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# ACOUSTIC SPREADING LOSS IN AN INHOMOGENEOUS MEDIUM

by

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Computation and Analysis Laboratory

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## TABLE OF CONTENTS

| | Page |
|---|---|
| ABSTRACT | ii |
| FOREWORD | iii |
| INTRODUCTION | 1 |
| I. Discussion of the Problem | 2 |
| II. Intensity Equations | 3 |
| A. Expression for Spreading Loss $N_s$ | 3 |
| B. Expressions for Y and Z | 8 |
| C. Initial Conditions | 9 |
| III. Application to a Two-dimensional Heat Source in an Ocean | 10 |
| A. Discussion of the Results | 11 |
| IV. Conclusions | 13 |
| V. Acknowledgment | 14 |

Appendices:

A. General Transformation of a Function with Independent Variables $(t, \theta_0)$ to $(s, \theta_0)$

B. An Approximate Formula for the Point of Origin of the Caustic Formed by a Heat Sink

C. References

D. Figures 1 - 7

E. Distribution

## ABSTRACT

A set of differential equations is derived for calculating the intensity loss due to geometrical spreading of planar sound rays in an inhomogeneous medium with a continuous index of refraction varying both in depth and horizontally. They are solved numerically for a mathematical model of an ocean with a heat source. Intensity contours are presented for rays in the vicinity of a heat source and a heat sink. The heat sink causes the rays to converge which results in the formation of a caustic. An approximate formula is derived for the point where this caustic originates.

## FOREWORD

This work was conducted under the Naval Weapons Laboratory Foundational Research Program. It was performed in the Computation and Analysis Laboratory as part of project (R360FR103/2101/R01101001).

The date of completion was 1 September 1965.

Approved for Release:

/s/ BERNARD SMITH
Technical Director

## INTRODUCTION

In two recent papers[1,2] a completely general technique for tracing sound rays through an inhomogeneous medium, together with some practical applications, were presented. These techniques were developed and used by the U. S. Naval Weapons Laboratory to study the effect of large-scale inhomogeneities on sound-ray patterns in the ocean. Before then the tracing of sound rays was usually restricted to media in which the speed of sound could be assumed to be a function of one variable only. In the case of the ocean this variable is the depth. In Ref. 1 two mathematical models of an ocean were constructed in which the index of refraction was a function of two space variables. One of these models represents a heat source, or heat sink, superposed on an otherwise homogeneous ocean. The other is a model of a thermal mixing zone designed to approximate the acoustical features of the Gulf Stream. With the aid of a high-speed digital computer (IBM 7030) the general eikonal equation of geometrical acoustics was solved, and sound rays were traced for various positions of a point sound-source for both of these models. The results showed that acoustic inhomogeneities of an ocean in the horizontal direction, such as presented by the Gulf Stream, or large circulating eddies with strong temperature gradients, can give a substantially different sound-ray pattern from that which

one would get if the index of refraction were assumed to vary with depth only. Among the results for the Gulf Stream model two were most conspicuous: (1) the ranges of surface shadow zones were found to be in error up to 50%, and (2) new shadow zones were found, which could not be predicted with a one-dimensional speed-of-sound gradient.

## I. DISCUSSION OF THE PROBLEM

It is the purpose of this paper to complement the results of the heat-source model by calculating the intensity loss due to geometrical spreading of the sound rays. The basic principle for calculating intensities in geometrical acoustics is simple: one considers a small cone of rays emanating from a point-source of sound. The total acoustic power transmitted along this cone is assumed to be constant. It then follows that the intensity at any point along the rays of that cone is inversely proportional to the cross-sectional area of the cone at that point. This will account correctly for the change in intensity which is due to the spreading or convergence of the rays. A further diminution of intensity will result in general due to dissipation, scattering, and other attenuation effects. This paper is concerned only with the former effect. The latter can be handled most effectively with a semi-empirical expression for an attenuation coefficient. For the

ocean such a coefficient can be found in Ref. 3.

Although the physical idea as outlined above for the calculation of spreading loss is exceedingly simple, its translation into an analytical expression for the change of the cross-sectional area of a bundle of rays along the cone has been accomplished so far for media with a speed of sound varying in only one space direction. A recent exception is Ref. 4, where a formal expression is derived for a general medium. Intensity calculations, therefore, were limited heretofore to media varying in only one space direction. Derivations of this can be found in any elementary textbook; see, for instance, Refs. 5 and 6. All such derivations express the intensity in terms of functions which depend on the explicit knowledge of the equations for the ray paths, and do not lend themselves readily for an extension to media where the speed of sound depends on more than one space variable.

G. Anderson et al[7] have recently presented a new approach, but still restricted to one-dimensional, continuous speed-of-sound profiles. They derive a set of differential equations for the functions appearing in the expression for intensity, so that an explicit knowledge of the ray equations is not necessary. We borrow their idea and extend it to include cases where the speed

of sound depends on two variables, restricted to those rays which always remain in one plane. This is just sufficient for the applications we wish to consider here, namely, the mathematical model of an ocean with a heat source in Ref. 1.

## II. INTENSITY EQUATIONS

### A. Expression for Spreading Loss $N_s$

In a medium with index of refraction n, the acoustic ray paths are given by the solutions of the eikonal equation, which in vector form can be written as[5,6]

$$\frac{d}{ds}(n\underline{r}') = \nabla n \quad , \tag{1}$$

with initial conditions

$$s = 0 : \quad \underline{r}(0), \ \underline{r}'(0) \quad . \tag{2}$$

Here s is the independent variable arc length, $\underline{r} = \underline{r}(s)$ is the position vector of the ray as a function of s, and $\underline{r}' = d\underline{r}/ds$ is the unit tangent vector. In a Cartesian coordinate system (x, y, z), for a medium in which n = n(y, z) is a function of y and z only, and for rays which start out initially in the y-z plane, Eq. (1) takes the form:

$$x = 0, \tag{3a}$$

$$y'' + b(y')^2 + cy'z' = b \quad , \tag{3b}$$

$$z'' + by'z' + c(z')^2 = c \quad , \tag{3c}$$

with

$$b = \frac{1}{n}\frac{\partial n}{\partial y} \ ; \ c = \frac{1}{n}\frac{\partial n}{\partial z} \quad . \tag{3d}$$

The spreading loss $N_s$, in db, is defined by

$$N_s = 10 \log_{10} |I_o/I| \quad , \tag{4}$$

where I is the intensity at a general point along a given ray, and $I_o$ is the intensity at unit distance from the source. For n = n(z) only, a suitable expression for $N_s$ has been derived by Anderson et al (Ref. 7, Eq. 5b), which in the present notation reads

$$N_s = 10 \log \left| \frac{y - y_o}{\cos\theta_o \; y'} Z_t \right| \quad . \tag{5}$$

In this equation $\theta_o$ is the initial inclination of the ray, and $Z_t = (\partial z/\partial\theta_o)_t$ where the subscript t indicates that the variable time is to be held constant in the partial differentiation. In this paper the appropriate independent variables are s and $\theta_o$. Equation (5), however, was derived in Ref. 7 with the time t and $\theta_o$ as independent variables. A straightforward transformation, (see appendix A, Eq. (A14)), from (t, $\theta_o$) to (s, $\theta_o$) as independent variables brings Eq. (5) into the following form:

$$N_s = 10 \log \left| \frac{(y - y_o)(y'Z - z'Y)}{\cos\theta_o'} \right| \quad , \tag{6}$$

where Y and Z are the corresponding partial derivatives with s held fixed:

$$Y = \left(\frac{\partial y}{\partial\theta_o}\right)_s \qquad Z = \left(\frac{\partial z}{\partial\theta_o}\right)_s \quad . \tag{7}$$

We wish to show now that Eq. (6) is also suitable within a good approximation for a two-parameter index of refraction

$n = n(y, z)$, for rays which remain in the y-z plane. To do this most clearly it is desirable to review briefly the derivation of Eq. (5) as given in Ref. 7. Figure 1 shows the geometry of a typical ray bundle determined by the angles $\Delta\theta_0$ and $\Delta\alpha_0$. The only difference between this figure and the corresponding figure in Ref. 7 is that $\Delta\alpha$ is not a constant any more. At the point 0, unit distance away from the source $(y_0, z_0)$, the cross-sectional area of this bundle is given by $\Delta A_0 = \cos\theta_0 \Delta\theta_0 \Delta\alpha_0$ . At the general point $P(y, z)$ the corresponding area is $\Delta A = r\Delta\alpha W \Delta\theta_0$ , where $W\Delta\theta_0 = Z_t \Delta\theta_0/\cos\theta$ is the length of the arc $\widehat{PP'}$. Note that $Z_t = (\partial z/\partial\theta_0)_t$ is the partial derivative evaluated at constant t. This must be so, since the cross-sectional area by definition must be congruent with a wavefront, which is a surface of constant phase. The ratio of the intensities in Eq. (4) now becomes:

$$\frac{I_0}{I} = \frac{\Delta A}{\Delta A_0} = \frac{r\ Z_t}{\cos\theta_0 \cos\theta} \frac{\Delta\alpha}{\Delta\alpha_0} \quad . \tag{8}$$

When $\Delta\alpha = \Delta\alpha_0$ remains constant, $r = y - y_0$, and Eq. (8) yields the result of Eq. (5). For the more general case we must now evaluate r and $\Delta\alpha/\Delta\alpha_0$ .

To get an expression for $\Delta\alpha$ in terms of $\Delta\alpha_0$ we take advantage of the fact that the index of refraction n does not depend on x.

The x-component of Eq. (1), therefore, yields the simple relation

$$nx' = n(0)x'(0) = \text{const} \quad . \tag{9}$$

Referring to Fig. 1b one sees that $x' = \sin(\frac{1}{2}\Delta\alpha)$, since $x'$ is the direction cosine of the ray with respect to the x-axis. The index of refraction can be defined to be unity at the source $s = 0$: $n(0) = 1$. It follows from Eq. (9) that

$$\sin(\tfrac{1}{2}\Delta\alpha)/\sin\tfrac{1}{2}(\Delta\alpha_0) = 1/n \quad . \tag{10}$$

In the limit $\Delta\alpha_0 \to 0$, $\Delta\alpha \to 0$ this reduces to

$$d\alpha/d\alpha_0 = 1/n \quad . \tag{11}$$

Considering Fig. 1b again, one sees the following relations between r, y, p and $\ell$:

$$(y - y_0)\Delta\alpha_0 + \ell \simeq r\Delta\alpha \quad , \tag{12a}$$

$$\ell \simeq p(\Delta\alpha - \Delta\alpha_0) \quad . \tag{12b}$$

Substituting Eq. (12b) into Eq. (12a) and using Eq. (11), we get

$$r \simeq (y - y_0)n + p(1 - n) \quad . \tag{13}$$

It is clear from the geometry of Fig. 1b that in the limit as $\Delta\alpha_0 \to 0$, p in general may remain of the order of r. The index of refraction n, however, is a slowly varying function about the value $n = 1$. For the model considered in this paper n remains within the range $0.94 < n < 1.06$. It is a very good approximation, therefore, to neglect the last term in Eq. (13). In that case Eqs. (8), (11) and (13) combine to give an expression for $N_s$ which is identical with Eq. (5).

A more rigorous expression for r can be obtained. Consider the point (x,y) in Fig.1b. An infinitesimal extension of the ray path at this point satisfies the relation $dx = \tan(\frac{1}{2}\Delta\alpha)dy \simeq \frac{1}{2}\Delta\alpha dy = (\Delta\alpha_0/2n)dy$. Also $r = x/\frac{1}{2}(\Delta\alpha) = nx/\frac{1}{2}(\Delta\alpha_0)$. In the limit these expressions become exact, and yield

$$r = n \int_{y_0}^{y} \frac{dy}{n} \quad . \tag{14}$$

We see then that the approximation made in the last paragraph is equivalent to replacing n in the integrand of Eq. (14) by its mean value, unity. Equation (14) substituted into Eq. (8) would give an exact result for $N_s$. However, in view of the good approximation, the much simpler Eq. (6) was used for the succeeding numerical calculations. A very conservative estimate shows that the results cannot be in error by more than ±0.2 db. By the same method it is easy to show that in the limit $\alpha_0 \to 0$, p assumes the following value

$$\lim_{\alpha_0 \to 0} p = \frac{n\left[y - \int \frac{dy}{n}\right]}{n - 1} \quad , \tag{15}$$

B. Expressions for Y and Z

There remains now to find suitable expressions for the partial derivatives Y and Z which appear in Eq. (6). From the identities $y' = \cos\theta$, and $z' = \sin\theta$, one obtains by differentiation with respect

to $\theta_0$, and an interchange of the $\theta_0$ and s derivatives

$$Y' = -z'\Theta \quad , \qquad Z' = y'\Theta \quad ; \tag{16}$$

where:

$$\Theta = \left(\frac{\partial\theta}{\partial\theta_0}\right)_s \quad , \tag{17}$$

To obtain the differential equation for $\Theta$ consider the eikonal equations (3b) and (3c). They can be rewritten, using the identity $(y')^2 + (z')^2 = 1$:

$$y'' = -(cy' - bz')z' \quad , \tag{18a}$$

$$z'' = (cy' - bz')y' \quad . \tag{18b}$$

The curvature of a ray, which in general is given by $\varkappa = \pm[(y'')^2 + (z'')^2]^{\frac{1}{2}}$, follows immediately:

$$\varkappa = \left(\frac{\partial\theta}{\partial s}\right)_{\theta_0} = \pm(cy' - bz') \quad . \tag{19}$$

By examining all possible cases one can readily verify that the lower sign in Eq. (19) is extraneous. Differentiating Eq. (19) with respect to $\theta_0$, and using the relations in Eq. (16), one obtains the equation which determines $\Theta$ :

$$\Theta' = (C - b\Theta)y' - (B + c\Theta)z' \quad , \tag{20}$$

where B and C are given by:

$$B = (\partial b/\partial\theta_0)_s = (\partial b/\partial y)Y + (\partial b/\partial z)Z \quad , \tag{21a}$$

$$C = (\partial c/\partial\theta_0)_s = (\partial c/\partial y)Y + (\partial c/\partial z)Z \quad . \tag{21b}$$

Equations (16) and (20) constitute a set of three simultaneous first-order differential equations, which for any given index of refraction n(y, z) can be integrated together with the eikonal Eqs. (3). It will be noticed that in order to calculate the spreading loss, $N_s$ in Eq. (6), only Y and Z are needed. Thus, one could easily eliminate $\Theta$ from the set of equations (16) and (20), reducing it to one second-order and one first-order equation. For numerical integrations, however, the present form is more convenient.

C. Initial Conditions

The initial conditions for the ray-trace system (3) are given by Eq. (2). At s = 0

$$\begin{aligned} y(0) &= y_0 \quad , \\ z(0) &= z_0 \quad , \\ \theta(0) &= \theta_0 \quad . \end{aligned} \tag{22}$$

Differentiation with respect to $\theta_0$ yields the initial conditions for the system of differential equations (16) and (20):

$$\begin{aligned} Y(0) &= Z(0) = 0 \quad , \\ \Theta(0) &= 1 \quad . \end{aligned} \tag{23}$$

These are identical with the corresponding initial conditions in Ref. 7. They are sufficient for a boundless medium with a continuous index of refraction. If the medium has a reflecting surface, or some other discontinuities such as interfaces where the

gradient of the speed-of-sound is not continuous, further conditions at such boundaries are necessary. These will be discussed in a forthcoming paper. For the purposes of this work, the theory presented so far is sufficient, since it will be applied to a medium without any reflecting surfaces and with an index of refraction which is everywhere continuous.

## III. APPLICATION TO A TWO-DIMENSIONAL HEAT SOURCE IN AN OCEAN

The preceding equations are now applied to the calculation of spreading loss in a horizontal plane of an ocean with a heat source. The quantitative description of this model together with actual ray paths are presented in Ref. 1 and 8. Here follows a brief summary.

The heat source is located at the origin, $r = 0$. The index of refraction is given by

$$n(y, z) = 1 + ke^{-r^2} \tag{24}$$

where $r^2 = y^2 + z^2$ in normalized coordinates, and k is a measure of the source strength, which is proportional to the temperature difference between the heat source and the ocean at a large distance from the origin. Numerical calculations with the Runge-Kutta method were made for two values of k, both corresponding to a temperature difference of 20°C:

$k = -0.0631$ : heat source,

$k = +0.0631$ : heat sink.

The point-source of sound was taken for both cases to be ten units from the heat source (sink). Numerical calculations were carried out within the ranges $-10 \leq y \leq 20$, $-4 \leq z \leq 4$. It was found that an integration step $\Delta s = 0.1$ yielded sufficient accuracy.

A. Discussion of the Results

Figures 2 and 3 show a few selected ray paths in the vicinity of a heat source and sink, respectively. It is seen that a heat source (sink) causes the rays to diverge (converge), but nothing more quantitative about the intensity can be inferred from these figures.

The intensity field can be presented in several ways. In its original form one gets the spreading loss $N_s$ from the solutions of Eqs. (6), (16) and (20) at every integration point $s_n$ on a given ray. This, however, is not the most useful form for presentation purposes. We chose to display steady state intensity contours instead. This required covering the whole (y-z) plane with a sufficient density of rays in order to be able to interpolate between these rays. For the present calculations 23 rays were used with the following initial angles: $\theta_0 = 0°$, $15'$, $30'$, $45'$, $1°$, $1°30'$, $2°$, $2°30'$, $3°$, $3°30'$, $4°$, $4°30'$, $5°$, $5°30'$, $6°$, $8°$, $10°$, $15°$, $20°$, $30°$, $45°$, $60°$, and $80°$. Only a few of these are shown in Figs. 2 and 3. At points where two rays intersect, as in Fig. 3,

one gets two different values for $N_s$. In reality, of course, the intensity at such points is uniquely determined by interference effects. One should, therefore, take into account the phase difference of the two rays at that point. A suitable method for doing this is indicated in Ref. 9. For steady-state conditions, however, one of the values of $N_s$ will usually be much smaller than the other if the two rays traverse different paths. So in our calculations we retained only the smaller value of $N_s$ at such points. An exception arises if the point of intersection lies on an envelope. In such a case the two paths become congruent in a limit. But this, of course, is precisely the condition for the formation of a caustic[6], where the intensity becomes infinite, $N_s = -\infty$.

The calculated intensity contours around a heat source and heat sink are shown in Figs. 4 and 5. The location of the heat source (sink) is marked by an asterisk. The point sound-source is off the figure at (-10, 0). Up to y = - 5 the contours are essentially circular, which implies spherical spreading. In the vicinity of the heat source (sink) and beyond, however, they become progressively more distorted. Figure 4 shows typical intensity contours produced by the heat source which diverges the rays in its vicinity. In Figure 5, however, the rays converge as they pass by the heat sink. This produces a caustic which is indicated by the heavy,

dashed line. It branches out on both sides of the y-axis beginning at approximately y = 9.5. The point on the y-axis where the caustic starts can be given approximately in closed form as a function of k and the distance L between the point-source of sound and the heat sink. This is done in appendix B with the result:

$$y_c = \frac{L}{2\sqrt{\pi}\, kL - 1} \quad . \tag{25}$$

For Figure 5, L = 10, Equation (25) predicts the value $y_c = 8.084$, which agrees fairly well with the calculated result. Equation (25) also shows that for a given sink-strength k there exists a minimum distance between the sound source and the heat sink for the formation of a caustic. Since $y_c$ must be positive, it follows that $L > 1/2\sqrt{\pi}\, k$ in order that a caustic may form.

## IV. CONCLUSIONS

Equations (3), (6), (16) and (20) are sufficient for calculating acoustic spreading losses for planar rays in an unbounded medium with a continuous index of refraction varying in two space directions. For a medium bounded by reflecting surfaces, and/or by interfaces across which the gradient of the index of refraction changes discontinuously, further boundary conditions are necessary to supplement these equations. This will be done in a future report, and the results will be applied to calculate intensity

contours for the mathematical model of the Gulf Stream described in Refs. 1 and 8.

## V. ACKNOWLEDGMENT

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# APPENDIX A

## APPENDIX A

### General Transformation of a Function with Independent Variables $(t, \theta_0)$ to $(s, \theta_0)$

In going from Eq. (5) to Eq. (6) it was necessary to transform the set of independent variables from $(t, \theta_0)$ to $(s, \theta_0)$. Specifically, it was required to have the partial derivative $Z_t = (\partial z/\partial\theta_0)_t$ at constant time in terms of corresponding derivatives at constant arc length s.

Let f be any function on a ray path $\theta_0$. It can be expressed either as a function of s and $\theta_0$, or as a function of t and $\theta_0$: $f(s, \theta_0)$ or $f(t, \theta_0)$. A general differential can then be written:

$$df = f'_{\theta_0} ds + F_s d\theta_0 = \left(\frac{\partial f}{\partial t}\right)_{\theta_0} dt + F_t d\theta_0 \quad . \tag{A1}$$

Here primes indicate differentiation with respect to s, capital letters indicate differentiation with respect to $\theta_0$, e. g.: $F_s = (\partial f/\partial\theta_0)_s$ , and the subscripts indicate the variable which is held fixed. At constant t the relation (A1) becomes:

$$F_t - F_s = f'_{\theta_0}\left(\frac{\partial s}{\partial\theta_0}\right)_t \quad . \tag{A2}$$

To obtain a suitable expression for $(\partial s/\partial\theta_0)_t$ consider s as a function of y and z:

$$s = s(y(\theta_0,t) \ , \ z(\theta_0,t)) \quad . \tag{A3}$$

Differentiating (A3) with respect to $\theta_0$ we get:

$$\left(\frac{\partial s}{\partial \theta_0}\right)_t = AY_t + BZ_t \quad , \tag{A4}$$

with the coefficients

$$A = \left(\frac{\partial s}{\partial y}\right)_z \quad ; \qquad B = \left(\frac{\partial s}{\partial z}\right)_y \quad . \tag{A5}$$

Again consider s with the following functional relationship:

$$s = s(y(\theta_0,s), \quad z(\theta_0,s)). \tag{A6}$$

Differentiation of (A6) with respect to s and $\theta_0$ yields:

$$1 = A\left(\frac{\partial y}{\partial s}\right)_{\theta_0} + B\left(\frac{\partial z}{\partial s}\right)_{\theta_0} \quad . \tag{A7}$$

$$0 = AY_s + BZ_s \quad . \tag{A8}$$

The coefficients of A and B in Eq. (A7) are just the direction cosines of the ray, $y' = \cos\theta$ and $z' = \sin\theta$, respectively. The solution of Eqs. (A7) and (A8) gives

$$A = -\frac{Z_s}{z'Y_s - y'Z_s} \qquad B = \frac{Y_s}{z'Y_s - y'Z_s} \quad . \tag{A9}$$

Thus Eq. (A4) becomes:

$$\left(\frac{\partial s}{\partial \theta_0}\right)_t = \frac{Y_s Z_t - Z_s Y_t}{z'Y_s - y'Z_s} \quad . \tag{A10}$$

Going back to Eq. (A2), let f be successively y and z. The results are:

$$\begin{aligned} Y_t &= Y_s + \frac{y'}{z'}(Z_t - Z_s) \quad , \\ Z_t &= Z_s + \frac{z'}{y'}(Y_t - Y_s) \quad . \end{aligned} \tag{A11}$$

Equations (A11) can be shown to be not independent, so that they are not sufficient to solve for $Z_t$ in terms of $Y_s$ and $Z_s$. An additional, independent relation between $Y_t$ and $Z_t$ is required. This relation is provided by the fact that t=const. describes surfaces of constant phase (wave fronts) which are normal to the rays. One such surface together with the position vector of the ray $\underline{r}$, its direction $\underline{r}' = (\partial r/\partial s)_{\theta_o}$, and the vector $\underline{R}_t d\theta_o$, are shown in Fig. 6, where

$$\underline{R}_t = \left(\frac{\partial \underline{r}}{\partial \theta_o}\right)_t = Y_t \hat{\underline{j}} + Z_t \hat{\underline{k}} \quad . \tag{A12}$$

The requirement that $\underline{R}_t$ be perpendicular to $\underline{r}'$ can be expressed as

$$\underline{R}_t \cdot \underline{r}' = Y_t y' + Z_t z' = 0 \quad . \tag{A13}$$

With this, Eqs. (A11) can finally be solved for $Z_t$ in terms of $Y_s$ and $Z_s$:

$$Z_t = y'(y' Z_s - z' Y_s) \quad . \tag{A14}$$

This is the expression which was used to transform Eq. (5) into Eq. (6).

# APPENDIX B

## APPENDIX B

### An Approximate Formula for the Point of Origin of the Caustic formed by a Heat Sink

In this section an approximate expression (Eq. (25)) will be derived from $y_c$ -- the point along the y-axis where a caustic is formed for sound rays in the vicinity of a heat sink.

A caustic[6] is defined in general as an envelope of the rays. Thus we wish to find the point on the y-axis in Fig. 3 where such an envelope starts. Specifically, we wish to determine the limiting point of intersection of the y-axis (0° ray), with a ray which starts out at an angle $\alpha$, in the limit $\alpha \to 0$. The situation is shown in Fig. 7. The equations of the ray paths are not known analytically. It is known, however,[1] that asymptotically, far away from the sink the rays are straight lines. It is a reasonably good approximation, therefore, to represent the rays by the two linear segments which coincide with their asymptotes. In Fig. 7 these are shown as the two straight lines AB and BC. We thus need

$$y_c = \lim_{\alpha,\ \eta \to 0} L' \quad . \tag{B1}$$

The total angle of deviation $\theta$ is derived under the same approximation in Ref. 1 (Eq. 22)), where it is found to be

$$\theta = 2\sqrt{\pi}\, k\eta e^{-\eta^2} \quad , \tag{B2}$$

where $\eta$ is the normal distance between the asymptote and the heat sink. From the two triangles in Fig. 7 the following relations are readily established:

$$\sin \alpha = \eta/L \quad , \tag{B3}$$

$$L' = \frac{\eta \sin \gamma}{\sin \beta} = \frac{\eta \cos \theta}{\sin(\theta - \alpha)} . \tag{B4}$$

Applying L'Hospital's rule to Eq. (B4) we find:

$$y_c = \lim_{\eta \to 0} \left[ \frac{\cos\theta - \eta \sin\theta (\partial\theta/\partial\eta)}{\cos(\theta-\alpha)(\partial\theta/\partial\eta - \partial\alpha/\partial\eta)} \right] \quad . \tag{B5}$$

From (B2) and (B3) one gets:

$$\lim_{\eta \to 0} \left( \frac{\partial\theta}{\partial\eta} \right) = 2\sqrt{\pi}\, k \quad , \tag{B6}$$

$$\lim_{\eta \to 0} \left( \frac{\partial\alpha}{\partial\eta} \right) = \frac{1}{L} \quad . \tag{B7}$$

Evaluation of the limit in Eq. (B5) with these values yields Eq. (25) in the text:

$$y_c = \frac{L}{2\sqrt{\pi}\, kL - 1} \quad . \tag{B8}$$

# APPENDIX C

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# APPENDIX D

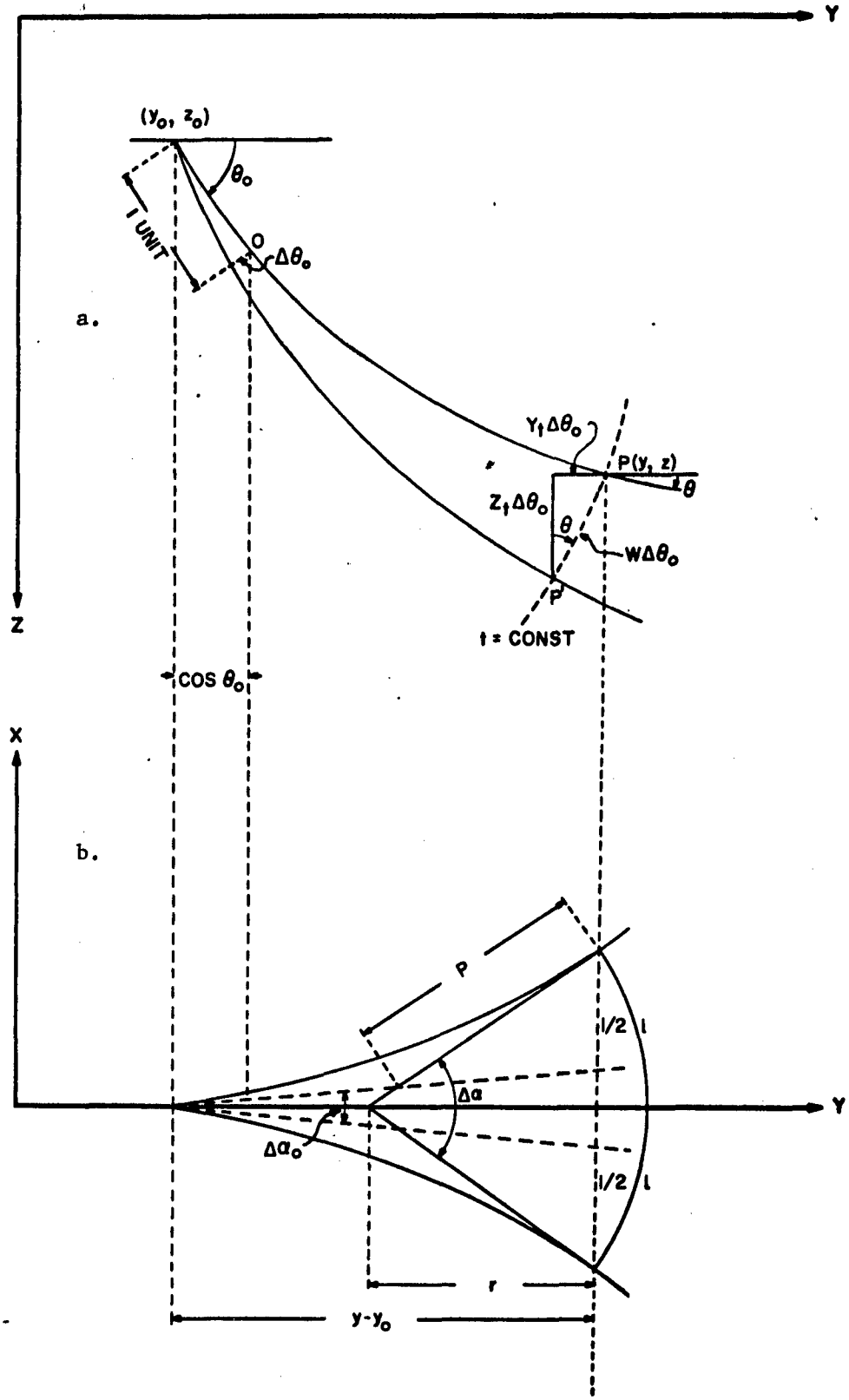


FIG. 1: a. Side View, and b. Top View of an Infinitessimal Ray Bundle

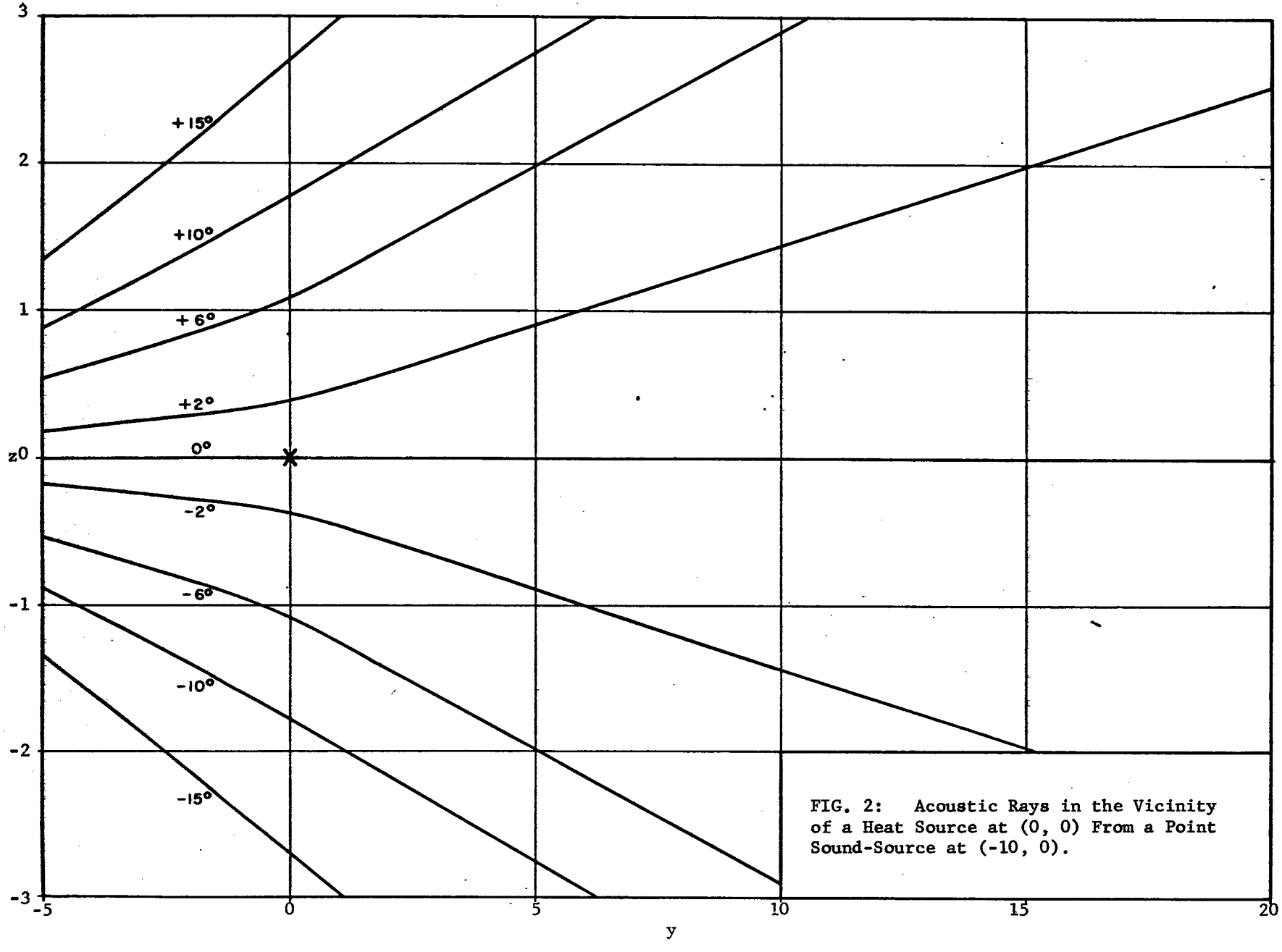


FIG. 2: Acoustic Rays in the Vicinity of a Heat Source at (0, 0) From a Point Sound-Source at (-10, 0).

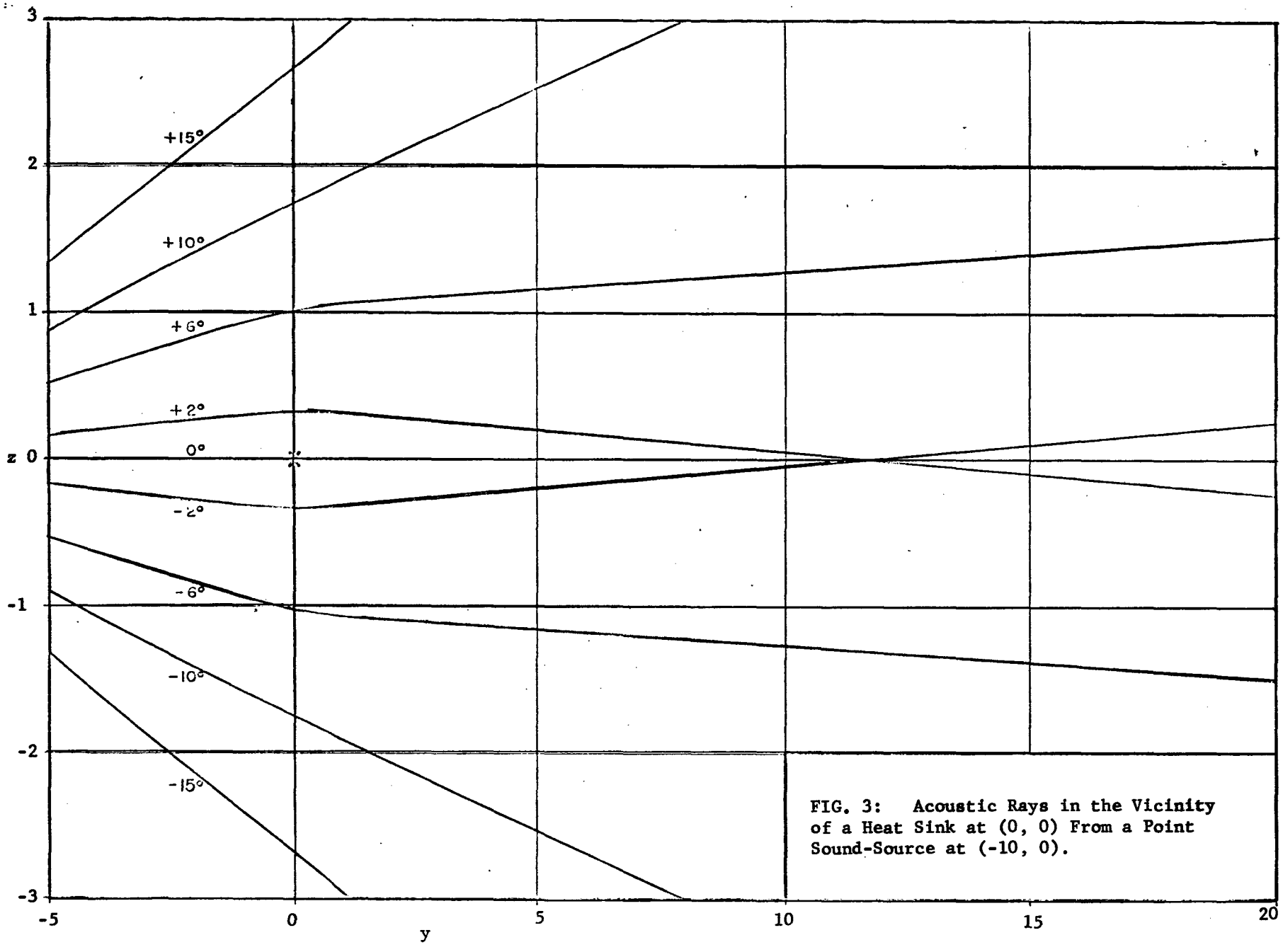


FIG. 3: Acoustic Rays in the Vicinity of a Heat Sink at (0, 0) From a Point Sound-Source at (-10, 0).

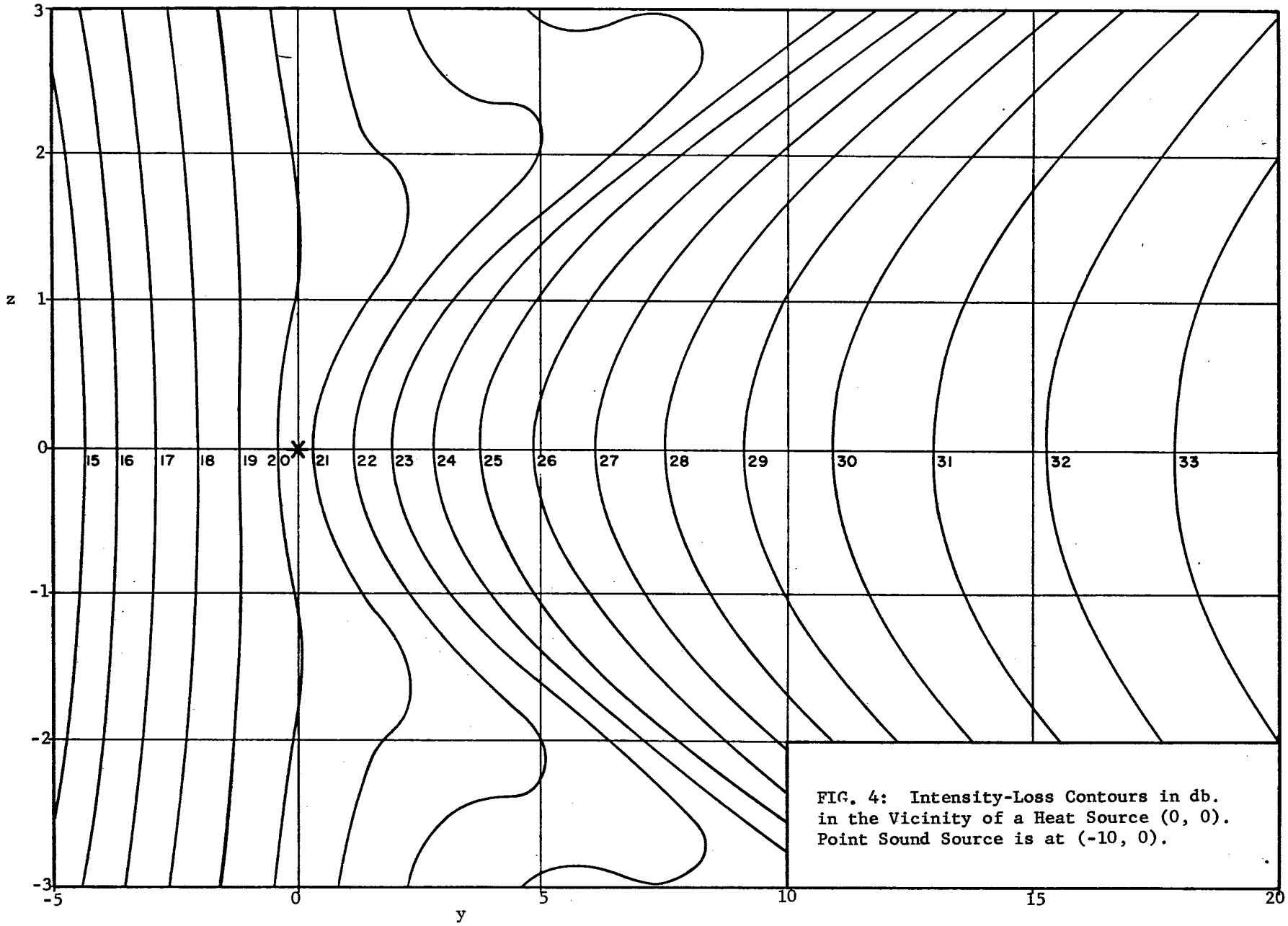


FIG. 4: Intensity-Loss Contours in db. in the Vicinity of a Heat Source (0, 0). Point Sound Source is at (-10, 0).

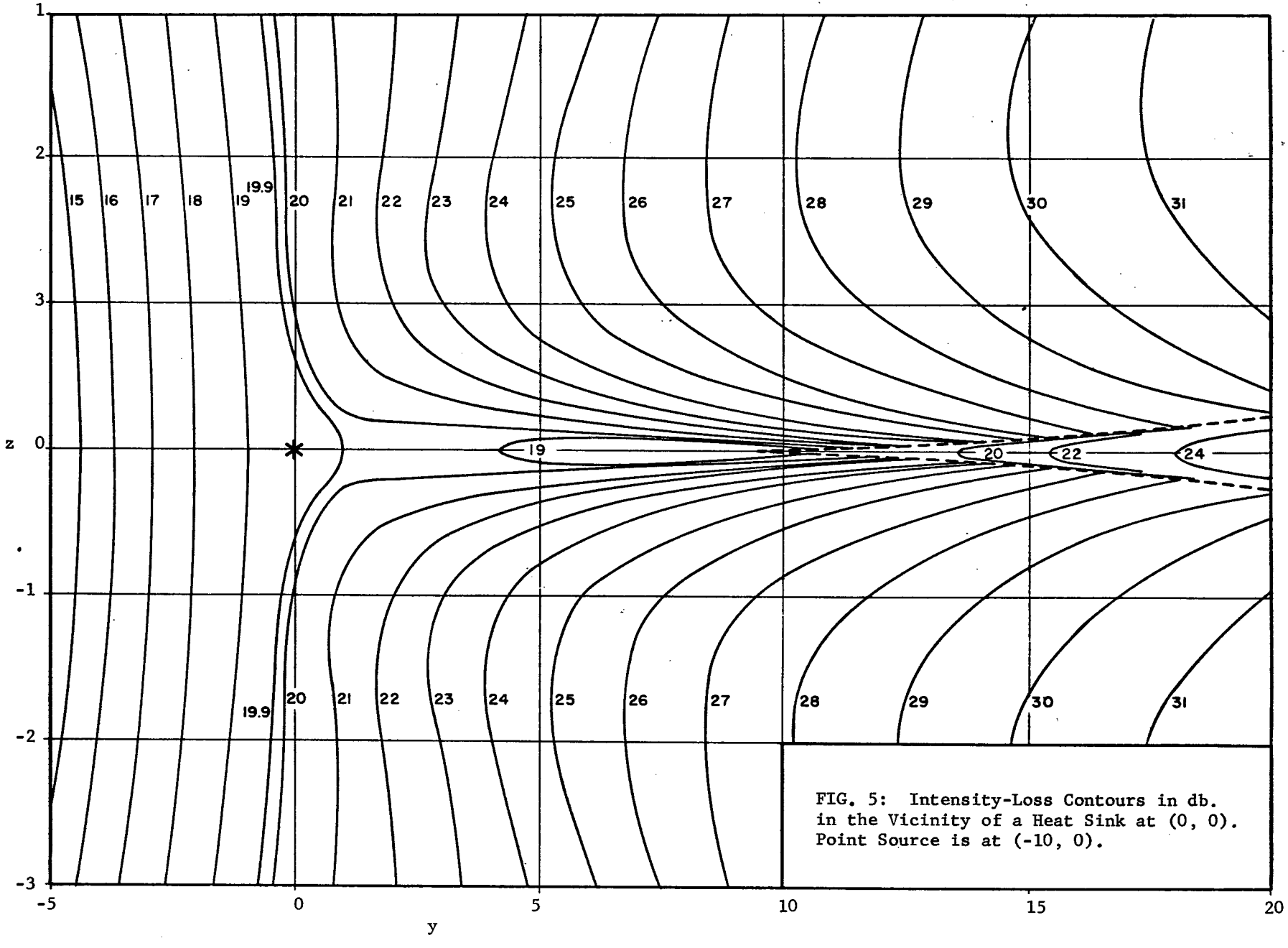


FIG. 5: Intensity-Loss Contours in db. in the Vicinity of a Heat Sink at (0, 0). Point Source is at (-10, 0).

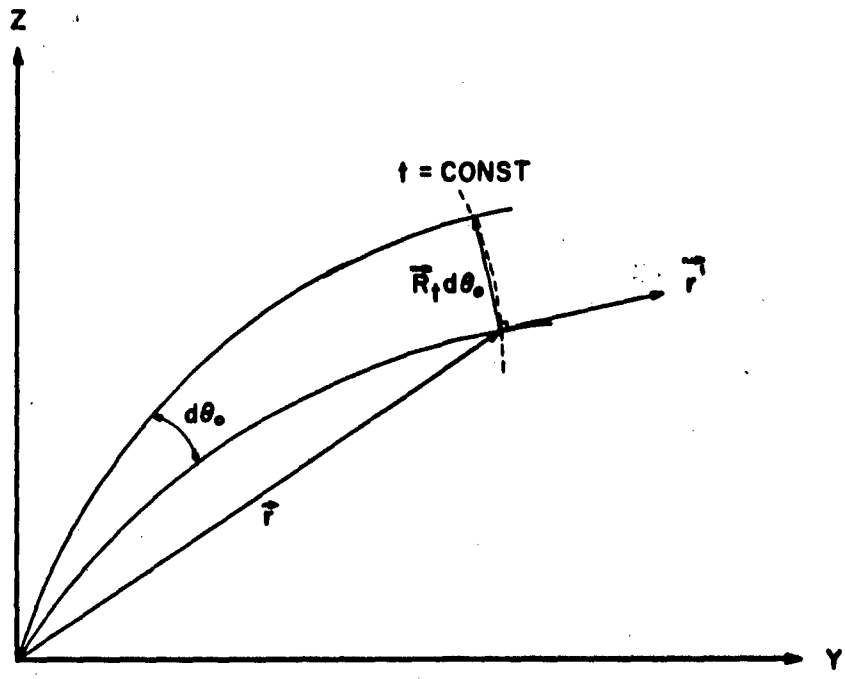


FIG. 6: Ray Geometry used in deriving Eq. (A13).

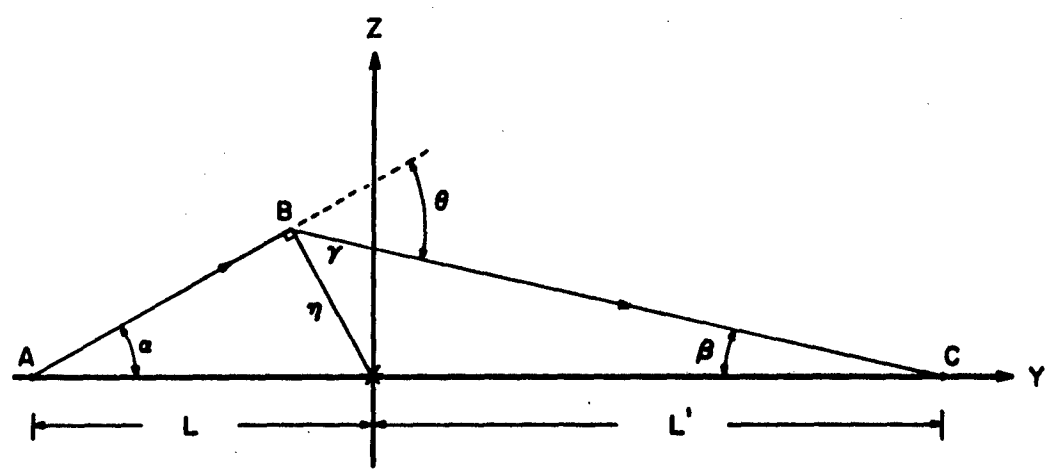


FIG. 7: Ray Geometry used in deriving Eq. (B8).

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A set of differential equations is derived for calculating the intensity loss due to geometrical spreading of planar sound rays in an inhomogeneous medium with a continuous index of refraction varying both in depth and horizontally. They are solved numerically for a mathematical model of an ocean with a heat source. Intensity contours are presented for rays in the vicinity of a heat source and a heat sink. The heat sink causes the rays to converge which results in the formation of a caustic. An approximate formula is derived for the point where this caustic originates.

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