Type of Report: Final Report

Project Title: A Research Program in Operations Research and Management Sciences

Institution: Georgia Institute of Technology


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Authors: Dr. Norman R. Baker  
Dr. John J. Jarvis  
Dr. Vernon E. Unger

Approved for public release; distribution unlimited. The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.
Research was conducted in three broad areas of Operations Research and Management Science. The specific problems studied and results obtained are discussed in the following reports:

(1) "The Development of a Quantitative Model for Resource Allocation Within the Exploratory Development Category of the Army Material Command"

(2) "A Model for Determining Skill Requirements in a Research Organization"

(3) "Information and Innovation in Research Organizations"

(4) "On a Two-Dimensional Resource Allocation Problem"

(5) "Maximal Flow with Gains Through a Special Network"
The scientific personnel supported in part by contract funds are summarized in Appendix A. They consist of three Georgia Institute of Technology faculty and a number of graduate students. The graduate students did not receive direct support from the contract, but did receive indirect support in travel and secretarial assistance and conducted their research under the guidance of one of the three faculty.

During the support period, 7 graduate theses were written, 4 papers were presented at National meetings of ORSA (Operations Research Society of America) or of TINS (The Institute of Management Science), and three have been accepted for publication. Each paper was forwarded to ARO, Durham at the time of submission. Abstracts from the theses and the papers are included in Appendix B.

Research was conducted in three broad areas of Operations Research and Management Science. The specific problems studied and results obtained are discussed in the following sections. The abstracts in Appendix B detail the specific problems and results.

**The Administration of Research, Development, and Engineering Endeavors**

A model of the resource allocation problem faced by research administrators in large organizations characterized by many hierarchial levels was developed (Appendix B1). For the special case where only three funding levels are considered for each alternative, e.g., minimal, anticipated, and maximal funding, the out-of-kilter algorithm of network theory can be used as a solution methodology. The results of this work are presently being evaluated for potential utilization by the Department of Army. A study team from Northwestern University (Prof. W. E. Souder, Project Leader) is evaluating a modification of the model with respect to the budgeting and allocation decisions at the Division and Branch Chief level in the Research, Development and Engineering Directorate of the Army Material Command, Headquarters. In addition, Captain C. R. Shumway will be examining the applicability of the model for utilization at the U. S. Army Air Mobility Research and Development Laboratory, Ames Research Center.

The Dean-Hauser advanced material systems planning model has been extended to include a manpower planning model and information system (Appendix B2). A paper was written which integrates recent behavioral research on creativity with manpower and technological planning (Appendix B3). The result is a model of a management information system which supports researcher innovative
behavior and which provides the input data for technical planning, project selection, and manpower planning. Since its presentation by Dr. Baker as an invited paper at the 38th National Meeting of ORSA, over 200 copies of the paper have been distributed and the paper has been accepted by Management Science for publication.

The following students have received advanced degrees for their research effort in this area:

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<thead>
<tr>
<th>Name</th>
<th>Degree Attained</th>
<th>Thesis Summary</th>
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<tbody>
<tr>
<td>James R. Freeland</td>
<td>M.S.I.E.</td>
<td>Appendix B2</td>
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<td>Major John M. Grimshaw</td>
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Network Theory/Analysis

There are many network situations in which the amount of flow into a node (activity) is not equal to the amount leaving the node. Examples of such networks include multi-level maintenance operations, information systems, and health systems. Such networks are referred to as networks with gains. The special structure of a (directed) acyclic network with positive gains was utilized to develop an algorithm to determine maximal flow (Appendix B5). A paper has been presented, and accepted by Operations Research, which reports on this research (Appendix B6).

Other networks arise which have the characteristic that all flow must pass through some common node, called a funnel-node. The transport and dissemination of ammunition in an established theater of operations is one such problem. Another such problem is a communications system within which all communications must pass through a common message center. The general concept of funnel-nodes and funnel-flows, the resulting mathematical model, and an algorithm for finding maximal flow were developed (Appendix B7). A paper was read at a national meeting of the Operations Research Society of America and accepted by Operations Research (Appendix B8).

In many applications of network theory/analysis, especially in transportation studies, a network exists. The research question then becomes one of how to best modify an existing network, perhaps in order to achieve some planned result, while simultaneously minimizing disruption to existing flows. Initial work has been accomplished on this problem (Appendix B9).

The following students have received advanced degrees for their thesis research effort in this area:
Analysis of Manpower Training Systems

Consider a manpower training system which accepts input of varying quality and which contains multiple training events all of which must be successfully completed by an individual in order for him to be certified as trained. The cumulative probability that an individual will pass an event is a function of the input (quality) group of which he is a member, and it is a monotonically increasing function of time spent (number of trials) in that event. If trainees are permitted to advance to the next training event as soon as the present event is passed, and if trainees are dropped (failed) from the system if success in an event is not achieved (after some fixed number of trials), then it is possible to improve system effectiveness by properly sequencing the training events.

The cost of operating such a system is measured as a function of time, the probability of failure, the magnitude and quality of input, and the number of instructor hours per time unit. The performance criterion is specified as instructor hours per graduate. A rule for optimally sequencing the events is proven and a general methodology for achieving the least cost, constrained output from an input of given size and quality is constructed.

Using data provided by the Human Resources Research Office, the proposed system was applied to Basic Combat Training. It appears that the proposed system may result in a 58 percent savings in Basic Combat Training instructor costs. However, the proposed system would require field testing before true savings or losses could be accurately determined (Appendix B10). The analytical results and solution algorithm were extended by subsequent research (Appendix B11).

Two advanced degrees were awarded for thesis research effort in this area:

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<td>M.S.</td>
<td>Appendix B11</td>
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APPENDIX A

List of Scientific Personnel

A. Faculty:
   Dr. Norman R. Baker
   Dr. John J. Jarvis
   Dr. Vernon E. Unger

B. Graduate Students

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<td>Appendix B1</td>
</tr>
<tr>
<td>Lt. Col. Anthony M. Jezior</td>
<td>M.S.</td>
<td>Appendix B5</td>
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<tr>
<td>Lt. Col. Duane D. Miller</td>
<td>M.S.</td>
<td>Appendix B7</td>
</tr>
<tr>
<td>Oscar Mejia</td>
<td>M.S.</td>
<td>Appendix B9</td>
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<tr>
<td>Major John E. Miller</td>
<td>M.S.</td>
<td>Appendix B10</td>
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<tr>
<td>Major David S. Grieshop</td>
<td>M.S.</td>
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APPENDIX B

PAPERS AND THESSES RESULTING FROM THE PROJECT


A comprehensive analysis is conducted of a decision problem existing within the early stages of military research and development (R and D). Three important areas are investigated: 1) a detailed description of the problem environment; 2) analysis of three general approaches to project/task evaluation; and 3) quantitative models of the decision problem and methods of solution.

The problem investigated is one of the allocation of resources at the Army Material Command (AMC) for the 6.2 exploratory development effort, particularly at the division and branch chief level. Three general phases of the problem are identified: long range technical planning, preliminary budget planning, and resource allocation. The decision problems are defined and major constraints are specified.

Three approaches to measuring the anticipated value of projects and tasks are considered: successive comparisons, multiple criteria scoring models, and planning-relevance trees. The advantages and limitations of each approach for use at AMC are clarified. Although the planning-relevance tree approach appears to have gained the widest recommendation within military R and D, recent advances in multiple criteria scoring models suggest decided advantages over the more popular planning-relevance tree approach.

The preliminary budget planning and resource allocation problems are structured and modeled mathematically. Dynamic programming (coarse grid approach) and a linear programming approximation are used to establish an upper and lower bound on the optimal solution for the grid size specified. An algorithm is constructed to search this bounded region for solutions which improve the feasible, non-optimal and optimal, non-feasible solutions thus determined. Information is generated with respect to trade-offs in value for alternative funding patterns. This information can be utilized during the budget specification process in order to identify and support unfunded requests.
Abstract of "A Model for Determining Skill Requirements in a Research Organization" by James R. Freeland, M. S. Thesis, Georgia Institute of Technology, June 1970

Skilled manpower is a critical resource in a research organization; however, few quantitative models have appeared which relate long-term organizational objectives and skill requirements. The thesis develops mathematical models which allocate skilled manpower to projects over time in order to maximize the expected contribution from the research effort.

The future organizational objectives can be related to projects, alternative technical approaches, and tasks by planning procedures such as the Department of Army's QMDO planning. The models developed in the thesis are extensions of and consistent with the work done by Dean, Hauser, and Roepcke. It is assumed that the success of a technical approach is dependent upon the success of all member tasks, but that the success of a project depends only on the success of at least one technical approach.

The overall model consists of four sequential models, called T, P, V, and M where models T, P, and V are solved by dynamic programming and M is solved by linear programming. Given an amount of support available for all projects, models T, P, and V identify the skill amounts for each task, technical approach and project which maximize total expected contribution. Model M allocates the skills to the projects in the pattern specified by the previous models so that project completion dates are met and the variation in the size of the workforce is minimized.

The overall model provides information which can be used by the research organization to develop viable and timely policies for maintaining the proper mix of research skills.


Empirical work in industrial research organizations has provided data sufficient to describe researcher behavior during innovation. The role of information during idea creation and idea submission is described based on these data. A model of a management information system is structured which is consistent with and supportive of researcher behavior and which includes technical planning, project selection, and manpower planning. The critical
problems of information search and dissemination are examined.


A two dimensional resource allocation problem is considered; for example, allocation over activities and time periods. The independence and separability of returns for a given activity-time period pair is assumed. The form of the model is essentially a generalization of a nonlinear capacitated transportation problem. Lagrange multipliers are introduced and the Lagrangian is shown to decompose into a sequence of bounded variable nonlinear knapsack problems, amenable to solution by dynamic programming. Several results are proven for piecewise convex return functions including (1) the piecewise convexity of the optimal payoff (knapsack) function and (2) the structure of the admissible policies. The result (1) accelerates the calculation of the knapsack functions. The result (2) leads to (3) a closed form solution under the assumption of equal allocation bounds for an activity across time.


The special structure of a directed acyclic network with positive gains is utilized to develop an extremely simple and powerful algorithm for maximal flow. The algorithm uses a variation of the Ford-Fulkerson labeling procedure in which each node receives at most one label during any iteration. Flow changes are carried along and require only "back-tracking" to the source node. Finiteness of the algorithm is achieved through a theorem characterizing basic feasible solutions for this special network. As a consequence of these results, the algorithm is more efficient than Jewell's algorithm for the general maximal flows with gains problem, i.e., not requiring an acyclic network with positive gains.

Several potential areas of application are identified. As an example consider a health system with several levels of treatment, e.g., neighborhood health stations, community health centers, and urban hospitals. Identifying the facilities at each level as nodes, the health system can be formulated as a network problem where the flow is the patient. The patients flowing through
the system would not all pass through all levels of the system. This characteristic can be represented by a "loss factor" which is positive. Thus the network is acyclic with positive gains. Other potential applications include multi-level maintenance operations where not all repairable items pass through all levels of maintenance, multi-level educational or training operations where not all students pass through all levels, and multi-level information systems where not all queries pass through all levels of information sources.


The special structure of a (directed) acyclic network with positive gains is utilized to develop an extremely simple and powerful algorithm for maximal flow. Finiteness of the algorithm is achieved through a theorem characterizing basic solutions for this special network.


The research generates from the following type of problem. Consider a logistical commander who is in a recently established theater of operations and who is receiving ammunition at an ocean terminal for immediate shipment to a forward area. The only trucks available to transport the ammunition are being received at another terminal for ultimate shipment forward. Since there exists a road network of limited capacity, the problem is to route the trucks to the ammunition terminal for loading and then to route the loaded trucks to the front in such a way that the maximum amount of ammunition is forwarded. Since all the trucks must pass through a common node (ammunition depot) which has neither source (truck depot) nor sink (the front), the network has a "funnel-node", namely the ammunition node. Obvious analogous problems exist in communications networks where all messages pass through a common message center.
The general concept of funnel-nodes and funnel-node flows is defined and the resulting mathematical model is formulated. An algorithm is presented which computes the maximal value of a funnel-node flow by three applications of a "single-commodity, max-flow" algorithm and which determines the maximal flow pattern by an additional application of a "single-commodity, max-flow" algorithm and the application of a chain flow decomposition algorithm. Several example problems are defined and formulated. The algorithm is illustrated by solving simple cases of the example problems.


The "Funnel-Node, Maximal Flow" problem is formulated, examined, and solved for an undirected network. The solution procedure thus derived requires only applications of the single-commodity flow algorithm, and is therefore extremely efficient. Several applications are also presented.


This thesis presents an efficient method for finding the dual variables associated with a new set of arcs for a given multicommodity network. Also, a method is presented for adding the "best" set of arcs to a given network. The methodology of this research utilizes the principles of linear programming, duality and the multicommodity flow algorithms.


Two performance criterion functions are developed and applied to a simulated manpower training system. The simulated system accepts inputs of varying quantity and quality. It contains multiple training events and may be operated such that each event has a probabilistic outcome. The cumulative probability of success for an event is a monotonically increasing function of time spent in that event and is independent of the order of the events.
As the amount of time in an event is constrained, the maximum achievable probability of success within an event is lessened. The cost of operating the system is presented as a function of time, the probability of failure, the magnitude and quality of input, and the number of instructor hours per time unit. The performance criterion for the simulated training system is instructor hours per graduate.

Two different training procedures are compared using the system performance criterion of instructor hours per graduate. Within each procedure general relationships between input quantity and quality, output quantity, and cost of operation are developed, proven and demonstrated.

A rule for optimally sequencing the events in one of the training procedures is developed, proven and demonstrated. A general methodology for achieving the least cost, constrained output from an input of given size and quality is developed and demonstrated.


Rules for efficiently improving the measure of system effectiveness for a sequential manpower training system are developed, proven, and demonstrated. The system permits trainees of varying aptitudes to train in a series of events at a pace commensurate with their abilities. The cumulative probability of success for an event is a monotonically increasing function of the number of times the event is attempted and is independent of the order of the event within the sequence of all events. An incremental cost is incurred each time an event is attempted and accrues until a trainee either successfully completes the system or is eliminated as a failure. The measure of system effectiveness, the efficiency index, is the expected cost per successful trainee.

An algorithm is presented where the decision variables are the number of times an event may be attempted and the sequence of the events. The cost of training and probabilities of success are considered fixed. A sufficient condition, which when satisfied insures the algorithm yields an optimal solution, is developed and proven. An alternative method for solution improvement is outlined when the sufficiency condition is not satisfied.