A Rapidly Converging Iterative Technique for Computing Wind Compensation Launcher Settings for Unguided Rockets

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A RAPIDLY CONVERGING ITERATIVE TECHNIQUE FOR COMPUTING WIND
COMPENSATION LAUNCHER SETTINGS FOR UNGUIDED ROCKETS

By

Louis D. Duncan
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ABSTRACT

This paper presents the development and evaluation of two algorithms for the determination of wind compensating launcher settings for unguided rockets. The algorithms are designed for use in an iterative trajectory simulation process and can be used to determine launch angles which will yield one of many possible trajectory objectives. One algorithm provides the first estimate as a function of the ballistic wind; the other controls the iteration.

Four test cases are presented to evaluate the technique. Sixteen different wind profiles were applied to four separate unguided rockets. Two different trajectory objectives were considered - nominal burnout attitude and nominal impact. The tests show that the process converges rapidly, usually requiring only one or two iterations.
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INTRODUCTION

The trajectory of an unguided rocket or projectile will be affected by the wind encountered during flight. This perturbation of the trajectory is commonly called wind effect. If one desires to obtain a predetermined value of a given trajectory objective (such as impact of a given booster, orientation of the velocity vector at burnout, etc.), it is necessary to determine a launcher setting which will compensate for the wind effect. The sophistication of the algorithms to determine these launcher settings depends not only on the specified trajectory objective but also on the rocket configuration and the nominal trajectory.

A first-order approximation of the wind effect may be obtained by a technique called "wind weighting." This technique is discussed in Appendix A. Several procedures [Lewis (1949), James and Harris (1960), Hennigh (1964), Duncan and Engebos (1966a, 1966b)] based on the wind weighting technique have been developed which give first-order approximations for the launcher settings required to compensate for the wind effect.

In the previous developments, the model was usually restricted to considering either a specific trajectory objective or a specific type of trajectory or both. This paper presents a procedure which allows for a much wider range of applicability.

The model presented herein has a twofold capability. An approximation for launcher settings based upon the wind weighting technique is developed. For missile projects which require more accurate computations, an iterative procedure is developed. The algorithms for the iteration converge rapidly; the wind-weighting-based model is used as a first approximation. The iterative procedure can also be used to refine the numerical values for the wind-weighting-based model. The use of the complete model as an operational tool requires a real-time computational capability (Duncan and Rachele, 1967).

COORDINATE SYSTEMS AND TRANSFORMATIONS

In this development it is assumed that the rocket trajectory is specified in a right-hand topocentric coordinate system (x,y,z). The positive x-axis points east, and the positive y-axis points north. The azimuth angle, α, and the elevation angle, θ, are defined in Figure 1.
The equations developed herein are based upon the angles $\theta_1$ and $\theta_2$ which are the components of the elevation angle in the y-z and x-z planes, respectively. There is, of course, a one-to-one correspondence between the pair $(\theta, \alpha)$ and the pair $(\theta_1, \theta_2)$. The transformation equations are

\[ \theta_1 = \tan^{-1}(\tan \theta \cos \alpha) \]  

\[ \theta_2 = \tan^{-1}(\tan \theta \sin \alpha) \]  

and

\[ \alpha = \tan^{-1}(\tan \theta_2 \cot \theta_1) \]  

\[ \theta = \tan^{-1}[\tan \theta_1 \cot^{-1}(\tan \theta_2 \cot \theta_1)] \]  

\[ = \tan^{-1}(\tan \theta_1 / \cos \alpha). \]
DISCUSSION

The launch angles required to compensate for wind effect depend upon the desired trajectory objective. These angles are determined by an iterative process of trajectory simulation. The first estimate is determined from the ballistic wind. A trajectory is computed using these angles. If the simulated objective is not within tolerance of nominal, a correction for launcher angles is determined from the error vector and a new trajectory is simulated. The iteration is continued until the tolerance is achieved. This procedure requires two algorithms, one for the first estimate and another for the iteration step.

The first estimate is based upon the ballistic wind. The algorithm for this estimate can be expressed in functional form by

$$\Delta \theta_1 = f_1(W_x, W_y)$$
$$\Delta \theta_2 = f_2(W_x, W_y)$$

(3)

where $W_x$ and $W_y$ are the East-West, North-South ballistic winds, respectively, and $\Delta \theta_1$ and $\Delta \theta_2$ are required changes from nominal settings.\(^*\)

The functional form for the iteration step is similar,

$$\Delta \theta_1 = g_1(\alpha, \beta)$$
$$\Delta \theta_2 = g_2(\alpha, \beta)$$

(4)

where $\alpha$ and $\beta$ are the component deviations from the nominal objective and $\Delta \theta_1$ and $\Delta \theta_2$ are the changes in the launcher settings required for the next iteration.

\(^*\)When the procedure is used in a real-time meteorological system where launcher settings are computed for consecutive (timewise) wind profiles, $W_x$ and $W_y$ are replaced by the changes in the ballistic winds from the last profile and $\Delta \theta_1$ and $\Delta \theta_2$ become changes from the settings obtained for that profile.
Although the development presented herein is directed toward either of the following trajectory objectives:

1) Orientation of velocity vector at burnout, and
2) The \(x, y\) coordinates of the impact of either the afterbody or a prescribed booster, it is apparent that the procedure is applicable to many other constraints.

**EXPRESSIONS FOR THE ALGORITHM FUNCTIONS**

The development of the algorithms requires the determination of suitable expressions for \(f_1, f_2, g_1\) and \(g_2\). Since there is little or no theoretical indication of the nature of these expressions, an empirical approach is indicated. Various types of expressions may be determined by a curve fitting procedure and then tested for accuracy and suitability. The empirical approach may, of course, yield entirely different functional forms for different missile configurations. The general procedure is, however, invariant. It consists of determining expressions for \(g_1\) and \(g_2\), using these expressions in the determination of expressions for \(f_1\) and \(f_2\), and finally, testing the algorithms.

The following paragraphs will discuss the expressions for \(f_1, f_2, g_1\) and \(g_2\) for the Athena missile. Other missile systems will be discussed in later sections.

To determine approximating expressions for \(g_1\) and \(g_2\), several trajectory simulations were performed. Each of the trajectories was computed for no-wind conditions; various changes in the launch angles were used for the simulations. The pertinent information (for the determination of \(g_1\) and \(g_2\)) was extracted from these simulations and is presented in Table I. An inspection of these results suggests that each of \(\Delta \theta_1, \Delta \theta_2, \Delta X,\) and \(\Delta Y\) is approximately linear in each of the independent variables and that the contributions of the independent variables are approximately additive.

This observation suggests a least-squares fit of the generic form

\[
g(\varepsilon_1, \varepsilon_2) = \alpha \varepsilon_1 + \beta \varepsilon_2 \quad (5)
\]

where \((\varepsilon_1, \varepsilon_2)\) represents either \((\Delta \theta_{1b}, \Delta \theta_{2b})\) or \((\Delta X, \Delta Y)\) as appropriate.
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The departures from linearity and additivity may be investigated by fitting the form

\[ g(e_1, e_2) = a e_1^2 + b e_1 + c e_2^2 + d e_1 e_2 + e e_1 e_2. \]  

(6)

Equation (5) will be referred to as the bilinear fit while (6) will be called quadratic. Each expression provides a good fit to the data in the sense that the residuals are small. The residuals for Eq. (6) are, as would be expected, somewhat smaller. Tests revealed that either of these forms provides a suitable algorithm for the iteration. These tests will be discussed in detail in a later section.

A similar procedure was used to determine expressions for \( f_1 \) and \( f_2 \). Trajectory simulations were made with various values of ballistic winds introduced into the calculations. The iterative procedure was used to determine the launcher angles required to compensate for the effect of winds. The results of these simulations are shown in Table II. As before, the quadratic and bilinear forms were fit to the data. The independent variables in this case were the components of ballistic wind, \((W_x, W_y)\). An examination of the residuals indicated that both forms provided a good fit to the data.

EVALUATION OF THE PROCEDURE

Since the equations developed in the preceding section cannot be verified by theoretical techniques, an empirical test of their accuracy is dictated. This section describes the tests and presents the results therefrom.

Sixteen different wind profiles were used in the evaluation. Some of these profiles were measured during missile support operations at White Sands Missile Range while the others are hypothetical. These wind data are shown in Figures 2, 3, 4 and 5. It should be observed that there is considerable variation among the profiles; hence, it is believed that, collectively, they provide a good test for the evaluation of the algorithms.

To show that the algorithms are not restricted to a given missile configuration and to investigate the possibility of using a missile-invariant form for the algorithm expressions, four different rockets were used in the evaluation. These were: two configurations of the Athena which have
### TABLE II.

CHANGE IN LAUNCH ANGLE REQUIRED TO COMPENSATE FOR SPECIFIED BALLISTIC WIND TO MATCH EITHER NOMINAL IMPACT OR BURNOUT ATTITUDE (REGULAR ATHENA)

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FIGURE 2. Wind Profiles Used in the Evaluation.
FIGURE 3. Wind Profiles Used in the Evaluation.
FIGURE 5. Wind Profiles Used in the Evaluation.
quite different nominal trajectories; the Aerobee-350; and the Ballistic Missile Target System (BMTS). A short description of each will be given in the following paragraphs.

Two independent iterations were performed. The first was to match nominal impact; the second was to match nominal burnout angles. A convergence tolerance of 1500 meters was selected for the impact iteration except for one case, the "regular" Athena where a tolerance of 300 meters was used. This smaller tolerance was used to show the speed with which the iteration converges. Compatible tolerances, which differed for each rocket, were used for the iterations to burnout angles.

The first and most exhaustive test was performed on the "regular" Athena. This is a typical configuration of several slightly different Athena missiles fired from Green River, Utah, to impact on White Sands Missile Range. The Athena is a two-stage (for the ascent trajectory) unguided rocket which is fired at a nominal elevation angle of approximately 13.5 degrees and achieves an apogee of approximately 250 kilometers and a range of 725 kilometers.

Both the quadratic and bilinear equations were used to perform the iterations; these were performed as independent cases. The results are shown in Tables III and IV. Two things should be observed from these results. First, both expressions provide rapid convergence. Second, there is no significant difference in the results obtained from the two expressions. This suggests the use of the simpler bilinear equations.

Only the results from the use of the bilinear equations are presented for each of the other three test cases. In every instance the iteration converged rapidly - frequently converging on the first estimate.

The Aerobee-350, used for the second test case, is a two-stage unguided high-altitude probe which is fired nearly vertical (elevation angles between 2 and 3 degrees). The nominal range and altitude vary between 100 and 150 km and 320 and 400 km, respectively. The results for this test are presented in Table V.

A special Athena trajectory requirement was the subject of the third test case. The nominal elevation angle for this trajectory is 18.534 degrees. This is near, but below, the critical angle of 17.6 degrees. (The critical angle is defined to be the elevation angle which results in maximum horizontal range under no-wind conditions.) The nominal range for this trajectory is 718.6 kilometers while the maximum (no-wind) range is 721.2 kilometers. The nominal impact could not be obtained for wind profiles 7, 8, 10, and 14. It was determined that the maximum range for these profiles was 715.6, 716.4, 715.6 and 713.1 kilometers, respectively. This decrease in maximum range is caused primarily by the
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increased drag and lift due to a head wind. The results of this test are presented in Table VI.

The final test case was for the BMTS vehicle. This is a two-stage unguided rocket fired at a nominal elevation angle of 30.2 degrees (for the case considered here), resulting in nominal range and altitude of 293 kilometers and 89.4 kilometers, respectively. The nominal elevation angle is considerably below critical angle. The results of this test are presented in Table VII.

CONCLUSIONS

A rapidly converging iterative technique to compute wind-compensated launcher settings for unguided rockets was developed and discussed. This technique is applicable to a wide range of possible trajectory objectives—specified attitude at a given time, space point, specified booster impact, etc.

The procedure consists of two algorithms. The first provides an initial estimate for the launcher setting as a function of the ballistic wind, and as such, it is easily adaptable to field operations. The second algorithm, the iterative step, converges rapidly (one or two passes) to the desired objective.

Sixteen wind profiles were used in four separate test cases to provide an evaluation of the procedure. Two different functional forms for the algorithms were investigated—a quadratic form and a bilinear form. Two specific trajectory objectives were considered in each test case—nominal burnout attitude and nominal impact. In each test the iteration converged rapidly to the desired objective. There appeared to be no significant difference between the results obtained from the quadratic and bilinear equations.
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Lewis, J. V., "The Effect of Wind and Rotation of the Earth on Unguided Rockets," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, 1949.


APPENDIX A

BALLISTIC WIND WEIGHTING

The technique of wind weighting is the usual method of determining first-order effects of the wind on the trajectory of an unguided rocket. One of the earliest discussions of this procedure was given by Lewis (1949) who also presented a procedure for calculating the information required by the weighting technique. Since Lewis' original paper, many improvements have been made in the trajectory simulation algorithms required to calculate the constants for the wind weighting. A discussion of the algorithms for simulation would require a lengthy discourse of flight dynamics and aerodynamics and will, therefore, be omitted. A partial bibliographical listing of the theoretical development of these algorithms is presented in the references.

This appendix will discuss the techniques, assumptions, and limitations of wind weighting and present the basic procedure for computing the required constants. The availability of a computer simulation program capable of determining the trajectory for a given set of conditions is assumed.

Wind velocity is ordinarily a function of altitude. A wind profile is defined to be the triple \((W_x(z), W_y(z), z)\) where \(z\) is altitude and \(W_x(z), W_y(z)\) are the \(x\) and \(y\) components of wind at height \(z\).

Wind weighting is based upon two fundamental concepts - the ballistic wind and the unit wind effect. The ballistic wind is that constant wind (as a function of altitude) which has the same first-order effect on the rocket (trajectory) as the actual wind profile. The unit wind effect is the wind effect of a constant unit wind. A concept closely associated with, and required for the computation of, the ballistic wind is the wind weighting curve. Let \(f(z)\) be the wind effect for a wind profile consisting of a uniform unit wind up to altitude \(z\) and a zero wind above \(z\). The wind weighting function, \(g(z)\), is \(f(z)\) divided by the unit wind effect; the graph of \(g(z)\) is the wind weighting curve. The derivative \(g'(z) = dg(z)/dz\) is the wind weighting factor function.

The ballistic wind is determined from the wind profile by the integrals:

\[
BW_x = \int W_x(z)g'(z)dz
\]

\[
BW_y = \int W_y(z)g'(z)dz.
\]
In practice the integrals are usually replaced by finite sums. The atmosphere is divided into horizontal strata (wind layers) wherein the wind is assumed constant; the average value over the layer is usually used. If the boundaries of the strata are \( z_1 < z_2 < \ldots < z_n \), then the ballistic wind is approximated by

\[
BW_x = \sum_{i=1}^{n-1} W_x [g(z_{i+1}) - g(z_i)]
\]

\[
BW_y = \sum_{i=1}^{n-1} W_y [g(z_{i+1}) - g(z_i)]
\]

The value \( g(z_{i+1}) - g(z_i) \) is often called the weighting factor or the ballistic factor for the \( i \)th layer.

The wind effect is calculated by multiplying the ballistic wind by the unit wind effect. These calculations inately assume

1. The effect of the wind in any particular stratum is directly proportional to the wind in that stratum.
2. The effects of the wind in the various strata are independent.

It is easy to take issue with these assumptions; in fact, one can produce results of trajectory simulations which contradict the assumptions. However, for determining first-order effects, which is the aim of the wind weighting technique, the assumptions are reasonable.

**DETERMINATION OF THE WIND WEIGHTING CURVE**

In the preceding section a definition of wind effect was omitted. Such a definition is unnecessary unless one is interested in quantitative results. If one considers the trajectory to be defined by a family of parameters, then wind effect can be broadly defined as the change in these parameters due to the wind encountered along the trajectory. Typical parameters are: first-stage impact, second-stage impact, orientation of velocity vector at burnout of a particular stage, etc.

The calculation of the wind weighting function requires the computation of a number of trajectories. Each trajectory simulation is based upon
the same set of initial conditions; however, different wind profiles are used for each simulation. These calculations yield a series of values for the trajectory parameters from which one determines the wind weighting function.

There are numerous procedures which can be followed in performing the calculations. The following procedure is somewhat typical and serves as a more precise explanation of the general procedure given above. For simplicity of explanation it will be assumed that one is interested in wind effect on impact (either afterbody or a given booster). Consider altitudes \( 0 = z_0 < z_1 < z_2 < \ldots < z_n \) and, for each \( 0 \leq i \leq n \), wind profiles \( P_i \) where each \( P_i \) consists of a zero wind above altitude \( z_i \) and a uniform wind (of constant nonzero magnitude, say \( C \), and constant direction) below \( z_i \). (The same value of the wind velocity is used in the nonzero portion of each profile.) Note that \( P_0 \) is the zero profile; the corresponding trajectory is usually called the no-wind (nominal) trajectory. Let \( R_i \) be the range, from launcher to impact, for the simulated trajectory which uses the \( i \)th profile. The unit wind effect is

\[
\sigma = (R_n - R_0)/C.
\]

Specific values for the wind weighting function, \( g(z) \), at the points \( z_i \) are

\[
g(z_i) = (R_i - R_0)/C.
\]

Observe that \( g(z_0) = 0 \) and \( g(z_n) = 1 \).

The wind weighting curve may be plotted from the values of \( g(z_i) \). Of course, one must choose the \( z_i \)'s sufficiently dense to describe the graph. The choice of \( z_i \) sometimes presents problems. This altitude must be such that any wind effect above \( z_n \) can be ignored. Apogee altitude will clearly satisfy this requirement.

**TWO-DIMENSIONAL REFINEMENTS**

In the general discussion of wind weighting, it was assumed that the weighting factor curve and the unit wind effect were one-dimensional in the sense that the magnitude of the wind effect for the uniform
profiles was assumed to be independent of wind direction. This assumption is quite good for near-vertical firings; however, for other cases, the results can sometimes be improved by extending the technique and using separate wind weighting curves and unit wind effects for the x and y wind components. The definitions are similar to those given earlier. The x-unit wind effect, \( a_x \), is the x-component of the wind effect for a profile with components \( W_x = 1, W_y = 0 \) throughout. For a given altitude, \( z \), let \( P(z) \) be the wind profile with components \( W_x = 1, W_y = 0 \) below \( z \) and \( W_x = W_y = 0 \) above \( z \). The wind weighting function for the x-component, \( g_x(z) \), is the wind effect for \( P(z) \) divided by \( a_x \). The functions \( g_y(z) \) and \( a_y \) are defined analogously.

The component wind effects and wind weighting curves are calculated similarly to those for the one-dimensional case and will not be explicitly outlined.
This paper presents the development and evaluation of two algorithms for the determination of wind compensating launcher settings for unguided rockets. The algorithms are designed for use in an iterative trajectory simulation process and can be used to determine launch angles which will yield one of many possible trajectory objectives. One algorithm provides the first estimate as a function of the ballistic wind; the other controls the iteration. Four test cases are presented to evaluate the technique. Sixteen different wind profiles were applied to four separate unguided rockets. Two different trajectory objectives were considered - nominal burnout attitude and nominal impact. The tests show that the process converges rapidly, usually requiring only one or two iterations.
1. Ballistics
2. Impact Prediction
3. Unguided Rockets
4. Wind Weighting
5. Trajectory Simulations