Mathematical Model of Skin Exposed to Thermal Radiation

Naval Air Systems Command
AirTask R01 101 01 (Task Problem No. RB-6-01)

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
Aerospace Medical Research Department

NADC-MR-6708 22 August 1967

Mathematical Model of Skin Exposed to Thermal Radiation

Naval Air Systems Command
AirTask R01 101 01 (Task Problem No. RB-6-01)

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Prepared by:

\[ \text{John A. Weaver} \]

Alice M. Stoll

Approved by:

\[ \text{Carl F. Schmidt, M.D} \]
Research Director
Aerospace Medical Research Department

Released by:

\[ \text{E. M. Wuzel, CAPT, MC, USN} \]
Director
Aerospace Medical Research Department
SUMMARY

Prediction of dermal injury resulting from exposure to thermal energy of any given intensity and duration depends entirely upon the resultant skin temperature-time history. Means are now available for assessing heat transfer by low temperature radiation, convection and conduction to the bare skin and through thin protective coverings of known physical properties. However, thermal effects of nuclear detonations constitute a special problem because much of the radiation lies in the visible range where the optical properties of the skin and its coverings, if any, greatly influence the heating pattern. Blackening of the skin eliminates effects due to its optical properties but enhances the ever-present variations in the thermal "constants" of the skin. The present report describes the utilization of a mathematical equation and computer techniques for extracting these variations from empirical data obtained at relatively low levels of radiation (<0.5 Cal/cm² sec), and applying extrapolations of these values in calculations of temperature-time histories at higher levels of irradiance where empirical data are lacking. This procedure is subject to validation by experimentation within a limited range of exposures. If validation is achieved in the blackened skin then the entire system may be utilized in the determination of optical properties of unblackened skin.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Summary</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Theory</td>
<td>2</td>
</tr>
<tr>
<td>Calculations and Data</td>
<td>7</td>
</tr>
<tr>
<td>Conclusions</td>
<td>22</td>
</tr>
<tr>
<td>References</td>
<td>24</td>
</tr>
</tbody>
</table>

---

## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comparison of Damage Rates Derived from Conductive and from Radiative Data.</td>
</tr>
<tr>
<td>2</td>
<td>The Heating and Cooling Pattern of Skin Exposed to a Square-Wave Pulse of Radiant Energy.</td>
</tr>
<tr>
<td>3</td>
<td>Experimental Variations of the Thermal Conductivity of Human Skin with Tissue Temperature.</td>
</tr>
<tr>
<td>4</td>
<td>Experimental and Theoretical Thermal Conductivity of Skin versus Tissue Temperature.</td>
</tr>
<tr>
<td>5</td>
<td>Experimental and Theoretical T-t History of Human Skin Exposed to Thermal Radiation.</td>
</tr>
<tr>
<td>6</td>
<td>Plot of the Thermal Conductivity versus Level of Radiation</td>
</tr>
<tr>
<td>7</td>
<td>Plot of Peak Temperatures Attained versus Level of Radiation.</td>
</tr>
<tr>
<td>8</td>
<td>Plot of Reciprocal of Tau versus Level of Radiation.</td>
</tr>
<tr>
<td>9</td>
<td>Theoretical T-t History of Human Skin Exposed to Thermal Radiation.</td>
</tr>
<tr>
<td>10</td>
<td>Comparison of Theoretical Data and Experimental Data for Various High Intensity Radiation Burn Studies.</td>
</tr>
<tr>
<td>Table</td>
<td>Title</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>I</td>
<td>Characteristics of Experimental Time Temperature Histories</td>
</tr>
<tr>
<td>II</td>
<td>Characteristics of Theoretical Time-Temperature Histories</td>
</tr>
</tbody>
</table>
INTRODUCTION

The basic principles underlying tissue injury by heating, as we understand them today, were enunciated by Henriques and Moritz in a series of papers based on studies of tissue properties and burns produced by conduction heating. They fitted their data to an equation expressing tissue damage as a temporal integral of rates of tissue injury depending upon the tissue temperature, and increasing logarithmically with this temperature (1). For this purpose the symbol $\Omega$ was chosen to represent damage and a value of $\Omega = 1.0$ was chosen arbitrarily to represent complete transepidermal necrosis. The resultant equation was of the rate process type as shown in Eq. 1:

$$\Omega = 1.0 = \int_{0}^{t} \frac{d\Omega}{dt} dt = \int_{0}^{t} Pe^{-\Delta E/RT_x} t \quad \text{(Eq. 1)}$$

where $d\Omega/dt =$ rate of damage

$t =$ time in seconds

$P =$ constant of integration

$R =$ gas constant

$\Delta E =$ energy of inactivation

$T_x =$ tissue temperature in °K at time $t$

Subsequent studies in this laboratory using thermal radiation heating showed that for short-term, high intensity exposure, the damage done during cooling became significant and increasingly important as the heating episodes shortened, so that as much as 35 per cent of the total damage produced with an irradiance of 0.4 Cal/cm² sec occurred during cooling, after cessation of the exposure (2). Thus, it became evident that the damage rates derived from the
relatively long-term conduction heating studies required modification in order to correlate accurately with tissue temperature. This adjustment was made as shown in Figure 1 and using these rates it was possible to apply the original equation as a composite of damage during heating and cooling to yield a sum equal to unity at the point of threshold blistering (3).

The procedure developed in these studies were satisfactory for their purpose but required direct measurements of skin temperatures during the irradiation, a difficult task at best and extremely so with very high-intensity energy. It was desired, therefore, to devise a procedure for the prediction of injury from a knowledge of the heating intensity and time alone which could be used for estimating first the effects of square-wave pulses and, finally, the effects of variable pulses of known patterns. The problem is complicated not only by the need for considering damage during cooling but also by the fact that the thermal conductivity changes with temperature and vasomotion in the skin itself (2, 4). For this reason the mathematical model developed in the present study was designed to provide for changes in the conductivity which would reflect the temperature-time heating patterns observed in the blackened skin of the earlier studies. It is understood that the blackening even though it is only 10μ thick must contribute to the value of the skin conductivity. However, for the present purpose this effect has been ignored and the average conductivity is treated as though it were that of the skin alone.

THEORY

The heating and cooling pattern of the skin exposed to a square-wave pulse of radiant energy is the familiar exponential rise and fall shown in
Figure 1. Comparison of Damage Rates Derived from Conductive and from Radiative Data.
Figure 2. It is characterized by a rise in temperature from some initial temperature, $T_0$, at time $t = 0$ when irradiation at a flux of magnitude $Q$ begins. The temperature continues to rise to some peak temperature $TPK$ at time $t = \tau$, the time at which the radiation ceases. The temperature then drops, rapidly at first, then more slowly, toward the initial value, $T_0$.

The literature contains no single equation to describe both the heating phase and the cooling phase continuously, although there are separate equations to describe either the heating phase or the cooling phase (5). The real part of the following equation suggested by Buettner (6) accomplishes the desired end:

$$T_x = \frac{Q}{k} \left[ \frac{2a\sqrt{t}}{\pi} e^{-\frac{x^2}{4a^2t}} - x \left( 1 - \theta \left( \frac{x}{2a\sqrt{t}} \right) \right) \right] + T_0$$

$$- \frac{Q}{\pi} \left[ \frac{2a\sqrt{\tau - t}}{\pi} e^{-\frac{x^2}{4a^2(t - \tau)}} - x \left( 1 - \theta \left( \frac{x}{2a\sqrt{\tau - t}} \right) \right) \right]$$

(Eq. 2)

where $T_x$ = tissue temperature at depth $x$ below the surface ($^\circ$C)

$Q$ = effective radiation on the surface of skin (Cal/cm$^2$ sec)

$k$ = heat conductivity (Cal/cm sec $^\circ$C)

$\rho$ = density (g/cm$^3$)

$c$ = specific heat (Cal/g $^\circ$C)

$a^2 = k/\rho c$ = temperature diffusivity (cm$^2$/sec)

$t$ = time (sec)
Figure 2. The Heating and Cooling Pattern of Skin Exposed to a Square-Wave Pulse of Radiant Energy.
\[ x = \text{depth below the surface (cm)} \]

\[ T_o = \text{initial surface temperature (°C)} \]

\[ \Theta(u) = \text{integral of the probability curve} = \frac{2}{\sqrt{\pi}} \int_0^u e^{-y^2} dy = \text{Error Function} \]

\[ \tau = \text{time at which thermal radiation ceases (exposure time) (sec)} \]

The equation can be derived directly from the general differential equation for heat conduction in one dimension:

\[ \frac{\delta T}{\delta t} = a^2 \frac{\delta^2 T}{\delta x^2} \quad \text{(Eq. 3)} \]

assuming a constant heat flow and an initial isothermal condition (vertical temperature gradient equals zero), with the heat absorbed at the surface of the skin transferred inward by conduction. The equation is obviously a two-part solution; first the solution during the heating phase \( t < \tau \) made up of the first two terms, the third term being imaginary or zero; second, the solution during the cooling phase \( t > \tau \) for which all three terms are real.

As time \( t \) approaches infinity, the first and third terms approach zero and thus \( T_x \) approaches \( T_0 \). The third term is a Laplace solution for a negative \( Q \) input in the interval \( \tau < t \leq \infty \) so that the effective heat input is zero, as required during the cooling phase; thus \( Q = 0 \) for \( t < 0 \) and \( t > \tau \) while \( Q \) is a positive quantity when \( t > 0 \) and \( t \leq \tau \).

Equation 2 cannot be solved explicitly for either the thermal diffusivity \( a^2 \) or the thermal conductivity \( k \). However, for any given experimental time-temperature history all other terms are known or can be assumed reliably and a simplification may be made in order to provide for solutions for \( k \). Since
the product of the density \( \rho \) and the specific heat \( c \) of human skin is very nearly one \( (\rho c = 1) \) (2) this approximation may be used to relate the conductivity to the diffusivity:

\[
\alpha^2 = \frac{k}{\rho c} \quad (\text{Eq. 4})
\]

substituting \( \rho c = 1 \)

\[
\alpha^2 = k
\]

\[
\alpha = \sqrt{k}
\]

Hence for any given experimental time-temperature history at any particular time-temperature point we know the values of temperature \( T_x \), time \( t \), intensity of radiation \( Q \), initial surrounding temperature \( T_0 \), time to peak temperature \( t_{\text{peak}} \), at the given depth \( x \). Using this information and a computer program, we are able to solve for a value of the thermal conductivity \( k \) of the skin at each time-temperature point in the experimental curve.

The method of measuring and obtaining the experimental time-temperature histories used in this analysis has been described (2). In the present analysis, the measured surface temperatures \( x = 0 \) were used as raw data and from each time-temperature point the value of thermal conductivity \( k \) of the skin at the surface was computed. Then assuming that the value of conductivity \( k \) of the skin remains constant from the surface inward, we found the value of the tissue temperature \( T_x \) at the depth of 80\( \mu \), the commonly assumed depth of the basal layer, the interface between the dermis and epidermis.

**CALCULATIONS AND DATA**

Nine experimental time-temperature histories were analyzed. Three were
for a level of radiation of 0.10 Cal/cm² sec, three for a level of 0.15 Cal/cm² sec, one history for 0.30 Cal/cm² sec, and two for a level of 0.40 Cal/cm² sec. Table I summarizes the main characteristics of the histories. The first column in the table is the level of the effective radiation, Q, in Cal/cm² sec where an absorptivity of 0.94 is assumed; column 2 is the time at which thermal radiation ceases (exposure time), τ, in sec; Column 3 is the total time of the episode, the time in sec at which the temperature at depth x, Tₓ, falls below the level of the injurious temperature, 44°C; Column 4 is the time interval between temperature points in sec; and Column 5 is the peak temperature attained at depth x, in °C.

Some examples of the results of these analyses are shown in the plots of Figure 3, where thermal conductivity of the skin (k) is plotted versus tissue temperature. It is seen that the greatest variations occur at temperatures below 44°C. Since little variation occurs at temperatures above 44°C, it was possible to average the values of conductivity in this range. The square root of this average value of conductivity during the heating phase for temperatures equal to or greater than 44°C and equal to or less than the peak Tₓ was found for each history and is listed in column 6 of Table I under the heading of Al. Similarly the average value of conductivity during the cooling phase for temperatures greater than the peak Tₓ was found for each history and the square root of this average value is listed in Column 7 of Table I under the heading of A2. Care should be taken to distinguish between the values Al and A2, the square roots of the average values of conductivity during the heating and cooling phases respectively, and k, the conductivity. The first two values, Al and A2, are computer variables and
<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>t</th>
<th>Total Time</th>
<th>At</th>
<th>Peak $T_x$</th>
<th>A1</th>
<th>A2</th>
<th>$\mu_H$</th>
<th>$\mu_L$</th>
<th>$\Omega_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.094</td>
<td>33.80</td>
<td>59.80</td>
<td>1.00</td>
<td>52.50</td>
<td>0.0294</td>
<td>0.0262</td>
<td>0.8546</td>
<td>0.1446</td>
<td>0.9792</td>
<td></td>
</tr>
<tr>
<td>.094</td>
<td>33.12</td>
<td>42.69</td>
<td>0.87</td>
<td>52.20</td>
<td>0.0278</td>
<td>0.0511</td>
<td>0.8656</td>
<td>0.0373</td>
<td>0.9029</td>
<td></td>
</tr>
<tr>
<td>.094</td>
<td>33.49</td>
<td>43.06</td>
<td>0.87</td>
<td>52.40</td>
<td>0.0279</td>
<td>0.0314</td>
<td>0.9047</td>
<td>0.0365</td>
<td>0.9411</td>
<td></td>
</tr>
<tr>
<td>.141</td>
<td>20.80</td>
<td>33.20</td>
<td>0.40</td>
<td>53.20</td>
<td>0.0318</td>
<td>0.0310</td>
<td>0.8121</td>
<td>0.1422</td>
<td>0.9543</td>
<td></td>
</tr>
<tr>
<td>.141</td>
<td>21.97</td>
<td>27.29</td>
<td>0.36</td>
<td>53.50</td>
<td>0.0332</td>
<td>0.0357</td>
<td>0.9447</td>
<td>0.1237</td>
<td>1.0684</td>
<td></td>
</tr>
<tr>
<td>.141</td>
<td>20.50</td>
<td>25.90</td>
<td>0.36</td>
<td>53.50</td>
<td>0.0306</td>
<td>0.0346</td>
<td>0.8919</td>
<td>0.0997</td>
<td>0.9916</td>
<td></td>
</tr>
<tr>
<td>.282</td>
<td>7.80</td>
<td>23.30</td>
<td>0.50</td>
<td>55.20</td>
<td>0.0361</td>
<td>0.0272</td>
<td>0.6020</td>
<td>0.2902</td>
<td>0.8923</td>
<td></td>
</tr>
<tr>
<td>.376</td>
<td>5.60</td>
<td>14.10</td>
<td>0.25</td>
<td>57.20</td>
<td>0.0375</td>
<td>0.0321</td>
<td>0.7802</td>
<td>0.3067</td>
<td>1.0869</td>
<td></td>
</tr>
<tr>
<td>.376</td>
<td>5.56</td>
<td>17.44</td>
<td>0.27</td>
<td>55.80</td>
<td>0.0365</td>
<td>0.0281</td>
<td>0.6916</td>
<td>0.3825</td>
<td>1.0741</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3. Experimental Variations of the Thermal Conductivity of Human Skin with Tissue Temperature.
the variable k is the actual physiological parameter. The relationship between the three is expressed by Equation 4 where $A_1$ and $A_2$ were chosen to represent the diffusivity, $a^2$, during heating and cooling respectively.

It is important to note that using a constant value of thermal conductivity of the skin during the heating phase, $A_1$, gave a satisfactory representation of the temperature rise at all levels of radiation; however using a constant value of thermal conductivity of the skin during the cooling phase, $A_2$, did not give a satisfactory representation of the temperature drop at the higher levels of radiation (0.30 Cal/cm$^2$ sec and 0.40 Cal/cm$^2$ sec), as it was first expected to do. Thus for all levels of radiation to achieve an empirical fit, the value of $A_2$ was incremented by $-10^{-4}$ after the calculation of the first time-temperature point during the temperature drop and was continually incremented by $-10^{-4}$ after the calculation of each successive time-temperature point on the temperature drop. Figure 4 is a plot of thermal conductivity of the skin (k) versus tissue temperature, where some of the experimental conductivity curves from Figure 3 are plotted along with the theoretical conductivity curves used to approximate the experimental curves. The theoretical curve is a straight line (k = constant value) during the heating phase and is a curved line \( \sqrt{K_n} = \sqrt{K_{n-1}} - 10^{-4} \) during the cooling phase. This theoretical function of conductivity gave a satisfactory representation of time-temperature histories for all experimental data considered at various levels of radiation. Figure 5 shows a plot of an experimental time-temperature history together with the calculated theoretical time-temperature history for a level of radiation 0.40 Cal/cm$^2$ sec.

Column 8 of Table I shows the amount of tissue damage done during the
Figure 4. Experimental and Theoretical Thermal Conductivity of Skin versus Tissue Temperature.
Figure 5. Experimental and Theoretical T-t History of Human Skin Exposed to Thermal Radiation.
heating phase designated \( \Omega_H \); Column 9 shows the amount of tissue damage done during the cooling phase designated \( \Omega_C \) and Column 10 shows the total tissue damage done during the entire episode, designated \( \Omega_T \). Notice that a larger portion of the tissue damage is done during the cooling phase at the higher levels of radiation as mentioned earlier. Also, it should be noted that for all nine histories, \( \Omega_T = 1.0 \), which represents a threshold exposure or damage just sufficient to cause complete transepidermal necrosis. The values of \( P \), the constant of integration, and \( \Delta E/R \), the energy of inactivation divided by the gas constant, used in Equation 1 to compute the value of \( \Omega_H \), \( \Omega_C \), and \( \Omega_T \) listed in Table I where

\[
P = 2.1850 \times 10^{124} \\
\Delta E/R = 93,534.9 
\]

for \( 44 ^\circ C < T_x < 50 ^\circ C \)

and

\[
P = 1.8230 \times 10^{51} \\
\Delta E/R = 39,109.8 
\]

for \( T_x \geq 50 ^\circ C \)

These values were determined by use of the plot of damage rates versus temperature shown in Figure 1. Values of \( d\Omega/dt \) and \( T_x \) were read from the plot and substituted into the following equation:

\[
\frac{d\Omega}{dt} = Pe^{-\Delta E/RT_x} 
\]

which yielded solutions for \( P \) and \( \Delta E/R \).

Theoretical time-temperature histories were adjudged satisfactory representation of experimental time-temperature histories if the characteristics
of the theoretical history (τ, Total Time, Peak Tₓ, A₁, A₂, Qₓ, Qₚ, and Qₜ) at a given level of radiation were within a reasonable range of the characteristics of the experimental histories for the given level. What constituted "a reasonable range" was not statistically defined since in some cases each experimental characteristic at any given level of radiation contained considerable variation due to the experimental nature of the data. However, final criterion for any given theoretical history generated was considered to be a value of Qₓ very nearly equal to 1.0 (±5%).

After suitable theoretical time-temperature histories had been generated to match the experimental time-temperature histories at various levels of radiation the values of the thermal conductivity during heating, A₁², the thermal conductivity during cooling, A₂², the peak temperature attained, TPK, and the length of time the skin is exposed to the radiation, τ, were plotted versus their levels of radiation. Figures 6 through 8 are these plots with extrapolations to the higher levels of radiation (Q > 0.40 Cal/cm² sec). It should be understood that the value of thermal conductivity during cooling read from Figure 6 for any given level of radiation is the value used to compute only the first time-temperature point after the peak temperature. This value is then decremented as mentioned above. However, the value of thermal conductivity during heating read from Figure 6 for any given level of radiation is the constant value used to compute every time-temperature point during the temperature rise.

Using these extrapolated values of A₁, A₂, TPK, and τ, theoretical time-temperature histories of skin exposed to high levels of thermal radiation were calculated. An example of these appear in Figure 9 where both the
Figure 6. Plot of the Thermal Conductivity versus Level of Radiation.
Figure 7. Plot of Peak Temperatures Attained versus Level of Radiation.
Figure 8. Plot of Reciprocal of Tau versus Level of Radiation.
Figure 9. Theoretical T-t History of Human Skin Exposed to Thermal Radiation.
surface time-temperature history (the curve with the larger peak) and the corresponding time-temperature history at a depth of 80μ (the curve with the smaller peak) are plotted. In comparison with Figure 2, the heating and cooling pattern of skin exposed to radiant energy, Figure 9, has two immediate differences. First, in Figure 9 after the peak temperature is attained and the radiation ceases, the surface temperature does not drop below the temperature at depth as time continues. Secondly, the peak of the temperature at depth curve is not followed by as sharp a drop off as is shown in Figure 2. Possible reasons for these differences are:

1. possible error in extrapolation to these much higher levels of radiation,
2. differences in the mathematics equation used to calculate tissue temperatures in Figure 2 (Equation 2) and equation used to calculate tissue temperatures in Figure 9 (Equation 2) or
3. the sensitivity of Equation 2 to the values of thermal conductivity used during calculations of temperatures during the cooling phase.

The values of $\theta_H$, $\theta_C$, and $\theta_T$ were computed for each theoretical history and it will be noted that each theoretical history calculated is a threshold exposure, that is, for any given level of radiation an exposure such as shown would be just sufficient to cause complete transepidermal necrosis. Table I' is similar to Table I in that it summarizes the main characteristics of the time-temperature history of skin exposed to thermal radiation except that Table II is for higher levels of radiation for which no experimental data exists. Table I summarizes experimental data; Table II summarizes theoretical calculations. The values indicated, considered in comparison with the sparse
### TABLE II
CHARACTERISTICS OF THEORETICAL TIME-TEMPERATURE HISTORIES

<table>
<thead>
<tr>
<th>Q</th>
<th>T</th>
<th>Total Time</th>
<th>Peak 1</th>
<th>a1</th>
<th>A2</th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>33.470</td>
<td>52.47</td>
<td>1.0000</td>
<td>52.78</td>
<td>0.0290</td>
<td>0.0285</td>
<td>0.8640</td>
</tr>
<tr>
<td>0.143</td>
<td>21.090</td>
<td>35.76</td>
<td>0.066</td>
<td>53.81</td>
<td>0.0327</td>
<td>0.0319</td>
<td>0.8253</td>
</tr>
<tr>
<td>0.282</td>
<td>8.300</td>
<td>18.63</td>
<td>0.335</td>
<td>56.02</td>
<td>0.0364</td>
<td>0.0336</td>
<td>0.7354</td>
</tr>
<tr>
<td>0.376</td>
<td>5.550</td>
<td>13.90</td>
<td>0.2500</td>
<td>57.13</td>
<td>0.0374</td>
<td>0.0345</td>
<td>0.7298</td>
</tr>
<tr>
<td>0.564</td>
<td>3.000</td>
<td>8.50</td>
<td>0.167</td>
<td>58.80</td>
<td>0.0375</td>
<td>0.0348</td>
<td>0.7092</td>
</tr>
<tr>
<td>0.752</td>
<td>1.550</td>
<td>6.20</td>
<td>0.1250</td>
<td>60.00</td>
<td>0.0375</td>
<td>0.0349</td>
<td>0.6870</td>
</tr>
<tr>
<td>0.940</td>
<td>1.410</td>
<td>5.01</td>
<td>0.1000</td>
<td>61.01</td>
<td>0.0375</td>
<td>0.0350</td>
<td>0.6910</td>
</tr>
<tr>
<td>1.128</td>
<td>1.080</td>
<td>4.16</td>
<td>0.0833</td>
<td>61.72</td>
<td>0.0375</td>
<td>0.0351</td>
<td>0.6703</td>
</tr>
<tr>
<td>1.316</td>
<td>0.862</td>
<td>3.56</td>
<td>0.0770</td>
<td>62.25</td>
<td>0.0375</td>
<td>0.0352</td>
<td>0.6435</td>
</tr>
<tr>
<td>1.504</td>
<td>0.713</td>
<td>3.17</td>
<td>0.0630</td>
<td>62.77</td>
<td>0.0375</td>
<td>0.0353</td>
<td>0.6326</td>
</tr>
<tr>
<td>1.692</td>
<td>0.603</td>
<td>2.84</td>
<td>0.0560</td>
<td>63.15</td>
<td>0.0375</td>
<td>0.0354</td>
<td>0.6099</td>
</tr>
<tr>
<td>1.880</td>
<td>0.522</td>
<td>2.62</td>
<td>0.0500</td>
<td>63.36</td>
<td>0.0375</td>
<td>0.0355</td>
<td>0.6034</td>
</tr>
<tr>
<td>2.068</td>
<td>0.459</td>
<td>2.44</td>
<td>0.0450</td>
<td>63.93</td>
<td>0.0375</td>
<td>0.0356</td>
<td>0.5978</td>
</tr>
<tr>
<td>2.256</td>
<td>0.409</td>
<td>2.30</td>
<td>0.0420</td>
<td>64.28</td>
<td>0.0375</td>
<td>0.0357</td>
<td>0.6011</td>
</tr>
<tr>
<td>2.444</td>
<td>0.367</td>
<td>2.19</td>
<td>0.0380</td>
<td>64.52</td>
<td>0.0375</td>
<td>0.0358</td>
<td>0.5902</td>
</tr>
<tr>
<td>2.632</td>
<td>0.333</td>
<td>2.08</td>
<td>0.0360</td>
<td>64.78</td>
<td>0.0375</td>
<td>0.0359</td>
<td>0.5779</td>
</tr>
<tr>
<td>2.820</td>
<td>0.303</td>
<td>1.95</td>
<td>0.0330</td>
<td>64.91</td>
<td>0.0375</td>
<td>0.0360</td>
<td>0.5456</td>
</tr>
</tbody>
</table>
experimental data reported for various high intensity radiation burn studies (7, 8) appear to be entirely feasible. Some of these are shown in Figure 10. Discrepancies, particularly those noted for very short exposure times, may well be due to differences in experimental techniques.

**CONCLUSIONS**

It is concluded that it is quite possible to develop a mathematical model that faithfully reflects the heating and cooling pattern of skin exposed to a square wave pulse. Whether or not application of this model in extrapolations to higher levels of intensity yields accurate results remains to be verified. Such indications as can be gleaned from existing data tend to support the validity of this procedure.

**NOTE:** The mathematics and computer program involved in generating the theoretical time-temperature histories are reported elsewhere (9).
Figure 10. Comparison of Theoretical Data and Experimental Data for Various High Intensity Radiation Burn Studies.
REFERENCES


6. Buettner, K., Professor, Department of Atmospheric Sciences, University of Washington, Seattle, Washington, 98105, Unpublished Data.


Mathematical Model of Skin Exposed to Thermal Radiation

Prediction of dermal injury resulting from exposure to thermal energy of any given intensity and duration depends entirely upon the resultant skin temperature-time history. Means are now available for assessing heat transfer by low temperature radiation, convection and conduction to the bare skin and through thin protective coverings of known physical properties. However, thermal effects of nuclear detonations constitute a special problem because much of the radiation lies in the visible range where the optical properties of the skin and its coverings, if any, greatly influence the heating pattern. Blackening of the skin eliminates effects due to its optical properties but enhances the ever-present variations in the thermal "constants" of the skin. The present report describes the utilization of a mathematical equation and computer techniques for extracting these variations from empirical data obtained at relatively low levels of radiation (<0.5 Cal/cm² sec.), and applying extrapolations of these values in calculations of temperature-time histories at higher levels of irradiance where empirical data are lacking. This procedure is subject to validation by experimentation within a limited range of exposures. If validation is achieved in the blackened skin then the entire system may be utilized in the determination of optical properties of unblackened skin.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Thermal Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Tissue Damage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Thermal Radiation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Mathematical Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Skin Time-Temperature History</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Computer Program</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**INSTRUCTIONS**

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (contractor author) issuing the report.

2. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

3. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

4. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b. **Gr. & Bd. PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBERS:** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBERS:** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY LIMITATION NOTICES:** Enter any limitations on further dissemination of the report other than those imposed by security classification, using standard statements such as:

    1. "Qualified requesters may obtain copies of this report from DDC."
    2. "Foreign announcement and dissemination of this report by DDC is not authorized."
    3. "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through "
    4. "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through "
    5. "All distribution of this report is controlled. Qualified DDC users shall request through "

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (funding) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designations, trade names, military project code names, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of limits, rules, and weights is optional.