SENSITIVITY ANALYSIS OF CIVIL DEFENSE SYSTEMS AND COMPONENTS

FINAL REPORT VOLUME III

R-OU-157

A GENERALIZED SENSITIVITY ANALYSIS OF CD SYSTEMS

Prepared for
Office of Civil Defense
United States Department of the Army
under
Office of Civil Defense Contract No. OCD-PS-64-56
OCD Subtask 4113E
RTI Project OU-157

Best Available Copy
A Generalized Sensitivity Analysis of CD Systems

by

John H. Neblett
and
Kenneth E. Willis

October 1, 1965

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Approved by:

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R. S. Titchen, Deputy Director

1 October 1965
FOREWORD

This is Volume III of three separately bound volumes in which are reported the research completed under the general terms of the Office of Civil Defense Subtask Number 4113E, "Sensitivity Analysis of Civil Defense Systems and Components."

The research of the authors was very ably supported by Mr. Herbert Hill, and Miss Mary B. Woodside. Mr. Hill assisted in the development of the models and Miss Woodside performed the calculations reported in several of the Appendixes.
This sensitivity analysis employs mathematical models which estimate the total dose, Maximum ERD, and probability of casualty or fatality for an individual exposed to a particular radiological environment. The objective of the analysis is to determine the relative importance of the parameters normally employed in evaluating Civil Defense systems; i.e., to determine those parameters which, through large variance or inaccurate estimates, will contribute most to erroneous evaluations of CD systems. The sensitivity of the dose (or probability of casualty) to variations in the input parameters defining the radiological environment is examined. The total dose model is analytical and the ERD model is computerized. The parameters examined are reference intensity, time of arrival, time outside in fallout, radiation decay exponent, ERD recovery fraction, ERD recovery rate, duration of fallout buildup, and protection factor. Sensitivity indices are calculated for each parameter. The sensitivity index is defined as the fractional change in dose (or probability of casualty or fatality) divided by the corresponding fractional change in the input variable. It is concluded that dose and casualty computations are quite sensitive to errors in the field decay exponent, and they remain sensitive over the examined range. Sensitivity to variations in fallout reference intensity and protection factor are high over the whole range of parameter values. Sensitivity of time of arrival of fallout can be quite high for the lower values of the parameter. Sensitivity of dose and casualty computations to the remaining parameters is low in most cases of interest. Expansion of the sensitivity analysis to include parameters other than fallout, which define the total casualties from a given attack on the United States, is necessary before further conclusions concerning a national vulnerability analysis can be drawn.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOREWORD</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>I. INTRODUCTION:</td>
<td>1</td>
</tr>
<tr>
<td>A. Purpose</td>
<td>1</td>
</tr>
<tr>
<td>B. Procedure</td>
<td>1</td>
</tr>
<tr>
<td>II. THE SENSITIVITY ANALYSIS</td>
<td>5</td>
</tr>
<tr>
<td>A. Selection of Parameters and Values</td>
<td>5</td>
</tr>
<tr>
<td>B. Model Basic Equations</td>
<td>6</td>
</tr>
<tr>
<td>C. Derivation of the Sensitivity Equation</td>
<td>7</td>
</tr>
<tr>
<td>D. Calculation of ERD and Probability of Fatality</td>
<td>8</td>
</tr>
<tr>
<td>E. Grouping of Cases and Calculations of Mean Values</td>
<td>9</td>
</tr>
<tr>
<td>F. Determination of the Principal Model Approximation</td>
<td>11</td>
</tr>
<tr>
<td>G. Calculation of the Sensitivity Indices</td>
<td>11</td>
</tr>
<tr>
<td>H. Total Dose Model</td>
<td>15</td>
</tr>
<tr>
<td>I. Validity of Ranking the Sensitivity Indices</td>
<td>18</td>
</tr>
<tr>
<td>J. Analysis of Time Required in Shelter</td>
<td>26</td>
</tr>
<tr>
<td>K. Evaluation of Significance of Sensitivity Indices in Shelter</td>
<td>28</td>
</tr>
<tr>
<td>Systems Analysis</td>
<td></td>
</tr>
<tr>
<td>III. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>32</td>
</tr>
<tr>
<td>A. Conclusions</td>
<td>32</td>
</tr>
<tr>
<td>B. Recommendations</td>
<td>34</td>
</tr>
<tr>
<td>REFERENCES TO VOLUME III</td>
<td>36</td>
</tr>
<tr>
<td>Appendix A - The Intensity Buildup of a Radioactive Fallout Field</td>
<td>A-1</td>
</tr>
<tr>
<td>Appendix B - The Mainline Model and Computer Program</td>
<td>B-1</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Values and Symbols for the Input Parameters Used in the Sensitivity</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Basic and Subgroup Divisions of the Mainline Program Cases</td>
<td>10</td>
</tr>
<tr>
<td>III</td>
<td>Mean Values of the Basic Parameters</td>
<td>12</td>
</tr>
<tr>
<td>IV</td>
<td>Principal Sensitivity Indices</td>
<td>15</td>
</tr>
<tr>
<td>V</td>
<td>Comparison of Sensitivity Indices</td>
<td>18</td>
</tr>
<tr>
<td>VI</td>
<td>Sensitivity Analysis of Time Required in Shelter</td>
<td>28</td>
</tr>
<tr>
<td>VII</td>
<td>Comparison of Sensitivity Indices and Uncertainty in Input Parameters</td>
<td>32</td>
</tr>
<tr>
<td>B-I</td>
<td>Program Symbols</td>
<td>B-11</td>
</tr>
<tr>
<td>B-II</td>
<td>Major Program Routines</td>
<td>B-12</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Research Steps in Sensitivity Analysis</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Sensitivity Index of PF as a Function of PF</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>Sensitivity Index ( S_{TA} ) as a Function of Time of Arrival</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>Sensitivity Index ( S_{TE} ) as a Function of Buildup Factor ( E )</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>Sensitivity Index ( S_{T_2} ) as a Function of Time in Fallout</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>Sensitivity Index ( S_{T_2} ) as a Function of Decay Constant ( Z )</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>Sensitivity Index ( S_{T_2} ) for Time in Fallout ( S_{T_2} ) as a Function of Decay Constant</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>Sensitivity Index ( S_{T_2} ) as a Function of ( T_0 )</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>Frequency of Occurrence of Stay Times vs. Days for Return of ERD to 80r ( N_2 )</td>
<td>29</td>
</tr>
<tr>
<td>A-1</td>
<td>Fallout Mass Buildup Functions</td>
<td>A-2</td>
</tr>
<tr>
<td>B-1</td>
<td>Decay of Radioactive Field</td>
<td>B-2</td>
</tr>
<tr>
<td>B-2</td>
<td>Particle Arrival Rate</td>
<td>B-3</td>
</tr>
<tr>
<td>B-3</td>
<td>Accumulation of Particles</td>
<td>B-3</td>
</tr>
<tr>
<td>B-4</td>
<td>Field Intensity vs. Time</td>
<td>B-4</td>
</tr>
<tr>
<td>B-5</td>
<td>Attenuation of Field Intensity</td>
<td>B-6</td>
</tr>
<tr>
<td>B-6</td>
<td>Dose Response</td>
<td>B-10</td>
</tr>
</tbody>
</table>
A Generalized Sensitivity Analysis of CD Systems

I. INTRODUCTION

A. Purpose

An analysis of the effectiveness of the civil defense shelter system must take into account many variables. These variables include some that determine the fallout environment (fallout reference intensity, its time of arrival, its rate of build-up, and the radioactivity decay rate); some that determine the protection against radioactivity (the gamma ray shielding characteristics of available shelter, and the time required for travel through fallout before shelter is reached); and some that influence physiological condition (recovery rate, and the fraction of radiation damage that can be repaired). The research reported in this volume is directed at evaluating the importance of uncertainty or inaccuracy in each of these variables in estimating effectiveness of a shelter system in a fallout environment. Effectiveness is measured in terms of Equivalent Residual Dose (ERD), probability of fatality, and time required in primary shelter for the ERD to return to a specified level.

The results from such an analysis can be applied in two ways: (1) determination of research priorities and requirements for increasing the validity of shelter systems analysis, and (2) improvement of the models or formulations that are used for systems analysis.

B. Procedure

1. General

The procedure followed in this volume is known as sensitivity analysis, which is a systematic determination of the effects of small changes in input variables. Such an analysis requires the utilization of an equation or a mathematical model that relates the input and output variables—such as reference intensity and estimates of casualties. Each input variable is varied, while all others are kept constant, so that the effects of these variations on the output are measured and compared.
2. **Specific**

Because of the number and nature of the variables that influence estimates of the effects of the radioactive fallout, the establishment of a model for sensitivity analysis is complex. The major steps that were followed in this analysis are given below and are covered in more detail in Section II, which follows. A generalized flow chart of the procedure is shown in Figure 1, Research Steps in Sensitivity Analysis.

**Step I. Selection of Parameters and Values.**

Several values for each of eight factors contributing to radiological effects were selected as input parameters. Combinations of these values were used later in equations (derived while performing Step II) to provide the cases upon which the analysis was based. (See Section II.A below.)

**Step II. Derivation of Basic Equations to Represent Fallout Effects.**

Equations were derived for determining ERD and corresponding probability of fatality ($P_f$) on the basis of the eight selected factors. (See Section II.B below.)

**Step III. Selection of Formulas for Determining Relative Sensitivity of Input Variables.**

The following relationship was established as the sensitivity index:

$$ S_i = \frac{\Delta Y}{Y} \frac{\Delta X_i}{X_i} $$

Where $S_i$ is the sensitivity index; $Y$ is the measure of effectiveness for a selected set of cases; $\Delta Y$ is an arbitrary value change in $Y$; $X_i$ is the value of any single input parameter in the same set of cases; and $\Delta X_i$ is the change in this input necessary to cause $\Delta Y$. (See Section II.C below.)

Although it would have been desirable to relate the equations in Step II directly to the sensitivity ratio in Step III, it was not possible to do this readily without making over-simplifying assumptions. Consequently, intermediate steps were required.

**Step IV. Compilation of Cases for Analysis.**

Several levels of each input variable were selected and the Maximum ERD and probability of fatality were computed for each combination of input.
I. Selection of Parameters and Values

II. Derivation of Basic Equations

III. Selection of Sensitivity Formulas

IV. Compilation of Cases of Interest (10,143)

A. Inline Computer Program

V. Grouping of Cases and Calculation of Mean Values

VI. Determination of Model Approximation Equation

TSAR Computer Program

VII. Determination of Principal Sensitivity Indices (Prob. of Fatality)

VIII. Verification of Sensitivity Indices

IX. Sensitivity Analysis of Indices (Shelter Stay Time)

X. Comparison with Expected Range of Input Values

Conclusions

Fig. 1. Research Steps in Sensitivity Analysis
variables. Using a CDC 1604 computer and the Mainline Program which was designed specifically for this purpose, approximately 43,000 cases were developed. After those cases were excluded in which the Maximum ERD was less than 100r or greater than 700r, a total of 10,143 cases were left for the sensitivity analysis. (See Section II.D below.)

**Step V. Grouping the Cases and Calculation of Mean Values.**

A tape containing the 10,143 cases was transferred to a general purpose data processing program written for the IBM 7072 [Reference 1] in order to group the selected cases into subsets for analysis. (See Section II.E below.)

**Step VI. Determination of Model Approximation Equation.**

Using the 10,143 selected cases and multiple regression, the same general purpose program [Reference 1] was utilized to determine a single equation (the principal model approximation) for the calculation of Maximum ERD as a function of the eight input variables. (See Section II.F below.)

**Step VII. Determination of Principal Sensitivity Indices.**

The effect of each input parameter on estimating fatalities was calculated using the sensitivity formula (Equation 1) stated in Step III, the mean values of the input variables determined in Step V, and the principal model approximation referred to in Step VI. (See Section II.G below.)

**Step VIII. Verification of the Principal Sensitivity Indices.**

The change in sensitivity indices was examined throughout a range of values using a Total Dose Model. (See Section II.H below.) These results were compared to those contained using the model approximation equation of Step VI. (See Section II.I below.)

**Step IX. Sensitivity Analysis of the Effect of Input Parameters on Shelter Stay Time.**

A subgroup of cases was used in an analysis of sensitivity in which the dependent variable was the shelter stay time required for the ERD to return to 80r. (See Section II.J below.)

**Step X. Evaluation of Significance of Individual Sensitivity Indices.**

The sensitivity indices of the various parameters were then evaluated in view of the accuracy with which these parameters could be expected to be established as inputs to any analysis of system effectiveness. (See Section II.K below.)
II. THE SENSITIVITY ANALYSIS

A. Selection of Parameters and Values

In order to obtain a representative sample of cases on which to base the analysis of sensitivity, a range of reasonable values was assigned to each of the input variables commonly used in shelter systems analysis. These values are summarized in Table I. It will be noted that in all cases except that for protection factor (PF), the values are evenly spaced.

**Table I**

Values and Symbols for the Input Parameters Used in the Sensitivity Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Intensity (r/hr)</td>
<td>$I_o$</td>
<td>300, 1500, 2700, 3900</td>
</tr>
<tr>
<td>Time of Arrival (hrs)</td>
<td>$T_A$</td>
<td>1, 4, 7</td>
</tr>
<tr>
<td>Buildup Duration Factor</td>
<td>$E$</td>
<td>1.13, 3.64, 6.15</td>
</tr>
<tr>
<td>Protection Factor</td>
<td>$PF$</td>
<td>2, 10, 40, 100, 500</td>
</tr>
<tr>
<td>Time in Fallout (hrs)</td>
<td>$T_2$</td>
<td>0.0, 0.3, 0.6</td>
</tr>
<tr>
<td>Field Decay Exponent</td>
<td>$Z$</td>
<td>-1.0, -1.2, -1.4</td>
</tr>
<tr>
<td>ERD Recovery Fraction</td>
<td>$F$</td>
<td>0.85, 0.90, 0.95</td>
</tr>
<tr>
<td>ERD Recovery Rate (Fraction/day)</td>
<td>$B$</td>
<td>0.020, 0.025, 0.030</td>
</tr>
</tbody>
</table>

Fallout hazard (Risk) data on 200 Standard Metropolitan Statistical Areas (SMSA's) were used as the basis for selection of values of reference intensity and time of arrival. These data were extracted from an OCD source. Times of arrival between one hour and seven hours occurred in about 50 percent of those cases; reference intensities were between 300 r/hr and 3900 r/hr for 94 percent of the cases. (See Volume II, Figs. 31 and 32.)

The buildup duration factor ($E$) is a factor by which time of arrival is multiplied in order to represent estimates of possible time between the beginning and cessation of fallout. Appendix A contains a discussion of buildup functions.

The protection factor (PF) values were based partially on National Fallout Shelter Survey data summarized in Reference [3]. Identified shelter spaces with PF's of 40 and above could accommodate roughly 20 percent of the United States population. PF's of 40, 100, and 500 were selected to represent the three levels of PF's between 40 and 99, between 100 and 250, and greater than 250, respectively. Factors of 10 and 2
were used as representatives of the PF of population in residential basements and in homes without basements, respectively.

Time in fallout, prior to shelter occupancy was set arbitrarily at 0.0, 0.3, and 0.6 hours.

In determining equivalent residual dose (ERD), values used for the radiation field decay exponent and for the ERD recovery fraction and ERD recovery rate were varied about the generally accepted values used in systems analysis.

B. Model Basic Equations

The following basic equations constitute the analytical formulation of the model. It was upon these equations that the Mainline Computer Program (see Appendix B) was developed.

1. Equivalent Residual Dose (ERD)

\[
\text{ERD} (n) = D_n + d_n
\]

\[
D_n = (1-B) D_{n-1} + F \int_{n-1}^{n} W(t) \, dt
\]

\[
d_n = d_{n-1} + (1-F) \int_{n-1}^{n} W(t) \, dt
\]

where:

- \( D_n \) = Recoverable dose remaining during time interval \( n-1 \) to \( n \) (roentgens).
- \( d_n \) = Nonrecoverable dose remaining during time interval \( n-1 \) to \( n \) (roentgens).
- \( B \) = Fractional rate of recovery for reparable damage (fraction/unit time).
- \( F \) = Fraction of dose which is reparable.
- \( (1-F) \) = Fraction of dose which is nonreparable.
- \( n \) = Number of time units after detonation. \( n = 1, 2, 3, \ldots \), when increments (\( n \)) in time are 24-hour days. (Time is incremented in minutes for the first 24 hours and in days thereafter.)
- \( t \) = Number of hours after detonation.
- \( W(t) \) = Field intensity in \( r/hr \) at time \( t \) after detonation.

and:

\[
W(t) = 0 \quad \text{for} \quad t \leq T_A
\]
\[ I(t) = \frac{I_0 t}{(PF)^6} \left[ 1 - \cos \frac{t - T_A}{T_A E} \pi \right] \]

for: \( T_A < t \leq T_A (1+E) \)

\[ I(t) = \frac{I_0 t^{-2}}{(PF)^6} \]

for: \( t > T_A (1+E) \)

where:

- \( T_A \): Time of arrival of the first measurable radioactive fallout
- \( PF \): Protection Factor of the primary shelter
- \( Z \): Field decay constant
- \( E \): Buildup duration factor
- \( T_2 \): Time outside in fallout before reaching shelter
- \( \delta = 0 \) for \( t \leq T_A + T_2 \)
- \( \delta = 1 \) for \( t > T_A + T_2 \)

The expression \( \frac{1}{2} \left[ 1 - \cos \frac{t - T_A}{T_A E} \pi \right] \) was established as a representation of the buildup duration factor. (See Appendix A.)

2. Probability of Fatality and Nonfatal Casualty

The following relationships were established for converting ERD to \( P_1 \) (the probability of fatality) or to \( P_2 \) (the probability of becoming a noneffective or a nonfatal casualty):

- \( P_1 = 0 \) when Max. ERD < 200r
- \( P_1 = \frac{(Max. \, ERD) - 200}{500} \) when 200r ≤ Max. ERD ≤ 700r
- \( P_1 = 1 \) when 700r < Max. ERD
- \( P_2 = 0 \) when Max. ERD < 100r
- \( P_2 = \frac{(Max. \, ERD) - 100}{200} \) when 100r ≤ Max. ERD < 200r
- \( P_2 = \frac{3 \, (Max. \, ERD) - 100}{1000} \) when 200r ≤ Max. ERD < 300r
- \( P_2 = \frac{700 - (Max. \, ERD)}{500} \) when 300r ≤ Max. ERD < 700r
- \( P_2 = 0 \) when 700r ≤ Max. ERD

C. Derivation of the Sensitivity Equation

The sensitivity equation explicitly states the general research objective of the project in an algebraic form. Each term of the equation is defined and developed so that numerical values can be substituted for the variables to give a numerical
sensitivity index for each input parameter.

We begin the mathematical development by first defining the sensitivity index as the ratio of the percent change in the output, or dependent variable, to the percent change in an input parameter [Reference 4].

Thus, the sensitivity index \( S_i \) of the \( i \)th input parameter, measured from mean values, is:

\[
S_i = \frac{\text{[Change in the dependent variable (\( \Delta Y \))]} \quad \text{[Mean value in the dependent variable (\( Y \))]} \quad \text{[Change in the \( i \)th input parameter (\( \Delta X_i \))]} \quad \text{[Mean value of the \( i \)th input parameter (\( X_i \))]} \]

or:

\[
S_i = \frac{\Delta Y}{\frac{Y}{X_i}} \quad \text{(4)}
\]

D. Calculation of ERD and Probability of Fatality

The model basic equations were used with each combination of input values in the Mainline Program\(^1\) on the CDC 1604 computer.

For each combination of input parameters, the time required for the buildup of the radiation field is calculated. Next, the dose a person would receive, if any, while outside the shelter and the dose at the end of the first day inside is computed and totaled. The dose is then split into a recoverable and nonrecoverable fraction and a test is made to determine if the probability of fatality equals "one." If so, the program returns to the initial state and selects another input combination.

The ERD is accumulated and updated by recoverable and nonrecoverable fractions and the following tests are made on the daily increments:

(1) If a Maximum ERD has been reached, its values and date of occurrence are recorded for output.

\(^1\) An additional independent variable, \( D_a \), was included to set three possible levels to which the ERD could return--80r, 50r, and 20r. There was a total of 42,740 cases for all combinations of input values.
(2) If a Maximum ERD has been reached and the nonrecoverable ERD has not exceeded the allowable dose, and the accumulated ERD is less than the allowable ERD, then the day is recorded for output.

Eventually the test conditions will be met and the program will branch out of the day-by-day calculation of the ERD. Once out of this routine, the program will take the Maximum ERD value and calculate the probability of fatality and other dependent variables.

The dependent variables calculated by this program are:

1. Maximum ERD.
2. Probability of fatality.
3. Probability of being a noneffective.
4. Day in which the Maximum ERD occurs.
5. Day in which allowable ERD occurs.
6. Nonrecoverable dose on that day.

Cases were excluded in which the Maximum ERD was greater than 700r or less than 100r, thereby leaving for analysis a group of 10,143 cases (Group 0 of Table II, Basic and Subgroup Divisions of the Mainline Program Cases). Thus, the sensitivity analysis was based on cases in which the interaction of input parameter values resulted in situations of maximum interest—that is, cases in which even small variations in input parameters could affect both casualties and fatalities.

E. Grouping of Cases and Calculations of Mean Values

Subgroups of Group 0 (which contained all cases between 100r and 700r Maximum ERD) were established for each of the four input values of reference intensity. In addition, it was desirable to combine two of these subgroups in order to have a subgroup with "moderately" high reference intensity (Subgroup 20 of Table II).

In order to examine the significance of the time-in-shelter data, another basic group of data was established (Group 1). Into this group were put cases in which the ERD would eventually reach 80r or less, because 80r is considered to be the threshold below which physical disability is unlikely to exist. Within this latter basic group, subgroups based on reference intensity, allowable ERD, and protection factor were established. Table II shows the basic and subgroup divisions.

The cases were transferred from the Mainline Program by a tape to the IBM 7072 computer. A general purpose data processing program (TSAR) at Duke University was
# TABLE II

**Basic and Subgroup Divisions of the Mainline Program Cases**

<table>
<thead>
<tr>
<th>BASIC GROUPS</th>
<th>SUB-GROUPS</th>
<th>NUMBER OF CASES</th>
<th>SORTING PARAMETER</th>
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</thead>
<tbody>
<tr>
<td>Group 0</td>
<td>10,143</td>
<td>Reference Intensity (r/hr)</td>
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<tr>
<td>Parameter Combinations with:</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>2,115</td>
<td>( I_0 = 300 )</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3,108</td>
<td>( I_0 = 1500 )</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3,431</td>
<td>( I_0 = 2700 )</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1,485</td>
<td>( I_0 = 3900 )</td>
<td></td>
</tr>
<tr>
<td>20 (sum of 7 &amp; 8)</td>
<td>4,920</td>
<td>( I_0 = 3063 )</td>
<td></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Group 1</th>
<th>3,481</th>
<th>Reference Intensity (r/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Cases which return to an allowable ERD ( \leq 80r ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>909</td>
<td>( I_0 = 300 )</td>
</tr>
<tr>
<td>10</td>
<td>1,041</td>
<td>( I_0 = 1500 )</td>
</tr>
<tr>
<td>11</td>
<td>1,058</td>
<td>( I_0 = 2700 )</td>
</tr>
<tr>
<td>12</td>
<td>473</td>
<td>( I_0 = 3900 )</td>
</tr>
</tbody>
</table>

| Allowable ERD (r) | |
|-------------------| |
| 2                 | 2,069     | \( D_4 = 80 \) |
| 3                 | 1,204     | \( D_4 = 50 \) |
| 4                 | 208       | \( D_4 = 20 \) |

| Protection | |
|------------| |
| 13         | 987       | \( PF = 2 \) |
| 14         | 1,291     | \( PF = 10 \) |
| 15         | 795       | \( PF = 40 \) |
| 16         | 321       | \( PF = 100 \) |
| 17         | 87        | \( PF = 500 \) |

| Multiple Sorting Parameters | |
|-----------------------------| |
| 18                          | 352       | \( D_4 = 80r \) |
| \( 0 < P_2 \leq .5 \)     | \( T_2 = 0 \) |
| 19                          | 240       | \( D_4 = 50r \) |
| \( 0 < P_2 \leq .5 \)     | \( T_2 = 0 \) |
used for the grouping of data into sets [Reference 1].

The same program was used for determining the mean values of the input parameters for the basic groups of cases and for each of the subgroups. Table III, Mean Values of the Basic Parameters, shows the pertinent means for each of these groups and subgroups.

F. Determination of the Principal Model Approximation

For purposes of computation it was desirable to have a single equation that approximates the Mainline Program computer model. Using a stepwise regression to obtain this, three basic functions (quadratic, logarithmic, and exponential) were fitted to the sets of data produced by that model. This was accomplished also by the TSAR general purpose program. Each function was fitted by the step procedure [Reference 1]. Equation 5 best fitted the data and was made the principal model approximation:

\[
f(X) = \text{Max. ERD} = e^{(-2.7468) I_0^{(.392 + .0001499PF - .0446F + 5.354B - .006493E)}} \\
T_A^{(.4346 - .7954Z)} P_F(-1.0949 + .04181 T_2 + .06268Z) \\
Z^{(-3.074 - .1161 B)} B^{(-1.1553)}
\]  

Equation 5

The statistical characteristics of the principal model approximation equation are:

1. Multiple correlation coefficient: 0.9991.
2. Standard error of the estimate: 0.02421 (i.e., c of ERD ± 6r).
3. Value of F-ratio: 886.785 (a further indication of the high significance of the equation).
4. Data Group 0 (Table II).

G. Calculation of the Sensitivity Indices

We now have a formula (Equation 5) relating the input variables to the dependent variable Maximum ERD, and a formula for the sensitivity index (Equation 4). To calculate the sensitivity indices at the means, Equation 4 is placed in differential form:

\[2/\] Identification of the variables is in Table I.
### TABLE III
Mean Values of the Basic Parameters

<table>
<thead>
<tr>
<th>Group Sort Index</th>
<th>Reference Intensity $I_o$</th>
<th>Time of Arrival $T_A$</th>
<th>Protection Factor PF</th>
<th>Max. ERD $D_3$</th>
<th>Day Max. ERD Occurred $N_1$</th>
<th>Allowable Dose $D_4$</th>
<th>Day Allowable Dose Occurred $N_2$</th>
<th>Nonrecoverable Dose on Day $N_2$ $D_2$</th>
<th>Fatality Probability $P_1$</th>
<th>Casualty (Non-Fatality) Probability $P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000</td>
<td>3.3</td>
<td>39.1</td>
<td>297.6</td>
<td>10.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.239</td>
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<tr>
<td>R</td>
<td>2000</td>
<td>3.3</td>
<td>3.4</td>
<td>284</td>
<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.217</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>3.5</td>
<td>3.4</td>
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<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.217</td>
</tr>
<tr>
<td>6</td>
<td>1500</td>
<td>3.5</td>
<td>3.4</td>
<td>284</td>
<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.217</td>
</tr>
<tr>
<td>7</td>
<td>2700</td>
<td>3.6</td>
<td>3.4</td>
<td>284</td>
<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.217</td>
</tr>
<tr>
<td>8</td>
<td>3900</td>
<td>3.6</td>
<td>3.4</td>
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<td></td>
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<td>35.1</td>
<td>250</td>
<td>8.8</td>
<td>65.7</td>
<td>125</td>
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<td>.146</td>
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<td>1934</td>
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<td>37.7</td>
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<td>3</td>
<td>1865</td>
<td>3.1</td>
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<td>238</td>
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<td>50</td>
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<td>25</td>
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<td>34.5</td>
<td>238</td>
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<td>50</td>
<td>133</td>
<td>25</td>
<td></td>
<td>.124</td>
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<tr>
<td>R</td>
<td>9</td>
<td>300</td>
<td>3.6</td>
<td>218</td>
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<td>62.5</td>
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<td>128</td>
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<td>.144</td>
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<td>45.0</td>
<td>262</td>
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<td>66.9</td>
<td>125</td>
<td>29</td>
<td></td>
<td>.144</td>
</tr>
<tr>
<td>12</td>
<td>3900</td>
<td>1.0</td>
<td>85.9</td>
<td>282</td>
<td>5.9</td>
<td>67.0</td>
<td>107</td>
<td>26</td>
<td></td>
<td>.144</td>
</tr>
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<td>PF</td>
<td>13</td>
<td>606</td>
<td>4.4</td>
<td>2</td>
<td>10.0</td>
<td>64</td>
<td>143</td>
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<td></td>
<td>.175</td>
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<tr>
<td>14</td>
<td>1910</td>
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<td>10</td>
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<td>9.2</td>
<td>65</td>
<td>136</td>
<td>30</td>
<td></td>
<td>.175</td>
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<td>15</td>
<td>2831</td>
<td>1.8</td>
<td>40</td>
<td>232</td>
<td>8.2</td>
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<td>112</td>
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<td></td>
<td>.105</td>
</tr>
<tr>
<td>16</td>
<td>3043</td>
<td>1.2</td>
<td>100*</td>
<td>178</td>
<td>6.5</td>
<td>68</td>
<td>66</td>
<td>16</td>
<td></td>
<td>.059</td>
</tr>
<tr>
<td>17</td>
<td>2782</td>
<td>1.0</td>
<td>500*</td>
<td>235</td>
<td>1.5</td>
<td>70</td>
<td>60</td>
<td>16</td>
<td></td>
<td>.126</td>
</tr>
<tr>
<td>18</td>
<td>1690</td>
<td>--</td>
<td>73</td>
<td>144</td>
<td>7.0</td>
<td>80</td>
<td>47</td>
<td>--</td>
<td></td>
<td>--</td>
</tr>
<tr>
<td>19</td>
<td>1635</td>
<td>--</td>
<td>55</td>
<td>146</td>
<td>6.7</td>
<td>50</td>
<td>80</td>
<td>--</td>
<td></td>
<td>--</td>
</tr>
</tbody>
</table>

* Individuals in 500 PF shelters only reached or exceeded the Maximum ERD if they receive appreciable dose outside. Their time-outside average was twice any other group.
Let:
\[ Y = f(X) \]
so:
\[ S_i = \frac{\partial f(Y)}{\partial X_1} (f(X))^{-1} \]  

Note that \( f(X) \) is the principal model approximation (See Equation 5):

\[ f(X) = K I_o T_A PF Z B \]  

where:
\[ C_1 = .932 + .0001499PF - .0446F + 5.354B - .006493E \]
\[ C_2 = .4346 - .7954Z \]
\[ C_3 = -1.0949 + .04181T_2 + .06268Z \]
\[ C_4 = -3.074 - .1161E \]
\[ C_5 = -1.1553 \]
\[ K = e^{-2.7468} \]

The partial derivatives of Equation 5 with respect to each independent variable \( X_i \) are:

\[ \frac{\partial f(X)}{\partial T_o} = K C_1 I_o C_1^{-1} T_A C_2 PF C_3 C_4 C_5 \]
\[ \frac{\partial f(X)}{\partial T_A} = K C_2 I_o T_A C_2^{-1} PF C_3 C_4 C_5 \]
\[ \frac{\partial f(X)}{\partial PF} = K K_3 I_o T_A PF C_3 C_4 C_5 \]
\[ \frac{\partial f(X)}{\partial Z} = K C_2 C_3 C_3^{-1} C_4 C_5 \]
\[ \frac{\partial f(X)}{\partial F} = K K_5 I_o T_A PF Z B \]
\[
\frac{\Delta f(x)}{\Delta B} = K_6 I_o C_1 T_A C_2 PF C_3 Z B^{-1}
\]
\[
\frac{\Delta f(x)}{\Delta E} = K_7 I_o C_1 T_A C_2 PF C_3 Z B^2
\]

where:

\[
K_2 = 0.00015 \text{ PF (ln } I_o) + C_3
\]

\[
K_3 = 0.0418 \text{ ln PF}
\]

\[
K_4 = -0.795 Z \text{ ln } T_A + 0.0627 Z \text{ ln PF} + C_4
\]

\[
K_5 = -0.0446 \text{ ln } I_o
\]

\[
K_6 = 5.35 B \text{ ln } I_o + C_5
\]

\[
K_7 = -0.00649 \text{ ln } I_o - 0.116 \text{ ln } Z
\]

Solving Equation 6 for the sensitivity indices, using the appropriate terms developed above, we get:

\[
S_{I_o} = C_1
\]

\[
S_{T_A} = C_2
\]

\[
S_{PF} = K_2
\]

\[
S_{T_2} = K_3 T_2
\]

\[
S_{Z} = K_4
\]

\[
S_{F} = K_5 F
\]

\[
S_{B} = K_6
\]

\[
S_{E} = K_7 E
\]

Taking the mean parameter values for data group (0) (see Section II.E), we get:

\[
I_o = 2008
\]

\[
T_A = 3.3
\]

\[
PF = 39.1
\]

\[
T_2 = 0.311
\]

\[
Z = 1.18
\]
\[ F = .9 \]
\[ B = .025 \]
\[ E = 2.57 \]

and using the Equation 4, we calculate the Sensitivity Indices \( (S_i) \).

**TABLE IV**

**Principal Sensitivity Indices**

<table>
<thead>
<tr>
<th>Index</th>
<th>Variable Denoted by Index Subscript</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_z )</td>
<td>Field Decay Exponent</td>
<td>-4.22</td>
</tr>
<tr>
<td>( S_{PF} )</td>
<td>Protection Factor</td>
<td>-0.96</td>
</tr>
<tr>
<td>( S_{Fo} )</td>
<td>Reference Intensity</td>
<td>+1.02</td>
</tr>
<tr>
<td>( S_{TA} )</td>
<td>Time of Arrival</td>
<td>-0.50</td>
</tr>
<tr>
<td>( S_F )</td>
<td>ERD Recovery Fraction</td>
<td>-0.30</td>
</tr>
<tr>
<td>( S_B )</td>
<td>ERD Recovery Rate</td>
<td>-0.14</td>
</tr>
<tr>
<td>( S_{SE} )</td>
<td>Buildup Duration Factor</td>
<td>-0.18</td>
</tr>
<tr>
<td>( S_{T2} )</td>
<td>Time in Fallout</td>
<td>+0.05</td>
</tr>
</tbody>
</table>

It is upon these values that the conclusions and recommendations will be developed.

The above sensitivity indices are obtained for Maximum ERD as the dependent variable. However, over the dose range where probability of casualty is neither 0 nor 1, the relation between probability sensitivity \( (PS_i) \) and dose sensitivity \( (S_i) \) is given by the relation:

\[ PS_i = S_i \left(1 - \frac{b}{p}\right) \]  \hspace{1cm} (9)

where \( p = \) probability of casualty = \( a \cdot \text{Dose} + b \). Hence, the relative rank of the \( S_i \) remains the same when evaluated at a given set of input variables.

**H. Total Dose Model**

The major computational difficulties in determining the above sensitivity indices arise because of the use of equivalent residual dose instead of total dose as a measure of biological effect. This section will describe a simpler model which
allows the calculation of the sensitivity indices analytically. In this model the dose rate buildup function is assumed to be linear, and total dose is used as the dependent parameter.

A new parameter, $T_0$, is introduced in this approach. $T_0$ is the time period from entrance into shelter until the cutoff of the total dose calculations. Thus, $(T_A + T_2 + T_0)$ represents the complete time period from time of detonation until completion of the total dose calculation. Using the parameters previously defined (see Table I) and the new parameter $T_0$, the total dose $D$ is given by the expression:

$$D = \frac{T_0}{PF} \left\{ \frac{1}{2T_A (1+E)^Z_E} \left[ (PF-1) T_2^2 + E^2 T_A^2 \right] \right. \right.$$

$$+ \left. \frac{1}{(1-Z)} \left[ (T_A + T_2 + T_0)^{1-Z} - T_A^{1-Z} (1+E)^{1-Z} \right] \right\}$$

(10)

The sensitivity indices can then be calculated directly:

$$S_{T_2} = \frac{\partial D}{\partial T_2} \cdot \frac{T_2}{D} = \frac{T_0}{PF} \left\{ \frac{(PF-1) T_2}{T_A (1+E)^Z_E} + \frac{(T_A + T_2 + T_0)^Z}{(1-Z)} \right\}$$

(11)

$$S_{T_A} = \frac{\partial D}{\partial T_A} \cdot \frac{T_A}{D} = \frac{T_0}{PF} \left\{ \frac{1}{2(1+E)^Z_E} \left[ (PF-1)(Z+1) \left( \frac{T_2}{T_A} \right)^2 + (Z-1) E^2 \right] \right\}$$

(12)

$$S_{PF} = \frac{\partial D}{\partial PF} \cdot \frac{PF}{D} = \frac{T_0}{2DPF} \left\{ \frac{1}{2T_A (1+E)^Z_E} \left( T_2^2 - E^2 T_A^2 \right) \right. \right.$$

$$- \left. \frac{2}{(1-Z)} \left[ (T_A + T_2 + T_0)^{1-Z} - T_A^{1-Z} (1+E)^{1-Z} \right] \right\}$$

(13)

$$S_{T_0} = \frac{\partial D}{\partial T_0} \cdot \frac{T_0}{D} = \frac{T_0}{PF} \left( T_A + T_2 + T_0 \right)^{-Z}$$

(14)
The total dose sensitivity indices ($S_i$) can be readily converted to probability of casualty sensitivity indices ($PS_i$), if desired. In the region of interest (probability neither zero nor one), the dose is related to probability of casualty ($p$) by a linear relation:

$$p = aD + b.$$  

Hence:

$$PS_i = \frac{\frac{\partial D}{\partial x_i}}{p} \cdot \frac{x_i}{p} = S_i \cdot (1 - \frac{b}{p})$$  

Only dose sensitivity will be discussed in the remainder of this analysis, since the relative rank of both $S$ and $PS$ are the same for a given set of values of the independent variables.

In Table V, Comparison of Sensitivity Indices, the values of the $S_i$ obtained from Equations 7 through 13 by evaluation at the mean values of the variables listed in section G above are compared to the values of $S_i$ listed in Table IV (calculated from the ERD model).
TABLE V
Comparison of Sensitivity Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Variable Denoted by Subscript</th>
<th>ERD Model Value</th>
<th>Total Dose Model Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_z$</td>
<td>Field Decay Exponent</td>
<td>-4.22</td>
<td>-4.36</td>
</tr>
<tr>
<td>$S_{PF}$</td>
<td>Protection Factor</td>
<td>-0.96</td>
<td>-0.99</td>
</tr>
<tr>
<td>$S_{I_o}$</td>
<td>Reference Intensity</td>
<td>+1.02</td>
<td>+1.00</td>
</tr>
<tr>
<td>$S_{TA}$</td>
<td>Time of Arrival</td>
<td>-0.50</td>
<td>-0.50</td>
</tr>
<tr>
<td>$S_E$</td>
<td>Buildup Duration Factor</td>
<td>-0.18</td>
<td>-0.25</td>
</tr>
<tr>
<td>$S_{T_2}$</td>
<td>Time in Fallout</td>
<td>+0.05</td>
<td>+0.02</td>
</tr>
</tbody>
</table>

In Figures 2 thru 6, the values of the sensitivity indices are plotted as a function of the indexed variable. While the rest of the independent variables are held constant at the mean values given previously, two values for PF (39.1 and 500) and two values of $T_o$ (96 and 240 hours) are plotted for each curve. Figure 7 shows the increase in the sensitivity index for $T_2$ (time in fallout) as $Z$ (the decay rate) increases. Figure 8 shows the dose sensitivity to $T_o$ as a function of $T_o$ for the two protection factors. This curve indicates that total dose is not particularly sensitive to $T_o$ when $T_o$ is measured after 100-200 hours, especially for high PF shelters. In fact, where Maximum ERD occurs within the first few days, it can be expected that the results from the total dose sensitivity analysis model will be essentially equivalent to the results from the ERD model.

I. Validity of Ranking the Sensitivity Indices

In Tables IV and V, the sensitivity indices were evaluated for one set of input variables. However, the relative values of these $S_i$ might change when evaluated for radiological environments other than the one defined by the mean values in section G. To investigate this problem, the following conclusions were drawn based on Figures 2 through 8 and other calculations of sensitivity indices:

1) $S_I$, sensitivity index for reference intensity ($I_o$), is 1 over the entire range of values; thus, $I_o$ remains a relatively important parameter in all cases.

2) $S_{PF}$ is essentially constant at 1 for most cases of interest; thus, protection factor (PF) is a relatively important parameter in all cases. (See Figure 2).

- 18 -
Fig. 2. Sensitivity Index of PF as a Function of PF
Fig. 3. Sensitivity Index ($S_{T_A}$) as a Function of Time of Arrival

Time of Arrival ($T_A$) in Hours

PF = 39.1
$T_o = 96$ hrs or 240 hrs

PF = 500
$T_o = 96$ hrs or 240 hrs
Fig. 5. Sensitivity Index \( S_{T_2} \) as a Function of Time in Fallout
Fig. 6. Sensitivity Index ($S_2$) as a Function of Decay Constant ($Z$)
Fig. 7. Sensitivity Index for Time in Fallout ($S_{T_2}$) as a Function of Decay Constant ($z$)
(3) $S_Z$ remains high for all cases, thus $Z$ is extremely critical in vulnerability analyses. (See Figure 6).

(4) For analysis observing only Maximum ERD reached while in primary shelter, ERD concepts and a total dose approximation of Maximum ERD yield essentially the same results in sensitivity analysis. (This does not imply that they yield the same results in casualty calculations.)

(5) $S_{T_A}$ can become quite large for short times of arrival of fallout ($T_A$) and high protection factors. In areas where this combination of environments exists, $T_A$ must be treated carefully in vulnerability analyses. (See Figure 3).

(6) In most cases of interest, the other sensitivity indices remain appreciably less than 1, though some change significantly as the parameter values are varied.

J. Analysis of Time Required in Shelter

1. Procedure

In the preceding analysis, the independent variable was Maximum ERD. It is also important to know how the input variables may affect the time required in shelter after the beginning of fallout.

In the analysis of time required in shelter, only those cases from the Mainline Program were considered in which the probability of a noneffective ($P_2$) was greater than zero and less than 0.5. Thus, the largest Maximum ERD to be considered was 200r and the smallest Maximum ERD was 100r. (See the dose response relationships in Section II.B.2.) Another condition was that the ERD returned to 80r. It was assumed that the reduction of ERD to this level would mark approximately the end of the continuous stay in shelters. Time in fallout outside shelter was also excluded from consideration.

The TSAR general purpose program was used to set up subgroups 18 and 19 (see Table II) and to develop an equation by multiple regression. The equation given below determines the number of days in shelter until ERD returns to 80r.

When: $100r \leq \text{Maximum ERD} \leq 200r$,
\[ f(X) = N_2 = 4.47 - 44Z + 2.26 \frac{I_o}{PF} + 1.6D_3 - 1.55(Z \times \frac{I_o}{PF}) + 926(Z \times B) - 0.7920(D_3 \times F) - 27(D_3 \times B) \]

* Multiple Correlation Coefficient: 0.9717
Value of F-ratio: 830

Where:
- \( N_2 \) = Duration of shelter phase until ERD falls to 80r
- \( Z \) = Field decay exponent
- \( \frac{I_o}{PF} \) = Reference intensity/protection factor
- \( D_3 \) = Maximum ERD prior to \( N_2 \)
- \( B \) = ERD recovery rate
- \( F \) = ERD recovery fraction

The mean value of stay time for the cases considered (\( \bar{Y} \) in the sensitivity formula) was found to be 47 days.

The sensitivity indices were calculated using Equation 20, the differential form of Equation 4:

\[ S_1 = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X_1}{X_1}} \]

\[ S_1 = \frac{\partial f(X)}{\partial X_1} \cdot \frac{\bar{X}_1}{47} \]

\[ \bar{N}_2 = \bar{Y} = 47 \text{ days.} \]

2. Results

The sensitivity indices for the input parameters are shown in Table VI, Sensitivity Analysis of Time Required in Shelter. It is seen that changes in the decay exponent (Z) are the most significant in affecting the length of stay in shelter.
TABLE VI

Sensitivity Analysis of Time Required in Shelter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean Value</th>
<th>Gradient</th>
<th>Sensitivity Index (Equation 21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\bar{X}_1$</td>
<td>$\frac{\partial f(X)}{\partial X_1}$</td>
<td>$S_1$</td>
</tr>
<tr>
<td>$Z$</td>
<td>1.260</td>
<td>-144.000</td>
<td>-3.860</td>
</tr>
<tr>
<td>$F$</td>
<td>0.914</td>
<td>-144.000</td>
<td>-2.220</td>
</tr>
<tr>
<td>$D_3$</td>
<td>144.000</td>
<td>0.373</td>
<td>1.140</td>
</tr>
<tr>
<td>$B$</td>
<td>0.025</td>
<td>-1700.000</td>
<td>-0.918</td>
</tr>
<tr>
<td>$RP$</td>
<td>79.600</td>
<td>0.331</td>
<td>0.560</td>
</tr>
</tbody>
</table>

The independent variables (which affect the length of stay in shelter) are ranked on the sensitivity index in decreasing importance as follows:

1. Field decay exponent, $Z$ = 3.9
2. ERD recovery fraction, $F$ = 2.2
3. Maximum ERD, $D_3$ = 1.1
4. ERD recovery rate, $B$ = 0.9
5. Reference intensity/protection factor ratio, $RP$ = 0.6

Of equal interest to the parameter sensitivity analysis is the actual timing of Maximum ERD and the number of days' stay in the primary shelter before recovery to an allowable dose is accomplished. For a shelter PF of 73, the Maximum ERD of 144r occurred on the 7th day after the attack. On the average, 47 days elapsed before the ERD recovered to the allowable dose level of 80r. If instead, an allowable level of 50r (Subgroup 19) were chosen, then the average length of stay would increase by 33 days (from 47 to 80).

K. Evaluation of Significance of Sensitivity Indices in Shelter Systems Analysis

1. General

The objective of shelter systems analysis is to predict as accurately as possible the adequacy of existing or potential protective measures against...
Fig. 9. Frequency of Occurrence of Stay Times vs Days for Return of ERD to 80% (N2)
(for Cases in Which Max ERD ≥ 20%)
fallout. In general, the better the parameters that determine the adequacy are known, the better the predictions will be.

The sensitivity indices derived in the above analysis give an indication of the effects of not knowing the various input parameters exactly; the analysis does not yet take into account how accurately these parameters might be known.

In the following sections, an attempt will be made to identify the approximate magnitude of uncertainty to be expected in the eight parameters considered.

2. **Reference Intensity and Time of Arrival**

   Of the eight parameters that were considered in the sensitivity analysis, reference intensity and time of arrival are largely dependent upon the choice of strategy in the attack; consequently, these parameters cannot be known accurately. Desired ground zeros and the occurrence of air or surface burst are the choice of the attacker.

   At any given point, the reference intensity and time of arrival are also affected by wind speed and direction.

   The RISK type programs are attempts to represent the combined effects of strategic decisions and wind conditions. In the choice of input variables for the sensitivity analysis (section II.A), it was seen that reference intensities between 300 and 3900 r/hr and times of arrival between 1 and 7 hours seem to represent the range of intensities with which fallout-only shelter systems are primarily concerned. Variations from the means of these input parameters of ±75% would seem to be a representative estimate of the probable range of values for reference intensity and time of arrival in an analysis of a shelter system.

3. **Buildup Factor**

   The values of the buildup multiplying factor used in the sensitivity analysis ranged from 1.13 to 6.15. These extreme values vary from their mean by about 70% and this is proposed as a range of values that could be expected in systems analysis.

4. **Protection Factors**

   The National Fallout Shelter Survey (NFSS) produced data on the present number and the protection factors of shelter spaces. However, there are various reasons why these data alone may not represent the actual shelter posture of the population.

- 30 -
First, based on a limited sample of shelter [Reference 5], it appears that procedures used in the Phase 1 shelter survey generally underestimated the actual protection factor by a considerable amount. The current updating of NFSS data is gradually improving the accuracy of information on shelter status, but a complete reevaluation of all NFSS shelters is not contemplated by OCD. At the present time the average potential protection factor of the United States population may be considerably more than presently represented in Phase 1 or Phase 2 data. However, even if the protection factors were accurately determined, there still would remain uncertainties as to shelter utilization that would influence the results of shelter analysis. The question of shelter utilization was presented in Volume II where it was considered in terms of actual SMSA's. (See Volume II, A Sensitivity Analysis of Selected Parameters Based on 8 SMSA's.)

Consequently, random variations of 80% about the mean can be considered reasonable in estimating the significance of the sensitivity index for protection factor.

5. **Time in Fallout**

Estimates of length of time outside in fallout are arbitrary. By assuming adequate warning, no one would need to be outside. However, there may also be cases where large numbers of the population are in fallout for periods exceeding the 0.6 hours arbitrarily used in the sensitivity analysis.

Although the sensitivity index for time in fallout is small, the importance of this parameter in systems analysis must be determined in conjunction with actual locations and categories of shelters. The combined effects of these are analyzed in Volume II.

6. **Decay Rate**

The decay rate may vary with the construction of the weapons and to a lesser degree with the nature of the surface materials at ground zero. Variations probably will not be more than plus or minus 25 percent of the 1.2 rate of decay, which is based on empirical data.

7. **Biological Factors**

The fraction of the dose that is reparable (F) and the rate of recovery (B) are important parts of the Blair Formula for calculating ERD. However according to Reference [6], a firm experimental basis appears to be lacking for the values presently used. The author in the same reference also questions
whether the constants are independent of time.

In this study, the average value of F, the recovery fraction, is taken as 0.90; variations of 10 percent would seem likely. The average value of B, the recovery rate, is 0.025 and probably could vary by 70 percent.

8. Comparison

We then may compare the sensitivity indices with the rough estimates of the accuracy to be expected in input variables. These are summarized in Table VII. An approximation of the relative importance of each parameter in contributing to total uncertainty in shelter analysis is obtained by multiplying the range of uncertainty by the sensitivity indices.

<table>
<thead>
<tr>
<th>Variable</th>
<th>S (ERD Model)</th>
<th>S (Total Dose Model)</th>
<th>Uncertainty</th>
<th>Approximate Relative Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Decay Exponent</td>
<td>-4.22</td>
<td>-4.36</td>
<td>25%</td>
<td>1.09</td>
</tr>
<tr>
<td>Reference Intensity</td>
<td>+1.02</td>
<td>+1.00</td>
<td>75%</td>
<td>0.75</td>
</tr>
<tr>
<td>Protection Factor</td>
<td>-0.96</td>
<td>-0.99</td>
<td>80%</td>
<td>0.80</td>
</tr>
<tr>
<td>Time of Arrival</td>
<td>-0.50</td>
<td>-0.50</td>
<td>75%</td>
<td>0.35</td>
</tr>
<tr>
<td>Buildup Duration Factor</td>
<td>-0.18</td>
<td>-0.25</td>
<td>70%</td>
<td>0.18</td>
</tr>
<tr>
<td>ERD Recovery Rate</td>
<td>-0.14</td>
<td>---</td>
<td>70%</td>
<td>0.10</td>
</tr>
<tr>
<td>Time Outside in Fallout</td>
<td>+0.05</td>
<td>+0.02</td>
<td>100%</td>
<td>0.05</td>
</tr>
<tr>
<td>ERD Recovery Fraction</td>
<td>-0.30</td>
<td>---</td>
<td>10%</td>
<td>0.03</td>
</tr>
</tbody>
</table>

III. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

Considering both the sensitivity of each parameter and the range of uncertainty that each is likely to have, it is possible to draw some conclusions as to their effect on the results of systems analysis.

In analyses for estimates of Maximum ERD, the following have the most significance: Field decay exponent ($Z$), reference intensity ($I_o$), and protection factor ($PF$). Having less significance are time of arrival ($T_A$), ERD recovery fraction ($F$), ERD recovery rate ($B$), and buildup duration factor ($E$).
Time in fallout \((T_2)\) has a low sensitivity index. However, extending the range of values that were assumed for it beyond 0.6 hours may have a considerable effect on the magnitude of this number. The importance of time in fallout was better shown in Volume II, where it was analyzed as a factor in shelter utilization.

In analysis of required time in shelter, field decay exponent \((Z)\) and ERD recovery rate \((B)\) are the most significant.

Of the above parameters, reference intensity and time of arrival are almost entirely dependent upon the strategic choice of the attacker and wind conditions. Consequently, better estimates of their effects in systems analysis is dependent upon military intelligence more than upon additional research and analysis.

The importance of the field decay exponent in determining both fatalities and shelter stay makes it important to have continuing analysis of the validity of the 1.2 value. There may be appreciable gains in accuracy possible since the exponent is primarily dependent upon the physics of the radioactive decay process.

Increased knowledge of the shelter protection factors available to the population is important for the making of valid systems analyses. The effect of underestimates of protection factors can be taken into account by using a multiplying factor. However, it is important to define the factor more closely than is possible now. This would not necessarily require large expenditures for obtaining new data or for correcting data that now exist. Statistical analysis based on the parts of the survey that have been updated or examined in detail could provide a much better estimate than evidently now exists of the overall shelter that would be available to the population.

Identification of the biological recovery process is particularly important in estimating time required in shelter. Possible approaches to reducing the range of uncertainty of this input are given in Reference [6].

Finally, efforts to define more closely the limits of any input variables should be viewed in the light of their relative effect on output data. For example, attempts to define protection factors accurately may make a change in the relative importance of that parameter that is small compared to the relative importance of reference intensity, which cannot be accurately predicted by any research means.

The sensitivity index of the buildup duration factor \((E)\) is based on "moderate" conditions where the time of arrival approximates 3.3 hours and \((E)\) approximates 2.6. In observing the value of the sensitivity index \(S_E\), it was found that the index is quite dependent on changes in \((E)\). Some further analysis of the effect of the buildup duration factor is discussed in Appendix A.

- 33 -
The analysis based upon return of ERD to 80r (the threshold of appearance of clinically detectable symptoms of radiation sickness), showed that shelter stay times of much more than two weeks could be necessary. It should be noted that this analysis included only cases in which the Maximum ERD was 200r or less. Without these restrictions, even longer required stay times would result. This is very significant in planning for the stocking of shelters, recovery planning, and for the establishment of policies for postattack management of supplies.

In summary the conclusions from the sensitivity analysis are:

1. Casualty calculations are quite sensitive to errors in the field decay exponent. The sensitivity of dose or casualties to the field decay rate remains at approximately -4.0 over the range examined. (The decay rate was varied from $Z = 1.1$ to $Z = 1.4$.)

2. Sensitivity to variations in fallout reference intensity and protection factor are high over the whole range of parameter values. Hence, precise knowledge of the fallout shelter posture and the fallout reference intensity is much more essential to accurate vulnerability analysis than the remaining parameters in most cases of interest.

3. Sensitivity to time of arrival of fallout can be quite high, in some radiological environments.

4. Sensitivity of casualty computations to the remaining parameters is low in most cases of interest.

5. Expansion of the sensitivity analysis to include parameters other than fallout, which define the total casualties from a given attack on the United States, is necessary before further conclusions concerning a national vulnerability analysis can be drawn.

B. Recommendations

1. Vulnerability analyses should employ protection factors computed by best available methods, and research and/or surveys to improve protection factor data should be encouraged.

2. Because of the sensitivity of systems analysis results to the field decay exponent, continuing analysis of the validity of the $t^{-1.2}$ decay law should be made.

3. Additional study is required to establish the sensitivity of fatalities, casualties, and dose to duration of shelter stay.

- 34 -
4. Sensitivity analysis should be extended to include the parameters defining the effects of blast and alternative measures of effectiveness (casualties by type, dose distribution of survivors, etc.).

5. Sensitivity analysis should be applied to identification of the important cost/effectiveness parameters used in the budget allocation model (see Volume I, A Cost/Effectiveness Computer Procedure for Optimum Allocation of Fallout Shelter System Funds Under Uniform or Variable Risk Assumptions).
REFERENCES FOR VOLUME III


and

Appendix A

The Intensity Buildup of a Radioactive Fallout Field

In calculating the radiation dose received at a point in the fallout field, it is necessary to determine the rate at which the fallout particles are deposited from the cloud. The easiest method of representing this buildup is to plot the fraction of total fallout mass deposited as a function of time. If $T_A$ is the time the first particles arrive, and $T_C$ is the time the last particles are deposited, then the duration of the buildup period, $T_C - T_A$, is approximated by a relation of the form

$$T_C - T_A = T_A \cdot E \quad (A-1)$$

where $E$ is the buildup duration factor referred to in this volume.

Figure A-1 presents the buildup function [Reference A-1] which was obtained from weapons tests. In addition, two approximations to this function are plotted in Figure A-1 for $T_A = 1$ and $E = 2.5$. The analytic form for these approximate curves is given by Equations A-2 and A-3.

$$F(t) = \frac{1}{T_A \cdot E} \left( t - \frac{1}{E} \right) \quad (A-2)$$

$$F(t) = \frac{1}{2} \left( 1 - \cos \frac{t - T_A}{T_A \cdot E} \right) \quad (A-3)$$

Equation A-3 gives a somewhat better fit of the curve from Reference [A-1], hence this buildup function was used in the computer model that calculated Maximum ERD. However, the total dose expressions cannot be integrated in closed form if Equation (A-3) is used, hence Equation (A-2) was used as the buildup function for the analytical total dose model. As might be expected from Figure A-1, little difference was found between sensitivity indices from the two models.

REFERENCE


- A-1 -
Fig. A-1. Fallout Mass Buildup Functions

\[ \frac{1}{2} \left( 1 - \cos \left( \frac{t - t_A}{T_A} \pi \right) \right) \]
Appendix B

The Mainline Model and Computer Program

I. THE OPERATIONAL TRANSFORM OR MAINLINE MODEL

A. Attack Environment Formulation

1. Introduction

The first step in formulating the Mainline Program is proper characterization of the attack environment. The interrelation between the initial intensity, field buildup, the field decay, and the field attenuation by shelter, are the four functions that predict the dose rate. To determine their relative importance, it becomes mandatory that this interrelation is accurately formulated.

The underlying assumptions throughout this section are:

(a) The Reference Intensity of the radioactive field (in roentgens per hour) will conform to the measurements given in the NREC RISK-type data (i.e., an intensity \( I_0 \) at \( H + 1 \) hours) as well as the distribution of intensity and arrival of fallout obtained also from NREC RISK-type data.

(b) The measure of protection obtained by the employment of shelters will be the Protection Factor (PF) obtained from "The Fallout Shelter Surveys" Phases 1 and 2.

2. Decay of Field Intensity

Since a radioactive particle decays with time, our problem is to determine the appropriate mathematical representation, when applied to residual fields used in Civil Defense Systems Analysis. The widely used function shown in Figure B-1 and Equation (B-1) will be used.

\[
\rho(t) = I_0 e^{-Z t}
\]  

(B-1)
where

\[ n(t) = \text{Field Decay Function, in } \text{r/hr.} \]
\[ I_o = \text{Reference Intensity, in } \text{r/hr.} \]
\[ t = \text{Time in hours.} \]
\[ Z = \text{Empirical Constant.} \]

![Graph showing decay of radioactive field intensity over time.]

Fig. B-1. Decay of Radioactive Field

The two important constants introduced with this function are: the empirically derived exponent (Z), and the reference intensity \( I_o \). The reference intensity is one of the two interface parameters connecting the fallout shelter analysis model with the "war games" which establish the probable attack environment.

3. **Buildup of Field Intensity**

a. **Introduction**

The problem now is to characterize analytically the field intensity during the buildup phase. The fact that there is no existing data within our files on which the usual methods of curve fitting can be applied, forced us to use a more general form of analysis. Our analysis uses what little significant information there is and employs as much intuition as seems necessary, useful, or feasible. Although there are two relevant functions

\[ t_p = 2T_A \] (B-2)

and

\[ t_e = 5T_A \cdot 7 \] (B-3)
they are incompatible over a wide range of times of arrival. The relation---
time of peak intensity equals twice the time of arrival---is taken as being 
more nearly valid. In the field buildup, two distinct phenomena are 
present. The first is the arrival (at ground level) of the radioactive 
particles. The second is the independent phenomena of the radioactive 
decay of these particles. Thus, characterizations of these two phenomena 
will be independent, and the field intensity represented by the product.

The hypothesis implies a particle buildup function of the following 
general form given in Figure B-2, which, when integrated, yields the time 
dependent arrival function given in Figure B-3.
The product of the functions represented in Figure B-1 and Figure B-3 would give the field intensity shown in Figure B-4.

![Graph showing field intensity over time with labels for Unit Buildup, Unit Decay, and Unit Field Intensity.]

Fig. B-4. Field Intensity vs. Time

b. Formulation

From several general forms a displaced Cosine function was chosen for simplicity and compatibility with the function \( \left(t - 2T_A\right) \) to represent the fallout particles accumulated (Reference Equation B-4).

\[
F(t) = \frac{1}{2} \left[1 - \cos \left(\frac{t - T_A}{\eta} \pi\right)\right] \quad (B-4)
\]

where

\[T_A \leq t \leq T_C\]

and

\[\eta = T_C - T_A\]

The field decay function (i.e., \(t^{-2}\)) times the buildup function times the reference intensity yields the final representation of the field intensity over time. For time between \(T_A\) and \(T_C\):

\[
I(t) = I_o \frac{t^{-2}}{2} \left[1 - \cos \left(\frac{t - T_A}{\eta} \pi\right)\right] \quad (B-5)
\]
Thus, the characterization of the field intensity is in two parts: the first during accumulation—from $T_A$ (time of arrival) to $T_c$ (time of cessation). The second, after all fallout has occurred (Reference Equation B-1), describes the field.

In order to comply with $T_p = 2T_A$, the value of $\gamma$ must be fixed. The following are values of $\gamma$ for three values of $Z$:

- $Z = 1.0$, $\gamma = 1.13T_A$
- $Z = 1.2$, $\gamma = 1.17T_A$
- $Z = 1.4$, $\gamma = 1.20T_A$

4. Attenuation of Field Intensity

a. The Problem

The population or an individual may move through a series of shelter conditions over time. The duration of each shelter phase, although nominally under operational control, has well-defined bounds based upon the attack environment. The initiation of the recovery phase depends upon minimizing the stay in shelter. Thus, the influence of shelters when modeled must take into account the operational requirement to minimize shelter stay time.

b. Formulation

Fallout shelters are classified by a protection factor (PF). The protection factor function operates on the unprotected radiation field to yield the attenuated field that exists within that shelter.

There are four major operational phases (Reference Figure B-5).

These are:

1. An initial unsheltered condition which corresponds with a person moving through a fallout field before entering shelter. This phase will be called "time outside." It is related to the efficiency of warning systems, drills and training, and shelter assignment plans.

2. Phase 2 is the primary shelter phase. In most current models it is the only one considered. Operational plans usually specify

\[ T_2 \]

The symbol used in this report for initial time outside in a fallout field is $T_2$. 

- B-5 -
a two-week stay, and they are stocked on that basis. The controllable variable is the length of stay in the phase. Our model considers the length of stay dependent upon an objective of the third phase and is subject to explicit calculation.

(3) Phase 3 is operationally associated with emerging from shelter to begin recovery. The objective in this period is to obtain productive effort (usually at a PF of 1) without any physiological radiation symptoms. To do this introduces a decision parameter (an allowable ERD), an operational parameter, and the equivalent PF required. The allowable ERD lies between the limits of clinical effects (25r) and symptomatic effects (100r).

(4) The fourth phase is the return to normal. It begins when the individual's recovery rate equals the dose received in the unsheltered field. It is characterized by the lack of control variables, decisions required, etc. In other words, the effects of normal biological recovery processes exceed the effects of the radiation fields.

The problem domain investigated in this volume extends only to Phase 3, time \( t_3 \). (Practical considerations precluded extensive investigation of Phases 3 and 4 under the present contract.)

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**Fig. B-5. Attenuation of Field Intensity**
In Fig. B-5, the solid line represents the sheltered point intensity. In Fig. B-5, the dotted line represents the field intensity if:

\[ PF_1 = PF_2 = PF_3 = 1. \]

where

- **Phase 1** \( PF_1(t) = 1, \ 0 < t \leq T_2 \)
- **Phase 2** \( PF_2(t) = PF, \ T_2 < t \leq T_3 \)
- **Phase 3** \( PF_3(t) = \text{Equivalent } PF(t), \ T_3 < t \leq T_4 \)
- **Phase 4** \( PF_4(t) = 1, \ T_4 < t \)

5. **Summary Attack Environment Model**

The attack environment radioactive field intensity at any time \( t \) is the product of:
- the Reference Intensity \( (I_o) \);
- the particle buildup function \( (F(t)) \);
- the field decay function \( (\eta(t)) \); and
- the protection factor function \( (PF(t)) \).

\[ \phi(t) = R \cdot F(t) \cdot \frac{\eta(t)}{PF(t)} \]  \hspace{1cm} (B-6)

Or by substitution:

\[ \phi(t) = \frac{I_o}{PF_k} \cdot \frac{t-Z}{2} \left[ 1 - \cos \left( \frac{t-T_A}{\eta} \pi \right) \right] \quad \text{when } T_A < t \leq T_A + \eta T_A \]

\[ \phi(t) = 0 \quad \text{when } t \leq T_A \]  \hspace{1cm} (B-7)

\[ \phi(t) = \frac{I_o}{PF_k} \cdot t-Z \quad \text{when } t > T_A + \eta T_A \]

where:

- \( \phi(t) \) = Field Intensity in r/hr at time \( t \) after the originating explosion event.
- \( I_o \) = The reference intensity in r/hr, one hour after the explosion event.
- \( t \) = Time in hours measured from \( H \) hour.
- \( T_A \) = Time of arrival measured from \( H \) hour.
- \( PF_k \) = Protection factor at time \( t \).
- \( Z \) = empirically derived field decay constant.
- \( \eta = T_C - T_A \) = fallout interval.
B. Biological Response

1. Introduction

The state, efficiency, and outcome of a shelter system analysis is measured by the biological impact, the usual criteria being the number of fatalities or lives saved. Fatalities are calculated by using a probability of response to the total radiation dose received.

At the present state of understanding, the equivalent residual dose model is the best available formulation of biological response to dose received.

2. Equivalent Residual Dose

To represent analytically the absorbed gamma radiation field in terms of a time dependent dose "state" requires taking into account the observed phenomena of biological recovery. The ERD model relates the field intensity and biological recovery rate to give a single "dose" value, approximating the physiological state of a human.

The ERD at any point in time consists of two parts. The nonrecoverable portion is the accumulative amount of the nonrecoverable dose received over time. The recoverable portion has two additive parts; one is a fraction of the field contribution within the unit time considered, and the second is the recoverable dose carried over from the previous period. The basic statement of ERD (recoverable dose \( D_n \) in time increment, \( n \)) is the sum of that not yet received from previous periods and the new contribution during the \( n \)th time period.

\[
D_n = (1-B)D_{n-1} + F \int_{n-1}^{n} \phi(t)dt
\]  
\( (B-8) \)

where

\( B \) is the percent per unit dose recovered in period \( n \), and \( F \) is the recoverable fraction.

Nonrecoverable dose \( (d_n) \) in time increment \( n \) is the accumulative sum of all nonreparable doses:

\[
d_n = d_{n-1} + (1-F) \int_{n-1}^{n} \phi(t)dt.
\]  
\( (B-9) \)

The practical problem of describing the function of the \( ERD(n) \) in a closed, well-behaved form requires that it be approximated by finite differences. Finite difference problems, which require recursive solutions, lead to computer
implementation. A solution is to use the basic difference equations with \( n \) in one day intervals and with \( d_n \) being computed daily, then added for the equivalent residual dose on the \( n^{th} \) day.

where

\[
\begin{align*}
D_n &= \text{Recoverable dose in time interval } n-1 \text{ to } n, \\
N_n &= \text{Nonrecoverable dose in time interval } n-1 \text{ to } n, \\
I_0 &= \text{Reference intensity,} \\
F &= \text{Fraction of dose which does reparable damage,} \\
(1-F) &= \text{Fraction of dose which does nonreparable damage,} \\
B &= \text{Daily fraction of dose repaired,} \\
T_A &= \text{The time of arrival of the first measurable radioactive fallout,} \\
Z &= \text{Decay constant,} \\
PF &= \text{Protection factor.}
\end{align*}
\]

3. **Probability**

The problem is now to relate the equivalent residual dose which an individual accumulates to the probable biological state. The usual specifications for radiological induced states are fatalities and casualties, with fatality being a type of casualty. Because this causes some confusion, we will introduce the term noneffective. For a given Maximum ERD, we will then have three probabilities:

1. Death.
2. Noneffective--The condition of being alive but physically unable to perform a productive task.
3. Normal and Marginal Effective--Individuals with acceptably low overt physiological symptoms (below 100r).

The following linear functions are used to approximate the empirically derived probability of casualty vs. ERD curves (reference Figure B-6).

\[
\begin{align*}
P_1 &= 0 \quad \text{when } \text{ERD} < 200r \\
&= \frac{(\text{Max. ERD} - 200)}{500} \quad \text{when } 200r \leq \text{ERD} \leq 700r \\
&= 1 \quad \text{when } 700r < \text{ERD.} \\
\end{align*}
\]

\[(B-10)\]
b. Probability of Noneffectiveness

\[ P_2 = 0 \quad \text{when} \quad \text{ERD} < 100r \]
\[ P_2 = \frac{(\text{Max. ERD})-100}{200} \quad \text{when} \quad 100r \leq \text{ERD} < 200r \]
\[ P_2 = 3 \frac{(\text{Max. ERD})-100}{1000} \quad \text{when} \quad 200r \leq \text{ERD} < 300r \]
\[ P_2 = \frac{700-(\text{Max. ERD})}{500} \quad \text{when} \quad 300r \leq \text{ERD} < 700r \]
\[ P_2 = 0 \quad \text{when} \quad 700r \leq \text{ERD}. \]

(B-11)

C. Mainline Program

1. Introduction

The models, previously described, are combined into a single model—a computer program. The finite difference method, particularly adaptable to computer programming, is necessary because of several functional discontinuities which preclude integrating the individual models simultaneously. In translating algebraic formulas into FORTRAN coding, a change in notation is required. To
assist in identifying corresponding terms, a Glossary of Symbols used in the programming, and the corresponding symbols found in mathematical formulation, are listed in Table B-I.

TABLE B-I

Program Symbols

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model Symbols</th>
<th>Programming Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Intensity</td>
<td>( I_0 )</td>
<td>( R )</td>
</tr>
<tr>
<td>Time of Arrival</td>
<td>( T_A )</td>
<td>( T_1 )</td>
</tr>
<tr>
<td>Time Outside</td>
<td>( T_2 )</td>
<td>( T_2 )</td>
</tr>
<tr>
<td>Allowable Dose</td>
<td>( D_4 )</td>
<td>( D )</td>
</tr>
<tr>
<td>Field Decay Constant</td>
<td>( Z )</td>
<td>( Z )</td>
</tr>
<tr>
<td>Recovery Fraction</td>
<td>( F )</td>
<td>( F )</td>
</tr>
<tr>
<td>Recovery Rate</td>
<td>( R )</td>
<td>( E )</td>
</tr>
<tr>
<td>Buildup Duration</td>
<td>( T )</td>
<td>( FTA )</td>
</tr>
<tr>
<td>Protection Factor</td>
<td>( P )</td>
<td>( P )</td>
</tr>
<tr>
<td>Protection Factor Categories</td>
<td>( K )</td>
<td>( K )</td>
</tr>
<tr>
<td>Maximum No. PF Categories</td>
<td>( K_{MAX} )</td>
<td>( K_{MAX} )</td>
</tr>
<tr>
<td>Time (the variable of integration)</td>
<td></td>
<td>( T )</td>
</tr>
<tr>
<td>Hours</td>
<td>( t )</td>
<td>( T )</td>
</tr>
<tr>
<td>Days</td>
<td>( n )</td>
<td>( N )</td>
</tr>
<tr>
<td>Total (ERD) Dose</td>
<td>ERD (n)</td>
<td>( D )</td>
</tr>
<tr>
<td>Recoverable Dose</td>
<td>( D_n )</td>
<td>( D_1 )</td>
</tr>
<tr>
<td>Nonrecoverable Dose</td>
<td>( d_n )</td>
<td>( D_2 )</td>
</tr>
<tr>
<td>Day Maximum ERD Occurred</td>
<td>( N_1 )</td>
<td>( N_1 )</td>
</tr>
<tr>
<td>Field Intensity (t)</td>
<td>( (t) )</td>
<td>RDN or FLT</td>
</tr>
<tr>
<td>Day Allowable Dose Occurred</td>
<td>( N_2 )</td>
<td>( N_2 )</td>
</tr>
<tr>
<td>Probability of Fatality</td>
<td>( F_1 )</td>
<td>( P_1 )</td>
</tr>
<tr>
<td>Probability of Noneffective</td>
<td>( F_2 )</td>
<td>( P_2 )</td>
</tr>
</tbody>
</table>

In the input sections of the program, if a programming symbol given in Table B-I is followed by L, H, or S, the number represented by the symbol is either the lowest, highest, or incremental value taken by the variable represented by the preceding letters (i.e., \( T_{AL} \) means the lowest value the time of arrival will take).
The coding statements are keyed to the flow diagram by the organization of the statement line number with the block numbers. Specifically, each of the coding line numbers are four digits; the first digit represents the major routine being calculated (see Table B-II); the second and third digits correspond to the flow diagram box number (xx); the fourth or units-position digit is "open" to allow for sequencing corrections, additions, and expansions (0).

**TABLE B-II**

<table>
<thead>
<tr>
<th>Major Program Routines</th>
<th>Code Number Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1xx0</td>
</tr>
<tr>
<td>Iterations &amp; Loops</td>
<td>2xx0</td>
</tr>
<tr>
<td>Calculation of Buildup</td>
<td></td>
</tr>
<tr>
<td>Duration in hours</td>
<td>3xx0</td>
</tr>
<tr>
<td>Initial Conditions</td>
<td>4xx0</td>
</tr>
<tr>
<td>Calculate 1st Day Doses</td>
<td>5xx0</td>
</tr>
<tr>
<td>Day by Day Doses</td>
<td>6xx0</td>
</tr>
<tr>
<td>Outcome Probabilities</td>
<td>7xx0</td>
</tr>
<tr>
<td>Formats &amp; &quot;Housekeeping&quot; Steps</td>
<td>8xx0</td>
</tr>
</tbody>
</table>

2. **Explanation of the Flow Diagrams**

The first section of the program (block numbers 1-36) initializes the input data and sets up the control loops for the variables. Block numbers 36 through 40 select the appropriate duration multipliers to calculate the duration of Particle Buildup (T3).

\[
T3 = (1.133 + ETA) \times TA \quad \text{when } Z = 1.
\]

\[
= (1.175 + ETA) \times TA \quad \text{when } Z = 1.2
\]

\[
= (1.200 + ETA) \times TA \quad \text{when } Z = 1.4
\]

the range of Buildup Duration Factor (ETA) is between zero (corresponding to the displaced sine function) and five (corresponding "th 5TA - TA function).

The program sets the initial conditions in block 47. Clock Time (T) begins at time of arrival (TA) and the Day Counter (N) begins at one, the time is incremented by 3 minutes or 0.05 hrs.
Field intensity (RDN) per unit of time is calculated in blocks 44, 45, 46 using the product of Reference Intensity (r/hr), Particle Arrival function, and Decay function.

\[
RDN = \frac{R}{2} \left[1 - \cos \left(\frac{T - TA}{T3}\right) \right] Z
\]

Limit on T3 is:

\[
0 \leq \frac{T - TA}{T3} < \pi
\]

The test, block 42, determines if there is any movement outside in fallout--or if T2 is positive.

Block 52 is similar to 47 except the field is attenuated by the protection factor. Block 53 tests to determine if buildup of particles is continuing. Block 54 increments time out of buildup phase so that field intensity is governed only by the decay function corresponding to the algebraic formula.

\[
\Theta = T^{-Z}.
\]

Block 56 tests to determine if the 1st day is completed. Block 55 increments time by 12 minutes during buildup. Blocks 49, 50, 51 are identical to 44, 45, 46 but are not linked because of a change in the time intervals.

FLD in block 59 is equivalent to RDN but is used to account for dose when the time step is by days.

\[
\phi(t) = \int_0^{24+TA} t^{-Z} dt
\]

\[
\phi(t) = \frac{t^{1-Z}}{1-Z} \left|_0^{25+TA} \right. \quad \text{when } Z \neq 1
\]

\[
\phi(t) = \frac{\left[ t^{1-Z} - (24+TA)^{1-Z} \right]}{Z-1}
\]

Block 60 does not have a time term since the integration in block 59 was over a finite time.

Blocks 64, 65, 66, 67 and 68 sets up transition from the first day, to day-by-day calculations. At blocks 44-65 total ERD division into recoverable and nonrecoverable fractions is made. Block 66 steps time by 12 hours. Block 67 sets maximum dose register. Block 69 solves equation for Field Intensity summed over 24 hours but corrected according to PF.

\[
FLD = \frac{R*T^{-Z}*(24 \text{ hrs.})}{PF(K)}
\]
Block 73 updates the recoverable dose. Block 71 accumulates the nonrecoverable dose. Block 72 tests for sure death. Block 74 tests the day's dose vs. Maximum ERD. If there is a new Maximum ERD, it is set in block 75, and the day and time are incremented in blocks 76 and 77. If the peak ERD (D3) has been passed (D < D3), then the next point of interest is whether or not the allowable level has been reached. This test is block 78. If the dose is above the allowable, block 79 tests to see if the allowable is reachable--i.e., nonrecoverable dose is less than the allowable. If the nonrecoverable dose exceeds the allowable, then the program is in a loop as the length of stay in shelter would approach infinity--therefore, the error message.

Blocks 80 through 92 calculate the probability of fatality and the probability of noneffectiveness.

The preceding explanation should give some understanding of the computer program and how it represents the mathematical formulation. This program, then, generates the possible data combinations that are analyzed in this volume.
Code Number Series 1XXO
D4 = D4 + D4S

D4 < D4H

T2 = T2 + T2S

D4 = D4L - D4S

T2 < T2H

TA = TA + TAS

T2 = T2L - T2S

TA < TAH

R = R + RS

TA = TAL - TAS

R < RH

R = RL - RS

Code Number Series 2XX0
CALCULATE
T3

36. Z < 1.1
   YES
   T3 = (1.1333 + ETA) TA

38. Z < 1.13
   NO
   T3 = (1.20 + ETA) TA

37. Z < 1.1
   NO
   T3 = (1.175 + ETA) TA

40. Z < 1.13
   YES
   T3 = (1.20 + ETA) TA

To Initial Condition

Code Number Series 3XX0
Set Initial Conditions

4

Outside

Increment Time by 3 min.
\[ T = T + .05 \]

43

\[ RDN = \frac{(T - T_A) \times 3.1416}{T_3} \]

44

\[ RDN = 1 - \cos(RDN) \]

45

\[ RDN = \frac{R \times RDN \times T}{2.0} \]

46

\[ D = D + RDN \times .05 \]

47

\[ T < T_2 + T \]

42

YES

NO

To In Shelter

4

\[ \text{SET} \]

Total ERD \( D = 0 \).
Recoverable dose \( D_1 = 0 \).
Non-rec. dose \( D_2 = 0 \).
Max. ERD \( D_3 = 0 \).
Day under calculation \( N = 1 \).
Day max. ERD occurred \( N_1 = 1 \).
Day ERD return to allowable \( N_2 = 1 \).
Time in hrs. \( T = T_A \).

Code Number Series 4XX0
In Shelter 1st Day

STEP TIME by 6 min.
\[ T = T + .1 \]

RDN = \( \frac{(T - TA) \times 3.1416}{T3} \)

RDN = \( .1 - \cos (RDN) \)

RDN = \( \frac{R \times RDN \times T^{- z}}{2} \)

\[ D = D + \frac{RDN \times .2}{PF(K)} \]

\[ T = T + .1 \]

Finish Time

Complete 1st Day

\[ Z = 1.0 \]

\[ FLD = \log (24. / T) \]

\[ D = D + \frac{1 - z - (TA + .24) \times 1 - Z}{Z - 1} \]

\[ D = D + \frac{R \times FLD}{PF(K)} \]

\[ D < 700 \]

Reset \( N1 = N \)

Day/Day

Reset \( N2 = -1.0 \)

To Death

Code Number Series 5XX0
7

Outcome Prob's.

KODE = 1

Error in non-rec > allow

N2 = N

D3 < 100

Calc. P2 = 0.0

D3 < 200

Calc. P2 = \frac{D3-100}{200}

D3 < 300

Calc. P2 = \frac{3*D3-100}{1000}

D3 < 700

Calc. P2 = \frac{700-D3}{500}

Set P1 = 0.0

Set P1 = 1.0

To Output

Code Number Series 7XX0
4. FORTRAN Listing

PROGRAM MAINLINE
DIMENSION PFN(10)
N=2
KODE=0
READ INPUT TAPE 5,1000,RL,RH,RS
READ INPUT TAPE 5,1000,TAL,TAH,TAS
READ INPUT TAPE 5,1000,T2L,T2H,T2S
READ INPUT TAPE 5,1000,D4L,D4H,D4S
READ INPUT TAPE 5,1000,ZL,ZH,ZS
READ INPUT TAPE 5,1000,FL,FM,FS
READ INPUT TAPE 5,1000,BL,BH,BS
READ INPUT TAPE 5,1000,ETAL,ETAH,ETAS
READ INPUT TAPE 5,1001,PFN(1),PFN(2),PFN(3),PFN(4),PFN(5),PFN(6),
PFN(7),PFN(8),PFN(9)
READ INPUT TAPE 5,1002,KMAX
1000 FORMAT(3F15.5)
1001 FORMAT(9F6.0)
1002 FORMAT(110)
WRITE OUTPUT TAPE 6,8933
8933 FORMAT(115H
1 ETA K P1 P2 D3 N1 D N D2
2110 R=RL-RS
2120 IF(K-RH)2130,2130,8820
2130 K=R+RS
2140 TA=TAL-TAS
2150 IF(TA-TAH)2160,2160,2120
2160 TA=TA+TAS
2170 T2=T2L-T2S
2180 IF(T2-T2H)2190,2190,2150
2190 T2=T2L+T2S
2200 D4=D4L-D4S
2210 IF(D4-D4H)2220,2220,2180
2220 D4=D4+D4S
2230 Z=ZL-ZS
2240 IF(Z-ZH)2250,2250,2210
2250 Z=Z+ZS
2270 F=FL-FS
2280 IF(F-FH)2280,2280,2240
2280 F=F+FS
2300 IF(B-BH)2310,2310,2270
2310 B=B+BS
ETA=ETAL-ETAS
2330 IF(ETA-ETAH)2340,2340,2300
2340 ETA=ETA+ETAS
2350 DO 9000 K=1,KMAX
3360 IF(Z-1.1)3370,3380,3380
3370 T3=(1.1333+ETA)*TA
GO TO 4000
3380 IF(Z-1.13)3390,3400,3400
3390 T3=(1.175+ETA)*TA
GO TO 4000
4000 CONTINUE
4410 D=0.
D1=0.
D2=0.
D3=0.
N=1
N1=1
N2=1
ZZ=-2

T=TA

4420 IF(T-(T2+TA))4430,5480,5480
4430 T=T+0.05
4440 RDN=(T-TA)*3.1416/T3
4450 RDN=1.0-COSF(RDN)
4460 RDN=(R*RDN)*((T*TZ)/2.0
4470 D=D+(RDN*0.05)
GO TO 4420
5480 T=T+0.1
5490 RDN=(T-TA)*3.1416/T3
5500 RDN=1.0-COSF(RDN)
5510 RDN=(R*RDN)*((T*TZ)/2.0
5520 D=D+(RDN*0.2/PFN(K))
5530 IF(T-(TA+T3))5550,5550,5550
5550 T=T+0.2
GO TO 5490
5540 T=T+0.1
IF(T-(TA+24.0))5570,5610,5610
5570 IF(Z-1.0)5590,5590,5590
5580 ARG=24.0/T
FLD=LOGF(ARG)
GO TO 5600
5590 ZZZ=1.0-Z
FLD=((T*TZ))-((TA+24.0)*ZZZ)/(Z-1.0)
5600 D=D+(FLD*R/PFN(K))
5610 IF(D=700.,6000,9000,9000
6000 CONTINUE
6640 D1=F+D
6650 D2=(1.0-F)*D
6660 T=T+12.0
6670 D3=D
6680 N=N+1
6690 FLD=(R*24.0)*((T*TZ)/PFN(K)
6700 D1=(1.-B1)*D1*(F*FLD)
6710 D2=D2+(1.0-F)*FLD
6720 D=D1+D2
6730 IF(D=700.,6740,9000,9000
6740 IF(D-D3)6750,6750,6750
6750 D3=D
6760 N1=N
6770 T=T+24.0
GO TO 6680
6780 IF(D-D4)7000,6790,6790
6790 IF(D-D2-D4)16770,7710,7710
7710 CODE=1
7000 CONTINUE
7800 N2=N
7810 IF(D3-100.,19000,7820,7820
7820 IF(D3-200.,1780,7830,7830
7830 IF(D3-300.,17890,7840,7840

B-24
7840 IF(U3-700.,17900,9000,9000)
7880 P2=(103-100.)/100.
7910 P1=0.
GO TO 8000
7890 P2=(13.0*D3)-100.}/1000.
GO TO 7920
7900 P2=(1700.-D3)/500.
7920 P1=(03-200.)/500.
8000 CONTINUE
8950 WRITE OUTPUT TAPE 6,8934,KODE,TA,T1,T2,T3,T4,T5,T6,T7,T8,T9
9000 CONTINUE
9260 KODE=0
GO TO 2330
8820 CONTINUE
END SENSprog
END
FINIS
This sensitivity analysis employs mathematical models which estimate the total dose, maximum ERD, and probability of casualty or fatality for an individual exposed to a particular radiological environment. The objective of the analysis is to determine the relative importance of the parameters normally employed in evaluating Civil Defense systems, i.e., to determine those parameters which, through large variance or inaccurate estimates, will contribute most to erroneous evaluations of CD systems. The sensitivity of the dose (or probability of casualty) to variations in the input parameters defining the radiological environment is examined. The parameters examined are reference intensity, time of arrival, time outside in fallout, radiation decay exponent, ERD recovery fraction, ERD recovery rate, duration of fallout buildup, and protection factor. Sensitivity indices are calculated for each parameter. The sensitivity index is defined as the fractional change in dose (or probability of casualty or fatality) divided by the corresponding fractional change in the input variable. It is concluded that dose and casualty computations are quite sensitive to errors in the field decay exponent, and they remain sensitive over the examined range. Sensitivity to variations in fallout reference intensity and protection factor are high over the whole range of parameter values. Sensitivity of time of arrival of fallout can be quite high for the lower values of the parameter. Sensitivity of dose and casualty computations to the remaining parameters is low in most cases of interest. Expansion of the sensitivity analysis to include parameters other than fallout, which define the total casualties from a given attack on the United States is necessary before further conclusions concerning a national vulnerability analysis can be drawn. (U)
Civil Defense Systems  
Sensitivity Analysis  
Radioactive Fallout  
Models  
Programming (Computers)  
Protection Factor  
Radiological Dosage

**UNCLASSIFIED**  
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