HOLOGRAPHIC TECHNIQUES FOR THE STUDY OF DYNAMIC PARTICLE FIELDS

J. D. Trolinger, R. A. Belz, and W. M. Farmer
ARO, Inc.

November 1968

This document has been approved for public release and sale; its distribution is unlimited.

ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE

PROPERTY OF U. S. AIR FORCE
F4060-59-0-0001
NOTICES

When U. S. Government drawings specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified users may obtain copies of this report from the Defense Documentation Center.

References to named commercial products in this report are not to be considered in any sense as an endorsement of the product by the United States Air Force or the Government.
HOLOGRAPHIC TECHNIQUES FOR THE STUDY
OF DYNAMIC PARTICLE FIELDS

J. D. Trolinger, R. A. Belz, and W. M. Farmer
ARO, Inc.

This document has been approved for public release and sale; its distribution is unlimited.
FOREWORD

The research reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), and supported by the Air Force Flight Dynamics Laboratory (AFFDL), AFSC, under Program Element 6240533F, Project 8219, Task 821907.

The results of research were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract F40600-69-C-0001. The work was performed from July 1967 through June 1968 under ARO Project No. 5819, and the manuscript was submitted for publication on September 5, 1968.

The authors wish to express appreciation to Mr. George Mycroft and Mr. Evard Flyn of the Flight Dynamics Laboratory for their assistance in these studies. Also, appreciation is expressed for the many ARO personnel who are involved in this program.

This technical report has been reviewed and is approved.

David G. Francis  
2nd Lieutenant, USAF  
Research Division  
Directorate of Plans  
and Technology  

Edward R. Feicht  
Colonel, USAF  
Director of Plans  
and Technology
ABSTRACT

A brief review of the techniques of small-particle holography is presented. These results are extended in an elementary way to show that multiple-exposure holography can be used to study the dynamic properties of particle fields. Experimental techniques for performing such a study are then described. Finally, as a typical example, a multiple-exposure hologram of an aerosol is presented with a portion of the data extracted from it. The hologram exhibits a vast amount of information including the determination of the velocity field, density field, size distribution, flow structure, and diffusion rate.
CONTENTS

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. SMALL-PARTICLE HOLOGRAPHY</td>
<td>4</td>
</tr>
<tr>
<td>III. MULTIPLE-EXPOSURE HOLOGRAPHY</td>
<td>5</td>
</tr>
<tr>
<td>IV. RECORDING AND RECONSTRUCTION</td>
<td>7</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX
Illustrations

Figure

1. Geometry of the Hologram Formation Process ... 11
2. Reconstructed Real Image of a Particle Field
   a. High Contrast. ... 12
   b. Low Contrast (Overexposed) ... 12
3. Image Reconstruction with Simultaneous Formation
   of the Secondary Hologram ... 13
4. Apparatus Used for Reconstructing and
   Analyzing the Three-dimensional Image ... 14
5. Reconstructed Real Image with High Pass
   Filtering ... 15
6. Direction Sensing of a Multiple Image by
   Controlled Exposure ... 16
7. Double-Exposure Hologram of an Aerosol Spray ... 17
8. Two Planes of the Reconstructed Image of a
   Doubly Exposed Hologram (Region Marked A
   in Fig. 7) ... 18
9. Reconstructed Image in a High Density Region
   (Region Marked B in Fig. 7) ... 20
10. Axial Speed Profile across an Aerosol Spray ... 21
SECTION I
INTRODUCTION

The visual observation of a volumetric array of dynamic elements, having individual dimensions much smaller than the volume dimensions, could rarely be achieved before the advent of holography. It is possible within the limits of holography to study such an array while, at the same time, observing the motion of all its elements. The extension of holography to the study of motion is in its very infancy, but for certain highly specialized instances, it can already be applied to practical problems. The purpose of this report is to discuss studies dealing with the determination of sizes, densities, and velocity fields associated with dynamic particle arrays.

SECTION II
SMALL-PARTICLE HOLOGRAPHY

The science of in-line holography of small particles owes its origin and most of its development to Thompson et al. (Ref. 1). A fairly complete treatment of their work and that of others on the subject is included in a recent book by De Velis and Reynolds (Ref. 2). The general approach to the theoretical problem concerns itself with the solution of the Helmholtz equation and subsequent fitting of the boundary conditions. A much simpler (but less complete) way of understanding the phenomenon is presented in this report.

To produce a hologram of an object, one needs a wavefront characterizing that object and a reference wave to interfere with the object wave at the surface of a photographic plate (or some other recording media). The in-line hologram of a small particle is perhaps the easiest of all hologram systems to describe theoretically and to record experimentally. It is most simply understood in terms of Babinet's principle, which is a direct result of the Fresnel-Kirchoff diffraction theory. Consider a plane wavefront of amplitude $U_o$ (Fig. 1, Appendix I) that passes through a plane, $\eta-\xi$, in which some object is placed. The resulting wavefront with amplitude $U(\vec{r})$ is recorded in the far field of the object in the x-y plane. Babinet's principle states that if $U(\vec{r})$ were added to the wavefront produced by the object's complement, $U'(\vec{r})$, then the resulting amplitude would be $U_o$. (For example, a screen with an aperture has as its complement an opaque disk with a diameter equal to that of the aperture.) Therefore

$$U_o = U(\vec{r}) + U'(\vec{r})$$  \hspace{1cm} (1)
It is of interest here to determine the intensity of light, \( U_p \), in the \( x-y \) plane that results when a disk (representing an opaque particle) is placed in the \( \eta-\xi \) plane. From Eq. (1) it is seen that

\[
U_p = U_o - U_a \tag{2}
\]

where \( U_a \) is the amplitude in the observation plane when the object plane is opaque except for an aperture of the same diameter as the disk. The resulting intensity in the recording plane is, therefore

\[
U_p^* = |U_p|^2 = + |U_o|^2 + |U_a|^2 - U_o U_a^* - U_o^* U_a \tag{3}
\]

Now, it will be shown that the intensity, \( U_p U_p^* \), actually forms a hologram of the aperture which is complementary to the disk. The simplest way to show this is to require that the photographic recording medium be related linearly to the intensity so that its amplitude transmission coefficient can be expressed as

\[
T = 1 - KI \tag{4}
\]

where \( K \) is a film constant and \( I \) is the light intensity striking the film. Even when the linear relation (Eq. (4)) is not obeyed, a hologram is formed but with less efficiency. This more complicated case will be considered later. When the processed photographic recording is illuminated by a plane wave, \( U_o \), the resulting wavefront emerging from the plate is \( U' \), where

\[
U' = U_o (1 - KI) = U_o (1 - KU_p U_p^*) \tag{5}
\]

From Eqs. (3) and (5)

\[
U' = U_o (1 - K|U_o|^2 - K|U_a|^2) + U_o U_k U_a^* + |U_o|^2 KU_a \tag{6}
\]

The last term in Eq. (6) is a wave exactly like the wave created by an aperture that is located behind the recording at the place where the particle had previously been. This is the wavefront of a virtual image. The second term represents a real-image wavefront in front of the photographic recording, and the first term represents a spatially modulated plane wave. The photographic recording has thus been shown to be a hologram of a screen with an aperture the shape of the disk. Figure 2a is a high-contrast, photographic recording of the real image reconstructed by such a hologram. The object, a glass slide, was covered with opaque paint particles by allowing the aerosol from a spray paint can to fall upon it. The hologram was formed by He-Ne laser illumination and Kodak SO-243 film. It is important to point out a few facts about Fig. 2 that are neglected in the literature. These will be discussed with reference to Figs. 2b and 3. When an image is reconstructed from an in-line hologram, as shown in Fig. 3, a new hologram
is formed, in addition to the image, and is superimposed upon the image. This "secondary" hologram is of dust particles in the reconstruction system, plus any information stored on the primary hologram and all out-of-focus images (real or virtual). The reference wave; acting to form holograms of these objects, is the first term in Eq. (6). In fact, if the image represented by the second term in Eq. (6) is in focus, then the secondary hologram of the third and first terms will be superimposed upon it.

To illustrate this point, the same reconstructed image shown in Fig. 2a was recorded photographically and overexposed (Fig. 2b) to emphasize the existence of the secondary hologram and the spatial modulation of the first term. Some interference between the conjugate and focused waves also exists, but only for large particles is this interference noticeable. To further illustrate these facts, it is of interest to remove most of the light caused by the first term. This can be accomplished by using a high pass (DC) optical filter in the reconstructing step as shown in Fig. 4. A lens is used in the reconstruction process, and a small opaque spot is placed at the focal point of the lens. This configuration will eliminate essentially all of the plane radiation. Therefore, returning to Eq. (6), and eliminating all plane wave terms, \( U' \) becomes

\[
U' = K U_o U_o U_o^* + K |U_o|^2 U_o
\]

The term, \( U_o \), can be further divided approximately into two terms, a scattered wave, \( U_{sa} \), and an unscattered wave. The scattered wave represents a tiny ring of light scattered at the edge of the aperture, and the unscattered wave represents light that passed far enough from the edge that it was not scattered. Since the unscattered light is plane radiation, it is removed by the high pass filter. The only portion of the wavefront that will be imaged, then, is the scattered radiation (the ring of light). Figure 5 is a photographic recording of the same reconstructed image shown in Fig. 2 except that the wavefront has been passed through a high pass optical filter. The secondary effects have been removed.

High pass filtering does cause new problems, however. Care must be taken in choosing the filter size since it can remove some low frequency information as well as plane (DC) radiation. One must, therefore, use high quality lenses and precise alignment to achieve the best results. Since the filter removes all paraxial radiation, it does improve the \( z \)-imaging sensitivity. In fact, as the diameter of the high pass filter is made larger, the accuracy of locating the exact image plane is
improved, and the depth-of-field of the image is decreased. This is equivalent to removing the central portion of a lens to decrease its depth of field.

The determination of particle sizes, shapes, and densities can be made by direct observation of the reconstructed image. In the reconstructed volume, one has essentially a frozen three-dimensional array characterizing the original subject and sitting in free space available for study by any optical instrument. The limitations are many, but since these are well-covered in the references, they will only be mentioned here. As the particle number density increases the hologram becomes poorer. As the particle size becomes smaller, it must be nearer to the hologram in the original construction (Ref. 3). (The present small-particle limits without premagnification are in the order of microns.) As the particle velocity increases, the hologram becomes poorer. The particle must not move more than a fraction of its diameter during the construction.

We have not found the usefulness of the technique to be specifically limited to the Fraunhofer diffraction region as suggested by Ref. 1. If the particle density is sufficiently low, high quality holograms can be formed even when they lie in the Fresnel zone. Figure 2a contains the reconstructed image of a 2-mm-diam particle located 50 cm from the hologram, which is well within the Fresnel zone.

SECTION III
MULTIPLE-EXPOSURE HOLOGRAPHY

The above discussion need be extended only slightly to encompass one method of velocity determination of elements in the array. A number of techniques have been proposed to accomplish this (Refs. 4 and 5). In fact, many of the usual photographic techniques for velocity determination can be extended to holography. These include motion-picture holography and stroboscopic holography. If the motion is slow, the extension is quite trivial. One simply makes a new hologram at different time intervals and then superimposes the various three-dimensional images to study motion of the elements during the intervals. As the motion becomes more dynamic, new requirements become evident. Exposure time for each hologram must be reduced, requiring higher optical power, and intervals between exposures must be shortened. The Q-switched ruby laser represents the present limit in high power and short exposure. One quickly finds that this laser is necessary for all but the slowest motion, particularly for small particles.
Multiple exposure of an object can be accomplished by using several lasers synchronized to fire at the desired interval. Fortunately, there are less expensive ways to achieve this. Multiple pulsing of a ruby laser can be accomplished with either active or passive Q-switches. This is possible because the firing lamp which pumps the ruby can be actuated considerably longer than the time required to energize and de-energize the ruby. The Q-switch is timed to open when the laser gain coefficient exceeds zero to allow lasing and then to close until the gain coefficient once more exceeds zero and so on. The process can continue as long as the flash lamp pumps the ruby. The time required between pulses varies with pumping rate, the ruby temperature, and other factors; these values range from about 10 to 300 μsec, whereas typical flash lamps can be excited sufficiently for times in the order of 1000 μsec. This means that one can achieve many pulses by multiple Q-switching with existing systems. In fact, with no Q-switching at all, a laser usually generates, in its normal firing mode, 20 or so pulses which are 10 to 20 nsec wide. Each pulse has a substructure also, but this substructure is of no particular interest here.

An active Q-switch can be timed to open and close electronically at the proper intervals. A passive Q-switch can also be adjusted so that it opens and closes spontaneously at various intervals but with much less control and repeatability. In the present experiments, passive Q-switches were used because of their lower cost and because they produce better holograms. When the techniques are carried beyond the research laboratory, one would almost certainly choose active Q-switching.

SECTION IV
RECORDING AND RECONSTRUCTION

Consider the case in which a particle passes before a photographic plate, and two extremely short exposures of the plates are made. If the emulsion continues to act linearly, according to Eq. (4), the resulting amplitude transmission coefficient will be given by Eqs. (3) and (4).

\[
T = 1 - K(|U_{o1}|^2 + |U_{o2}|^2 + |U_{a1}|^2 + |U_{a2}|^2) + KU_{o1}^*U_{a1} + KU_{o2}^*U_{a2} + KU_{o1}^*U_{a2}^* + KU_{o2}^*U_{a1}^* \tag{8}
\]

Now, if the hologram is illuminated by a plane wave, \( U_{o1} \), a transmitted wave, \( U' \), will have the form
\[ U' = U_{oi} \left\{ 1 - K(|U_{oi}|^2 + |U_{o2}|^2 + |U_{a1}|^2 + |U_{a2}|^2) \right\} \]
\[ + KU_{oi}(U_{oi}U_{a1}^* + U_{o2}U_{a2}^*) + K(|U_{oi}|^2U_{a1} + |U_{o2}|U_{o2}U_{a2}) \] (9)

Equation (9) leads one to the conclusion that the real and virtual images of the particle are produced in both exposures without any degradation. It must be remembered that this can only happen when the assumption of linearity (Eq. 4) holds true. The transmission versus exposure curve can be expanded in terms of linear and higher order terms. The linear terms lead once again to the reconstruction of a double image. The higher order terms cause interference and degradation of the hologram. Even so, these are often tolerable even when the film nears saturation.

In a double exposure hologram it would be convenient if images constructed by the second exposure could be differentiated from the first to give a sense of direction. That this can be achieved is shown, for example, by the third term in Eq. (9) (virtual-image term). If the product \(|U_{oi}||U_{o2}|\) differs from \(|U_{oi}|^2\), then the intensity of the reconstructions will differ according to the ratio. Figure 6 is a photograph of the reconstructed image of a doubly exposed hologram that was made with the second exposure being twice the first. This illustrates the potential of the technique for direction sensing. A more accurate determination of the intensity ratio and additional discussion of nonlinearities has been presented by Wyant and Givens (Ref. 6).

Figure 7 is the positive of a double-exposed, in-line hologram taken near the nozzle of a high pressure atomizer. A 60-micron-diam wire, with a 300-micron disk attached to its center, was stretched across the field of view normal to the direction of flow. The hologram was made on an Agfa-Gevaert Scientia plate with a Korad K1 ruby laser. The Q-switch was a passive cell filled with a cryptocyanine dye solution adjusted for double pulsing.

It would serve no purpose to present all of the data extracted from this hologram since only the technique is under discussion here. The hologram contains a vast amount of data, a few of which are presented. Figures 8a and b are photographs of the volume reconstructed from the area marked A in Fig. 7 with the camera focused on planes that are 1 cm apart. Some of the in-focus particle pairs are designated. The particles shown in the reconstruction are water droplets with diameters varying between 20 and 150 microns. The velocities vary up to 15 m/sec. Figure 9 shows a reconstruction of the area marked B. This region is difficult to analyze because the density of particles was high, but even so, a considerable amount of information was extracted from this region.
Figure 10 is the axial speed profile taken across the center of the flow near the reference wire. Such phenomena as turbulence, droplet oscillation, break up, collisions, acoustic phenomena, instabilities, and diffusion can be observed in this hologram.

Because of the amount of data contained in a hologram of the type under discussion, it would be highly desirable to develop optical reduction techniques for processing the data into final form. The hologram would then serve as the input to an optical computer which would be programmed to extract the desired data.

Other areas in which double exposure holography has been applied successfully include electrolysis, bubble formation, water jets, and other arrays of scattering centers. It shows promise in the study of seed and contaminant materials in various types of flows, in the study of rocket exhausts, chemical reactions, impact, and other dynamic phenomena.

REFERENCES


APPENDIX
ILLUSTRATIONS
Fig. 1 Geometry of the Hologram Formation Process
Fig. 2 Reconstructed Real Image of a Particle Field
Fig. 3 Image Reconstruction with Simultaneous Formation of the Secondary Hologram
Fig. 4 Apparatus Used for Reconstructing and Analyzing the Three-dimensional Image
Fig. 5 Reconstructed Real Image with High Pass Filtering

\[ d = 250 \mu \]
Fig. 6  Direction Sensing of a Multiple Image by Controlled Exposure
Fig. 7 Double-Exposure Hologram of an Aerosol Spray
Fig. 8 Two Planes of the Reconstructed Image of a Doubly Exposed Hologram (Region Marked A in Fig. 7)

\[ z = 15 \text{ cm} \]
Fig. 9 Reconstructed Image in a High Density Region (Region Marked B in Fig. 7)
Fig. 10 Axial Speed Profile across an Aerosol Spray
A brief review of the techniques of small-particle holography is presented. These results are extended in an elementary way to show that multiple-exposure holography can be used to study the dynamic properties of particle fields. Experimental techniques for performing such a study are then described. Finally, as a typical example, a multiple-exposure hologram of an aerosol is presented with a portion of the data extracted from it. The hologram exhibits a vast amount of information including the determination of the velocity field, density field, size distribution, flow structure, and diffusion rate.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ROLE</td>
<td>ROLE</td>
<td>ROLE</td>
</tr>
<tr>
<td></td>
<td>WT</td>
<td>WT</td>
<td>WT</td>
</tr>
<tr>
<td>Holography</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lasers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Particle fields, dynamic</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>