TEST SECTION SIZE INFLUENCE ON MODEL HELICOPTER ROTOR PERFORMANCE

By
August F. Lehman
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U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
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This report has been reviewed by the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory and is considered to be technically sound. The purpose of this study was to attempt to define the influence of tunnel cross section and model height above the tunnel floor on the wake and inflow characteristics for a given rotor. This program was conducted under the technical management of Mr. Patrick A. Cancro of the Aeromechanics Division of this Directorate.
## Test Section Size Influence on Model Helicopter Rotor Performance

### Author(s)
August F. Lehman and Jeffrey A. Besold

### Description
This study indicates that the successful testing of model helicopter rotors in tunnels when in the hover and transitional flight regimes requires not only Reynolds number scaling, where the Reynolds number is recognized as controlling the rotor's maximum section lift coefficient, but also a scaling of a characteristic defined here as the wake energy dissipation pattern. The dissipation of the energy from the rotor into the surrounding fluid is a function of the viscosity of the test fluid; thus, operation of a reduced scale model in the same fluid as that in which the full-scale vehicle operates makes appropriate scaling of this wake characteristic difficult if Reynolds number scaling is to be maintained. These conflicting factors have led to the requirements of relatively large models and correspondingly large wind tunnel test sections when rotor studies involving hovering or transitional flight modes are of interest. The use of water as the test fluid for scale model studies permits an easier maintaining of Reynolds number scaling while having the scale model produce a wake energy dissipation pattern similar to that of the full-scale vehicle because of the advantages water possesses over air in terms of kinematic and dynamic viscosities. While a wake energy dissipation scaling factor is not expressly defined, a test procedure is suggested which should permit meaningful studies of model helicopter rotors at hover, or in the transitional flight regimes, in either wind or water tunnels.
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**Flow Visualization**

**Helicopter Rotor**

**Rotor Flow Pattern**

**Aerodynamics**

**Water Tunnel Tests**

**Tunnel Test Section Size**
TEST SECTION SIZE INFLUENCE ON MODEL HELICOPTER ROTOR PERFORMANCE

Oceanics Report No. 70-76

by

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Jeffrey A. Besold

Prepared by

Oceanics, Inc.
Plainview, New York

for

EUSTIS DIRECTORATE
U. S. ARMY
AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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ABSTRACT

This study indicates that the successful testing of model helicopter rotors in tunnels when in the hover and transitional flight regimes requires not only Reynolds number scaling, where the Reynolds number is recognized as controlling the rotor's maximum section lift coefficient, but also a scaling of a characteristic defined here as the wake energy dissipation pattern. The dissipation of the energy from the rotor into the surrounding fluid is a function of the viscosity of the test fluid; thus, operation of a reduced scale model in the same fluid as that in which the full-scale vehicle operates makes appropriate scaling of this wake characteristic difficult if Reynolds number scaling is to be maintained. These conflicting factors have led to the requirements of relatively large models and correspondingly large wind tunnel test sections when rotor studies involving hovering or transitional flight modes are of interest. The use of water as the test fluid for scale model studies permits an easier maintaining of Reynolds number scaling while having the scale model produce a wake energy dissipation pattern similar to that of the full-scale vehicle because of the advantages water possesses over air in terms of kinematic and dynamic viscosities. While a wake energy dissipation scaling factor is not expressly defined, a test procedure is suggested which should permit meaningful studies of model helicopter rotors at hover, or in the transitional flight regimes, in either wind or water tunnels.
FOREWORD

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INTRODUCTION

The testing of models is a most valuable tool in the design of aircraft; therefore, the desire for complete compatibility between wind tunnel test data and subsequent flight data is understandable. The discrepancies which exist between wind tunnel data and subsequent full-scale performance are caused by tunnel influences not present in full flight. These influences include such considerations as Reynolds number effect, model support system interference, test section static pressure gradients, tunnel-model size ratios, and the configuration and location of the tunnel walls relative to the model. This last item is generally identified with the term wall influence or wall constraint and is the subject of this investigation.

The effectiveness of the use of a water tunnel for qualitative determination of rotor tip vortex wake characteristics was initially established in 1967 by Lehman in Reference 1 and was subsequently corroborated through independent studies in 1969 by Werle and Armand in Reference 2. In addition to a documentation of the rotor tip vortex wake characteristics over an equivalent speed range from hover to 120 knots, the Reference 1 study included measurements of rotor lift. This present study extends that initial work in an attempt to define the influence of tunnel cross section and model rotor height above the tunnel floor (for a given rotor) on the rotor lift, wake and inflow characteristics.

This investigation has contributed to a better understanding of the nature of sharply deflected wakes, in particular with respect to meaningful model studies in tunnels, by uncovering the importance of the test fluid viscosity in maintaining a proper scaling of the rotor wake pattern during scale model studies. The results indicate the necessity of more accurately representing the details of the wake field in the rotor vicinity when tunnel wall influence or constraint is of interest than has been done in most mathematical analyses performed in the past. An accurate wake representation is, of course, most necessary at the hover condition and in the transitional flight regimes.
TEST FACILITIES AND EQUIPMENT

TUNNEL

The tests were run in the Oceanics water tunnel. This tunnel is a recirculating, closed jet type with both the water velocity and the test section static pressure as controllable variables. The cross section of the basic test section is a "square" of 28 inches on a side with corner fillets having a 2.36-inch radius. The usable length of the test section is approximately 12 feet. Viewing windows are on all four sides of the test section, thus permitting excellent observation of the model undergoing test. To create test sections of other cross-sectional dimensions, 6-foot-long Plexiglas panels were inserted into the basic section. The corner fillet radius of every test section size was scaled to the fillet radius of the basic section according to the ratio of the diagonals between the reduced and basic sections.

In the settling section, just ahead of the nozzle, there are two honeycombs which improve the flow before the water enters the nozzle and passes into the test section. The upstream honeycomb is an egg-crate type consisting of square elements 1.12 inches on each side and having a length of 7 inches. The honeycomb nearest the test section is also an egg-crate type with the individual square elements having a side dimension of 0.96 inch and a length of 16 inches.

MODEL

The model helicopter rotor blades were originally used in the Reference 1 study. They were machined from Armco 17-4 Ph stainless steel and consisted of scaled versions of the Bell UH-1D rotor. The increase in thickness at the root of the blade was achieved in the model by a uniform linear increase in profile thickness to the appropriate grip thickness rather than through a true scaling of the doubler plates employed on the actual rotor.

The model rotor blades were 5.42 inches long and had an NACA 0012 profile. The uniform twist from the root of the blade to the tip was 9 degrees 50 minutes (corresponding to a twist from the rotor center to the tip of the blade of 10 degrees 54 minutes). The model rotor blades were rigidly attached to the hub; this method of attachment is a deviation from the full-scale rotor. The model rotor is shown in Figure 1. The blades, which were mounted into the
hub at a precone angle of 4 degrees, were set at the desired collective pitch angle through the use of the jig shown in Figure 2. This jig permitted setting the rotor blade to within ±0.10 degree of the desired collective pitch angle.

The test assembly consisted of a variable-speed electric drive motor, a lift measuring balance, and a drive shaft surrounded by a streamlined pylon. A sketch of the test assembly is shown in Figure 3.

**ROTOR LIFT AND RPS MEASUREMENT**

The lift of the rotor was sensed in the direction along the drive shaft by utilizing a balance inserted (as a coupling) between the motor and the drive shaft itself. Calibration of the assembly was performed using a rig which incorporates dead weights certified to National Bureau of Standards Tolerance Class C. The calibration was performed with the test assembly mounted in the tunnel, submerged in water, and with the drive shaft (less rotor blades) operating at the test rotational speed. In this manner, any influence of the bearings in the pylon support was accounted for during the calibration procedure. The output from the balance transducer assembly was fed into an X-Y plotter, thus permitting the pen deflection, as measured from the trace of the X-Y plot, to be associated with known weights. A calibration was performed at the start of each test day. The deviation of the daily calibrations values was less than 0.25 percent from the beginning to the end of the test program.

The rotational speed of the drive motor was electrically controlled. The rotational speed was measured with an electronic counter with the input consisting of pulses originating from a toothed wheel attached to the rotor shaft and sensed by a reluctance pickup.

The output from the pickup could also be fed to a time base memoscope, and with the time base on the X-Y plotter also activated, it was possible to obtain an indication of the rotor lift as a function of rotor rotational speed and of time. The value of being able to obtain the lift-rps-time data in this manner will be discussed in a later section.
TEST PROGRAM

The experimental program consisted of testing the 2-bladed, 12-inch-diameter rotor in test sections having six different cross-sectional dimensions. For each of the test section configurations, the rotor lift, wake and inflow characteristics were examined for different rotor heights above the tunnel floor. The test section sizes and rotor heights are presented in Table I. For each arrangement of test section size and rotor height, tunnel velocities equivalent to hover, 10, 15, 25, 35, 70 and 110 knots were introduced. For each velocity, vertical loads equivalent to 6,000 pounds and 8,000 pounds were examined.

The tip vortex wake patterns were made visible through the use of the air bubble technique developed for the Reference 1 study and described in Appendix I. The inflow to the rotor and the mass (or momentum) flow through the rotor disk was identified by injecting neutral-density dye from probes located upstream of the rotor. From an overhead viewing position, the dye probes were in line with the rotor rotational axis and at locations 0.75 rotor radii to either side. From a side viewing position, dye was emitted at spatial positions both in line and above and below the rotor head. A sketch of the dye injection system is shown in Figure 3.

Documentation of the tip vortex wake patterns and the inflow to the rotor, as identified by the dye streaks, was provided through visual stroboscopic illumination, high-speed 16mm black and white movie clips, and still photographs. Photographic documentation was obtained from both the side and overhead viewing positions. The program is documented by some 1000 50-foot reels of 16mm film and 1600 photographs. All of these data are at USAAVLABS.
TABLE I. TEST SECTION SIZE AND ROTOR LOCATION

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<tr>
<th>Test Section Size</th>
<th>Rotor Height ($R_h$) Above the Floor</th>
<th>$R_h / H$ (in.) (ratio)</th>
<th>$R_{H/H}$ (in.) (ratio)</th>
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<td>Height (H) (in.)</td>
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5
GENERAL TECHNICAL CONSIDERATIONS

ROTOR FLOW FIELD

The flow field of a hovering rotor can be characterized as containing two major elements. One element consists of the mass flow passing through the rotor disk, which is reminiscent of the flow through a nozzle. This mass flow is encircled by a helix trail of tip vortices, with this tip vortex trail identified as the second element. If the spanwise loading distribution of a rotor is uniform, it would follow that the mass flow through the rotor disk would be analogous to that of a circular uniform jet. The synthesis of this type of flow is shown in Figure 4. However, the spanwise loading distribution for a helicopter rotor is not uniform, and a typical loading is one which is more heavily loaded toward the tips. Thus, for a realistic spanwise rotor loading, the mass flow through the rotor disk could be considered more analogous to that of an annular nozzle. The synthesis of this type of flow is sketched in Figure 5.

With an awareness of these analogies, and because this study concerns tunnel wall influences, some insight as to the effect of rotor height above the tunnel floor on the rotor lift and wake characteristics can be gleaned by examining studies involving impinging uniform and annular jets. This approach was followed by reviewing the work of Brady and Ludwig in Reference 3 for impinging uniform jets, that of Chaplin in References 4 and 5 for annular jets, and that of von Glahn in Reference 6 for both annular and circular jets.

Two major characteristic differences separate the annular from the uniform jet. One difference concerns augmentation which does not occur with a uniform jet; the other concerns the shape or boundary of the jet surface, with both factors strongly influenced by the proximity of a ground plane. In the cited reference, it is clearly established that the reaction on the nozzle and the shape of the jet boundary produced by a circular uniform jet is considerably different from the nozzle reaction and jet boundary produced by an annular jet. By relating nozzle reaction to rotor lift and jet shape to rotor wake, it seems reasonable to conclude that the spanwise loading of the rotor should play an important role in determining the influence that a ground plane has on a rotor's lift and wake characteristics. Therefore, in establishing testing limits, the relationships of tunnel test section size, rotor size, rotor location, and realistic spanwise rotor loading distributions must be employed if useful...
The degree of matching of the model and full-scale Reynolds number is always of concern, particularly when testing lifting models. This concern is related to the fact that while the basic slope of the lift coefficient-section attack angle curve may not be significantly altered with increasing Reynolds number, the maximum lift coefficient which can be obtained before the initiation of stall is strongly dependent on Reynolds number.

The prototype Reynolds number, derived from the rotor chord length and for a tip velocity of 800 fps, is of the order of $7.9 \times 10^6$. For the water tunnel tests, while the model rotor is quite small, the difference in the kinematic viscosity between air and water is advantageous and the model Reynolds number is of the order of $9.1 \times 10^6$. Thus, there is a difference in the magnitude of the full-scale and model Reynolds number. However, as indicated, the matter of concern, when the lift and wake characteristics of a rotor are of interest, lies primarily in the maximum lift coefficient obtainable (as a function of radial location) before stall occurs. In the case of rotor studies then, the section attack angle at different radial stations must be examined, along with the Reynolds number existing at the specific radial station, to determine the differences between the model and the full-scale lift characteristics.

In Appendix II, such a comparison calculation is made for a rotor operating at Reynolds numbers comparable to the full-scale and model values of this study. This comparison shows that on the basis just described, the calculated difference in the total disk loading at the two Reynolds numbers is of the order of 2 percent (for the hover case) at a rotor collective pitch angle corresponding to a load of 7,000 pounds, i.e., the mid-point for these studies which examined 6,000- and 8,000-pound lift values. In other words, the calculated lift performance of a rotor operating at the full-scale and the model Reynolds numbers is remarkably close when considering only the Reynolds number influence.

**COMPRESSIBLE VS INCOMPRESSIBLE TEST DATA**

The full-scale rotor was assumed to have a constant tip
velocity of 800 fps; thus, compressibility effects will influence its performance. On the other hand, the model rotor is operating in water; therefore, its operating regime is fully incompressible. A reasonable approximation for estimating the compressibility effects on incompressible data (when the lifting surface is not in a stall condition) is to employ the Prandtl-Glauert factor, which is \( \sqrt{1-M^2} \). Payne, in Reference 7, shows that the thrust coefficient obtained with an incompressible (non-stalling) flow situation, divided by the Prandtl-Glauert factor, provides good agreement with compressible flow thrust coefficient data. The Mach number \( (M) \) is taken as that occurring at the 0.7 radial section of the rotor.

BASIC FULL-SCALE MODEL DATA CALCULATIONS

The full-scale UH-lD rotor and fluid property values employed were:

- Rotor radius \( R = 24 \) ft
- Rotor chord = 21 in.
- Rotor tip speed \( V_T = 800 \) fps
- Rotor rotational speed \( n = 5.308 \) rps
- Rotor advance ratio \( \mu = \frac{\text{forward velocity}}{\text{rotor tip velocity}} \)
- Air density \( \rho = 0.00218 \) slug/ft\(^3\)
- Kinematic viscosity \( \nu = 1.77 \times 10^{-4} \) ft\(^2\)/sec
- Acoustic velocity \( c = 1166 \) fps
- Mach number (at 0.7 radial section) = 0.480
- Prandtl-Glauert factor = 0.877

\[
C_T = \frac{\text{Thrust}}{\rho V_T^2 \pi R^2}
\]

For 6,000 pounds vertical lift, \( C_T = 0.00238 \)
For 8,000 pounds vertical lift, \( C_T = 0.00317 \)

The model rotor and fluid property values employed were:

- Rotor radius \( R = 6 \) in.
- Rotor chord = 0.437 in.
- Rotor tip speed \( V_T = 25.21 \) fps
- Rotor rotational speed \( n = 8.0 \) rps
- Water density \( \rho = 1.937 \) slugs/ft\(^3\)
- Kinematic viscosity \( \nu = 1.02 \times 10^{-5} \) ft\(^2\)/sec

For 6,000 pounds vertical lift, \( C_T = 0.00209 \) (incompressible value)
For 8,000 pounds vertical lift, $C_T = 0.00278$
(incompressible value)

Model rotor lift values measured along the rotor axis were converted to vertical lift values through trigonometric considerations.
EXPERIMENTAL PROGRAM - RESULTS, ANALYSIS, AND DISCUSSION

The bulk of the data obtained during the experimental portion of this study consists of a voluminous number of high-speed movie clips and still camera photographs. All of these data are at USAAVLABS. In this report, the thrust of the major comments will be substantiated by typical photographs or plots of test data.

WAKE FORM

The tip vortex trail as a concentric helix occurs at hover and is shown in Figure 6. As forward velocity is introduced, the tip vortex helix decomposes and the trail now consists of a pair of trailing vortices reminiscent of a fixed wing; lying between, or joining, these trailing vortices are segments of the tip vortex shed when the rotor tip is in the downstream sector of each rotor revolution. This tip vortex trail situation, which occurs over the range of forward velocities from about 10 to 70 knots, is illustrated in Figure 7. At forward velocities higher than the order of 70 knots, the pair of trailing vortices no longer form and the tip vortex trail consists only of a highly skewed helix. This condition is shown in Figure 8.

ROTOR-INDUCED RECIRCULATION-REINGESTION

With a rotor operating in a closed tunnel at hover or low forward velocities, the wake of the rotor impinges on the floor of the tunnel, travels along the floor and up the walls, and then can be reingested by the rotor. This action of recirculation-reingestion has been suspected as producing an increase in rotor lift. This reingestion influence alone decreases rotor lift although the net rotor lift may be increased over what would occur in a free, unbounded state by the presence of the tunnel floors, walls and ceiling. The conclusion that reingestion alone produces a decrease in rotor lift was reached by two nonrelated pieces of evidence.

It will be recalled that the technique employed in determining the rotor lift permitted the obtaining of the lift value as a function of time. The rotor speed could also be obtained as a function of time. With time = 0 associated with the initiation of rotor rotation, it was then possible to relate rotor lift with rotor rotational speed. It was found that all rotor lift-time curves
followed an easily recognizable trend when they were examined as a function of forward velocity. Figure 9 depicts typical lift-time histories for a variation in forward velocity. It is emphasized that all lift-time histories followed this same trend regardless of test section size or rotor height above the tunnel floor. Also shown on Figure 9 is a trace of a typical rotor rotational speed-time history as obtained from photographs of the screen of the oscilloscope recording the rotor rotational speed history. Figure 10 is a direct copy of a hover lift-time history trace taken from the X-Y plotter records. Also shown on this figure is the associated rotor rotational speed-time history. The rotor reached its operational speed approximately 2 seconds before the maximum rotor lift was obtained. After reaching operational rotor speed, the rotor operated at a steady rotational condition for some 2 seconds or 16 revolutions before the occurrence of maximum lift. Since each rotor revolution is equivalent to about 86 rotor chord lengths at the tip or 60 rotor chord lengths at the 0.7 radial station, it is believed that a steady or quasi-steady circulation pattern about the rotor had been achieved. On this basis, it is believed that the maximum lift value obtained was a "true" measure of the rotor lift and that the subsequent decrease in rotor lift occurred with the reingestion of the rotor wake. This belief was further substantiated by the lift-time traces, which show no "peaking" of lift-time curves after forward velocities of 15 knots are achieved, and also with observations of the high-speed movies with dye injection showing that reingestion of the rotor wake occurs only with forward velocities less than the order of 15 knots.

While the material just presented is fairly solid evidence of what does occur, Figure 11 presents the background for another approach showing that reingestion of a rotor's own wake will result in a decrease in rotor lift. For the case of the unconfined rotor at hover, the effective attack angle for the section under consideration is the difference between the geometric attack angle and the angle associated with the induced velocity. With rotor confinement, the recirculation-reingestion contribution can be considered as an additional \( A \)-velocity increase through the rotor disk. Since the geometric attack angle and the rotor rotational speed are nonchanging quantities, it then follows that the effective attack angle must decrease as the downward component of velocity increases and, consequently, the rotor lift must decrease.

From an experimental standpoint, the rotor (at hover) can be
considered as a propeller under static load. A tunnel mounting arrangement sketched in Figure 12 is equivalent to an operating rotor (at hover) bounded at the sides but with no floor or ceiling. The A-velocity increase above the rotor resulting from recirculation-reingestion (occurring when a floor and ceiling are present) can be simulated by starting flow through the tunnel. From the depicted testing arrangement, it must be immediately obvious to anyone familiar with testing aircraft or marine propellers that, for a constant rotational speed and given positive blade pitch, the maximum thrust can be obtained with no tunnel flow. The propeller thrust will decrease, with the introduction of increasing tunnel flow, until the propeller reaches a zero thrust condition. Further increases in tunnel flow velocity (while maintaining a constant rotational speed) will result in a drag rather than a thrust condition. The model helicopter rotor was tested in the manner just described, and the thrust (lift) results are presented in Figure 13.

It is believed that the "true" rotor lift values are those obtained before recirculation-reingestion occurs provided recirculation-reingestion exerts an influence on the lift value. In other words, based on the test techniques employed in this study, lift data for conditions below about 15 knots should be obtained utilizing methods permitting the rotor rotation to be established (and stabilized) when starting from a nonrotating condition for a time period equivalent to 8 to 12 rotor revolutions before recirculation-reingestion occurs. Most model rotors do not permit the "across the line starting" procedures used in these water tunnel studies, and therefore most of the data presented here applies to conditions where any recirculation-reingestion which may occur has been well established. The techniques employed in obtaining the data have been identified.

ROTOR LIFT PERFORMANCE

In an earlier section, the question of Reynolds number effect was raised. It was shown that a calculation (Appendix II) indicated that only a slight discrepancy in the (non-dimensionalized) total lift should exist between the model and full-scale rotors based on Reynolds number considerations (at the equivalent 7,000-pound load). This was conditionally verified experimentally in the following way. A number of initial tests consisted of determining the rotor lift coefficient as a function of rotor collective pitch angle for
variations both in the mast angle and equivalent forward velocity. This information was then used in selecting the collective pitch angles which were employed when examining the equivalent 6,000-pound and 8,000-pound load situations for the various test section size-rotor location conditions. If the slope of the rotor lift coefficient-collective pitch angle curves were constant, it would indicate that stall was not occurring. For a normal nontwisted wing, stall is associated with an abrupt falloff in the lift coefficient value. However, because the helicopter rotor blade has a uniform twist from the root to the tip, the inner radial sections can be in a stall condition while the outer radial section need not be. The occurrence of such stall for the inboard sections is shown in the calculations presented in Appendix II. For rotors, however, the inner radial sections contribute such a small percentage to the total rotor lift that it would be difficult to assess the degree of inboard localized stall from experimental lift data.

Figure 14 presents the equivalent rotor lift-collective pitch angle data. These data show that up to equivalent lift values corresponding to 8500 pounds, the lift slope curves are generally straight lines. Only when equivalent lift values correspond to lifts of the order of 9500 pounds is there an indication that rotor blade stall is beginning to exert a significant influence on the equivalent lift value. The fact that the data does appear linear up to and beyond the value shown in the calculations implies that the method of calculation presented in Appendix II is reasonable.

TEST SECTION SIZE INFLUENCE ON LIFT

In this study involving model helicopter rotors, the approach initiated in Reference 1 is continued. This approach postulates that if rotor lift and wake performance is to be examined, the hovering rotor (free, unbounded) is the logical starting point. The hovering rotor is nothing more than a propeller operating under static load. Since propeller theory has been undergoing refinement for a considerable period of time, it seems reasonable to assume that the theoretically predicted performance for propellers under static load (or hovering rotors) should inherently possess a greater degree of accuracy than when forward velocity is introduced to the rotor.

While this approach stresses model/full-scale lift agreement at hover as the basis for establishing tunnel testing limits, test configurations having lesser wake deflection angles do
permit meaningful studies with test section size, rotor size and rotor location arrangement configurations other than those permitted for hover testing. The growing complexity of the wake structure, as forward velocity is introduced, apparently does not lend itself to the establishment of a simplistic formula for selecting such test arrangements once a test section size-rotor configuration is accepted which does not permit meaningful hover studies.

An examination of flight test data for the UH-1D helicopter in Reference 8 shows that the power coefficient versus thrust coefficient curves stabilize at a distance above the ground of about 1 rotor diameter. Changing the rotor height above the ground from 52 to 72 feet results in an additional 1 percent change. It therefore follows that with the rotor slightly more than 1 rotor diameter above the ground, there is essentially no ground effect on rotor thrust or powering performance. Heyson's work presented in References 9 and 10 on deflected wake flows indicates that the presence of the ground plane dominates any influence that tunnel boundaries may have on deflected wake characteristics and their subsequent effect on rotor lift performance. Because of this dominance by the ground plane, it seemed reasonable, in undertaking the initial rotor studies, to assume that if the rotor were approximately centered in an equal-sided (rounded corners) tunnel and were separated from the floor by a distance approximating a rotor diameter, little interference of the tunnel boundaries should exist. The correctness of that supposition is further supported by the present study.

It would appear that the most straightforward way of examining the influence of test section cross-sectional shape on rotor lift would be to establish the model collective pitch and the model mast angle* corresponding to specified full-scale values, record the model data, and then analyze it. Unfortunately, the actual situation is not quite that cooperative. Table III shows that a change in collective pitch of 1 degree results in a change of lift at hover of

*Since this study involved only the main rotor (no tail rotor was involved), the blade pitch variation around the azimuth includes only collective pitch and fore and aft cyclic pitch variations, as lateral pitch variations need not be introduced. Fore and aft cyclic, mast tilt, and fuselage orientation (in the pitch direction) can all be properly introduced through a given mast tilt setting, with collective pitch set on each individual blade.
the order of 1,430 pounds and at 70 knots of the order of 1,900 pounds. The rigging tolerance on the UH-1D main rotor is ±1/2 degree. Now include the error introduced by the instrumentation accuracy tolerance and additional variations in rotor performance resulting from center of gravity limits, and even with the best field data, it is doubtful if the absolute collective pitch angle information can be better than the order of ±1 degree. Yet, it has just been shown that a 1-degree change in collective pitch produces a change in lift of the order of 1,500 pounds (at hover) which, in turn, is 25 percent of the 6,000-pound vertical load case under examination! In addition, computer predictions appear to be valid only at hover and at velocities above the order of 35-40 knots. Therefore, there exist no valid criteria to which the model data obtained during the transitional flight regime can be compared. Computer predictions are questionable and flight data lack the required absolute accuracy. However, because the collective pitch and mast angle combinations were held constant for all test section configurations in this study, and because only the absolute collective pitch angle (of field data) is believed subject to significant error, not the differential angle required for two different lift values, the influence of test section size on rotor lift can be examined by comparing the change in rotor collective pitch required to produce a given change in lift.

The data presented in Tables II and III obtained from USAAVLABS (Reference 1) for UH-1D operation permits a determination of the change in rotor collective pitch required to produce an increase in the vertical load from 6,000 to 8,000 pounds as a function of forward velocity. Plotting the lift coefficient values obtained during this experimental study as a function of tested rotor collective pitch angle permits a determination of the slope of the lift coefficient curve as a function of the rotor collective pitch angle, and that in turn permits a calculation of the change in the rotor collective pitch angle required to increase the rotor lift from 6,000 to 8,000 pounds. It is permissible to assume that the lift coefficient–collective pitch angle curves for the model studies are linear in this lift range because tests performed to establish the relationship of model rotor lift as a function of model rotor collective pitch angle covered a larger equivalent lift value range than was required to satisfy the program objectives. These data indicated no significant departure from a linear relationship for the lift value range under investigation. The full-scale data is also linear. (See Table II.)
The experimentally determined collective pitch angle change required to produce a change in lift from 6,000 to 8,000 pounds vertical load is compared with the USAAVLABS (Reference 1) data in Tables IV, V, and VI, for rotor height to tunnel height ratios of 0.50, 0.65 and 0.80 respectively. No corrections or adjustments have been made to the experimental data other than adjusting the incompressible lift coefficient value to the equivalent compressible value through the use of the Prandtl-Glauert factor mentioned earlier. (The calculations presented in Appendix II indicated good total rotor lift agreement at the model and full-scale Reynolds number values (7,000-pound vertical load situation); thus, no correction for Reynolds number influence was necessary.)

A study of these three tables shows that only the equal height-width sections with rounded corners and with the rotor centered in the test section result in rotor collective pitch angle changes which agree well with full-scale values over the entire forward velocity range. Other test section sizes and other rotor heights result in rotor collective pitch angle changes agreeing with the full-scale values over certain limited velocity ranges, but none of the test section size-rotor location configurations examined surpass the agreement obtained with the centered rotor and the equal-sided test section arrangement.

Why do the data obtained with a rigid model rotor, in this study configuration, agree so well with full-scale rotor values? It is believed that there are a number of reasons why this is so. Total rotor lift is being measured. Therefore, while the lift produced by a particular radial section of an actual rotor may vary considerably from that produced by the model rotor at specific azimuthal locations, it is believed that the integrated lift value will not vary significantly as long as section stall is not encountered. In addition, it has been shown that the influence of Reynolds number on total rotor lift at the model and full-scale values is negligible. However, other factors which have been considered and are satisfied in these studies must be considered when studying rotor operation in tunnels; these factors may be the key to successful tunnel testing of rotors at hover and low forward velocities. These factors will now be discussed.

Since the rotor studies are being performed in a real fluid, the rotor disk loading, the fluid viscosity, the fluid density, and a length dimension, perhaps the rotor diameter, must be scaled in some manner. The Reynolds number does
<table>
<thead>
<tr>
<th><strong>6,000 Pounds Gross Weight</strong></th>
<th><strong>10,000 Pounds Gross Weight</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{kt} )</td>
<td>Collective Pitch Angle ( (\text{deg}) )</td>
</tr>
<tr>
<td>0</td>
<td>+13.4</td>
</tr>
<tr>
<td>10</td>
<td>12.4</td>
</tr>
<tr>
<td>20</td>
<td>11.7</td>
</tr>
<tr>
<td>25</td>
<td>11.4</td>
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<tr>
<td>35</td>
<td>11.4</td>
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<tr>
<td>70</td>
<td>12.0</td>
</tr>
<tr>
<td>90</td>
<td>12.9</td>
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<tr>
<td>110</td>
<td>14.6</td>
</tr>
<tr>
<td>120</td>
<td>16.0</td>
</tr>
</tbody>
</table>

*To change grip collective pitch angles to center of rotation collective pitch angles, add 0.8 degree to the values in the table.

**Mast tilt represents the angle between the tip path plane and the free stream, the minus sign signifying nose down. The mast tilt angle accounts for the aggregate of cyclic pitch, flapping, fuselage tilt, and mast tilt with respect to fuselage.

Additional points for 4,000 pounds and 8,000 pounds gross weight may be computed on a linear basis.
<table>
<thead>
<tr>
<th>V_{kt}</th>
<th>Grip Collective Pitch Angle (deg)</th>
<th>Δ- Collective Pitch Angle Change (deg) for a 2,000-Pound Lift Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.4</td>
<td>16.2</td>
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<tr>
<td>10</td>
<td>12.4</td>
<td>14.8</td>
</tr>
<tr>
<td>15</td>
<td>12.0</td>
<td>14.5</td>
</tr>
<tr>
<td>25</td>
<td>11.4</td>
<td>14.1</td>
</tr>
<tr>
<td>35</td>
<td>11.4</td>
<td>13.9</td>
</tr>
<tr>
<td>70</td>
<td>12.0</td>
<td>14.1</td>
</tr>
<tr>
<td>110</td>
<td>14.6</td>
<td>15.2</td>
</tr>
<tr>
<td>Test Section Size</td>
<td>Velocity - Kt</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hover</td>
<td>10</td>
</tr>
<tr>
<td>USAAVLABS DATA *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 x 20</td>
<td>1.40</td>
<td>1.20</td>
</tr>
<tr>
<td>20 x 14</td>
<td>1.43</td>
<td>1.36</td>
</tr>
<tr>
<td>28 x 28</td>
<td>1.20</td>
<td>1.60</td>
</tr>
<tr>
<td>20 x 28</td>
<td>1.44</td>
<td>1.27</td>
</tr>
<tr>
<td>28 x 20</td>
<td>1.47</td>
<td>1.51</td>
</tr>
<tr>
<td>28 x 20</td>
<td>1.27</td>
<td>1.37</td>
</tr>
</tbody>
</table>

*See Table III
TABLE V. REQUIRED COLLECTIVE PITCH ANGLE CHANGE TO INCREASE THE VERTICAL LIFT FROM 6,000 POUNDS TO 8,000 POUNDS GROSS WEIGHT - ROTOR HEIGHT ABOVE THE FLOOR 0.65 OF TUNNEL HEIGHT

<table>
<thead>
<tr>
<th>Test Section Size</th>
<th>Velocity - Kt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hover</td>
</tr>
<tr>
<td>Height x Width</td>
<td></td>
</tr>
<tr>
<td>USAAVLABS DATA *</td>
<td>1.40</td>
</tr>
<tr>
<td>14 x 20</td>
<td>1.24</td>
</tr>
<tr>
<td>20 x 14</td>
<td>1.28</td>
</tr>
<tr>
<td>28 x 28</td>
<td>1.67</td>
</tr>
<tr>
<td>20 x 28</td>
<td>1.35</td>
</tr>
<tr>
<td>28 x 20</td>
<td>1.35</td>
</tr>
</tbody>
</table>

*See Table III
<table>
<thead>
<tr>
<th>Test Section Size</th>
<th>Velocity - Kt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hover 10 15 25 35 70 110</td>
</tr>
<tr>
<td></td>
<td>1.40 1.20 1.25 1.35 1.25 1.05 0.30</td>
</tr>
<tr>
<td></td>
<td>1.24 1.25 1.13 1.55 1.70 1.56 0.56</td>
</tr>
<tr>
<td></td>
<td>1.50 1.41 1.28 1.30 1.33 1.69 0.63</td>
</tr>
<tr>
<td></td>
<td>1.14 1.39 1.05 1.21 1.13 1.38 0.66</td>
</tr>
<tr>
<td></td>
<td>1.21 1.28 1.37 1.40 1.31 1.58 -</td>
</tr>
<tr>
<td></td>
<td>1.21 1.42 1.08 1.39 1.29 1.64 0.73</td>
</tr>
</tbody>
</table>

*See Table III*
consider these characteristics but, as usually calculated, is concerned with the lift and drag characteristics of the airfoil section, not the characteristics of wake dissipation which are believed to be of at least equal importance in studies involving tunnel constrain':. Perhaps the easiest way to grasp the significance of this line of reasoning is to reconsider the hovering rotor case. The hovering rotor can be operated as a propeller in a tunnel (see Figure 12). With this arrangement (to the rotor), there is no floor or ceiling, only boundary walls. Through tests of the rotor with this arrangement, it was determined, with the use of tufts which spanned the test section, that the absolute projection distance of the wake, as determined from observing the motion of the tufts, was a function of the absolute rotor load per unit area. Figure 15 presents the model wake projection distance, for a hovering rotor, observed in the tunnel for different absolute rotor loads but with the same rotor lift coefficient. From this figure, it follows that if the model rotor were mounted in the usual manner, i.e., the rotor shaft perpendicular to the tunnel floor, rather than aligned with the tunnel test section longitudinal axis, the distance the tunnel floor must be separated from the rotor to avoid wake impingement is a function of the absolute loading of the rotor rather than the rotor lift coefficient value. The situation can be considered as one where the rotor is injecting energy into the fluid stream which is then being absorbed by the viscous action of the fluid. It is believed that the wake dissipation pattern, created in the model tests and with the model test medium, must scale the wake energy dissipation created by the full-scale vehicles when in its operating medium if similar wake patterns are to be achieved. It follows that when considering tunnel test section size influence on rotor performance, rotor lift coefficient agreement is not sufficient. The absolute rotor load and the viscosity of the fluid should be considered. The importance of the absolute load when testing models was recognized by South in Reference 11, but the equally important aspect of fluid viscosity was not included, probably because wind tunnel studies do not entail significant differences in the test fluid viscosity compared with the viscosity in which the prototype operates. Thus, this inability to vary the test fluid viscosity may be considered a disadvantage when considering the proper scaling of model studies performed in air with model studies performed in a more viscous fluid (water).

It is believed that some measure of the prototype rotor load, in conjunction with the prototype fluid viscosity, should be scaled or represented in the model studies since these two
factors will influence the wake projection distance. The influence of the absolute rotor load on the wake projection distance of the model rotor, in the hover mode, was shown in Figure 15. Wake projection distance may not be of significant concern in the study of isolated rotors but is of concern when considering complete aircraft models. For example, in the study of main rotor/tail rotor interactions, model wake projection similarity with full-scale wake projection must be maintained. Rotor lift coefficient scaling alone is not sufficient since the manner in which the wake projects away from the main rotor tip path plane plays an important part in how the main rotor wake will interact with the tail rotor wake.

In the present study, the rotor load per unit area was 3.87 and 2.97 pounds per square foot for the full-scale and model rotors respectively for the 7,000-pound vertical load situation. The average through-disk velocities were of the order of 42 and 1.2 feet per second respectively, with air something like 200 times less viscous than water. With these data, any number of dimensionless scaling parameters can be derived, but the significance of these parameters on rotor lift and wake form characteristics could not be accurately determined since there was not enough data to establish such a relationship. Therefore, this area needs additional investigation. At the present time, all that can be said is that some measure of wake scaling, which involves the wake dissipation distance, must be considered when testing model rotors in tunnels. It is further believed that the water tunnel studies performed at Oceanics have quite accurately represented the free-air wake characteristics based on the following:

1. Reynolds number differences of the order existing between the model and full-scale values indicate little change in the calculated total rotor lift value.

2. The combination of the absolute load per unit area of the model rotor and the fluid viscosity of the model test medium (water) produced a wake which was essentially dissipated in a distance approximating one rotor diameter. This distance duplicated the ground clearance distance indicated by full-scale data as sufficient to prevent the ground from influencing rotor lift performance.

3. If a tunnel cross section has equal sides and
the rotor is centered, and if the rotor is separated from the floor by a distance such that the wake is essentially dissipated by viscous action in traveling to the floor, it follows that the influence of the floor on rotor performance should be negligible and, since the magnitude of the influence of the floor dominates the magnitudes of the additional influence of the walls and ceiling, the influence of the walls and ceiling on rotor performance should be negligible also, except for the recirculation-reingestion which can occur up to equivalent forward velocities of the order of 15 knots.

4. Recirculation-reingestion influences the absolute value of the rotor collective pitch angle required to obtain a specific lift value, although it does not influence the change in the rotor collective pitch angle required to produce a given increase in lift. By utilizing techniques discussed in this report, the absolute collective pitch angle required for a specific lift can be determined.

5. The total rotor load developed by a nonrigid rotor in one revolution is reasonably well represented by a rigid rotor, although details of the rotor load as a function of azimuthal location undoubtedly differ.

6. Prandtl-Glauert corrections for incompressible-compressible data applied at the 0.7 radial section appear to be adequate.

The preceding discussion is associated with the procedures believed to be most reliable in ascertaining the influence of test section size and rotor location on rotor lift and wake performance. The procedures are believed to be reliable because of the recognized weakness in accurately determining the absolute rotor collective pitch angle, the rotor tip path plane orientation, and the blade cyclic variations. However, for the sake of completeness, a direct comparison of the USAAVLABS and water tunnel data will now be made.

In Figures 16 and 17, the rotor collective pitch angles as measured from the rotor center of rotation are presented as a function of forward velocity. These figures present the USAAVLABS (Reference 1) data and the model data obtained in
both the 28-inch by 28-inch and the 20-inch by 20-inch test sections. The rotor height above the floor of the tunnel is 0.50 of the tunnel height. At hover, the degree of correlation between all sets of data is excellent. With the introduction of forward velocity, the values obtained with the 28-inch by 28-inch and the 20-inch by 20-inch test sections agree well, but there is a difference between the model test values and the USAAVLABS (Reference 1) values. The difference in the collective pitch values between the two sets of data is of the order of 1 degree for the velocity range tested.

In view of the recognized difficulties in determining the absolute values of the rotor collective pitch angle from full-scale flight data, coupled with the model test procedures of lumping together the mast angle, fuselage orientation, and fore and aft cyclic occurring in the real vehicle into a single mast angle value for model tests, the degree of agreement existing between the model and the USAAVLABS (Reference 1) data is about all that could be expected. However, it is believed that the agreement must be better for truly meaningful model studies.

It is believed that perhaps the model data may actually be more representative of the actual or "true" rotor collective pitch values than the USAAVLABS (Reference 1) data.

The USAAVLABS (Reference 1) collective pitch angle values were obtained by plotting a considerable amount of flight data and then fairing a curve through this data to select specific collective pitch and mast angle values for given forward velocities. If there are many data points equally spread across a band of data scatter, the tendency is to fair the curve at the mid-point of the data scatter limits. This method of interpreting data was much the same as the procedures employed for the Oceanics model test data presented in Reference 1 where a particular test condition was repeated until at least two lift coefficient values fell within a 3-percent data scatter band (of each other) for the collective pitch angle and equivalent lift value under consideration. However, the spread of the 3 to 6 test points normally obtained for each test situation was of the order of 5 to 6 percent. The faired model data agreed better with the USAAVLABS (Reference 1) data than did the data presented in this report. (See Table III of Reference 1.) However, the rotor collective pitch values presented in this report were obtained with an improved technique, in that a trace of the rotor lift-time history was used in determining the lift coefficient value associated with each particular
test situation. The procedure of obtaining lift coefficient values from the lift-rps-time history trace produced from an X-Y plotter trace is believed to be more accurate than the earlier procedure of recording lift-rps values at single points in time.

The data presented here are believed to be a better representation of the true full-scale operating values than the USAAVLABS data (Table III of Reference 1) for another reason. Simple strip theory indicates that the change in the rotor collective pitch angle required when flight conditions change from hover to a forward velocity of 10 knots should be of the order of 0.1 degree. This model data indicates pitch angle changes of that order of magnitude, but the full-scale data of Reference 1 shows pitch angle changes of the order of 1 degree. It does not seem reasonable that strip theory predictions should be in error by a factor of 10, but it does seem reasonable to recognize that measurements of the full-scale rotor collective pitch angle under flight conditions cannot be obtained to an accuracy better than the order of 1 degree. Therefore, this apparent discrepancy between model and full-scale data exists.

TEST SECTION SIZE INFLUENCE ON INFLOW AND WAKE PATTERNS

As mentioned in an earlier section, dye was emitted from positions located upstream of the rotor. This injection of neutral-density dye permitted a visualization of the stream paths through the rotor.

In Reference 12, Hough and Ordway present a consideration of the streamlines associated with the perturbation field of a propeller. They point out that this perturbation contribution can be regarded as a crude "first approximation" to the flow field which occurs with a propeller operating in the static condition. Figure 18 presents the perturbation mean flow streamlines associated with a representative blade circulation distribution approximating Goldstein's optimum, together with a photograph of the model rotor inflow pattern as illustrated by dye streaks at the hover condition. Note the striking similarity between the calculated streamline pattern and the dye streak pattern. Hough and Ordway point out in discussing Figure 18a, "Perhaps the most striking feature that we notice, though, is the appearance of a 'natural hub' created within the flow field. This is caused by the local inner trailing vortex system. These vortices are of opposite sign to those further outboard and so induce a flow 'upstream'. As a result, we expect that the hub for a real propeller probably does not disturb the flow as much as might be thought before-
The appearance of this natural hub, as illustrated by dye streaks from these studies, is shown in Figure 19. This figure also shows the large amount of fluid entrained with the starting vortex.

A characteristic of interest when testing flows involving sharply deflected wakes is the angular relationship of the wake with some reference plane. This angular relationship is commonly referred to as the skew angle when measured from the axis of rotation of the helicopter rotor and the deflection angle when measured from the rotor tip path plane.

Determining what constitutes the angle under consideration is a simple matter for the hovering rotor case or for cases above forward velocities of the order of 50 knots, because for these cases the wake can be accurately represented solely by a cylindrical configuration (hover case) or a skewed cylindrical configuration (forward velocity above the order of 50 knots). For the cases in between these velocity limits, the determination of what constitutes "the" wake angle becomes a more difficult task.

In the transition range, the wake consists of two elements, one element identified as the mass flow through the rotor disk and the other element identified as a rolled-up portion of the wake which is reminiscent of the pair of trailing vortices shed by a fixed wing. Photographs of these two wake elements shown in Figure 20 illustrate the difference in the two skew angles. Because of the existence of these two wake elements, each with their own wake angle, the question is raised as to which element of the wake should be identified as "the" rotor wake angle. Since the mass flow through the rotor disk is always present, while the rolled-up portion reminiscent of the trailing vortex system of a fixed wing is not, it is believed that the angle associated with the mass flow wake must be the angle examined if the wake angle is to be used as a characteristic in determining the test section size-rotor location influence on rotor performance.

The wake angle discussed in these studies is the angle existing between the mass flow path through the rotor disk as depicted by the dye streak photographs and a reference plane associated with the longitudinal axis of the tunnel test section. The rotor tests were run with the introduction of a mast tilt angle as forward velocity was introduced. Therefore, the mass flow wake angle presented here includes the mast tilt angle.
The measured mass flow wake angles indicated surprisingly less variation with changes in test section size or rotor location than might be expected. In general, the equal width-height test sections resulted in wake deflections that were somewhat more strongly downward than did sections that had unequal height-width dimensions. This variation is illustrated in Figure 21. Increasing the rotor height from its centered position slightly reduced the deflection angle at the hover and lower forward velocities. This effect is shown in Figure 22 for the 28-inch by 28-inch test section. Changing the equivalent vertical load from 6,000 pounds to 8,000 pounds did not produce any noticeable change either. Figure 23 presents data for the 6,000-pound and 8,000-pound cases with the 20-inch by 28-inch test section and a rotor height above the floor of 13 inches. The data presented in Figures 21, 22 and 23 are typical; thus, all data plots are not included in this report.

The conclusion from the wake angle observations made during these investigations is that the wake angle is not strongly influenced by test section size or rotor location. However, it must be strongly emphasized that all of these investigations were made under test conditions wherein the rotor height above the floor of the tunnel was such that the wake was essentially dissipated by the viscous action of the test fluid before reaching the floor of the tunnel. It therefore follows that under these conditions (essential wake dissipation before encountering a boundary), the wake angle should not be expected to be influenced to a significant degree by a changing of the boundary locations or the position of the rotor above the floor of the tunnel for the range investigated in these studies.
CONCLUSIONS

The most important conclusion gained from this study is the recognition of the importance of dynamic viscosity when testing models having sharply deflected wakes in tunnels. Reynolds number scaling is not sufficient. Specifically:

1. The manner in which the wake energy is absorbed by the surrounding fluid must be appropriately duplicated in model size if proper scaling between the model and full-scale flow regimes is to be maintained.

2. Because the absorption of the wake energy by the surrounding fluid is a function of the viscosity of the fluid, it follows that if a reduced scale model is tested in the same medium as that in which the full-scale vehicle operates, appropriate scaling of the wake energy absorption pattern in the model studies is extremely difficult because Reynolds number scaling must also be maintained. This requirement has led to the correct conclusion by both analytical and experimental investigators that relatively large test sections are required for model studies performed in air when rotors operating in the hover or transitional flight regimes are being investigated.

3. Testing helicopter models in water rather than air offers two significant advantages:
   a. Reynolds number scaling is easily obtained because of the kinematic viscosity advantage of water over air.
   b. The wake energy dissipation pattern can be easily scaled because of the dynamic viscosity advantage of water over air.

Other conclusions are:

4. The total rotor lift capability must be determined by examining the lift coefficient capability of the blade sections location. The inboard sections of a blade operating at the model Reynolds number may encounter stall sooner than they would when
operating at the full-scale Reynolds number because the maximum obtainable lift coefficient is a function of the Reynolds number. However, the total rotor lift may only be slightly affected since the inboard sections make only a small contribution to the total lift.

5. Testing of rigid model rotors in an incompressible fluid with appropriate corrections for incompressible-compressible correlations appears satisfactory if total rotor load and general wake configurations are of interest. A more precise scaling of the model and the Mach number may be required for studies involving rotor drag and torque variations, the rotor blade slap phenomena, etc.

6. For these studies, the rotor was positioned at a distance above the floor to permit the wake to effectively dissipate before encountering the floor. Therefore, the fact that the wake angles observed for the different test sections and rotor heights showed little change is to be expected.

7. The model data obtained with the equal-sided test section configurations and with a centered rotor agreed best with the comparable full-scale values over the entire velocity range examined. Model data obtained with certain other test section configurations and rotor heights also agreed well with full-scale values, but only over limited velocity ranges. Therefore, the equal-sided test section with a centered rotor appears to be the best overall test configuration, provided, of course, that the other criteria listed here are also satisfied.

8. Until further definition of the wake energy dissipation scaling factor is obtained, the following test procedures are suggested. These procedures should permit meaningful model tests of helicopter rotors from hover to any desired forward velocity. It is believed that these suggested techniques are valid for either wind or water tunnel investigations. The basic assumption in formulating these procedures is that helicopter rotors operate out of ground effect when the ground clearance is approximately one rotor diameter.
a. Select a model rotor size believed to be appropriate in terms of eventual Reynolds number scaling.

b. Operate the model rotor as a propeller, i.e., on the longitudinal axis of the test section at the appropriate collective pitch angles but without flow in the tunnel, i.e., hover condition.

c. Through the use of tufts spanning the test section and located at various stations downstream of the rotor, determine how far downstream the rotor wake projects after the rotor has reached a stabilized rotational speed and the speed is such as to satisfy the Reynolds number scaling criteria.

d. The distance that the wake projects downstream from the rotor tip path plane is the distance that the tunnel floor must be separated from the rotor when the rotor is centered in an equal-sided test section.
Figure 1. Photograph of Model Rotor Blades.
Figure 2. Jig Used in Setting the Collective Pitch Angle.
Figure 3. Sketch of Tunnel Test Installation.
Figure 4. Sketch Illustrating How a Uniform Rotor Blade Circulation Distribution Approximates the Flow of a Uniform Circular Jet.
Figure 5. Sketch illustrating how a typical rotor blade circulation distribution approximates the flow of an annular circular jet.
Figure 6. Sketch of the Rotor Wake and a Photograph of the Model Tip Vortex Wake at Hover.
Figure 7. Sketch and Photographs Illustrating the Nature of Model Rotor Tip Vortex Wake Over an Equivalent Velocity Range of the Order of 10-70 Knots.
Figure 8. Sketch and Photographs of the Nature of the Model Rotor Tip Vortex Wake Occurring at Equivalent Velocities Above the Order of 70 Knots.
Figure 9. General Form of Typical Model Rotor Lift-RPS-Time Histories.
Figure 10. Model Rotor Lift-RPS-Time History at Hover.
Figure 11. Sketch Illustrating How Recirculation-Reingestion (Alone) Reduces Rotor Lift by Reducing the Effective Attack Angle of the Section.
The Tufts Located on the Test Section Vertical Centerline Were Employed in Determining the Wake Projection Distance for the Hovering Rotor as a Function of Absolute Rotor Load. (Tufts Are Shown Extended. Without Flow, the Tufts Hang Straight Down.)

Figure 12. Sketch of Tunnel Test Arrangement Employed To Determine Tunnel Recirculation-Reingestion Effect on a Hovering Rotor.
Test Arrangement as in Figure 12
Zero Velocity = Hover Case

Lift as Percent of Lift at Hover

Δ-Tunnel Velocity Imposed on Model Rotor - Equivalent Knots

Figure 13. Plot Illustrating Nature of Hovering Rotor's Lift Decrease
With the Introduction of a Δ-Velocity Through the Rotor Disk
(Analogous to Tunnel Recirculation-Reingestion at Hover).
Data Taken With Recirculation and at Mast Angle Associated With Each Specific Velocity (See Table II)

Figure 14. Model Rotor Collective Pitch Angles as a Function of Equivalent Full-Scale Lift.
Test Arrangement as in Figure 12

\[ \text{Absolute Rotor Load - lb/ft}^2 \]

0 0.5 1.0 1.5 2.0 2.5 3.0

Wake Projection Distance in Rotor Diameters

\( O \) = Point corresponding to full-scale 6,000-pound load. The wake projection distance corresponds well with out-of-ground-effect distance for full-scale rotor.

\( 1 \) = Curve defining tunnel floor clearance distance believed adequate, i.e., only very slight motion of the fluid at this distance.

\( 2 \) = Curve defining maximum disturbance distance.

Figure 15. Curves Illustrating the Model Wake Projection Distance of a Hovering Rotor as a Function of Absolute Rotor Load per Unit Area.
Figure 16. Comparison of the Model Collective Pitch Angle as a Function of Forward Velocity With USAAVLABS Data for a 6,000-Pound Vertical Load.
Figure 17. Comparison of the Model Collective Pitch Angle as a Function of Forward Velocity With USAAVLABS Data for an 8,000-Pound Vertical Load.
Figure 18. Sketch and photograph of the flow field of a hovering rotor.
a) Sketch from Reference 12
b) Photograph of Model
Figure 19. Photographs Illustrating the "Natural Hub" in the Flow Field of a Hovering Rotor.
Figure 20. Photographs Illustrating the Difference in the Wake Angles Associated With the Two Elements of Rotor Wakes: the Rolled-Up Portion and the Mass Flow Portion.
Figure 21. Plot of the Mass Flow Wake Deflection Angles as a Function of Forward Velocity for Different Test Section Sizes.

Rotor Centered in the Tunnel
Equivalent Vertical Load 6,000 Pounds
Data Taken With Tunnel Recirculation
Figure 22. Plot of Mass Flow Wake Deflection Angles as a Function of Forward Velocity for a Change in Rotor Height Above the Tunnel Floor.
Figure 23. Plot of the Mass Flow Wake Deflection Angles as a Function of Forward Velocity for a Change in Vertical Load.

- ○ - 6,000-Pound Equivalent Load
- △ - 8,000-Pound Equivalent Load

20-Inch-High x 28-Inch-Wide Test Section
Rotor Height 13 Inches Above the Floor
Data Taken With Tunnel Recirculation
LITERATURE CITED


6. von Glahn, Uwe, H., EXPLORATORY STUDY OF GROUND PROXIMITY EFFECTS ON THRUST OF ANNULAR AND CIRCULAR NOZZLES, Lewis Flight Propulsion Laboratory, Cleveland, Ohio, Technical Note 3982, National Advisory Committee for Aeronautics, April 1957.


APPENDIX I

THE AIR BUBBLE TECHNIQUE USED IN VISUALIZING TIP VORTEX TRAILS

The disturbances which are created in incompressible airflow by an object will also exist in water. With the flow disturbances existing in water, the presence and location of the disturbed flow (vortex patterns) can be made visible in several ways. For the case of tip vortex patterns, cavitation and air bubble injection offer the greatest advantages. Cavitation is the local vaporization of the water due to a local pressure reduction brought about by local velocity increases. The presence of the vapor- or gas-filled void offers an excellent way of viewing a vortex trail since nothing is added to the test medium. The vaporization of the water in the cores of the vortices is brought about by decreasing the ambient pressure in the water tunnel until this phenomenon (cavitation) occurs.

Under certain test conditions, while tip vortices exist, it may not be possible to reduce the ambient pressure in the tunnel to produce a vaporous core in the tip vortex trail before causing cavitation on the upper portion of the lifting surface. If cavitation occurs, the lift and drag characteristics of the lifting surface may be altered to such a degree that they no longer adequately represent the normal operation of the lifting surface. For example, should the upper surface of the wing begin to cavitate during water tunnel tests involving wing tip vortex trail studies, the cavitation region can be roughly compared to a "stall" condition, so that flow conditions over the wing no longer represent performance before the occurrence of the "stall". If that situation occurs, the ambient pressure can be raised to prevent the unwanted surface cavitation and, through proper instrumentation in the wing, air can be injected into the core of the tip vortex. Because of the centrifuging action of the vortex and the lesser density of the air bubbles, the air bubbles are centrifuged to the center of the vortex. The air bubbles remain "trapped" in the core of the vortex until the circulation strength of the vortex reduces to a level less than the bouyant force of the bubble. When this occurs, the bubbles leave the vortex formation. Air bubbles not remaining in the vortex will interfere with visual observation, so that the proper amount of air injection is important. However, it should be emphasized that the air bubbles trapped in the vortex core patterns do not distort the vortex pattern, just as the vaporous cavities do not distort it.
Reynolds number scaling is a well recognized requirement for meaningful model studies. However, below the stall, blade lifting characteristics are virtually independent of Reynolds number and the primary effect of Reynolds number is to limit the maximum lift that can be achieved at any radial section. For a helicopter rotor operating at a full-scale Reynolds number all sections are likely to be unstalled, while for operation at a model-scale Reynolds number some sections may have stalled out. However, if, at model scale, only the sections near the hub are stalled, the effect on the overall lift will be small since these sections contribute very little to the overall lift whether they are stalled or not. Consequently, if the model tests are carried out at a Reynolds number for which only inboard sections are stalled, such tests should still give good predictions of full-scale total lift characteristics.

The relative insensitivity of rotor total lift characteristics to Reynolds number is illustrated in Table VII, where strip theory at two different Reynolds numbers has been applied to a 48-foot-diameter UH-1D rotor having a chord of 21 inches and an NACA 0012 airfoil. The twist in the rotor blade is 0.454 degree per foot. The tip speed of the rotor is 800 feet per second.* The collective pitch angle of the rotor is 14.1 degrees, which corresponds to the setting for a total lift of 7,000 pounds. The blade was divided into 10 equal strips, and the true sectional lift coefficient at the appropriate Reynolds number was applied in calculating the lift of each section. It can be seen that the total lift obtained in this way for the widely disparate tip Reynolds numbers of $7.9 \times 10^6$ and $9.1 \times 10^6$, respectively, differ by less than 2 percent despite the fact that the inboard sections of the rotor at the smaller Reynolds number are stalled.

*Although compressibility effects are likely to be important for determining true rotor lift at this speed, for simplicity, incompressible flow has been assumed in making the present Reynolds number comparisons.
TABLE VII. COMPARISON OF CALCULATED TOTAL ROTOR LIFT AT TWO REYNOLDS NUMBERS; HOVER CONDITION; 48-FOOT-DIAMETER ROTOR; 800-FOOT-PER-SECOND TIP SPEED; INCOMPRESSIBLE FLOW ASSUMED; ELEMENT AREA OF 2.45 SQUARE FEET; AIR DENSITY OF 0.00218 SLUG PER CUBIC FOOT

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\[
\Sigma = 3494.7 \\
\text{lb per blade}
\]

\[
\Sigma = 3427.3 \\
\text{lb per blade}
\]

Difference = 1.93 percent