AUTOMATIC METHOD OF CREATING A MATHEMATICAL BASIS FOR MAPS

A. I. Martynenko
EDITED MACHINE TRANSLATION

AUTOMATIC METHOD OF CREATING A
MATHEMATICAL BASIS FOR MAPS

By: A. I. Martynenko
This document is a machine translation of Russian text which has been processed by the AN/GSQ-16(XW-2) Machine Translator, owned and operated by the United States Air Force. The machine output has been post-edited to correct for major ambiguities of meaning, words missing from the machine's dictionary, and words out of the context of meaning. The sentence word order has been partially rearranged for readability. The content of this translation does not indicate editorial accuracy, nor does it indicate USAF approval or disapproval of the material translated.
Electronic digital computers are now being used to calculate cartographic projections. However, the points are entered and the lines are drawn by hand. In order to raise labor productivity, eliminate labor consuming manual operations, and improve the working conditions of the mappers, the present author has developed an automatic method for the creation of a mathematical basis for maps. This method is based on the use of an electronic digital computer and an automatic coordinate graph. A programmed computer controls the operations of the graph. The method proposed is not restricted to the application described, but may be used to solve any problem on the conversion of digital information into graphic form and the reverse. The method may be used primarily for the conversion of the coordinates of one projection into another, taking into account the preserved scale. It may also be used to speed up the search for new geographic projections for the reproduction of grids and curves. Orig. art. has: 4 tables and 5 figures.
<table>
<thead>
<tr>
<th>Block</th>
<th>Italic</th>
<th>Transliteration</th>
<th>Block</th>
<th>Italic</th>
<th>Transliteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A a</td>
<td>A a</td>
<td>A, a</td>
<td>R r</td>
<td>R r</td>
<td>R, r</td>
</tr>
<tr>
<td>B B</td>
<td>B, t</td>
<td>B</td>
<td>S s</td>
<td>S, s</td>
<td>T, t</td>
</tr>
<tr>
<td>G g</td>
<td>G, g</td>
<td>G</td>
<td>T t</td>
<td>T t</td>
<td>U, u</td>
</tr>
<tr>
<td>D d</td>
<td>D, d</td>
<td>D</td>
<td>F f</td>
<td>F f</td>
<td>F, f</td>
</tr>
<tr>
<td>E e</td>
<td>Ye, ye; E, e</td>
<td>Ye, ye; E, e</td>
<td>X x</td>
<td>Kh, kh</td>
<td></td>
</tr>
<tr>
<td>Zh zh</td>
<td>Zh, zh</td>
<td>Zh, zh</td>
<td>Ts ts</td>
<td>Ts ts</td>
<td></td>
</tr>
<tr>
<td>Z z</td>
<td>Z, z</td>
<td>Z</td>
<td>Ch ch</td>
<td>Ch ch</td>
<td></td>
</tr>
<tr>
<td>I i</td>
<td>I, i</td>
<td>I</td>
<td>Sh sh</td>
<td>Sh sh</td>
<td></td>
</tr>
<tr>
<td>Я Я</td>
<td>Я, я</td>
<td>Я</td>
<td>Shch shch</td>
<td>Shch, shch</td>
<td></td>
</tr>
<tr>
<td>K k</td>
<td>К, k</td>
<td>К</td>
<td>Y, y</td>
<td>Y, y</td>
<td></td>
</tr>
<tr>
<td>L l</td>
<td>L, l</td>
<td>L</td>
<td>Ь Ь</td>
<td>Ь Ь</td>
<td></td>
</tr>
<tr>
<td>М м</td>
<td>М, m</td>
<td>М</td>
<td>З з</td>
<td>E, e</td>
<td></td>
</tr>
<tr>
<td>Н н</td>
<td>Н, н</td>
<td>Н</td>
<td>Юю</td>
<td>Yu, ya</td>
<td></td>
</tr>
<tr>
<td>О О</td>
<td>О, о</td>
<td>О</td>
<td>Я я</td>
<td>Я, я</td>
<td></td>
</tr>
<tr>
<td>П п</td>
<td>P, p</td>
<td>P</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* ye initially, after vowels, and after ъ, ъ; e elsewhere. When written as є in Russian, transliterate as ye or є. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.
FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

<table>
<thead>
<tr>
<th>Russian</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin</td>
<td>sin</td>
</tr>
<tr>
<td>cos</td>
<td>cos</td>
</tr>
<tr>
<td>tg</td>
<td>tan</td>
</tr>
<tr>
<td>ctg</td>
<td>cot</td>
</tr>
<tr>
<td>sec</td>
<td>sec</td>
</tr>
<tr>
<td>csec</td>
<td>csc</td>
</tr>
<tr>
<td>sh</td>
<td>sinh</td>
</tr>
<tr>
<td>ch</td>
<td>cosh</td>
</tr>
<tr>
<td>th</td>
<td>tanh</td>
</tr>
<tr>
<td>cth</td>
<td>coth</td>
</tr>
<tr>
<td>ech</td>
<td>sech</td>
</tr>
<tr>
<td>csech</td>
<td>csch</td>
</tr>
<tr>
<td>arc sin</td>
<td>sin⁻¹</td>
</tr>
<tr>
<td>arc cos</td>
<td>cos⁻¹</td>
</tr>
<tr>
<td>arc tg</td>
<td>tan⁻¹</td>
</tr>
<tr>
<td>arc ctg</td>
<td>cot⁻¹</td>
</tr>
<tr>
<td>arc sec</td>
<td>sec⁻¹</td>
</tr>
<tr>
<td>arc csec</td>
<td>csc⁻¹</td>
</tr>
<tr>
<td>arc sh</td>
<td>sinh⁻¹</td>
</tr>
<tr>
<td>arc ch</td>
<td>cosh⁻¹</td>
</tr>
<tr>
<td>arc th</td>
<td>tanh⁻¹</td>
</tr>
<tr>
<td>arc cth</td>
<td>coth⁻¹</td>
</tr>
<tr>
<td>arc ech</td>
<td>sech⁻¹</td>
</tr>
<tr>
<td>arc csech</td>
<td>csch⁻¹</td>
</tr>
</tbody>
</table>

ret       | cwr1
lg        | log
AUTOMATIC METHOD OF CREATING A MATHEMATICAL BASIS FOR MAPS

A. I. Martynenko

Recently in the calculation of cartographic projections, electronic digital computers are being used. But along with such highly productive calculation, the plotting points on the base and construction of lines of frames and grids are still produced by hand. For the purpose of increasing the labor productivity, eliminating labor consuming manual operations, and improving working conditions of the cartographers there has been developed an automatic method for the creation of the mathematical basis for maps, in which it is assumed to use a computer and an automatic coordinatograph.

The essence of the method consists in the following. By the assigned nomenclature of one sheet or a group of map sheets on the machine, into the memory of which was introduced a general programming program, there are calculated corresponding particular programs controlling the work of the coordinatograph during construction of the mathematical basis for the maps.

It is anticipated to use the universal computer, for example, "Minsk-2," "Ural-2," or "Razdan-2." A precision electrocoordinatograph with program control is used, which in contrast to the hand
The coordinatograph is endowed with drafting and engraving functions. By an assigned program it draws lines and points with a root-mean-square error of 0.05 mm. The coordinatograph assembly includes a coordinatograph and equipment for program control: linear code translator, an instrument for recording and control, panel of program control and reverse step motors.

The coordinatograph consists of a frame, screen, carriages of axes x and y with step motors and two heats of executing device—revolving engraving and drafting. With the simultaneous operation of both heads there are prepared two originals of mathematical basis of the map—compiling and publishing.

The linear code translator is an interpolator intended for readout of information about reference points from punch tape and conversion of it into digital command pulses. The quantity of the pulses depends on the value of the increase in coordinates for each reference point. Through the control units pulses are transmitted by step motors rotating actuating screws of the coordinatograph and imparting forward movement to the carriage with the engraving head. As a result of this, the cutter set in engraving head moves between the reference points.

The program control panel serves for the readout of information from the magnetic tape, formation and amplification of current pulses and the feeding of them to the reverse step motor. The last ones permit converting each control pulse into a fixed linear movement of the actuating device without a feedback transducer, which considerably simplifies the instrument control circuit and increases its reliability.

Information processed in the interpolator can be directly fed through control units to the step motors or entered on the magnetic tape. Such magnetic tapes with programs of engraving of mathematical basis for maps can be prepared in a centralized location (with the use of computer, linear code translator, and recording and control
panel) and sent to cartographic enterprises, where it will be sufficient to have a panel of program control. The control circuit of the coordinatograph is shown in Fig. 1. Programs for operation of the coordinatograph are composed automatically on the computer by a special programming program, which is a universal program, providing not only automatic computation but automatic construction of the mathematical basis for maps of different scales and in any projections. By this program it is anticipated to obtain:

a) geometric elements of mathematical basis for maps (segments of a straight line, arcs of circles and curves of the second order, dashes and points);

b) conditions of detecting geometric elements (crossing at the set angle, assigned distance from each other);

c) necessary technological indications (initial and end point of movement of the cutter, direction and magnitude of this movement assigned by the number of pulses in the frame; operating conditions of the cutter, i.e., its lowering and raising, acceleration and braking, speed; location of frames and time of their operation; accuracy of the approximation).

Fig. 1. 1 - automatic coordinatograph; 2 - program control panel; 3 - linear translator; 4 - recording and control panel; 5 - digital computer.
Information is transmitted to the program control, which provides automatic operation of the cutter with engraving of elements of the mathematical basis of the map and its standard formulation. Coordinates of the reference points are calculated in sequence corresponding to the marked route of the cutter and are recorded on punched tape in the code of the interpolator, for example, the linear code translator. With this there are considered technological peculiarities connected with the sequence of frames during readout of the punched tape and with operating conditions of the engraving head.

For programming of elements of a mathematical basis all maps were grouped by projection and scale. These criteria are assumed as a basis of the decisive algorithm. In general the algorithm can be expressed by the relation

\[ F_i(x, y, z, u, t) = 0, \]

where \( x, y \) and \( z \) are controlling variables, \( u \) — parameters of the controlled actuating device, \( t \) — time.

The programming program is constructed taking into account the ruling and nomenclature of Soviet maps. It consists of separate blocks, each of which constitutes a program of solution of one definite problem. Here the main role is played by the leading scale blocks, united into groups by form of the projection. For each such group there is its program, "Deciphering," which by assigned nomenclature determines what control from the leading scale blocks should be transmitted. Here the standard technological blocks servicing all the leading scale blocks are found. Such a structure permits introducing a change into the separate blocks and adding new ones, not disturbing the entire system as a whole.

A block diagram of the programming program for automatic construction of elements of the mathematical basis for maps of scales 1:50,000—1:40,000,000 is shown in Fig. 2. It consists of
Fig. 2.
five leading scale blocks, one leading block of geodesic points and auxiliary technological blocks. The number of blocks can be increased with the connection of blocks of other scales, which will require the addition of programs of appropriate projections. The program of every leading scale block organizes the calculations of geodesic and rectangular coordinates of reference points of the geographic (rectangular) grid correct to one pulse.

Let us consider the programming program in more detail.

The initial information is the coding nomenclature of the map sheet. For this the letters and Roman figures of nomenclature are replaced by Arabic figures. Then the nomenclature of the map sheet are recorded in the form of a ninth digit decimal number $a_1a_2a_3a_4a_5a_6a_7a_8a_9$. In places where elements of the nomenclature are absent zeroes are placed. With this in coded nomenclatures for maps at 1:500,000 and 1:200,000 in the fifth digit of the code there is placed the number 5 or 2 in order to distinguish these scales from 1:100,000. Coding examples of nomenclature are given in Table 1. With the coding of doubled and quadrupled sheets elements of nomenclature referring to the western sheet are indicated.

The coded nomenclature is introduced into the machine by the operator by means of its set on keys of the control panel. Such a method is convenient during calculations for one map sheet. For several sheets the nomenclature is more conveniently introduced from punched tape, or by special algorithm one can produce ordered sorting of nomenclatures of the sheets.

Table 1.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Nomenclature</th>
<th>Coded nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1,000,000</td>
<td>N-34</td>
<td>143 400 000</td>
</tr>
<tr>
<td>1:500,000</td>
<td>N-34-B</td>
<td>143 450 000</td>
</tr>
<tr>
<td>1:200,000</td>
<td>N-34-XXIV</td>
<td>143 452 400</td>
</tr>
<tr>
<td>1:100,000</td>
<td>N-31-106</td>
<td>143 411 500</td>
</tr>
<tr>
<td>1:50,000</td>
<td>N-34-106-B</td>
<td>143 410 630</td>
</tr>
</tbody>
</table>
Deciphering of the nomenclature consists in the determination of the scale and geodesic coordinates of summit of the southwest angle of the frame of the map sheet. The initial information here will be the coded nomenclature collected on the control panel. As a result of operation of the block - "Deciphering" program - there are calculated the following:

1) Quantity of joined sheets k by the formulas:

\[(10a_1+a_2)<16, \text{ if } k=1;
\]
\[16 \leq (10a_1+a_2)<20, \text{ if } k=2;
\]
\[20 \leq (10a_1+a_2), \text{ if } k=4.
\]

where a is one of digits of the nomenclature.

2) Latitude B of the southern side of the frame of the map sheet determined by the formulas given in Table 2.

3) Longitude l of the western side of the frame of the map sheet from the axial meridian of the zone enumerable by the formulas given in Table 3.

4) The number of scale i if scale M = 1:200,000, then i = 0; if M = 1:100,000, then i = 1; if M = 1:50,000, then i = 2.

### Table 2.

<table>
<thead>
<tr>
<th>Map scales</th>
<th>Formulas for calculation of latitude B</th>
<th>Values of in formulas</th>
</tr>
</thead>
</table>
| 1:1,000,000 | \(B = (10a_1 + a_2 - 1) \cdot 4'\) | \(b = \{\begin{align*}
0, \text{ if } &a_1 > 2 \\
1, \text{ if } &a_1 \leq 2
\end{align*}\} \) |
| 1:500,000  | \(B = (10a_1 + a_2 - 1) \cdot 4' + b \cdot 2'\) | \(b = \{\begin{align*}
0, \text{ if } &a_1 > 2 \\
1, \text{ if } &a_1 \leq 2
\end{align*}\} \) |
| 1:200,000  | \(B = (10a_1 + a_2 - 1) \cdot 4' + b_1 \cdot 40'\) | \(b_1 = \left\lfloor \frac{6 - 10a_1 - a_2}{6} \right\rfloor \text{ whole part} \) |
| 1:100,000  | \(B = (10a_1 + a_2 - 1) \cdot 4' + b_2 \cdot 20'\) | \(b_2 = \left\lfloor \frac{12 - 100a_1 - 10a_1 + a_2}{12} \right\rfloor \) |
| 1:50,000   | \(B = (10a_1 + a_2 - 1) \cdot 4' + b_3 \cdot 10'\) | \(b_3 = \{\begin{align*}
0, \text{ if } &a_1 > 2 \\
1, \text{ if } &a_1 \leq 2
\end{align*}\} \) |
Table 3.

<table>
<thead>
<tr>
<th>Scales</th>
<th>Formula for calculations of longitude ( l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:500,000</td>
<td>( l = l_0 + \nu \cdot 3' )</td>
</tr>
<tr>
<td>1:200,000</td>
<td>( l = -3' \cdot s' + 3' )</td>
</tr>
<tr>
<td>1:100,000</td>
<td>( l = -3' + s', 30' )</td>
</tr>
<tr>
<td>1:50,000</td>
<td>( l = -3' + s', 30' + s', 15' )</td>
</tr>
</tbody>
</table>

Value \( l_0 \) and \( \nu \) in formulas:

\[
\begin{align*}
l_0 &= \begin{cases} 
3', & K < 4 \\
-6', & K = 4 
\end{cases} \\
\nu &= \begin{cases} 
0, & s_{e} - \text{odd} \\
1, & s_{e} - \text{even} 
\end{cases} \\
s' &= 60 + (10a_k + a_1) - 31 \\
s'_1 &= 12s_1 + (100a_k + 10a_1 + a_1) - 133 \\
\nu_2 &= \begin{cases} 
0, & s_{e} - \text{odd} \\
1, & s_{e} - \text{even} 
\end{cases} 
\end{align*}
\]

Note. Quantities \( s \) and \( s' \) in these formulas constitute the number of columns and zones in the ruling system determined by the number of the sheet.

To determine the scale there are used the following criteria:

- for 1:1,000,000... \( a_1 + a_2 + a_1 = 0 \).
- for 1:500,000... \( a_1 = 3 \).
- for 1:200,000... \( a_1 = 2 \).
- for 1:100,000... \( a_0 = 0 \).

The latitude and longitude of points obtained by the program "Deciphering" permit crossing over to the calculation of rectangular coordinates of these points. For brevity we will give a description of only two leading blocks.

I. Initial data for operation of the leading block (scales 1:50,000, 1:100,000 and 1:200,000) will be the number of the scale \( i \), quantity of sheets \( k \), latitude \( B \) and longitude \( l \) of summit of southwest angle of the map sheet. The order of operation of the block is the following. At first there are calculated the

- quantity of reference points \( n \) on the southern side of the internal frame

\[
\begin{align*}
n &= 1 - s_1 \\
&= 4, s_1 = 2, s_2 = 1;
\end{align*}
\]

- their rectangular coordinates

\[
\begin{align*}
x_i &= (B_i, L_i), y_i = y(B_i, L_i) + 500,000, \\
B_i &= B, L_i = l, l_{180} = l_{180} = l_{180} + 15', B_{x+} = B_i + \Delta B_i, \\
\Delta B_i &= 60', \Delta B_1 = 20', \Delta B_2 = 10';
\end{align*}
\]

where

\[
\begin{align*}
x_1 &= (B_1, L_1), y_1 = y(B_1, L_1) + 500,000, \\
B_1 &= B, L_1 = l, l_{180} = l_{180} = l_{180} + 15', B_{x+} = B_1 + \Delta B_1, \\
\Delta B_1 &= 60', \Delta B_1 = 20', \Delta B_2 = 10';
\end{align*}
\]
- quantities
\[ \bar{x} = \min(x_i), \quad 1 \leq i \leq n, \quad \bar{y} = \min(y_i, y_{i+1}) ; \]

- coordinates of reference point (in pulses)
\[ \bar{x} = 1920 \left[ \frac{x}{a_i} \right], \quad \bar{y} = 1920 \left[ \frac{y}{b_1} \right], \]

where \( A_1 \) - the step of the coordinate grid \( (A_0 = 1000, A_1 = 2000, A_2 = 4000) \);

- coordinate of reference points (in pulses)
\[ x_i = x_i \cdot m_1 - \bar{x}, \quad y_i = y_i \cdot m_1 - \bar{y}, \]

where \( m_1 \) - scale factor \( (m_0 = 0.48, m_1 = 0.96, m_2 = 1.92) \).

Then preparation for operation of the program "Division of straight line" is made, and the frame of lowering the cutter is issued. After that there are consecutively calculated and issued on punched tape coordinates of reference points of the external and internal line of the external frame and internal frame and external and internal line of the minute frame.

From reference points of the internal frame and internal and external lines of the minute frame the following four contours shown in Fig. 3 will be formed.

1. \((n+1) \rightarrow A \rightarrow (n) \rightarrow (n-1) \rightarrow H \rightarrow (2n) \).
2. \(1 \rightarrow D \rightarrow (n+2) \rightarrow \ldots \rightarrow (2n-1) \rightarrow E \rightarrow n \).
3. \(C \rightarrow B \rightarrow \ldots \rightarrow (n-1) \rightarrow G \rightarrow F \).
4. \(B \rightarrow C \rightarrow (n+2) \rightarrow \ldots \rightarrow (2n-1) \rightarrow F \rightarrow G \).

To determine the vertical line RR' (see Fig. 3) of the coordinate grid in the first contour there are taken two neighboring points \((j \text{ and } j + 1)\), satisfying the condition \( y_j < y_R < y_{j+1} \) \( (A) \), where \( y_R = 1920 \cdot R \).

Coordinate \( x_{R} \) is found by the formula
\[ x_{R} = x_{j+1} + \frac{x_j - x_{j+1}}{y_j - y_{j+1}} (y_R - y_{j+1}) . \]
Found in the same way is coordinate x point $R'$ in the secondary contour, contour, and the output of coordinates of points of straight line $RR'$ (or $R'R$) is produced. Calculations for $R = 1, 2, 3, \ldots$ continue until the first and secondary contours conditions (A) is satisfied.

By method analogous to that used for the vertical line of the grid in the third contour a segment is detected, and there are calculated coordinates $y_R$ and $y'_R$ of crossing points of the horizontal line of coordinate grid $x_R = 1920$ with sides of the internal or minute frames. In the fourth contour coordinates of points of the crossing of contour with horizontal lines of the coordinate grid $x_R = 1920R$ ($R = 2, 3, \ldots$) are calculated until condition $x_j > x_R$.

Upon completion of the calculations the conversion frames move to the reference point of the cutter and its rise, and then there is produced an imprint of the engraving time and stop of the machine.

II. With operation of the leading block for the scale 1:1,000,000 initial data will be the same as for earlier examined scales. Rectangular coordinates of reference points of the frame are calculated by the formulas

$$x_j = x(R, j); \quad y_j = y(R, j),$$

where $j = 1, 2, 3, \ldots, 2n(n = 7)$;
\[ B_1 = B, \quad B_{n+1} = B_1 + \Delta B_n, \quad \Delta B_n = 4'; \]
\[ \Delta I = \Delta I_n - k, \quad I_{n+1} = I_1 - h \cdot \Delta I_n, \quad \Delta I_n = -3', \quad \Delta A = 1'. \]

Then coordinate of reference points (in pulses) are converted to the reference point (summit of the southwest angle)

\[ x_j = m_1 \cdot (x_j - x_1), \quad y_j = m_1 \cdot (y_j - y_1), \quad m_1 = 0.096. \]

After preparation for operation of the program "Division of straight line," there is issued the frame of lowering of the cutter, and there are calculated and issued coordinates of the reference points: lines of external and internal frames, meridians passing through two points (1) and \((1+n)\); internal parallels \(s = 1, 2, 3, \ldots\), not being borders of the sheet

\[ x_j = x_1 + \frac{s}{4} (x_{1n} - x_1), \quad y_j = y_1 + \frac{s}{4} (y_{1n} - y_1). \]

Further, just as in the first block, the conversion frames move to the reference point of the cutter and its rise, and then the imprint of engraving time and stop of the machine are carried out.

In the leading scale blocks the program of calculations of appropriate projections are used. Rectangular coordinates of points of the internal frame of the map sheet are calculated by the program "Gaussian projection" with operation of the leading blocks for scales 1:50,000-1:500,000 and by the program "Modified polyconical projection" (stereographic) with operation of the leading block for the scale 1:1,000,000. The initial information for operation of the first program is obtained in latitude \(B\) and longitude \(L\) (from axial meridian) of points. Calculation is produced by the formulas:

1. \[ N = \sqrt{e^2 + e \cdot \sin^2 B}. \]

   where \( e = -0.006983422 \).

2. \[ X = b_2 + \sin B \cos B (b_2 + \sin^2 B (b_2 + b_1 \sin B)), \]

   where \( b_1 = -0.006983422 \) and \( b_2 = 0.096 \).

3. \[ P = \sin(1 + P(C_2 + \sin^2 B(\sin B + C_2 + C_2 \sin^2 B + P(-C_2 + C_2 \sin^2 B))). \]

11
The initial information for operation of the second program is obtained in latitude $B$, longitude $\ell$ and longitude $\ell_0$ of the meridian on which the scale is not distorted. Calculation is conducted by the formulas:

$$f_1(u) = b_1 \cdot u - \sin u \cdot \cos u (b_3 \cdot \sin^2 u (b_5 + b_6 \sin^2 u)), $$
$$f_2(u, v) = \rho \cdot \sin \delta, \ f_3(u, v) = \rho (1 - \cos \delta),$$

where

$$\rho = \frac{a_1 \cdot \cos u}{\sin u \sqrt{1 + a_2 \sin^2 u}}, \ \ \ k = u \sin u.$$ 

$$x = f_3(B, \ell) - f_3(B, 4) + \sqrt{(f_3(B, 4))^2 - (f_3(B, 4) - f_3(0, 4))^2}, $$
$$y = f_3(B, \ell).$$

If with calculations it will appear that $\rho = a$, then

$$f_1(u, v) = a_1 \cdot v, \ f_3(u, v) = 0.$$ 

For calculation of coordinates of reference points there is used an auxiliary program, according to which coordinates of points of an equidistant angle are determined. The initial information in this case will be coordinates of points 1, 2, and 3 (Fig. 4) forming the angle and distance $b$ up to the equidistant angle, where for external bypass of the angle $b < 0$, and the internal, $b > 0$. 

Coordinate of intersection points of external frames with the internal frame 4, 5, and 6 are calculated by the formulas:

$$\Delta x_i = x_i - x_a, \ \Delta y_i = y_i - y_a, \ \Delta x_i = x_i - x_a, \ \Delta y_i = y_i - y_a;$$

$$r_i = \frac{b}{\sqrt{\Delta x_i^2 + \Delta y_i^2}}, \ x_i = r_i \cos \gamma, \ y_i = r_i \sin \gamma;$$

$$r_i = r_i, \ x_1 = x_1 + x_a, \ x_2 = x_2 + x_a, \ y_1 = y_1 + y_a;$$

$$x = x_1 + x_2 + x_3 + x_4 + x_5, \ y = y_1 + y_2 + y_3 + y_4 + y_5.$$
If $r_1 = \omega$ then $x_5 = x_2 + B, y_5 = y_2$. In this case coordinates of points 4 and 6 are not calculated. Coordinates of points of the equidistant angle are necessary for operation of the program of calculation of coordinates of reference points of the external frame. Besides these points the initial information for calculation is obtained in the quantity of points $n$ on the southern and northern sides of the frames, coordinate of points $2n$ of the internal frame and distance to frame $b$ (Fig. 5). Coordinates of points of the external frame and coordinates of points of the crossing of it with continuation of sides of the internal frame are calculated.

Operation of the program consists in the consecutive output by the machine of coordinates of points grouped in threes $(n + 1, 1, 2), (1, 2, 3), \ldots, (n - 2, n - 1, n), (n - 1, n, 2n)$ in the standard cell and the calculation of coordinates of equidistant points. Coordinates of mean points are the sought coordinates of points $1', 2', \ldots, n'$. In exactly the same way there are calculated coordinates of points $(n + 1)', (n + 2)', \ldots, (2n)'$, but with this the internal bypass of the contour is assigned.

With calculation of angular points $1', n', (n + 1)'$, $(2n)'$, besides their coordinates, the machine stores also coordinates of intermediate points from $A$ to $H$ and from $B$ to $E$. 
Rectangular coordinates of reference points of external frames are used for obtaining the route of the cutter. The initial information for their position will be coordinates of points along the northern and southern sides of frames, coordinates of points of crossing of the internal and external frames and the quantity of points along the horizontal side of the frame.

The output of coordinates of points of frames in sequence, marked by the route of movement of the cutter, is produced in the following order: by internal frame - of points B-2-3 ... -(n - 1)-G-H-E-F-(2n - 1) ... -(n + 2)-C-D-A, by external frame -1'-2'- ... -n'-(2n)'-(2n - 1)'- ... -(n + 1)'-1' (see Fig. 5).

In projecting the route of movement of the cutter, the programmer provides for the least number of no-load operations.

Information on the route of movement of the cutter, because of technical characteristics of the interpolator, does not find room in one frame of the program. The linear code translator, for example, LKP-2-60, can read from one frame a maximum of 9999 pulses. Therefore, segments of the straight line, having a length in impulses more than that indicated, should be divided into part not exceeding 9999 pulses. This process is fulfilled by the program "Division of the straight line."

At first the cutter is set in the initial position \((x_0, y_0)\), then increases in coordinates between the initial \((x_0, y_0)\) and final \((x, y)\) points of the straight line are divided into frames taking into account the criteria of acceleration and braking of the cutter which will be discussed below. After that the machine issues coordinates of the straight line, and by formation of the frame of lowering or raising the cutter by magnitude \(d\) (in pulses) during the time \(t > 1\) s there is indicated the operating conditions, i.e., the operation or idling of the cutter. Then on punched tape increases \(\Delta x, \Delta y\) (and \(\Delta z\)) and minimum time of frame operation \(t\) are recorded. Under all conditions there is produced an imprint of
coordinates of the initial point of the cutter \( \frac{x_0}{c} \) and \( \frac{y_0}{c} \) (in millimeters), where \( c = 96 \).

To determine the criteria of starting \( \Pi_s \) and braking \( \Pi_b \), there are produced calculations by the formulas

\[
\Pi_s =\begin{cases} 
1, & \text{starting required} \\
0, & \text{starting not required},
\end{cases}
\]

\[
\Pi_b =\begin{cases} 
1, & \text{braking required} \\
0, & \text{braking not required},
\end{cases}
\]

\[
q_{p.T.} =\begin{cases} 
1, & \text{if } M \leq \Pi_1 \\
2, & \text{if } M > \Pi_1.
\end{cases}
\]

where \( q_{p.T.} \) is the number of frames of starting or braking.

\( q_{p.T.} = 1 \) denotes the straight line is so short that there is no need in two frames of starting (braking). \[ M = \max(|\Delta x|, |\Delta y|), \quad \delta_x = x - x_0, \quad \delta_y = y - y_0, \quad \Pi = \Pi_1 + \Pi_2; \]

\[
\phi_x = \left\lfloor \frac{M - \Pi(d_1 + d_2)}{d_3} \right\rfloor + 1, \quad \text{if } M > \Pi(d_1 + d_2).
\]

where \( q_{\delta_x} \) is the number of frames of the basic movement.

The formation of each frame is carried out by the formulas:

\[
\delta_x = \lfloor \delta_x \rfloor - \text{whole part}, \quad \delta_x = \delta_x - \delta_x, \quad x_0 = x_0 + \delta_x, \quad M = \max(|\Delta x|, |\Delta y|);
\]

\[
\delta_y = \lfloor \delta_y \rfloor - \text{whole part}, \quad \delta_y = \delta_y - \delta_y, \quad y_0 = y_0 + \delta_y, \quad I = \max(|\Delta x|, |\Delta y|).
\]

In the composition of the program according to the given formulas certain conditions are considered (Table 4).

As a result of operation of the leading blocks using auxiliary and technological blocks, programs are obtained on paper punched tape in the interpolator code which control the engraving of elements of a mathematical basis of map sheets.
Table 4.

<table>
<thead>
<tr>
<th>(n)</th>
<th>Conditions</th>
<th>Magnitude of formation of frames</th>
<th>(\Delta x)</th>
<th>(\Delta y)</th>
<th>(\Delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(q_1 = 1)</td>
<td>first frame of starting</td>
<td>(\Delta x)</td>
<td>(\Delta y)</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>2</td>
<td>(q_2 = 2)</td>
<td>first frame of starting</td>
<td>(\Delta x\cdot \frac{d_1}{M})</td>
<td>(\Delta y\cdot \frac{d_1}{M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>3</td>
<td>(q_2 = 2)</td>
<td>second frame of starting</td>
<td>(\Delta x\cdot \frac{M - \Pi \cdot d_1}{M})</td>
<td>(\Delta y\cdot \frac{M - \Pi \cdot d_1}{M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>4</td>
<td>(q_2 = 2)</td>
<td>second frame of starting</td>
<td>(\Delta x\cdot \frac{d_2}{M})</td>
<td>(\Delta y\cdot \frac{d_2}{M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>5</td>
<td>(q_2 \neq 0)</td>
<td>(q_{k-1}) frames</td>
<td>(\Delta x\cdot \frac{M - \Pi \cdot (d_1 + d_2)}{q_{k-1} \cdot M})</td>
<td>(\Delta y\cdot \frac{M - \Pi \cdot (d_1 + d_2)}{q_{k-1} \cdot M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>6</td>
<td>(\Pi_1 = 0)</td>
<td>last frame of basic movement</td>
<td>(\Delta x\cdot \frac{M - \Pi \cdot (d_1 + d_2)}{q_{k-1} \cdot M})</td>
<td>(\Delta y\cdot \frac{M - \Pi \cdot (d_1 + d_2)}{q_{k-1} \cdot M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>7</td>
<td>(\Pi_1 \neq 0)</td>
<td>first frame of braking</td>
<td>(\Delta x\cdot \frac{d_1}{M})</td>
<td>(\Delta y\cdot \frac{d_1}{M})</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>(\Pi_1 \neq 0)</td>
<td>second frame of braking</td>
<td>(\Delta x)</td>
<td>(\Delta y)</td>
<td>(\Delta)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Technological parameters can have, for example, the following values: \(d_1 = 500, d_2 = 1000, d_3 = 9999, f_1 = 200, f_2 = 350, f_3 = 500\), where \(d_1, d_2, d_3\) are lengths of path in pulses, \(f_1, f_2, f_3\) are frequencies in cycles per second.

In conclusion let us note that work on the automatic creation of a mathematical basis for maps is expediently produced in two stages: 1) centralized preparation of working programs and 2) engraving by the program.

In the first stage into the computer memory there is introduced a program of input and treatment, standard constants, table of memory distribution of the machine and units of the programming program recorded on punched tape. Initial information, for example, coded nomenclature of the map sheet (or group of sheets), is introduced from the panel of the machine (or is dispatched with the help of punched tape). Starting is produced, and in several seconds the machine produces a count and in several minutes issues punched tape – the working program for the coordinatograph. The program
obtained on the machine is copied on magnetic tape by the interpolator and recording and control panel. Owing to the use of cyclical electronic counter installed in the interpolator, on this tape there is additional information recorded about standard elements of shaping of the frames and grids - equidistant points, lines of separating segments, etc., which are inexpedient to program on the computer.

In the second stage of operation the magnetic tape is set on the panel of program control, where later on the information is read, and control signals are sent to step motors of the coordinatograph. For one or two hours the cutter automatically engraves on the surface of the plastic, covered by a lightproof layer, the mathematical basis of the sheet of topographic or special map.

With the carrying out of test works the programming program for the automatic plotting of elements of mathematical basis for maps of scales 1:50,000-1:40,000,000 was formulated, debugged and tested on the computer, "Minsk-2" and "Ural-2". Its total length is 1417 commands. Finally there were obtained (in the form of punched tapes) tens of working programs by which on the coordinatograph there were engraved mathematical elements of maps of the mentioned scale series.

On the basis of comparative analysis of labor costs by manual and automatic plotting (engraving) of mathematical elements of maps the conclusion can be made that application of the automatic method of creating a mathematical basis for maps of scales 1:50,000-1:1,000,000 increases labor productivity 6-10 times and for small-scale special maps and complex grids, 10-15 times. Mathematical elements engraved on plastic possess high quality and accuracy owing to the precision coordinatograph used and complete replacement of manual subjective labor by automatic not dependent on personal errors of the performer.

The field of application of the automatic method of creation of
The mathematical basis for maps is not limited to that examined. It is possible to solve any problem by conversion of digital information into graphic form and conversely. In the latter case as an automatic transducer of information it is expedient to use reader.

The method examined can be used first of all for conversion of coordinates from one projection into another. In this case coordinates of points and lines of the initial cartographic material are taken by the reader and transmitted to the computer input unit. In the use of the program, considering the projection of the source material, the computer calculates geodesic coordinates which serve as information for the program of calculation of coordinates in the projection of the compiler original. As a result of the interaction of separate blocks of the programming program, the machine produces a working program by which the coordinatograph plots on the compiler original points and lines corresponding to the initial map material. Conversion of the initial projection into a new one is produced by taking into account the assigned scale.

The method can be used also for the acceleration of a search for new cartographic projections, for the reproduction of grids of the position (hyperbolic, stadiometric, and others), position lines (routes of artificial earth satellites, orthodromes, rhumb lines and others) and arbitrary curves, for example, horizontals. With the automatic plotting of grids and position lines considerable gain in time as compared to existing manual methods is provided.

Footnotes

1 Subsequently, for brevity, automatic coordinatograph will be simply called coordinatograph.

2 The coordinatograph can operate from punched tape which is made manually. However, manual programming requires great labor cost, lowering the effect of application of the coordinatograph. Hence automation of the actual process of programming appears necessary.

3 Information for operation of the coordinatograph can be directly issued by the computer. However, such use of the machine is connected with the unjustified loading of it (approximately up to 90%) by the simplest operations in interpolation of the cutter route. Therefore it was required to use an additional calculating electronic device - the interpolator.
Reference points are initial and end points of the segment of the line and also points of the change in curvature of the line. Coordinates of such points serve for automatic computation by the interpolation formula of coordinates of intermediate points, giving trajectory of the movement of actuating device. The reference points can serve as nodes, i.e., the crossing of meridians and parallels.

The value of one pulse is 0.01 mm.

The frame is the part of punched tape with numerical data determining the program of action of the actuating device between two reference points.

Linear code translator can be replaced by another interpolator, in particular, a linear circular. With the use of the linear interpolator all curves are approximated by rectilinear segments in accordance with the assigned accuracy, which considerably increases the volume of the working program. Application of the linear circulator interpolator decreases the quantity of information issued by the computer.

Latitude \( B \) and longitude \( L(t_0) \) are arguments of functions \( u \) and \( v \) respectively.

This program fulfills the unification of separate blocks into the system, which serves for the input (with control) of arrays from punched tape onto an arbitrary place in immediate-access memory of the machine and for carrying out analysis of codes of arrays.