ON THE ELECTROMAGNETIC FIELD RADIATED ABOVE THE TREE TOPS BY AN ANTENNA LOCATED IN A FOREST

T. Tamir

June 1971
On the Electromagnetic Field Radiated above the Tree Tops by an Antenna Located in a Forest

Theoretical investigations of the field in forest environments have usually considered the case where both the transmitting antenna and the receiving terminal were located within or very close to the vegetation. The present work examines the case where one of the two antennas is located inside the forest while the other is situated above the tree tops. The mechanism of wave propagation and its effect on the field variation are discussed for heights that start at the forest-air interface and extend vertically to large elevations. It is shown that the field just above that interface and up to a critical height $H_c$ is dominated by a lateral wave. Above the height $H_c$, the field is given by a refracted "line of sight" wave whose amplitude is subject to a strong height gain effect. This gain continues up to a height $H_m$ above which the field starts decreasing monotonically. Both vertical and horizontal polarizations are examined and typical results are given for several representative forest varieties. While the present results extend certain conclusions obtained from previous work, they also represent an important first step in solving communication problems involving "mixed paths" which cross partly through vegetation and partly through air or clearings.
NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The citation of trade names and names of manufacturers in this report is not to be construed as official Government endorsement or approval of commercial products or services referenced herein.

Disposition

Destroy this report when it is no longer needed. Do not return it to the originator.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forest Environment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conductive Slab</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Dipole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lateral and Space Wave</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HF and VHF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixed Path Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radio Loss</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna Height Gain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variation of Field with Distance</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ON THE ELECTROMAGNETIC FIELD RADIATED ABOVE THE TREE TOPS
BY AN ANTENNA LOCATED IN A FOREST
by
T. Tamir
(Polytechnic Institute of Brooklyn)
Duke University Consultant Services
(Contract DA31-124-ARO-D-399)
JUNE 1971
DA Subtask No. 1H6 62701 A350-01-31
DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited.
US ARMY ELECTRONICS COMMAND
FORT MONMOUTH, NEW JERSEY
# TABLE OF CONTENTS

Abstract .......................... ii
1. BACKGROUND AND INTRODUCTION .......................................... 1
2. FIELD DESCRIPTION .................................................................. 3
3. THE CROSSOVER HEIGHT $H_c$ .................................................. 5
4. THE HEIGHT GAIN EFFECT ...................................................... 6
5. COMPARISON WITH EXPERIMENTAL RESULTS ............................ 10
6. DISCUSSION AND CONCLUDING REMARKS .............................. 11
REFERENCES ................................................................. 14

# LIST OF FIGURES

1. Schematic outline of a forest environment. ......................... 15
2. Geometry of the forest environment:
   (a) The slab model. ...................................................... 16
   (b) The half-space model. ............................................. 16
3. Variation of the cross-over height $H_c$ versus frequency for the three forest varieties. .................................................. 17
4. Variation of the relative attenuation factor $\alpha$ versus frequency for the three forest varieties. .................................................. 18
5. Variation of the basic height gain $R_0(H)$ versus height $H$ for a "thin" forest, at a distance $x = 1 \text{ km}$. .......................... 19
6. Variation of the basic height gain $R_0(H)$ versus height $H$ for an "average" forest, at a distance $x = 1 \text{ km}$. .......................... 20
7. Variation of the basic height gain $R_0(H)$ versus height $H$ for a "dense" forest, at a distance $x = 1 \text{ km}$. .......................... 21
8. Variation of radio loss versus range: Comparison of experimental and theoretical results. .................................................. 22
9. Geometry of a bounded forest slab showing the different propagation regions due to the multiple boundaries. .......................... 23
ABSTRACT

Theoretical investigations of the field in forest environments have usually considered the case where both the transmitting antenna and the receiving terminal were located within or very close to the vegetation. The present work examines the case where one of the two antennas is located inside the forest while the other is situated above the tree tops. The mechanism of wave propagation and its effect on the field variation are discussed for heights that start at the forest-air interface and extend vertically to large elevations. It is shown that the field just above that interface and up to a critical height $H_c$ is dominated by a lateral wave. Above the height $H_c$, the field is given by a refracted "line of sight" wave whose amplitude is subject to a strong height gain effect. This gain continues up to a height $H_n$ above which the field starts decreasing monotonically. Both vertical and horizontal polarizations are examined and typical results are given for several representative forest varieties. While the present results extend certain conclusions obtained from previous work, they also represent an important first step in solving communication problems involving "mixed paths" which cross partly through vegetation and partly through air or clearings.
1. BACKGROUND AND INTRODUCTION

The propagation of radio waves in forest environments has been extensively studied during recent years and the dissipation losses associated with the presence of vegetation have been examined from experimental as well as theoretical points of view. However, most of the work was concerned with situations where both the source (transmitter) and the field probe (receiver) terminals were located within or very close to the foliated regions. Much less effort has been devoted to situations wherein one of the two terminals was outside the forest and, in particular, the theoretical studies associated with this question have been rather limited in scope.

It is by now well accepted that, when both the receiving and the transmitting antennas are within a forest, communication within the frequency range of 2 to 200 MHz is effected primarily by means of a lateral wave, which travels between the two terminals along a path that closely follows the treetop contour. This communication mechanism was shown to hold well for distances larger than about 1 km. Other wave varieties may sometimes be stronger than the lateral wave, but they seem to occur more as an exception rather than the rule. Thus, a ground wave may be predominant at distances of a few kilometers at the lower frequencies, or a sky wave may provide better communication conditions at 5-15 MHz for distances larger than a few kilometers.

Although the lateral wave provides the dominant field component inside the foliated region, the exterior air region represents a domain wherein a lateral wave cannot, in general, offer a strong field contribution. This is due to the fact that the lateral wave is relatively strong at the boundary between two different media (in this case, the air-forest interface) but decreases away from that boundary, so that the predominant field contribution in the lossless (air) medium is usually provided by other wave varieties. Consequently, it is of great interest to understand well the propagation mechanism along a "mixed path," i.e., between two points situated so that the line joining the transmitter and the receiver traverses partly through the vegetation and partly through the air region.

To put the present work in proper perspective, it is pertinent to refer to Fig. 1 where a schematic outline of a rather general forest environment is shown. A transmitting antenna is assumed to be located at point T within the vegetation and a receiving antenna is considered at one of the four points I to IV in Fig. 1. When the receiver is located at I, both the transmitting and receiving terminals are in the forest - a situation that has been investigated extensively in the past. For a point such as II, the receiver is located above the tree tops and the only boundary that effectively intervenes between the transmitter and receiver is the forest-air interface. In this case, the forest edge (i.e., the boundary that separates the forest itself from the clear area to the right of the forest) is of secondary importance because it does not affect the field at point II unless that point happens to be close to the forest edge. On the other hand, the presence of the forest edge boundary cannot be neglected for points...
such as III or IV because this boundary occurs roughly along a vertical plane that intervenes between the transmitter at T and the receiver at III or IV. The distinction between the two points III and IV is that the latter is relatively close to the ground whereas the former is at an altitude that is higher than the forest height.

In view of the above classification, it is evident that communication with point I occurs essentially along a homogeneous path; communication with points II, III, or IV, on the other hand, occurs along a mixed path that includes both a foliated region and an air region. It is recalled that theoretical work in the past \cite{2-4,6,7} has successfully represented a forest in terms of a lossy dielectric layer bounded by ground below and by air above. To find the field for point I, the lossy slab was assumed to extend to infinity along its lateral x and y dimensions, as shown in Fig. 2(a); this approximation is well justified if neither of the two antennas is too close to the edge of the forest. The infinite slab model is still adequate for examining the electromagnetic field at point II because any discontinuity effects due to the presence of the forest edge may then also be neglected. However, for points III and IV, the forest edge discontinuity cannot be ignored and it may, in fact, be quite crucial in determining the electromagnetic field; in those cases, the forest slab may, in general, no longer be assumed to be infinite.

The present work consists of an extensive analysis of the field for the first mixed path (point II) situation discussed above, i.e., when one antenna is within the forest while the other one is in the air region above the treetops, but both antennas are sufficiently far from the forest boundaries. Because the infinite slab model may still be used in that case, the present study may be viewed as an extension of previous work \cite{4}, which considered the field within the forest layer only. A thorough understanding of the field along an entire vertical plane for the case of an infinite slab model is expected to provide a powerful and essential tool in dealing with mixed path situations for which the field discontinuity at the forest edge is important. Hence, the present study represents a necessary and useful step towards the solution of the general mixed problem.

The range of parameters considered in this work is the same as that already discussed in previous work. \cite{4,7} This includes a frequency spectrum between 2 and 200 MHz and a horizontal separation between the two antennas of 1 km or larger. In addition, the vertical range \(H\) for the antenna in the air region is assumed to start from the average tree height \(h\) and to extend indefinitely upwards. Although the range of \(H\) considered here may extend to infinity, quantitative results are limited for practical purposes to a maximum height \(H\) of 5 km.

The results show that the electromagnetic field in the air region above, but close to the treetops, is produced by a lateral wave in a manner which is quite similar to that obtained when both antennas are inside the forest layer. At a certain value \(H_c\) of the height \(H\) above the treetops, the lateral wave field becomes equal in magnitude to a refracted "line-of-sight" wave. While the lateral wave changes slowly, the refracted wave increases
rapidly as the height \( H \) rises, so that the latter wave provides the dominant contribution above the cross-over height \( H_c \). The field amplitude continues to increase for a considerable range in \( H \) and reaches a maximum at a height \( H_m \) which is of the order of the horizontal separation between the two antennas. After this maximum is reached, the field decreases at higher heights; this decrease is quite rapid for vertical polarization and moderately slow for horizontal polarization.

While the above summarizes the main results, additional more detailed aspects have also been considered and discussed in the present report. These include the field behavior as a function of the horizontal separation between the two antennas, the behavior of different varieties of forests (with sparse or dense vegetation), the effect of lowering the antenna located within the foliage, and other features. On the other hand, the influence of the ground plane has been neglected since it is of secondary importance, as discussed and explained in the next section.

2. FIELD DESCRIPTION

The slab geometry representing the forest environment is shown in Fig. 2(a) where, as usual, \( z \geq 0 \) the vegetation is assumed to be spread uniformly and to occupy a layer of height \( h \) bounded by ground below and by air above. A transmitting antenna \( T \) is assumed to be located within the layer at a height \( z_0 \) above ground and an observation point \( R \) at a height \( H \) above the forest-air interface is being considered. As is already well known from previous work, \( 4,7 \) the physical picture of the field for the forest slab model shown in Fig. 2(a) is somewhat complicated by the presence of the ground plane. A greatly simplified representation is obtained if this plane is removed by letting it recede to \( z \rightarrow -\infty \), in which case one obtains the half-space forest model shown in Fig. 2(b) wherein the forest medium fills the entire \( z < h \) region. This simplification serves as a very good approximation if both the transmitting and receiving antennas are not too close to the ground \( 4,7 \); since in the present case the receiving antenna is in the air region, the half-space model is expected to be adequate whenever the height \( z_0 \) is sufficiently large.

The antenna at both \( T \) and \( R \) terminals in Fig. 2 are assumed to be small dipoles and their orientation may be either vertical or horizontal. However, the two antennas are taken to be parallel to each other (cross-polarization is not being considered) and, in the case of horizontal polarization, the two dipoles are assumed to lie along the \( y \) direction, i.e., their axis is normal to the plane of the paper. The field detected at \( R \) then contains a component \( E_s \) which may be described by a refracted geometric-optical ray TAR. Since it is assumed here that the distance \( AR \) is much larger than the total height \( h \) of the forest, this ray-optical field is essentially given by the "space wave"

\[
E_s = j\mu c \frac{120\pi}{\lambda_0} \frac{p \cos^2 \theta}{r \cos \theta + \sqrt{n^2 - \sin^2 \theta}} e^{-jk_0 \left[ r + \left( h - z_0 \right) \sqrt{n^2 - \sin^2 \theta} \right]} \tag{1}
\]
where \( \mathbf{I} \) denotes the current dipole moment of the antenna and a time dependence \( e^{-j\omega t} \) was assumed and suppressed. The polar coordinates \( r \) and \( \theta \) are shown in Fig. 2(b) and \( k_0 \) is the free-space wavenumber

\[
k_0 = \omega \sqrt{\mu_0 \varepsilon_0} = \frac{2\pi}{\lambda_0}
\]

while \( n \) denotes the refractive index of the forest medium. In view of the vegetation losses, this refractive index is complex and given by

\[
n^2 = \varepsilon_1 - j\sigma_1 \lambda_0
\]

where \( \varepsilon_1 \) and \( \sigma_1 \) refer, respectively, to the relative permittivity and the conductivity of the medium which characterizes the forest slab. The parameters \( p \) and \( m \) depend on the particular polarization and are given by:

\[
p = \begin{cases} 
\sin^2 \theta, & \text{for vertical polarization;} \\
1, & \text{for horizontal polarization.}
\end{cases}
\]

\[
m = \begin{cases} 
n^2, & \text{for vertical polarization;} \\
1, & \text{for horizontal polarization.}
\end{cases}
\]

It should be noted that, if \( n \to 1 \), \( E_S \) yields a result which corresponds to the field of a dipole in free space, as expected.

In addition to the refracted field \( E_S \), a diffracted component \( E_L \) is present due to a lateral wave which skims just above the forest-air interface, as shown by the ray trajectory TBC in Fig. 2(b). This wave extends somewhat into the air region and accounts for a field that is often termed a "ground wave" in the literature, as given by

\[
E_L = \frac{100\pi}{k_0 \lambda_0 r^2} \frac{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}} e^{-jk_0 [r + (h - z_0) \sqrt{n^2 - \sin^2 \theta}]} e^{j\kappa [r + (h - z_0) \sqrt{n^2 - \sin^2 \theta}]}
\]

(6) is accurate only if \( k_0 r \gg 1 \) and if \( |n| \) is not too close to unity. The latter question was already discussed by Tamir who showed that this condition holds for the parameters which describe typical forests.
The importance of the lateral wave $E_L$ is evidenced by the fact that the refracted field $E_0$ vanishes at the forest-air interface since then $\cos \theta = 0$ in Eq. (1). However, at sufficiently large heights $H$ above the interface, the lateral wave is of lesser importance since the amplitude of $E_L$ varies with distance as $-2$ and, therefore, decreases much faster than the amplitude of $E_0$ whose variation is given by the $r^{-1}$ spherical divergence. In view of the strong dependence of the amplitudes of $E_L$ and $E_0$ on both $\theta$ and $r$, the two types of waves tend to occupy different spatial domains, as discussed in the next section.

3. THE CROSSOVER HEIGHT $H_C$

As was shown in previous work, the lateral wave is the dominant field within the forest layer and, as discussed above, it extends somewhat further into the air region. However, it is expected that at a certain height $H_c$, the lateral wave $E_L$ will be equal in magnitude to the refracted wave $E_0$; the latter will thereafter provide the dominant field at greater heights.

Comparing Eqs. (1) and (6) and noting that

$$x = r \sin\theta,$$
$$z - h = H = r \cos\theta,$$  \hspace{1cm} (7)

it is evident that the amplitudes of $E_0$ and $E_L$ will be equal when $H = H_c$ such that

$$H_c = \frac{m \sin\theta}{\lambda_0 2\pi \sqrt{n^2 - \sin^2 \theta}}.$$  \hspace{1cm} (8)

Recalling that we are concerned with distances $x \gg 1$ km, we anticipate that $H_c/x \ll 1$ so that $\sin \theta \approx 1$. In that case, the critical height is given by

$$H_c \approx \frac{1}{2\pi} \frac{m}{\sqrt{n^2 - 1}}.$$  \hspace{1cm} (9)

This result is tabulated in Fig. 3 which displays the variation of $H_c$ for both polarization and for three typical forest parameters. These typical forests are those already considered by Dence and Tamir; they comprise a variety denoted as "thin" which refers to a sparsely foliated region, another variety denoted as "dense" with relatively thick vegetation, and a third "average" variety whose properties are in-between the two other kinds. The parameters of interest in the present work are $\epsilon_1$ and $\sigma_1$ which are given below for these three forest types.
<table>
<thead>
<tr>
<th>Type of Forest</th>
<th>$\varepsilon_i$</th>
<th>$\sigma_i$ (mhos/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Thin&quot; forest</td>
<td>1.03</td>
<td>$3 \times 10^{-5}$</td>
</tr>
<tr>
<td>&quot;Average&quot; forest</td>
<td>1.1</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>&quot;Dense&quot; forest</td>
<td>1.3</td>
<td>$3 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Returning to Fig. 3, it is pertinent to observe first that the values of $H_o$ are all below 50 m and therefore for $x > 1$ km, the assumption $\sin \theta \approx 1$ is well justified in arriving at Eq. (9). Another important consideration is that the cross-over height $H_o$ is then independent of the horizontal distance $x$ between receiver and transmitter. Hence, the region $z < H_o + h$ wherein the lateral wave is dominant is independent of this horizontal distance. In fact, one may extend the concept of the forest height $h$ to an effective forest height $h' = h + H_o$ which describes the entire region characterized by a lateral wave; evidently, it is only at greater heights ($z > h'$) that the refracted wave $E_r$ is predominant. It is interesting also to observe that, although Fig. 3 indicates that $H_o < 1$ in all cases, nevertheless $H_o$ may be several times larger than the actual forest height $h$ at frequencies around 10 MHz and lower. On the other hand, $H_o$ is quite small at frequencies around 50 MHz and higher and, in fact, its value is then comparable to the irregularities encountered in the heights of individual trees. Hence, the effective layer $h'$ wherein the lateral wave mechanism predominates is essentially given by the actual forest height $h$ at those higher frequencies.

4. THE HEIGHT GAIN EFFECT

The investigations concerned with the field inside the forest layer have recognized that the field amplitude may increase considerably as either of the two antennas is raised. This amplitude increase was termed "height gain" and, from the discussion in the preceding section, it is evident that the height gain effect will be present also when one of the two antennas is in the air region. To assess the magnitude of this effect, it is appropriate to compare the field magnitude $E = E(x, H)$ at some arbitrary height $H$ with the field amplitude $E = E(x, 0)$ at the forest-air interface. Since we are concerned with the height variation only, the horizontal distance $x$ will be assumed to be constant so that $E(x, H)$ will be abbreviated to $E(H)$ henceforth. The comparison of $E(H)$ with respect to $E(0)$ for $x = \text{const.}$ implies that one is interested in the ratio

$$R(H) = R(x, H) = 20 \log \left| \frac{E(H)}{E(0)} \right| = R_o(H) + S(\theta) \quad (10)$$
where $E(H)$ obviously includes the total field, namely:

$$E(H) = E_s(H) + E_L(H)$$

wherein, by substituting Eqs. (1), (6), and (11) into Eq. (10), one identifies

$$R_0(H) = 20 \log \left[ \frac{n^2 - 1}{m} \frac{p \sin \theta}{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \left( k_o H - j m \sin^2 \theta \right) \right],$$

$$S(\theta) = 8.686 k_o (h - z_o) \text{Im} \left( \sqrt{n^2 - 1} - \sqrt{n^2 - \sin^2 \theta} \right)$$

where $\text{Im}(u)$ refers to the imaginary part of $u$.

The basic gain function $R_0(H)$ measured in dB represents the gain at a height $H$ with respect to the field at the interface, in the case where the transmitting antenna is located at that air-dielectric interface ($z_o = h$). Hence, $S(\theta)$ represents the additional gain available if the transmitting antenna is located at a depth $d = h - z$ below the air-dielectric interface. It should also be observed that $\sin \theta$ and $\cos \theta$ are functions of $x$ and $n$, as given by Eq. (7).

It is worthwhile to consider the term $S(\theta)$ first and recognize that the maximum additional gain that this term allows will occur when $\theta = 0$ ($H = h$), in which case one obtains

$$S(0) = 8.686 k_o (h - z_o) \text{Im} \left( \sqrt{n^2 - 1} - n \right) = \alpha_r (h - z_o)$$

where $\alpha_r$ refers to the additional gain $S(0)$ for one meter of vegetation above the transmitting antenna. As defined, it is recognized that $\alpha_r$ is identical to the same expression used by Tamir in his Eq. (17). If $\alpha_r$ is plotted for the frequency range of interest, as displayed in Fig. 1, it is noted that it is never larger than 0.5 dB/m. Since $h$ is of the order of 10 meters, the additional gain $S(\theta)$ is never more than a few decibels, and, furthermore, it occurs only at great heights ($\sin \theta$ small), so that its effect is of secondary importance. We shall therefore neglect $S(\theta)$ henceforth and regard it, at most, as a small correction term to the more significant basic height gain $R_0(H)$.

The basic gain $R_0(H)$ is zero at $E = 0$ since, by definition, the gain in dB is expressed with respect to the field at the air interface ($H = 0$, $\sin \theta = 1$). As $H$ increases, a gain close to 3 dB is achieved when $H = H_c$, where $H_c$ is the crossover height discussed in the preceding section. The reason that $R_0(H_c)$ is not exactly 3 dB stems from the fact that the two terms in the round brackets in Eq. (12) are not exactly in quadrature so that, although the magnitude of $k_o R_c$ may be equal to the magnitude of the
term containing the j factor, nevertheless, the magnitude of their sum is generally different from \(2^{1/2} k_0 H_0\).

In general, the gain \(R_0(H)\) depends on the horizontal distance \(x\) and, for \(x = 1\) km, its functional variation is shown in Figs. 5, 6, and 7 for the cases defined as thin, average, and dense forests, respectively. To examine the basic properties of \(R_0(H)\) and to extend the results to distances \(x\) different from 1 km, it is pertinent to note the functional features discussed below.

The term \(k_0 H\) in Eq. (12) refers to the space wave contribution \(E_s(H)\), whereas the second term in the round brackets refers to the lateral wave component \(E_l(H)\). The quantity \(E_s(H)\) decreases slowly with \(H\), while \(E_l(H)\) increases linearly with \(H\). Hence, as \(H\) increases, the relative importance of the lateral wave becomes quite insignificant soon after the crossover height \(H_c\) has been overtaken. This means that, if \(H > H_c\), we may approximate Eq. (12) by retaining the term pertinent to the space wave only so that:

\[
R_0(H) = 20 \log \left| \frac{n^2 - 1}{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}} k_0 H \right|.
\]  

(15)

The last result implies that \(R_0(H)\) is directly given by the radiation pattern associated with the space wave since \(H\) may be replaced with \(r \cos \theta\). In the present case, however, we are concerned with the field variation as \(x = r \sin \theta\) is held constant rather than with a radiation pattern wherein \(r\) itself is constant. When \(x\) is constant and \(H\) increases, it is evident from Eq. (15) that \(R_0(H)\) also increases as long as the trigonometric expression in Eq. (15) does not change too rapidly. For large values of \(H\), however, the term \(\sin \theta\) decreases rapidly with \(H\) so that any further increase in \(H\) will, nevertheless, reduce the total value of the factor \(H \sin \theta\); the net result is, therefore, a decrease in \(R_0(H)\) for larger values of \(H\). This decrease will be faster for vertical polarization than for the horizontal one since the former is augmented by the factor \(p = \sin^2 \theta\) which reduces \(R_0(H)\) much more drastically than in the case of horizontal polarization.

The above behavior is well illustrated in Figs. 5, 6, and 7, which reveal that the increase of \(R_0(H)\) for small values of \(H\) and its subsequent decrease for large values of \(H\) is quite general. Although the above curves were calculated for a distance \(x = 1\) km, the variation they indicate is typical also for larger distances. In all cases, a maximum value is obtained for \(R_0(H)\) at some value \(H_m\) of \(H\). The main difference between the curves of \(R_0(H)\) at distances other than \(x = 1\) km, is that both \(H_m\) and the maximum gain \(R_0(H_m)\) increase with the distance \(x\), as discussed below.

Owing to the complexity of the expression in Eq. (15) for \(R_0(H)\), it is rather difficult to obtain an explicit result for either \(H_m\) or \(R_0(H_m)\). However, one may get a good approximation for both these values by noting that Eq. (15) may be rewritten as:
The first term in Eq. (16) is a constant for any value of $x$, while the second term is a function of $\theta$ only. It turns out that, for the values of $n$ that are being considered here, the second term peaks at a value $\theta_m$ of which is not much different from $45^\circ$. If one then approximates $p \propto m \sim n^2 \sim 1$, a greatly simplified expression is obtained for the largest value $R_0(H_m)$ of $R_0(H)$ in the form:

$$R_0(H_m) \approx 20 \log \left| \frac{n^2 - 1}{m} \right| k \cdot x$$

Hence, the maximum available height gain is given approximately by Eq. (17) which indicates that this gain increases with the horizontal distance $x$ between the transmitter and receiver. The height $H_m$ for which this maximum gain occurs is given approximately by $H_m = x$ since, as already mentioned above, $\theta_m$ is roughly $45^\circ$.

The above considerations provide a good evaluation for the effect of the distance $x$ on the height gain $R_0(H)$. In fact, relation (17) together with the curves in Figs. 5, 6, and 7 may be used to advantage to obtain a good indication for the behavior of $R_0(H)$ for distances larger than 1 km. As an example, if a distance of $x = 10$ km is considered, the peaks $R_0(H_m)$ will rise approximately 20 dB higher than those shown in Figs. 5, 6, and 7, and the heights $H_m$ will take on values that are roughly 10 times higher than those in the above figures. Hence, the effect of increasing the distance $x$ is to stretch the curves along both the vertical and horizontal directions in the respective figures.

To conclude the present section, it is also appropriate to observe that the maximum available height gain $R_0(H_m)$ is larger for denser forests than for thinner ones. This is explained easily by noting that the field $E(H)$ at large heights $H$ is, to first order, independent of the exact composition of the forest medium since, as a refracted ray, the field travels mostly in the air region. However, $E(0)$ is very strongly dependent on the vegetation density since this field is due to the lateral wave, which skims just above the treetops. In fact, the amplitude of $E(0)$ takes on smaller values for denser forests as a result of increased losses. Consequently, the magnitude of $E(H)/E(0)$ that defines the height gain $R(H)$ will be larger for denser forests. The dependence of $R_0(H)$ on the density of vegetation, as shown in Figs. 5, 6, and 7, is therefore consistent with physical intuition.
5. COMPARISON WITH EXPERIMENTAL RESULTS

As already mentioned previously, experimental results for mixed paths are very scant, so that there are only few data to compare the theoretical results obtained here with measurements on actual sites. The relatively small amount of information for communication along a path such as TR in Fig. 2(a) is given in Vol. II of the Jansky and Bailey report, as described below.

The measurements carried out on mixed paths by Jansky and Bailey include such points as II, III, or IV in Fig. 1. For the points of type II discussed here, their work consists of field strength measurements by means of a helicopter that was flying at a fixed altitude of 500 feet \((\approx 150 \text{ m})\) and receiving signals radiated by an antenna in the vegetation. A typical plot of their results is given in Fig. 8 where the shaded region indicates the spread of the measured data.

In view of the relatively large height \(H = 150 \text{ m}\) for the measurements shown in Fig. 8, the field at the receiver is due to a refracted line-of-sight wave whose amplitude is expressed by \(E_s\) in Eq. (1). To find a theoretical result to compare with the experimental data of Fig. 8, it is necessary to find an expression for the radio loss \(L\) between the receiver and the transmitter. Owing to the fact that both antennas are sufficiently far away from the ground plane, the total path loss may be found by neglecting the effect of the ground loss \(L_g\). Hence, \(L\) in the present case is given simply by \(L_\infty\), which represents the radio loss for the half-space model shown in Fig. 2(b).

In agreement with the above considerations, the radio loss is given by

\[
L = L_\infty = 20 \log \left( \frac{2 \pi R_0}{\lambda E_s} \right) \quad (18)
\]

where \(R_0\) is the antenna resistance of a small dipole element, namely:

\[
R_0 = 80 \left( \frac{\pi \lambda}{\lambda_0} \right)^2 \quad (19)
\]

Inserting Eqs. (1) and (19) into Eq. (18), it is found that

\[
L = 20 \log \left\{ \frac{4\pi}{3} \frac{H}{\lambda_0} \left[ \frac{m + \sqrt{1 + \left( \frac{n^2 - 1}{2} \right) \sec^2 \theta}}{p} \right] \sec \theta \right\} \exp \left\{ \frac{4\pi}{\lambda_0} \frac{h - z}{\cos \theta} \cdot \left( \sqrt{1 + \left( \frac{n^2 - 1}{2} \right) \sec^2 \theta} \right) \right\} \quad (20)
\]
For the data shown in Fig. 8, a realistic value for the refractive index is given by

\[ n^2 = 1.04(1 - j0.05) \]  

(21)

By introducing this value into Eq. (20) and carrying out a numerical evaluation, the pertinent theoretical curve for \( L \) is found, as shown by a solid line in Fig. 8. It is observed that this curve is well within the shaded area that indicates the spread of the measured results.

Although Fig. 8 refers to an operating frequency of 25.5 MHz and horizontal orientation of the antennas, similar curves were obtained for other frequencies in the range 25.5 < \( f \) < 400 MHz and for both horizontal and vertical polarization. In all cases, a judicious choice for the refractive index \( n \) yields good correspondence between the measured data and the theoretically-predicted results. Whereas, this implies a large amount of confidence for the present theoretical study, it must be kept in mind that the experimental data available involve a limited amount of verification because only the horizontal range \( x \) was varied and the field was probed along only one (relatively high) altitude.

6. DISCUSSION AND CONCLUDING REMARKS

The foregoing study of the electromagnetic field radiated in the air region above the treetops by an antenna located within a forest layer has revealed several important and interesting features. A first result is that, up to a height \( H_c \) above the forest-air interface, the field continues to behave similarly to the field within the vegetation. This means that the electromagnetic field both inside the forest layer, as well as outside and up to a total effective height \( h' = h + H_c \) above ground, is dominated by a lateral wave.

The implication of the above result is that the field near the tree tops decreases rather rapidly with distance since the lateral wave power is proportional to \( r^{-2} \), where \( r \) denotes the distance away from the transmitter. The effective height \( h' \), which delimits the lateral wave domain, may be considerably larger than the average forest height at frequencies below 10 MHz. However, since \( H_c \leq \frac{\lambda}{2} \) at all frequencies, the crossover height \( H_c \) is quite small above 50 MHz and then the effective height \( h' \) is only slightly larger than the average forest height \( h \).

At heights above \( h' \), the electromagnetic field is dominated by a geometric-optical refracted wave. This wave accounts for a field which, for sufficiently large heights \( H \), progresses closely along a "line-of-sight" path. Consequently, the field above the effective height \( h' \) behaves like a spherical wave whose power diverges with distance like \( r^{-2} \); furthermore, the field amplitude is subjected to a radiation pattern modification which depends on the elevation angle \( \theta \). This angular variation accounts for a height gain which may yield a large ratio between the field magnitude at a
certain height $H$ as compared to that at the forest-air interface. In fact, it was shown here that the field amplitude increases constantly with height, up to a maximum height $H_m$, after which the amplitude decreases at greater heights. The value of $H_m$ is of the order of the horizontal separation $x$ between transmitter and receiver, and this estimate holds for both horizontal and vertical polarizations. However, the height attenuation for vertical polarization is much stronger than that for horizontal polarization at heights larger than the critical height $H_m$.

The qualitative and quantitative results obtained here indicate that the forest half-space and slab models are actually applicable to a wide class of situations that include several types of "mixed paths." To appreciate these conclusions, it is worthwhile to consider the geometry shown in Fig. 9, which depicts a forest layer that extends up to a boundary located in a vertical plane denoted by $x = x_0$. The boundaries at $x = x_o$ and $z = h'$, together with the ground plane, then specify four different regions denoted by I to IV in Fig. 9. Viewed in this fashion, Fig. 9 represents an idealized geometry for the general situation shown in Fig. 1, the points I to IV in Fig. 1 being now clearly associated with corresponding domains I to IV in Fig. 9. If we wish to examine a "mixed path" situation, one of the two antennas will be located in the forest region I whereas, the other antenna will be located in one of the three remaining air regions. The electromagnetic field is expected to behave in the following manner:

a. If the second antenna is placed in region II, i.e., above the tree tops, the results of the present work apply directly and the field consists of a refracted "line-of-sight" wave as discussed herein, provided $z > h' = h + H_c$. However, within a layer $h < z < h'$, the main contribution to the field is given by a lateral wave. It therefore follows that field calculations within points such that $0 < z < h'$ involve an augmented forest layer whose effective height is $h'$ rather than $h$. The antenna boundary at the plane $x = x_0$ will have little effect, as discussed below.

b. If the second antenna is in region III, it is no longer located in a region above the tree tops. Nevertheless, the construction shown in Fig. 2(b) for the refracted wave TAR still holds and it is, therefore, also indicated by TA'R' in Fig. 9. The latter construction is justified because the refracted wave that is being extended from region II occurs above the critical height $h'$, so that the lateral wave is relatively insignificant in that region. It therefore follows that one may extend the trajectory TAR in Fig. 2(b) well beyond the plane $x = x_0$ in Fig. 9 and, thus, obtain a path such as shown by TA'R' in Fig. 9. Hence, the field at $R'$ may be calculated by using the results in the present report, as if the forest did not end at the plane $x = x_0$ but rather extended to infinity through region IV.

c. If the second antenna is in region IV, the above path construction is no longer possible and the forest edge discontinuity at the plane $x = x_0$ cannot be disregarded. This last situation, therefore, remains to be investigated and could serve as the subject of a future study.
The above cases cover almost all the varieties of "mixed path" communication. One exception occurs when one antenna is just above the tree tops in region II, while the other is just above ground in region IV; in that case, a direct line joining the two antennas would still pass through vegetation although both antennas are outside the forest. Except for such rather special situations, the cases wherein both antennas are in air would consist of a communication path along a simple line of sight that stays clear of vegetation. The field in those cases could be found by conventional methods which, to a good approximation, may neglect the presence of vegetation. Of course, a more complicated situation may arise when a second forest edge occurs in addition to and to the left of the one shown at \( x = x_0 \) in Fig. 9. It is, however, evident that the complete understanding of the effect of a single boundary is essentially and basically sufficient to the understanding of the more complicated cases involving two forest edges.

In conclusion, it is noted that the present study has generalized considerably the results of previous work and that, in particular, a large amount of "mixed path" situations may henceforth be handled by simple models. A future investigation of the discontinuity that occurs at a forest boundary would complete the technique of estimating communication conditions in forest environments by means of simple and easily understood models.
References


Fig. 1. Schematic outline of a forest environment.
Fig. 2. Geometry of the forest environment:
(a) The slab model;
(b) The half-space model.
Fig. 3. Variation of the cross-over height $H_c$ versus frequency for the three forest varieties.
Fig. 4. Variation of the relative attenuation factor $\alpha_r$ versus frequency for the three forest varieties.
Fig. 5. Variation of the base height gain $R_0(H)$ versus height $H$ for a "thin" forest, at a distance $x = 1$ km.
Fig. 6. Variation of the basic height gain $R_o(H)$ versus height $H$ for an "average" forest, at a distance $x = 1$ km.
Fig. 7. Variation of the basic height gain $R_0(H)$ versus height $H$ for a "dense" forest, at a distance $x = 1$ km.
Fig. 3. Variation of radio loss versus range: Comparison of experimental and theoretical results.

- $f = 25.5$ MHz
- $z_0 = 4$ m.
- $H = 150$ m.  $h = 25$ m.
- Horizontal polarization

Range of experimental results

Theoretical curve for:

- $\epsilon_1 = 1.04$
- $\tan \delta_1 = 0.05$
Fig. 9. Geometry of a bounded forest slab showing the different propagation regions due to the multiple boundaries.