agency
ARPA Order No. 1733

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SEL-72-041
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Enhanced plasma line
Ionospheric heating
Ion-acoustic parametric instability
Ion-acoustic electron-plasma parametric instability
Plasma waves
DuBois-Goldman instability
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Defense Advanced Research Projects Agency
(ARPA Order No. 1733; Program Code No. 2E20)
through the Office of Naval Research

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ABSTRACT

Recent results from ionospheric modification experiments at the Arecibo Ionospheric Observatory show an enhanced plasma line temperature of approximately 4000°K due to excitation of the ion-acoustic parametric instability. Recent theories on the saturation spectrum of the plasma waves excited in this experiment predict that the bulk of the wave energy is associated with waves propagating in a very narrow angular cone centered on the geomagnetic field. The intensity of the plasma waves responsible for the scattering, propagating parallel to the probing radar beam i.e. at 40° to the geomagnetic field, is orders of magnitude weaker than the intensity of the parallel propagating plasma waves. It is shown here that it is still sufficient to account for the experimental results.
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The author wishes to thank F. W. Crawford, M. V. Goldman, G. Meltz, F. W. Perkins, S. A. Self, and O. G. Villard, Jr., for helpful discussions during the course of this study. The research in this paper was supported by the Defense Advanced Research Projects Agency and monitored by the Office of Naval Research Contract No. N00014-67-A-0112-0066.
I. INTRODUCTION

There has been considerable interest recently in phenomena accompanying the excitation of parametric interactions in ionospheric heating experiments [Utlaut and Cohen, 1971]. One of the most interesting of these is the enhancement of the plasma line in the radar backscatter spectrum whose frequency lies very near that of the HF transmitter [Carlson et al., 1972]. The enhanced plasma line temperature has been observed to be in the order of 4000°K for a 100 kHz bandwidth.

It is well known that the plasma waves responsible for the backscattering must be propagating parallel to the diagnostic beam. Since the angle between the radar beam and the geomagnetic field is approximately 40° at Arecibo, it is clear that the plasma waves responsible must also be propagating at 40° to the geomagnetic field, B0.

Several theories have been advanced recently which predict the spectral density of these plasma waves [Besserides and Weinstock, 1972; Du Bois and Goldman, 1972a and b; Valeo et al., 1972]. These theories predict that the spectral density of the waves propagating nearly parallel to B0 is orders of magnitude greater than for the off-angle waves. Furthermore these waves are contained in a cone-angle which is exponentially small.

The following problem thus presents itself: How can such strong enhancements of the plasma line occur at 40° to B0 when most of the plasma waves are propagating parallel to B0? It will be the purpose of this paper to show that the off-angle waves, though much weaker than the parallel-propagating waves, are still sufficiently strongly excited to produce back-scattering in agreement with the Arecibo results, both with respect to intensity and bandwidth. We do this by application of theories already available [DuBois and Goldman, 1972a and b; Valeo et al., 1972].
II. THEORY

It has been shown [DuBois and Goldman, 1972a, Valeo et al., 1972] that the spectral density of the plasma waves propagating at an angle to the pump wave electric field (and geomagnetic field) is given by

$$I_1 = \frac{4\pi k T_e}{(1+x^2)(1-\mu^2)\epsilon k}$$

where $k_D$ is the Debye wavenumber, $k$ is the wavenumber of the plasma waves, $\cos^{-1}\mu$ is the angle between the pump field and the propagation direction of the plasma waves, $T_e$ is the electron temperature, $\kappa$ is Boltzmann's constant, and

$$\alpha = (m_e/m_i)^{1/2}$$

$$E_0^2 = \frac{1}{16} \frac{E_0^2}{4\pi n k T_e}$$

$$\chi = \frac{\omega_0 - \omega_L(k) - \omega_a(k)}{\gamma_a}$$

Here, $E_0$ is the pump electric field, $n$ is the electron number density, $\omega_p$ is the plasma frequency, $\gamma_e$ and $\gamma_a$ are the damping rates of the electron plasma and ion-acoustic waves, $\omega_L$ is the electron plasma wave frequency given by

$$\omega_L = \omega_p \left(1 + \frac{3k^2v_e^2}{2\omega_p^2}\right)$$

$\omega_a$ is the ion acoustic wave frequency given by

$$\omega_a = \alpha k v_e \approx \gamma_a$$

and $v_e$ is the electron thermal velocity.

The power scattered by the enhanced plasma line is given by

[DuBois and Goldman, 1972b]
\[
\frac{dP}{dV} = -\frac{J}{\kappa T^2} \frac{1}{e} \frac{A_r}{\lambda} P_1 \ dV,
\]

where \( dV \) is the scattering volume, \( \sigma_T \) is the Thomson backscattering cross-section, \( A_r \) is the receiver aperture area, \( P_1 \) is the incident radar power flux, and \( h \) is the distance from the ground station to the heated region. Integrating over the scattering volume and applying the formula

\[
P_1 \ dV = \frac{2Hd\omega}{\omega p},
\]

where \( z \) is the vertical height, \( P_T \) is the transmitted radar power, and \( H \) is the electron density scale height, yields the equation

\[
P = \frac{n \sigma_T H A_r P_T}{2\pi \omega p h^2} \left( \frac{k^2}{k_D^2} \right) \int \frac{I_1}{\kappa T e} d\omega p.
\]

From Eqs. (4) and (5) we obtain the expression

\[
d\omega p = \gamma_a \ dx.
\]

Substituting this and Eqs. (1) and (6) into Eq. (9) yields the equation

\[
P = 2 \frac{n \sigma_T H A_r P_T E^2}{h^2} \left( \frac{k^2}{k_D^2} \right) \left( \frac{\mu^2}{1 - \mu^2} \right) \left( \frac{\mu^2}{1 - \mu^2} \right).
\]

Little error is made by integrating between \(-\infty < x < \infty\), and we obtain

\[
P = 2 \frac{\pi n \sigma_T H A_r P_T E^2}{h^2} \left( \frac{k^2}{k_D^2} \right) \left( \frac{\mu^2}{1 - \mu^2} \right).
\]

corresponding to a plasma line temperature

\[
T = 2 \frac{\pi n \sigma_T H A_r P_T E^2}{h^2 \kappa B} \left( \frac{k^2}{k_D^2} \right) \left( \frac{\mu^2}{1 - \mu^2} \right),
\]

where \( B \) is the receiver bandwidth.
The bandwidth of the enhanced line is given from Eq. (10) as
\[ \Delta f = \frac{\gamma_a}{2\pi} \Delta x. \] (14)

The width, \( \Delta x \), is determined by the width of the function \((1+x^2)^{-1}\) in Eq. (11). If we take this width to be 2, then the bandwidth is
\[ \Delta f = \frac{\gamma_a}{\pi}. \] (15)

The spectrum of plasma waves responsible for the scattering [given by Eq. (1)] arises essentially from induced scattering of the pump wave by ions and from the mixing of the pump wave with spontaneous emission at the low ion-acoustic frequency. Although Eq. (1) is valid only for the case where threshold is exceeded for waves propagating along the direction of the pump field, the mechanism is still operative in the subthreshold region. The appropriate modification of Eq. (13) for the subthreshold condition is obtained by substituting \((1 - E_{\mu}^2)^{1/2}\) for \((1 - \mu^2)\) in the denominator.

**Numerical Evaluation of the Enhancement Ratio and Bandwidth**

In order to carry out our calculations, we use the following values for the experimental parameters, obtained from Carlson et al. [1972] and related sources:

- \( A_r = 3 \times 10^3 \) m
- \( B = 1.0 \times 10^5 \) Hz
- \( H = 1.0 \times 10^5 \) m
- \( P_0 = 1.0 \times 10^5 \) watts
- \( P_T = 2.5 \times 10^6 \) watts
- \( T_e = 1200^\circ \) K
- \( h = 2.0 \times 10^5 \) m
- \( k = 18 \) m\(^{-1}\)
- \( k_D = 2.6 \times 10^2 \) m\(^{-1}\)
- \( n = 3.92 \times 10^{11} \) m\(^{-3}\)
- \( \Omega = 0.096 \) steradian
- \( \gamma_e = 650 \) Hz
- \( \mu = \cos^{-1} 40^\circ \)
- \( \sigma_T = 7.94 \times 10^{-30} \) m\(^2\)
- \( \omega_p = 2\pi \times 5.62 \) MHz
- \( \omega_a = \gamma_a = 2\pi \times 4 \) kHz

\(\text{(16)}\)
The transmitted HF power is given by

$$P_0 = h^2 \Omega \frac{cE_0^2}{8\pi S}$$  \hspace{1cm} (17)

where $S$ is the peak swelling factor [Ginzburg, 1964] given by

$$S = 4 \left( \frac{H}{Z} \right)^{1/2} = \frac{4kD}{\sqrt{3k}} = 3.4$$  \hspace{1cm} (18)

and $Z$ is the width of the slab between the planes where the HF frequency equals the plasma frequency and $x = 0$.

Substituting into Eq. (5) yields

$$E^2 = \frac{\omega P}{7e 8n^{\gamma} T_e^2 c h^2 \Omega} = 3.1$$  \hspace{1cm} (19)

We then calculate the plasma line temperature from Eq. (13) to be

$$T = 5400^\circ K.$$  \hspace{1cm} (20)

This is reasonable agreement with the experimental value of $4000^\circ K$ [Carlson et al., 1972] in light of the approximate nature of the theory.

The bandwidth is calculated from Eq. (15):

$$\Delta f = \frac{\gamma a}{\pi} = 10 \text{ kHz}.$$  \hspace{1cm} (21)

This is to be compared to the experimental values of 10-20 kHz.
III. SUMMARY

The results of the previous section show that the spectral intensity of plasma waves lying outside the high intensity $B_0$-field aligned cone are sufficient to explain the plasma line enhancement observed [Carlson et al., 1972]. This holds even though these intensities are orders of magnitude lower than the peak spectral intensities.

It is doubtful that such a narrow high intensity cone of $B_0$-field aligned waves predicted by the theory could indeed occur. As suggested recently by Perkins [1972], inhomogeneity probably broadens the cone out to about 20°. Even with this broadening of the cone, however, one would still expect much higher intensities within the cone than those corresponding to the 40° waves discussed in this paper. This result suggests that experiments with the radar beam oriented more nearly parallel to the geomagnetic field would yield plasma line enhancements considerably in excess of those reported up to now.

Finally, since the calculations indicate that the pump field is very near, and possibly below threshold, and the theory presented is relatively insensitive to the relation of the pump field to threshold, whether above or below, the arguments used here do not establish conclusively the validity of this theory vis-a-vis other above-threshold theories.
REFERENCES


