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MAXIMAL TWO-WAY FLOWS

by

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The most familiar network flow problem is that of finding the maximal integer flow from a source $s$ to a sink $t$ in a network $G$. In this paper we discuss the problem of simultaneous flows from $s$ to $t$ and from $t$ to $s$. The main result of this paper is a max-flow min-cut theorem for this type of problem. The method of proof used indicates a procedure for finding the maximal flows. Finally, the problem of feasibility is discussed.

Introduction.

The most familiar network flow problem is that of finding the maximal integer flow from a source $s$ to a sink $t$ in a network $G$ with integer capacities (see [1]). In this paper we discuss the problem of simultaneous flows from $s$ to $t$ and from $t$ to $s$. For example, one might wish to know how much traffic a specific system of roads could carry between two points going in either direction. If the network is undirected (e.g., there are no one-way streets), then this problem is identical with the problem of finding a maximal flow from $s$ to $t$. The answer to this is the well known Max-flow Min-cut theorem (see [1]). When one has some directed arcs, however, the problems are no longer equivalent. Here we obtain a Max-flow Min-cut theorem for two-way flows in certain special networks, called Euler networks, which have some arcs directed and some undirected. They are defined below. The method of proof used here indicates a procedure for actually constructing a maximal flow. It is the analogue to the cross path method used in [2] for the one-way flow.

We consider here mixed networks, that is networks with both
directed and undirected arcs. For convenience in what follows, we con-
sider all arcs to have capacity one, but we permit multiple arcs between
any two nodes. For two nodes a, b in the network G, an a → b path is
a path consisting possibly of both directed and undirected arcs such
that the directed arcs occur in the direction from a to b along the
path. We permit nodes to be used more than once in a path, but arcs may
only be used once. A flow is a collection of paths no two having any arc
in common. An a → b flow is a flow consisting of a → b paths; a b → a
flow is a flow consisting of b → a paths. A two-way flow or a ↔ b flow
is a flow consisting of a → b paths and b → a paths. An a → b (respec-
tively b → a) cut-set is a collection of arcs (again these may be directed
and undirected together) such that their removal from G eliminates all
a → b paths (respectively b → a paths). An a ↔ b cut-set is a collec-
tion of arcs whose removal eliminates both a → b and b → a paths.

A network is said to be Euler if at each node there is an even num-
ber of undirected arcs and there is the same number of incoming directed
arcs as outgoing directed arcs. A circuit (i.e., closed path) or a col-
lection of paths or a flow is called Euler if the subnetwork consisting
the arcs and nodes of the circuit or collection of paths or flow respec-
tively is Euler. A network G is called an Euler directed network if it
is Euler and all arcs are directed. Similarly, it is called an Euler un-
directed network if it is Euler and all its arcs are undirected. Clearly
the subnetwork of G consisting of all directed arcs and the nodes to
which they are attached is an Euler directed subnetwork D. Similarly,
all the undirected arcs of G determine an Euler undirected subnetwork U.
G = U + D. We note that neither D nor U need be connected, even if G
is connected. By a connected network we mean one such that if all directed
arcs are replaced by undirected arcs, the resulting network is connected as
an undirected network. A connected component of a network is a maximal connected subnetwork.

Main Result.

Lemma 1 Let \( G \) be a connected Euler network. Then if \( u \) and \( v \) are any two nodes of \( G \), there is an Euler circuit containing them both.

Proof. Let \( G = D + U \) as above. Since both \( D \) and \( U \) are Euler, each of their connected components must be Euler also. Then by [2] 3.1.1 and 3.1.3, each of these components can be considered to be a single circuit. Thus since \( G \) is connected, each of the components shares some node with some other component. Hence all of \( G \) can be considered to be a single circuit. In particular, it is clearly an Euler circuit containing \( u \) and \( v \). We note that there may certainly be Euler circuits containing \( u \) and \( v \) which do not exhaust \( G \) also.

Definition. Let \( P_1, \ldots, P_k \) be an \( a \leftrightarrow b \) flow in \( G \). Let \( C_1, \ldots, C_n \) be a family of Euler circuits of \( G - (P_1 + \ldots + P_k) \), where subtraction means just that the arcs of \( P_1 + \ldots + P_k \) are deleted from \( G \). Assume that no two of the \( C_j \) have any arcs in common. We do not exclude the degenerate case of any of the \( C_j \) being simply a single node. Then \( C_1, \ldots, C_n \) is called a system of alternating circuits with respect to the flow \( P_1, \ldots, P_k \) if:

1. \( C_1 \) contains \( a \) and some node \( v_1 \) in one of the \( P_i \), call it \( R_1 \).
(2) For \( j > 1 \), \( C_j \) contains a node \( u_{j-1} \) on the path \( R_{j-1} \) which is at least as close to \( a \) on \( R_{j-1} \) as \( v_{j-1} \) is.

(3) \( C_j \) contains a node \( v_j \) on one of the \( P_i \), say \( R_j \). \( R_j \) need not be distinct from previous \( R_i \)'s.

See Figure 1. (In Figure 1 we do not indicate which are the directed arcs, nor which direction each path has.)
Lemma 2  Let $P_1, \ldots, P_k$ be an $a \leftrightarrow b$ flow containing $a \to b$ paths and $k- b \to a$ paths. Let $C_1, \ldots, C_n$ be a system of alternating circuits associated with $P_1, \ldots, P_k$, and assume $v_n = b$. Then $P_1 + \ldots + P_k + C_1 + \ldots + C_n$ forms an $a \leftrightarrow b$ flow of $k + 2$ consisting of $i + 1 a \to b$ and $k - i + 1 b \to a$ paths.

Proof  We use induction on $n$. If $n = 1$, then we are done because $C_1$ contains $a$ and $b$ and has no arcs in common with any of the $P_i$. So assume that the Lemma is true for $n < r$, and let $n = r > 1$.

We may assume that among those $U_i$ having $R_i = R_1$, none lie closer to $a$ on this path $R_1$ than $v_1$ does. Otherwise, $C_1, C_{i+1}, C_{i+2}, \ldots, C_n$ is a system of alternating circuits, and we are done by induction. Thus we have the situation of Figure 2, where for any $i$ with $R_i = R_1$, $u_i$ must lie in $E$.

![Figure 2](image_url)
Let $A$ be the part of $C_1$ going from $a$ to $v_1$ if $G + F + E (= R_1)$ is an $a - b$ path. If $G + F + E$ is a $b - a$ path, then choose $A$ to be the part of $C_1$ which goes from $v_1$ to $a$. Then in either case, $A + E$ is a path going the same direction as $R_1$, while $B + F + D + C + G$ is a circuit. Then replacing $R_1$ by $A + E$, and $C_1$ and $C_2$ by $B + F + D + C + G$, we have a new flow with the same number of paths in each direction as we started with, but with a system of circuits with $n-1$ members. This system we claim is alternating with respect to the new flow. For (1) is satisfied by $B + F + D + C + G$ (with $R_2$ replacing $R_1$), and (2) and (3) are satisfied since all the $u_i$ and $v_i$, $i > 1$ are left unchanged. Also, $v_n$ still is $b$, so the hypotheses of the lemma are satisfied. Hence we can apply induction and conclude that this new flow and system of circuits form a flow of $\kappa+1$ $a \rightarrow b$ paths and $\kappa-\lambda+1$ $b \rightarrow a$ paths. But since the new paths and circuits use exactly the same arcs as the original ones, this $k + 2$ $a \leftrightarrow b$ flow is precisely the desired one. Q.E.D.

Now there may be many different systems of alternating circuits associated with a given set of paths. Consider those systems consisting only of Euler circuits. Let $v$ be any node on, say, $P_i$. If for some alternating system of Euler circuits $C_1, \ldots, C_n$ we have one of the $v_j$ on $P_i$ and at least as far from $a$ on $P_i$ as $v$ is, then we call the node $v$ accessible (with respect to the given paths $P_1, \ldots, P_k$). (See Figure 3.)
Lemma 3 Let $G$ be an Euler network, and $P_1, \ldots, P_k$ an Euler $a \leftrightarrow b$ flow. Suppose $x$ is an accessible node of $P_1$, and $y$ is a node of $P_i (1 \leq i \leq k)$ such that in $G - (P_1 + \ldots + P_k)$ there is an Euler circuit containing $x$ and $y$. Then $y$ is accessible.

Proof Since $x$ is accessible, we can let $C_1, \ldots, C_n$ be an alternating family of Euler circuits with $v_n$ on $P_1$ and $x$ on $P_1$ no farther from $a$ than $v_n$. Let $C$ be an Euler circuit containing $x$ and $y$. (See Figure 4.)
Then if \( C \) is in a different component of \( G - (P_1 + \ldots + P_k) \) from all of the \( C_i \), we have \( C_1, \ldots, C_n, C \) forming a system of alternating Euler circuits, and \( y \) is accessible. If \( C \) is in the same component \( K \) of \( G - (P_1 + \ldots + P_k) \) as any of the \( C_i \), let \( i_0 \) be the minimal \( i \) such that \( C_i \) is in the component. As \( G \) is Euler, and \( P_1 + \ldots + P_k \) is Euler, \( G - (P_1 + \ldots + P_k) \) is Euler, and hence \( K \) is Euler. By Lemma 1, there is an Euler circuit containing \( U_{i_0-1} \) and \( y \), say \( C_0 \). Then \( C_1, \ldots, C_{i_0-1}, C_0 \) is a system of alternating circuits, and \( y \) is accessible. Q.E.D.

**Lemma 4** Let \( m \) be the minimum size of an \( a \leftrightarrow b \) cut-set in an Euler network \( G \). Let \( P_1, \ldots, P_k \) be an Euler collection of paths forming an \( a \leftrightarrow b \) flow. Then \( b \) is accessible if \( k < m \).

**Proof** Assume \( b \) is not accessible. By beginning at \( a \) and moving along a path \( P_i \) to \( b \), (this may be in the same direction or the opposite direction from the direction of \( P_i \)), we must reach a last node on \( P_i \) which is accessible, and all nodes beyond this must not be accessible. Let \( e_i \) be the arc of \( P_i \) connecting the last accessible node of \( P_i \) with the first inaccessible one, \( i = 1, 2, \ldots, k \). Then since \( k < m \), \( e_1, \ldots, e_k \)
does not form an $a \leftrightarrow b$ cut-set. Hence in $G - (e_1 + \ldots + e_k)$ there is an $a \rightarrow b$ path or a $b \rightarrow a$ path $P$. Traveling from $a$ to $b$ along this path, there is a first inaccessible node $u$. Then there is a last accessible node before $u$, say $v$. Let $R$ be the part of the path $P$ between $v$ and $u$. Then $R$ lies in $G - (P_1 + \ldots + P_k)$. For suppose it didn't. Then some arc $e$ of $R$ would lie on one of the $P_i$, and thus connect either two accessible nodes, two inaccessible nodes, or one of each. But it can't be one of each, since then it would have to be $e_1$, and $R \subseteq G - (e_1 + \ldots + e_k)$. It can't be two accessible nodes, by definition of $R$. Similarly, it can't be two inaccessible nodes by definition of $R$. This is a contradiction. So $R \subseteq G - (P_1 + \ldots + P_k)$. But since $G$ is Euler, and $P_1 + \ldots + P_k$ is Euler, $G - (P_1 + \ldots + P_k)$ is Euler, and so is the component of $G - (P_1 + \ldots + P_k)$ containing $R$. Thus by Lemma 1, $u$ and $v$ lie on an Euler circuit of $G - (P_1 + \ldots + P_k)$. Now Lemma 3 implies that $u$ is accessible. This is a contradiction, so $b$ is accessible. Q.E.D.

Theorem 1. If $G$ is an Euler connected network, and if $m$ is the minimal size of an $a \leftrightarrow b$ cut-set, then $m$ is even, and there is an $a \leftrightarrow b$ flow of $m$ in $G$ such that its constituent paths are an Euler collection of paths.

Proof. Lemma 1 implies that there are at least two paths, one from $a$ to $b$, and one from $b$ to $a$ which form an Euler collection. Starting with these two, we may apply Lemma 4 if $2 < m$, and then Lemma 2 to obtain 4 paths forming an Euler collection. Continuing in this way, we may keep adding 2 paths at a time until we have an Euler collection of $k$ paths, and $k = m$. $m$ must be even since we began with $2$. 

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**Feasibility**

We say that a flow of \((s,t)\) is feasible if there is a flow with \(s - b\) paths and \(t - a\) paths. Certainly if \((s,t)\) is feasible, then \((x,y)\) is feasible for \(x \leq s, y \leq t\). So the interesting question is: What \((s,t)\) are feasible for \(s + t = m\)? We can answer this for the case where the \(m\) paths are an Euler flow.

The proof of Theorem 1 guarantees that a flow of \((m/2, m/2)\) is feasible, since we can start with \((1,1)\) and add two at a time, obtaining successively \((2,2), (3,3), \ldots\). But, of course, if somehow we obtained a flow of \((x,y)\) where the \(x + y\) paths were an Euler collection, then just as in the proof of Theorem 1, repeated applications of Lemmas 2 and 4 imply that \((x + 1, y + 1), (x + 2, y + 2), \ldots\) are feasible. Thus we would ultimately reach a flow of \((s,t)\) with \(s + t = m\) and \(s - t = x - y\). As \(m\) is even, \(s - t\) must also be even.

Now suppose we could obtain a flow of \(2k\) consisting entirely of undirected paths. Then this could be considered either as a \((2k, 0)\) or a \((0, 2k)\) flow or anything in between. It is clearly an Euler collection of paths. Hence we could obtain a flow of \((s,t)\) with \(s + t = m, |s - t| = 2k\). What we show now is that the converse of this is also true.

**Theorem 2** Suppose there is an Euler collection of paths forming an \((x,y)\) flow. Then we have seen that \(|x - y|\) must be even, so let it be \(2k\). Let \(P\) be the subnetwork consisting of these paths. Then there is a flow of \(2k\) in \(P\) between \(a\) and \(b\) consisting entirely of undirected paths.

**Proof.** We define a critical \(a - b\) cut-set in \(P\), or just critical cut-set, to be a set of arcs such that their removal leaves \(a\) and \(b\) in
different components, and such that no subset of these arcs has this property. Clearly, every critical cut-set separates the network into exactly two components, and each arc in it has one end in each component. Let \( E \) be a critical cut-set in \( P \), and let the two components of \( P - E \) be \( C \) and \( D \), where \( a \in C \) and \( b \in D \). Now consider \( C + E \). Each node of \( C \) here has as many incoming directed arcs as outgoing directed arcs, by the Euler condition on \( P \). Summing over all nodes in \( C \), the total number of incoming arcs is equal to the total number of outgoing arcs. Each directed arc in \( C \) contributes one to each of these sums. Each directed arc in \( E \), however, contributes to only one of the sums. Thus, since the sums are equal, there must be exactly as many arcs in \( E \) directed from \( C \) to \( D \) as there are from \( D \) to \( C \). (Of course, there may also be other arcs which are undirected.)

Let \( P_1, \ldots, P_x \) be the \( a \rightarrow b \) paths, and \( P_{x+1}, \ldots, P_{x+y} \) the \( b \rightarrow a \) paths constituting \( P \). Traveling along \( P_i \) (in the \( a \rightarrow b \) direction for \( i \leq x \), and in the \( b \rightarrow a \) direction for \( i > x \)) we use some of the arcs of \( E \) to get from \( C \) to \( D \), and some to get from \( D \) to \( C \). Let \( f_i \) be the number of directed arcs of \( E \) used by \( P_i \) to get from \( C \) to \( D \), and let \( u_i \) be the number of undirected arcs of \( E \) used by \( P_i \) to get from \( C \) to \( D \). Similarly, let \( b_i \) be the number of directed arcs of \( E \) used by \( P_i \) to get from \( D \) to \( C \), and \( v_i \) the number of undirected arcs of \( E \) used by \( P_i \) to get from \( D \) to \( C \). Then we have:

\[
\begin{align*}
(1) \quad f_i + u_i &= b_i + v_i + 1 \\
(2) \quad f_i + u_i + 1 &= b_i + v_i
\end{align*}
\]

This is because the \( a \rightarrow b \) paths cross from \( C \) to \( D \) exactly one more time than from \( D \) to \( C \), and similarly for \( b \rightarrow a \) paths.

Summing (1) from \( i = 1 \) to \( x \) and (2) from \( i = x + 1 \) to \( x + y \),
and adding these results, we get:

\[
\sum_{i=1}^{x+y} f_i + \sum_{i=1}^{x+y} u_i + y = \sum_{i=1}^{x+y} b_i + \sum_{i=1}^{x+y} v_i + x
\]

Now since all the arcs of \( E \) are used by the \( x + y \) paths by assumption, we know that \( \sum_{i=1}^{x+y} f_i = \sum_{i=1}^{x+y} b_i \) by what we observed above. Thus

\[(4) \quad |y - x| = |\sum u_i - \sum v_i| \leq \sum u_i + v_i\]

The expression \( \sum u_i + v_i \) is just the total number of undirected arcs in \( E \). Thus we have shown that for any critical \( a - b \) cut-set \( E \), there are at least \( |x - y| = 2k \) undirected arcs in it.

Now we claim that there is a flow of \( 2k \) undirected paths between \( a \) and \( b \) in \( P \). For let \( P = D + U \), the directed and undirected parts. Then if there is no flow of \( 2k \) in \( U \) alone between \( a \) and \( b \), by the Max-flow Min-cut Theorem [1], there is a minimal \( a - b \) cut-set \( E_u \) in \( U \) with fewer than \( 2k \) arcs. Now adjoin one at a time (in any order) the arcs of \( D \) until adjoining any more would result in \( a \) and \( b \) no longer being in different components. Let \( E_D \) be the remaining arcs of \( D \). Then clearly \( E = E_u + E_D \) is a critical cut-set for \( P \). But \( E \) has only as many undirected arcs as \( E_u \) and this is fewer than \( 2k \), contradicting (4). Thus there must be a flow of \( 2k \) undirected paths between \( a \) and \( b \) in \( P \). Hence the Theorem is proved.

We note that any collection of \( 2k \) undirected paths between \( a \) and \( b \) is an Euler collection. Thus we have a procedure or algorithm for finding all feasible Euler maximal flows. Namely, for \( G = D + U \) as usual, find a maximal flow in \( U \) between \( a \) and \( b \). Then if this is a \( 2k \) flow, assign the \( 2k \) paths either \( a \rightarrow b \) or \( b \rightarrow a \) directions. Then apply the method of Theorem 1 (i.e., repeated application of Lemma 2)
to obtain the maximal two-way flow.

Now none of the original undirected paths may be intact when the procedure terminates, but the initial difference between the numbers of \( a \rightarrow b \) paths and \( b \rightarrow a \) paths among them is preserved. Figure 5 is an example of an Euler network where the maximal undirected flow is 2, the maximal two-way flow is 6, and no maximal two-way flow contains any undirected path.

![FIGURE 5](image)

Remarks

Now although we know how to find all Euler flows in an Euler network, we know nothing about non-Euler flows in an Euler network. Figure 6 is an example of an Euler network with no undirected paths between \( a \) and \( b \), and a maximal double flow of 6. Thus the only Euler two-way flow is \((3,3)\). But also a \((4,2)\) flow is feasible (not an Euler one, of course).

![FIGURE 6](image)
The Max-flow Min-cut Theorem [1] holds for any undirected network. It does not need to be Euler. By Theorem 1 above, for Euler directed networks, the maximum two-way flow equals the minimum two-way cut-set. We might guess that if \( G = D + U \), as usual, and \( D \) is an Euler directed subnetwork, while \( U \) is not necessarily Euler, then the maximum two-way flow equals the minimum two-way cut-set. Figure 7 gives a counter-example for this, where the maximum flow is one, but the minimum cut-set is two.

![Figure 7](image)

Finally, we observe that we can generalize the above results slightly as follows. Consider a network \( G \) with \( A \) and \( B \) two disjoint sets of nodes of \( G \). Call \( G \) almost Euler if for all nodes except those of \( A \) and \( B \) we have the number of undirected arcs there even, and the number of incoming directed arcs equal to the number of outgoing directed arcs there. Similarly, a flow is almost Euler if the arcs used by it determine an almost Euler subnetwork. By using the standard method of introducing two super-nodes \( a \) and \( b \), as indicated in Figure 8,
we can obtain from Theorems 1 and 2 their analogues, replacing \( a \leftrightarrow b \) flows and cut-sets by \( A \leftrightarrow B \) flows and cut-sets, and the Euler conditions by almost Euler conditions. (The conditions of evenness are not true for these analogues.)
References


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