A CORRELATION OF THE BASE DRAG OF BODIES-OF-REVOLUTION WITH A JET EXHAUSTING THROUGH THE BASE

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INTRODUCTION

To be effective, missile system development must be based on performance analysis; and effective performance analysis, in turn, must be based on accurate estimates of the inertial, propulsive, and aerodynamic forces influencing the missile trajectory. Of the aerodynamic forces, one of the more difficult to estimate with the desired degree of accuracy is the power-on base drag force. During low acceleration or cruise phases, as much as 50 to 70 percent of the total drag may be due to base drag, however no technique has been developed for estimating power-on base drag over a range of parameters necessary for missile design or development studies. An improved estimation technique is essential to effective performance analysis -- a technique which will simplify allowances for base drag influences in consideration of the complex relationship among drag, stability, and propulsive factors in missile configuration selection.

Base drag can be defined as the pressure drag induced by flow separation from rearward facing steps such as body bases, protuberances, and blunt wing and fin-tailing edges. It is influenced by the geometry of the steps and by each of the properties of the flow approaching the steps. Addition of a rocket exhaust to a step complicates the phenomenon by effectively adding a second stream with its associated step geometry and approaching flow properties. The separation of the two streams at the step, with the resulting flow mixing and interaction, produces a very complex fluid mechanics problem. The lack of a complete understanding of this complex flow phenomenon leads to difficulty in estimating power-on base drag.

Many experimental investigations have been made of power-on base drag. However, the results do not form a systematic matrix of information sufficient to allow interpolative estimates of base drag, and estimates based on experimental data must rely on interpretive
techniques. In this study, an empirical estimation technique has been formulated on the basis of a successful correlation of experimental data.

**CORRELATION OF DATA**

Base pressure data exhibit a common three-phase variation with thrust which is illustrated in Fig. 1. When thrust increases from the power-off condition, base pressure increases slightly to a maximum value, drops off to the minimum value, and then increases at a steadily decreasing rate. This investigation is concerned with thrust levels above that at which minimum base pressure occurs. Correlation of the experimental data has been accomplished on the basis of a relationship between the ratio of base pressure to free-stream static pressure \( \left( \frac{P_b}{P_\infty} \right) \) and the ratio of the momentum flux of the jet to the momentum flux of the equivalent body stream tube \( \left( R_{mf} \right) \) where

\[
R_{mf} = \frac{(\mathbf{mV})_j}{(\mathbf{mV})_\infty} = \frac{\gamma_j P_j A_j M_j^2}{\gamma_\infty P_\infty A_\infty M_\infty^2}
\]

For this analysis, momentum flux value is calculated using one-dimensional flow theory. It is felt that momentum flux values based on flow surveys would yield an improved correlation.

**JET PARAMETERS:** Nozzle Diameter. The first parametric influence to be considered is the influence of nozzle diameter. Experimental data illustrating the influence of nozzle diameter on base pressure (1) are presented in Fig. 2a as a function of jet pressure ratio. These data, which were obtained for a family of geometrically similar nozzles, appear to indicate an influence of nozzle diameter. However, presenting the same data as a function of momentum flux ratio (Fig. 2b) shows that base pressure is independent of nozzle diameter, and is some nonlinear function of the momentum flux ratio. It is interesting to note that jet momentum flux \( (\mathbf{mV})_j \) is the predominant term in a thrust equation, and the free-stream momentum flux \( (\mathbf{mV})_\infty \) is equal to \( 2q_0A \). Thus the momentum flux ratio, in aerodynamics terminology, closely approximates a thrust coefficient \( C_T \).

Nozzle Exit Angle. The nozzle exit angle has been shown to exert a very small influence on base pressure (1,2). It is felt that this influence is due to a combination of vector and viscous factors. Nozzle exit angle accounts for only a small reduction in the axial component of jet momentum flux -- about three percent for a 20-degree exit angle. Nozzle exit angle also influences the nozzle boundary layer characteristics due to the variation in nozzle length and axial pressure gradient associated with variation in nozzle angle. For most practical considerations, the combined vector and viscous effects due to nozzle exit angle may be ignored.
Jet Mach Number. The next parametric influence to be considered is the influence of jet Mach number. Experimental base pressure data (2) for jet Mach numbers of 1.00, 1.78, and 2.70 are presented as a function of momentum flux ratio in Fig. 3a. Examination of these data shows a mutual proportionality which can be stated as follows:

\[
\left( \frac{p_b}{p_\infty} \right)_{M_j = 1.00} \propto \left( \frac{p_b}{p_\infty} \right)_{M_j = 1.78} \propto \left( \frac{p_b}{p_\infty} \right)_{M_j = 2.70}
\]

Efforts to define the proportionality factor in terms of meaningful flow parameters have not been completely satisfactory. Part of the difficulty might be due to the large variation in nozzle boundary layer conditions inherent in jet Mach number variation. A factor which tends to describe the proportionality trend is the inverse of the nondimensional velocity \( \mathcal{M}^* \) where

\[
\left( \frac{p_b}{p_\infty} \right)_{M_j > 1.00} = \frac{\mathcal{M}^*_j}{\mathcal{M}^*_j > 1.00} = \frac{1}{(\mathcal{M}^*_j > 1.00)}
\]

For this analysis the \( \mathcal{M}^* \) expression is of more significance in correlating experimental data than in practical application to base pressure estimates, since, for propulsive efficiency, missile rocket motors will tend to operate at conditions where the rate of change of \( \mathcal{M}^* \) with jet Mach number is very small.

Nozzle Position. Nozzle position relative to the body base has a significant influence on base pressure which is illustrated by the experimental data presented in Fig. 4a(2). Examination of these data reveals an incremental influence which is a function of position and independent of thrust level. Other experimental data indicate that the incremental influence of position varies with freestream Mach number. A summary of position and Mach number effects is shown in Fig. 4b. This influence can be approximated by the expression

\[
\Delta \left( \frac{p_b}{p_\infty} \right)_{x_j} = 0.047 \left( 5 - M_\infty \right) \left( 2x_j + x_j^2 \right)
\]

For \( M_\infty = 1.2 \) to 4.5

\( x_j = -0.2 \) to +0.6
The remaining nozzle parameters to be considered are jet stagnation temperature \( T_0 \) and ratio of specific heats \( \gamma_j \). Experimental data indicate that the effects of these parameters are small; however, the data are not sufficient for formulation of an expression describing the influences. Theoretical investigations, based on the base flow model formulated by Korst (3), also indicate that the influence of these two parameters is relatively small. Typical theoretical predictions of the influence of jet stagnation temperature are presented in Fig. 5, and of ratio of specific heats in Fig. 6.

**EXTERNAL PARAMETERS: Free-Stream Mach Number.** No further consideration will be given to the influence of free-stream Mach number, since it is felt that this effect was adequately accounted for in the momentum flux term. Useful experimental data are available only for Mach numbers of 1.0 and above. However, these data in conjunction with trends observed in a very few subsonic data points lead to the conclusion that no discontinuities exist in the transonic region. Therefore the technique is assumed to be continuous and applicable to a full Mach number spectrum.

**Body Geometry.** Only one other external stream parameter seems to exert a noticeable influence on power-on base drag. That parameter is body geometry. Experimental data illustrating the influence of boattail geometry are presented in Fig. 7a. (2) Although base diameter, boattail length, and boattail angle are varied in these data, the primary influence appears to be that of base diameter. This trend is apparent in other experimental data for both boattail and flare configurations. Analysis of the data shows that for a given base diameter the base pressure maintains a constant ratio to the base pressure of a cylindrical body. Values for this ratio, based on analysis of a large amount of experimental data (2, 4, 5, 6), are presented as a function of the ratio of base area to body area in Fig. 7b. Although it is felt that a discontinuity exists between value for boattail configurations and flare configurations, a linear approximation allows convenient formulation of the following expression:

\[
\left( \frac{p_b}{p_\infty} \right)_{\text{non-cyl}} = \left( \frac{p_b}{p_\infty} \right)_{\text{cyl}} \left( \frac{3.5}{1 + 2.5 \frac{A_b}{A_B}} \right)
\]

**Body Boundary Layer.** No further parametric influences are apparent in the experimental data, which leads to a question regarding the considerable influence generally attributed to the body boundary layer. All the experiments considered were conducted with a turbulent body boundary layer and thus with similar boundary layer characteristics. Therefore consideration of the major influences of body boundary layer is inherent in formulation of the empirical
expressions. Small differences in boundary layer characteristics are probably responsible for a small amount of scatter when comparing the various experimental data.

**EMPIRICAL TECHNIQUE**

Formulation of an expression to describe the influence of momentum flux ratio $R_{mf}$ may now be accomplished by using the formulated expressions to normalize experimental data in the following manner:

$$F = \begin{bmatrix} \frac{P_b}{P_\infty} - 0.047 \left(5 - M_\infty\right) \left(2x_j + x_j^2\right) \end{bmatrix} \left(\frac{1 + 2.5 A_b/A_B}{3.5}\right) = f(R_{mf})$$

Analysis of a large amount of experimental data in normalized form yields a variation of $F$ with $R_{mf}$ which is approximated in Fig. 8 and can be described by the following expression:

$$F = 0.19 + 1.28 \left(\frac{R_{mf}}{1 + R_{mf}}\right)$$

It should be noted that this expression describes a continuous curve through minimum base pressure to a hypothetical intercept of the base pressure axis at zero thrust. Neither the minimum value of base pressure nor the thrust at which it occurs are predicted. With the influence of momentum flux ratio described, an empirical equation relating power-on base drag to all of the influencing parameters, except $T_0$ and $\gamma_j$, may now be formulated by combining the expressions to yield:

$$\frac{P_b}{P_\infty} = \left(\frac{1}{M_\infty}\right) \left(\frac{3.5}{1 + 2.5 A_b/A_B}\right) \left(0.19 + 1.28 \left[\frac{R_{mf}}{1 + R_{mf}}\right]\right)$$

$$+ \left(0.047 \left[5 - M_\infty\right] \left[2x_j + x_j^2\right]\right)$$

The base drag coefficient is defined as:

$$C_{D_b} = \frac{1}{\gamma_\infty M_\infty^2} \left(1 - \frac{P_b}{P_\infty}\right) \left(\frac{A_b}{A_B}\right)$$

This empirical equation yields results which compare very favorably with those obtained from an extensive amount and range of experimental data.
CONCLUSIONS

In spite of the many parameters involved in the complex fluid mechanics phenomenon associated with power-on base drag, a small number of expressions can be formulated through systematic correlation of experimental results to accurately describe the combined effects of all parameters. The expressions formulated lead to a better understanding of parameters which must be considered for selection of optimum missile configuration. Combining the expressions yields an empirical equation for estimating base drag which requires only a slide-rule or a desk calculator for solutions, and which is amenable to direct inclusion in computerized missile trajectory analysis. It is felt that this technique will lead to better analytical definition of missile afterbody-propulsion configuration, thus limiting experimental investigations to verification rather than developmental studies. Future experimental investigation to be conducted for the Army Missile Command (Advanced Systems Laboratory, R&DD) are planned to yield expressions describing the influence of jet stagnation temperature and ratio of specific heats, and to verify the applicability of the technique at subsonic Mach numbers. Future investigations will also be directed toward methods of estimating the minimum value of base pressure and the base pressure at very low thrust levels.

SYMBOLS

\( a^* \) Speed of sound where local Mach number is 1.0
\( A \) Area
\( C_D_B \) Base drag coefficient
\( C_J \) Jet thrust coefficient, \( T/q_A_B \)
\( D \) Diameter
\( F \) Normalized base pressure data
\( m \) Mass flow rate, \( \rho AV \)
\( M \) Mach number
\( M^* \) Dimensionless velocity, \( V/a^* \)
\( P \) Pressure
\( q \) Dynamic pressure, \( \gamma p A M^2/2 \)
\( P_{mf} \) Momentum flux ratio, \( (mV)_j/(mV)_\infty \)
\( T \) Absolute temperature, Rankine

\( V \) Velocity
\( X_j \) Axial distance from plane of body base to plane of nozzle exit (positive aft)
\( x_j \) Dimensionless distance, \( X_j/D_B \)
\( \gamma \) Ratio of specific heats

Subscripts

\( ^\infty \) Free-stream static conditions
\( o \) Free-stream stagnation conditions
\( j \) Jet static conditions at plane of nozzle exit
\( c \) Jet stagnation co.
\( b \) Body base
\( B \) Body maximum
REFERENCES


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Figure 2
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Figure 8