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THEORY OF BOUNDARY LAYER NOISE

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Washington, D. C. 20007

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THEORY OF BOUNDARY LAYER NOISE

by

J. E. Ffowcs Williams

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I. INTRODUCTION

During the present contract work of a fundamental nature has been conducted into several aspects of the theory of boundary layer induced sound and vibration. Each of the major subject areas has proved to provide tractable theoretical problems which have led to technical reports. Accordingly, those topics will only be discussed very briefly and the abstract of the technical report included in the text. The final technical report to come out of the effort has not yet been submitted for publication because of lack of time. That report is reproduced here in its entirety as Section VI of this final report. Permission is sought to have this report submitted for journal publication. In summarizing the work accomplished during the contract period we must inevitably recapitulate on some of the earlier progress reports but the latter stages of the work have not previously been reported to the David Taylor Model Basin.

The first stage of the research program was a study of the sound radiated by turbulent flow formed on infinitely large homogeneous plane surfaces whose response to the surface pressure could be varied by changing the structural properties. Several interesting points have emerged from that analysis. Quite contrary to our initial expectation, there are no simple sources or surface dipoles associated with the problem, within the framework of the acoustic analogy. We know this to be the case when the surface is rigid, for the surface then acts as a passive reflector. But more surprising is the result that when the surface is perfectly limp and appears as a pressure relief condition, we again get perfect reflection of the volume quadrupoles. But this time the images have precisely the opposite strength, a result familiar in purely acoustical situations, but rather surprising in this instance where an intense hydrodynamic field excites the sound.

At intermediate surface conditions the general principle is still valid that no simpler and more efficient sources than those present in the turbulence are required to represent the total radiation field. In fact the surface acts as a passive reflector but the reflection coefficient is complex and varies with the angle of incidence of the reflected wave. The reflection coefficient is precisely that relevant to reflection of plane acoustic waves so that the boundary does not appear as a real source of radiation. This work is written up in a recently published report (Journal of Fluid Mechanics, June 1965) and the summary is included in the text of this report.

The second item that has been studied under this current contract is the nature of the sound radiated by the boundary stresses if these stresses are computed on the basis of the incompressible flow equations. A large, if not a dominant, fraction of the wall pressure arises from the interaction of turbulent eddies with the large mean shear of the boundary layer flow. The sound radiated by this type of pressure field has been studied in detail. The wall pressure appears to be equivalent to a distribution of lateral quadrupoles of strength proportional to the mean shear. One cannot draw defininte conclusions from this work at present, but it appears possible that the lateral quadrupoles Lighthill showed to dominate the turbulent field whenever the mean velocity gradients were high, whose strength increased in direct proportion to the mean shear, is being opposed by the image-like lateral quadrupoles equivalent to the mean shear induced surface pressure field. If this is the situation, it may be that the reflection criterion is already established even though the surface pressure, a near field phenomenon in this instance, is an extremely local one.

This is only one of the points described in a general survey Monograph on the sound generated by turbulent boundary layers. That monograph is published as AGARDograph 90 and its summary is included in this present text. It is pertinent to say that reviewers' comments on this work have, to date, been very complimentary describing it as a work likely to remain a standard reference for many years to come.

As part of the effort aimed at clarifying the role of incompressible flow arguments in the theory of acoustic radiation from turbulent boundary layers, an extensive study of how compressibility effects influenced the wall pressure was undertaken. The main conclusion was that it was precisely those usually insignificant features of the pressure field affected by compressibility that determined the radiation properties of the wall pressure field. Consequently it is essential to consider the influence of finite Mach number effects when considering radiation from turbulent flow or plane surfaces. The outcome of the work was reported on two occasions and appears published in papers that are similar but differ in detail, the latter publication being more rigorous. The first report appears in the published proceedings of the second international conference on acoustical fatigue and the abstract of the paper is included in the text. The second and more carefully considered report will appear in the July 1965 issue of the Journal of Fluid Mechanics and again the abstract of that work is included.

The final significant study to be undertaken under this contract was to study the effect of idealized supports on the sound radiated from an otherwise uniform homogeneous structure. The aim here was to maintain the exact approach made possible by the

choice of a suitably simple model structure to study how the support extracted energy from the turbulence that enabled the surface terms to represent a fundamentally more efficient radiator then was present in the unsupported surface. The mechanism is evidently a wave scattering process whereby components of the surface response that do not normally contribute to the radiation field, are coupled to the radiation by the point scatterer. This process destroys any significant correlation between the volume and surface sources so that no subtle cancellation effect that might limit the usefulness of approximate calculations need be expected. It was the neglect of precisely such a cancellation that led to the erroneous 'dipole' computations of boundary layer noise that were, until quite recently, prevalent in the literature. A report on this work is now ready for submission to a technical journal. Section VI of this final report is that report.

Finally, this report is concluded by brief reference to other topics that have been considered under this programme but which have not led to significant new conclusions. That is followed by a passing comment on some of the questions that remain outstanding in this field.

II. MECHANISM OF NOISE GENERATION IN A TURBULENT BOUNDARY LAYER

[Preface to AGARDograph 90]

The subject matter treated here is a comparatively new and rapidly expanding one. The bulk of the published works on boundary layer noise appeared in the last four or five years. Under these circumstances the writer of a monograph faces the risk of becoming rapidly out-dated. Recognizing this situation, the authors did not try to cover all the available literature on boundary layer noise production in detail - except by giving an extensive list of references - but rather they attempted to pick out those works that in their opinion were most instrumental in generating new ideas and approaches to this difficult problem. The arrangement of the chapters reflects this point of view.

Chapters 1, 2 and 3 are introductory ones: the first chapter briefly states the problem and gives its historical development; the second reviews some basic notions of classical acoustics with special emphasis on the sound field produced by elementary sound sources, while the third one contains the generalized wave equation governing the pressure field radiated by a nonuniform, nonstationary flow. The subsequent chapters describe then the methods proposed by various authors for finding a solution to the radiation problem.

Undoubtedly, the theory that exerted the most profound influence on the subject is Lighthill's acoustic analogy. Although the theory has been described in a number of papers by Lighthill himself and by several subsequent workers, it has often been misinterpreted and misused. Chapter 4 attempts to restate again

the analogy, summarizing all the assumptions involved and endeavors to point out the advantages and disadvantages of the approach. In the same chapter Ribner's interpretation of the source term in the analogy is touched upon, but only briefly.

Chapter 5 describes the application of the analogy to the boundary layer problem. The importance of the size of the boundary surface compared to the radiation wavelength is emphasized.

Chapter 6 introduces a new approach proposed by Phillips to treat the radiation problem for high convection velocities. Some comparisons are made between his theory and the consequences Ffowcs Williams drew from Lighthill's analogy applied to higher convection speeds.

A separate section, Chapter 7, is devoted to a third method of attack to study aerodynamic noise first proposed in an unpublished work by Liepmann. This approach, although virtually unexplored, offers a possibility of relating the radiated noise to flow parameters familiar in the study of incompressible flows. It is this aspect, with its promise of straight-forward experiments, that led the authors to devote to it a complete chapter, although much of the chapter is of a general illustrative nature.

The last chapter deals with the experimental investigations concerning boundary layer noise. It is to be noted that the considerable wealth of information on fluctuating velocity field within a boundary layer and on the pressure fluctuations over

the solid surface adjacent to the layer obtained at subsonic speeds have not been included. There are several indications, both theoretical and experimental, that such subsonic fields radiate very small noise indeed and have no practical significance. For this reason the chapter concentrates primarily on the supersonic boundary layer problem.

The monograph is designed for those who are familiar with classical acoustics and fluid mechanics in addition to some basic notions of turbulence: the concepts of correlation and spectrum functions. Wherever a more detailed treatment of a particular question exists in the literature, an attempt was made to give adequate references. The bibliography does not include all the papers on aerodynamic noise but only those restricted to boundary layers. In this sense it is hopefully fairly complete.

III. VIBRATION INDUCED BY BOUNDARY LAYER TURBULENCE

[Abstract of Paper presented at the Second International Conference on Acoustical Fatigue]

The vibration of a large homogeneous panel excited by a uniform turbulent boundary layer is described in detail. The simplicity of this situation allows one to appreciate the principal features of the boundary layer flow that bring about structural vibration. The large-surface result is then used as a guide to the study of smaller panels. It is shown how important features of the flow assume a different relative significance in studying the response of inhomogeneous structures. A general classification of the important vibration regimes is outlined and the experimental turbulence data useful in determining the vibration levels in these regimes are summarized.

IV. SURFACE-PRESSURE FLUCTUATIONS INDUCED BY BOUNDARY-LAYER FLOW AT FINITE MACH NUMBER

[Abstract of a Paper to appear in the July 1965 issue of the Journal of Fluid Mechanics]

A theory describing boundary-layer surface-pressure fluctuations on a rigid surface is presented in a form that illustrates the main effect of compressibility. The most significant effect is that the correlation area is proportional to the square of mean-flow Mach number so it does not vanish in flow of finite compressibility. Modifications of the wave-number and frequency spectra by this effect are described, and the results applied to the computation of large plate response. That computation incorporates the effect of fluid loading, which enters the response equations as a dissipative term for components at supersonic phase velocity but merely as an added loading for subsonic components.

V. SOUND RADIATION FROM TURBULENT BOUNDARY LAYERS FORMED ON COMPLIANT SURFACES

[Abstract of Paper from J. Fluid Mech. (1965), Vol. 22, Part 2, pp. 347-358]

The paper considers the effect of turbulence-induced surface response on the sound radiated by a turbulent boundary layer. The analysis is confined to an infinite plane homogeneous surface and the conclusions may not be a good indication of the behavior of more realistic structures. The main result of the analysis is that no fundamentally more efficient source of sound is introduced by the surface motion. The radiation remains quadrupole in character. The surface merely accounts for a reflection of the turbulence-generated sound, with the reflection coefficient being identical to that of plane acoustic waves. Dissipation in the surface reduces the magnitude of the image system. A brief discussion of the effect on the particular quadrupoles to be found in a turbulent boundary layer concludes the paper. There it is argued that the radiation will probably be increased by surface motion, but not by an order of magnitude.

VI. THE INFLUENCE OF SIMPLE SUPPORTS ON THE RADIATION FROM TURBULENT FLOW NEAR A PLANE COMPLIANT SURFACE

ABSTRACT

The paper deals with an extension of previous work on the radiation properties of turbulent flow formed on compliant surfaces. The effect of simple supports is shown to be acoustically equivalent to an extended dipole system of strength equal to the support stress. The dipole radiation is reduced by a transmission factor below that radiated into an uniform environment. A particular example is worked out in detail. That example deals with the case of a single point support on an otherwise homogeneous surface excited by boundary layer turbulence.

1. The Dipole Equivalent of Simple Supports.

The influence of surface vibration on the radiation from turbulent flow near a homogeneous plane surface has recently been treated by Ffowcs Williams (1965). There, it was shown that surface response did not introduce sources of high efficiency, and that any surface effect could be accounted for by a straightforward reflection coefficient for plane acoustic waves. The influence of simple supports can be treated in a similar way. Again, nonlinear terms in surface response are neglected, as are the viscous terms. The equations that describe the radiation field are those given by Powell (1961). There are two complimentary equations, one for a real flow with turbulence stress tensor T_{ij} distributed over the volume v_+, and one for a hypothetical image flow with a specular reflection of the turbulence in the volume v_-.

$$\left.\begin{aligned}
p(\underset{\sim}{x},t) &= T_+ + p_r + p_v \\
0 &= T_- - p_r + p_v \\
T_\pm &= \frac{1}{4\pi}\frac{\partial^2}{\partial x_i \partial x_j}\int_{v\pm} [T_{ij}]\,\frac{d\underset{\sim}{y}}{r}\,; \\
p_r &= \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [p]\,\frac{d\underset{\sim}{y}}{r}\,; \\
p_v &= -\frac{1}{4\pi}\int_s \bar{\rho}\,\frac{\partial[v_n]}{\partial t}\,\frac{d\underset{\sim}{y}}{r}
\end{aligned}\right\} \qquad (1.1)$$

The brackets, [ ], indicate that the integrals should be evaluated at retarded time, $(t-r/a_o)$, r being the distance separating the source point $\underset{\sim}{y}$ from the observer at $\underset{\sim}{x}$, and a_o the speed of sound. n is the outward normal from the real flow through the plane bounding surface s. $p(\underset{\sim}{x},t)$ is the pressure radiated to the point $(\underset{\sim}{x},t)$ and comprises four distinct source terms. The quadrupoles in the real flow induce a pressure T_+ while those in the image flow induce a pressure T_-. Should the surface be rigid, surface sources would account for a pressure p_r, while a pressure p_v would be radiated by surface terms if the surface were perfectly limp and could support no stress. More general surfaces radiate sound in a way that is determined by Eq. 1.1 so that the problem is reduced to one of relating the two pressures p_r and p_v through some knowledge of the surface response.

We now suppose that the previously considered homogeneous plane surface is supported by an inhomogeneous stress system induced by a distribution of simple supports. These stresses will be denoted by q, a positive q implying a force acting on unit area of the surface in the direction, -n. The stresses are related to the response velocity, v_n, through the response equation,

$$p - q = F(v_n) \quad , \tag{1.2}$$

F being a collection of differential or integral operators representing a linear integro-differential equation with constant coefficients. The surface pressure is now eliminated from 1.1 by use of the response equation.

$$p_r = \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [p]\,\frac{dy}{r} = \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [q]\,\frac{dy}{r} + \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [F(v_n)]\,\frac{dy}{r} \; . \tag{1.3}$$

This relation reduces the current problem, by analogy, to that of the unsupported homogenecus surface. This becomes clear when we regard the support stress, q, as the strength of a distribution of externally applied acoustic dipoles whose total strength we do not expect to be generally zero. These external dipoles are essentially different from the remaining surface terms, representing real sources of radiation and not, in general, accounting for a reflection property which is the sole role of the other terms. This point will become clear in what follows but we anticipate it by combining the total effect of the applied stress fields into one term. That term represents the sum of the pressure induced by the turbulent flow and that induced by the support dipoles, a value we denote by S. As before, we have a field due to the real and image source systems.

$$S_+ = T_+ + \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [q]\,\frac{d\underset{\sim}{y}}{r}$$

$$S_- = T_- - \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [q]\,\frac{d\underset{\sim}{y}}{r} \quad . \tag{1.4}$$

S_- is the pressure induced by the specular reflection of the real source system that generates the pressure S_+, the reflection of the dipole term merely requiring a change of sign.

Equation 1.1, 1.3 and 1.4 can be combined in a form that makes clear the analogy with the earlier problem.

$$p(\underset{\sim}{x},t) = S_+ + \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [F(v_n)]\,\frac{d\underset{\sim}{y}}{r} - \frac{1}{4\pi}\int_s \bar{\rho}\,\frac{\partial[v_n]}{\partial t}\,\frac{d\underset{\sim}{y}}{r}$$

$$0 = S_- - \frac{1}{4\pi}\frac{\partial}{\partial x_n}\int_s [F(v_n)]\,\frac{d\underset{\sim}{y}}{r} - \frac{1}{4\pi}\int_s \bar{\rho}\,\frac{\partial[v_n]}{\partial t}\,\frac{d\underset{\sim}{y}}{r} \quad . \tag{1.5}$$

This system of equations is precisely that treated by Ffowcs Williams (1965) in considering the problem of turbulent flow formed on an unsupported homogeneous surface. In fact the only change induced by the supports is that the turbulent sources are reinforced by surface dipoles so that S replaces T. Conclusions can therefore be based directly on that analysis. The most important point is that the surface integrals account for simple reflection of the source system S_+. However the reflection coefficient changes with direction of radiation and frequency. The analysis is particularly simple for the distant radiation field

where, $p^*(\underset{\sim}{x},\omega)$, the component of radiated pressure at frequency ω is given by the sum of direct and reflected fields.

$$p^*(\underset{\sim}{x},\omega) = S_+^* + R\, S_-^* \quad . \tag{1.6}$$

S_+^* and S_-^* are the component of sound pressure radiated by the real and image source systems at frequency ω, and R is the reflection coefficient for plane acoustic waves at that frequency.

The pressures S_+^* and S_-^* consist of a superposition of the fields induced by quadrupoles acoustically equivalent to the turbulent flow and dipoles whose strength density equals the supporting stresses. Equation 1.6 has an interesting special case when there is negligible turbulence so that both T_+ and T_- are zero. Then S_+^* is entirely due to an externally applied stress and S_-^* is its exact opposite, being the field of an image dipole. The radiated pressure is then given by,

$$p^*(\underset{\sim}{x},\omega) = (1-R)\, S_+^* \quad . \tag{1.7}$$

(1-R) is familiar as the transmission coefficient for waves passing from the fluid into a region with impedance equal to the normal impedance of the surface. This result, that the radiation from an externally excited surface is equal to that induced by dipoles of strength density equal to the applied stresses multiplied by the transmission coefficient for plane acoustic waves, seems an obvious one but does not appear to be readily available in the literature.

It is clear that if the supporting stresses were uniformly distributed over the plane their effect could be accounted for by a modified surface response equation. The previous conclusion that the radiation would be purely quadrupole would then be valid. It seems that it is essentially the inhomogeneous nature of the supports that induces the dipole component. If the plane surface were composed of several regions of locally homogeneous material, but material that differed from region to region, one could conclude from the foregoing analysis that within individual regions the effect of surface motion would be accounted for by the local reflection coefficient and that the radiation would be quadrupole. However at the interfaces there would be discontinuities in the response equation that would account for dipole terms which must be more efficient radiators of sound. The situation is completely analogous to that treated by Maidanik (1962) who showed how most of the sound radiated from a large finite plate appeared to emanate from the periphery of the plate.

The total dipole strength is the net applied force and the most effective radiation results from the force being concentrated on to an area of typical dimension small in comparison with an acoustic wavelength. The concentrated point support is then an important example in establishing an upper limit on the strength of the radiation field induced by a known supporting force. When the excitation originates in boundary layer turbulence the supporting force is not known a priori, unless by experiment. The example then offers no specific upper limit on the radiation but can still serve as a useful result for developing more pertinent models of real system. That example is considered below.

2. Point Supported Surface Under a Turbulent Boundary Layer

The simplest of the inhomogeneous support system is that in which the stress distribution q is concentrated at one point. Let that point be the origin of co-ordinates and let the applied force have a value Q.

$$q(\underset{\sim}{y},t) = Q(t)\ \delta(\underset{\sim}{y}) \quad . \tag{2.1}$$

This force induces a dipole radiation according to Eqs. 1.4 and 1.5, where the real and image fields become:

$$\begin{aligned} S_+ &= T_+ + \frac{1}{4\pi}\frac{\partial}{\partial x_n}\left[\frac{Q}{r}\right] \\ S_- &= T_- - \frac{1}{4\pi}\frac{\partial}{\partial x_n}\left[\frac{Q}{r}\right] \end{aligned} \quad . \tag{2.2}$$

In the distant radiation field, the differentiation with respect to x_n applies only to the retarded time so that the dipole term may be rewritten as,

$$\frac{1}{4\pi r}\frac{\sin\theta}{a_o}\frac{\partial Q}{\partial t} \quad . \tag{2.3}$$

$\sin\theta$ is written for $-\frac{\partial r}{\partial x_n}$, θ being the radiation angle measured from the surface. The spectral decomposition is achieved through Fourier transformation. We shall denote transformed quantities by an asterisk, p* being the component of p at frequency ω, T_+^* the component of T, etc.

$$p(\underset{\sim}{x},t) = \int p^*(\underset{\sim}{x},\omega)e^{i\omega t}\ d\omega \quad . \tag{2.4}$$

The particular case of Eq. 1.6 is then,

$$p^*(\underset{\sim}{x},\omega) = T_+^* + R\,T_-^* + \{1-R\}\frac{i\omega \sin\theta}{4\pi\, a_o r}\, Q^* \quad . \tag{2.5}$$

It is easy to verify that the dipole term is precisely that worked out by (Maidanik and Kerwin, Jr.) to be the radiation from a point driven plate, but its interpretation as the sum of direct and reflected dipole fields seems to be new. A point of considerable interest is the question of how large is the dipole field in comparison with the sound radiated by the surrounding surface? To progress with that issue, the value of the externally applied force, Q, must first be found.

Suppose the support to have some impedance z_q, so that Q^* is related to the velocity at the support point v_{nq}^*, through the relation:

$$Q^* = z_q\, v_{nq}^* \quad . \tag{2.6}$$

The applied force has modified the velocity at its point of application, from a value v_{no}, which would have occurred in an unsupported structure, to its current value v_{nq}. The force is related to this velocity change through z_p, the point impedance of the structure, an impedance that includes any influence of fluid loading.

$$Q^* = -\, z_p(v_{nq}^* - v_{no}^*) \quad . \tag{2.7}$$

Combining this relation with Eqs. 2.5 and 2.6 we obtain expressions for the applied force and radiated sound in terms of the velocity response of an unsupported structure.

$$Q^* = \frac{z_p}{\left\{1 + \frac{z_p}{z_q}\right\}} v^*_{no} \tag{2.8}$$

$$p^*(\underset{\sim}{x},\omega) = T^*_+ + R\, T^*_- + (1-R) \frac{i\omega \sin\theta}{4\pi\, a_o\, r} \frac{z_p\, z_q}{(z_p + z_q)} v^*_{no} \quad . \tag{2.9}$$

The influence of the support can now be interpreted as a wave scattering process. Sound radiation from an unsupported surface occurs only from those spectral components that match both frequency and wave number of the distant acoustic wave. But in this instance, components of response velocity at all wave numbers contribute to the term v^*_{no}, so that the supported structure acts like a sounding board. That is, energy is converted from a reactive to a radiative regime by a wave scattering process. This feature destroys any significant correlation between the dipole and quadrupole terms making the mean square radiation the sum of the individual mean square values. The power spectral density of the radiated pressure field $P^*(\underset{\sim}{x},\omega)$, can then be expressed as the sum of that due to the combination of real and image quadrupoles T^*, and that due to the dipole which is proportional to the power spectral density of the response velocity in an unsupported panel, V^*_o.

$$P^*(\underset{\sim}{x},\omega) = T^* + |1-R|^2 \frac{\omega^2 \sin^2\theta}{16\pi^2\, a_o^2\, r^2} \left|\frac{z_p\, z_q}{z_p + z_q}\right|^2 V^*_o \quad . \tag{2.10}$$

V^*_o is simply related to the three dimensional Fourier spectrum of the pressure field acting on a rigid surface through the surface

and wave impedances, z and z_w respectively. The rigid-surface pressure field at wave vector $\underset{\sim}{k}$ and frequency ω, $p^*_{rs}(\underset{\sim}{k},\omega)$, must balance both the structural response force, $z\, v^*_n(\underset{\sim}{k},\omega)$, and the force induced by fluid motion, $z_w\, v^*_n(\underset{\sim}{k},\omega)$. Therefore:

$$p^*_{rs}(\underset{\sim}{k},\omega) = (z + z_w)\, v^*_n(\underset{\sim}{k},\omega) \quad , \tag{2.11}$$

where $v^*_n(\underset{\sim}{k},\omega)$ is the three dimen ıonal Fourier transfrom of the surface velocity. The three dimensional spectral functions, which we denote $P^*_{rs}(\underset{\sim}{k},\omega)$ and $V^*_n(\underset{\sim}{k},\omega)$, are formed from the product of this equation with its complex conjugate

$$P^*_{rs}(\underset{\sim}{k},\omega) = |z + z_w|^2\, V^*_n(\underset{\sim}{k},\omega) \quad . \tag{2.12}$$

The frequency spectrum of the surface response velocity is simply the integral of the three dimensional spectrum over all wave number space so that $V^*_o(\omega)$ is given by a straightforward integral.

$$V^*_o(\omega) = \int \frac{P^*_{rs}(\underset{\sim}{k},\omega)}{|z + z_w|^2}\, d\,\underset{\sim}{k} \quad . \tag{2.13}$$

For a homogeneous structure the impedance function depends only on the magnitude of the wave vector so that a change of coordinates to a polar system is suggested. We let $\underset{\sim}{k}$ be defined by (k,ϕ) and rewrite $d\underset{\sim}{k}$ as $k\,dk\,d\phi$.

$$V^*_o(\omega) = \int_0^\infty \frac{k\, dk}{|z + z_w|^2} \int_0^{2\pi} P^*_{rs}(\underset{\sim}{k}(k,\phi),\omega)\, d\phi \quad . \tag{2.14}$$

In a highly resonant structure, the integral over wave number can be approximated in a way that considerably simplifies the analysis. The approximation rests on the assumption that both the pressure spectrum and the real part of the impedance $(z + z_w)$, $(z_R + z_w)$, (z_w being necessarily real for radiating waves) remains fairly constant over the effective bandwidth of the 'resonance peak,' a peak assumed to occur at wave number k_p. In that event, the integration over wave number is straightforward.

$$V_o^*(\omega) = \frac{2\pi}{[z_R + z_w]z_p} \int_0^{2\pi} P_{rs}^*(\underset{\sim}{k}(k_p,\phi),\omega)\, d\phi \quad . \tag{2.15}$$

Most turbulent flows of practical interest display convective features that are apparent in the spectrum function as a tendency for the energy to be concentrated at the eddy passage frequency. This property can be important in the response problem and is best dealt with by re-expressing the pressure spectrum in terms of that measured by an observer in uniform motion with the most coherent eddy structure. That spectrum we will denote by $P_m^*(\underset{\sim}{k},\omega)$.

$$P_{rs}^*(\underset{\sim}{k},\omega) = P_m^*(\underset{\sim}{k},\omega - \underset{\sim}{k}\cdot\underset{\sim}{U}_c) \quad . \tag{2.16}$$

U_c is the convection velocity which we normalize with respect to the free surface wave speed, c_p. $|U_c| = c_p M$. Then, by setting the origin of ϕ coincident with the direction of convection and noting that $\omega = k_p c_p$, we can rewrite Eq. 2.15 in a form that displays the convective effects more clearly.

$$V_o^*(\omega) = \frac{2\pi}{[z_R+z_w]z_p} \int_0^{2\pi} P_m^*(\underset{\sim}{k}(k_p,\phi),\ \omega\{1 - M\cos\phi\})d\phi. \tag{2.17}$$

At low flow velocities, particularly in underwater applications, interest centers on situations where the number M is negligible. Then it is convenient to assume that space and time variables are separable in the moving reference frame so that:

$$P^*_m(\underset{\sim}{k}(k_p,\phi),\ \omega\{1 - M\cos\phi\} \simeq p_h^2\ P^*_{\underset{\sim}{k}}(\underset{\sim}{k})\ P^*_\omega(\omega) \quad . \tag{2.18}$$

p_h is written for the r.m.s. pressure level active on a rigid boundary, $P^*_{\underset{\sim}{k}}$ is the wave number spectrum and P^*_ω is the moving axis frequency spectrum. Both these spectral functions are normalized to integrate to unity.

Before going on to evaluate the integral at low values of M, it is worth pointing out the equivalence that exists between this theory of vibration induced by convected pressure fields and that of aerodynamic sound generation by convected turbulence. (Lighthill 1962, Ffowcs Williams 1963). In the aerodynamic case, M is the Mach number of eddy convection, and radiation frequencies differ from those of the source by the Doppler factor (1-M cosϕ). It is apparent from Eq. 2.17 that this feature is also a property of the vibration problem and that we might expect an analogue of the Mach wave radiation at values of M in excess of unity. This is evident from the alignment that occurs whenever (1-M cosϕ) approaches zero of the dominant spectral component in the surface pressure field with the response frequencies. The spectrum P^*_ω being chosen so that its maximum occurs at zero frequency. Then, by analogy with the aerodynamic problem, only the uniformly convected components induce response. Consequently the response would be expected to be relatively intense for those waves radiating at the "Mach angle" on account of the strong tendency to uniform convection of

many pressure fields. The situation is illustrated quite effectively by assuming the moving axis frequency spectrum to be a unit delta function. Then Eq. 2.17 can be evaluated to give the response velocity spectrum typical of structural excitation by high speed flows where the convection velocity exceeds the phase speed of free waves.

$$V_o^*(\omega)_{(1-M\cos\phi)=0} = \frac{2\pi\, p_h^2\, P_{\underset{\sim}{k}}^*(k_p, \phi = \cos^{-1} M^{-1})}{\sqrt{M^2-1}\ \{z_R+z_w\}\ z_p} \quad . \tag{2.19}$$

In underwater problems the low speed situation is more relevant, where, from Eqs. 2.17 and 2.18,

$$V_o^*(\omega) = \frac{2\pi\, p_h^2\, P_\omega^*(\omega)}{\{z_R+z_w\} z_p} \int_0^{2\pi} P_{\underset{\sim}{k}}^*(\underset{\sim}{k}(k_p, \phi))\, d\phi \quad . \tag{2.20}$$

The integral over ϕ is the correlation area $A(k_p)$ used by Ffowcs Williams and Lyon (1963) so that it is a previously estimated property of boundary layer flows. The values for $A(k_p)$ are illustrated in Fig. 1.

$$2\pi \int_0^{2\pi} P_{\underset{\sim}{k}}^*(\underset{\sim}{k}(k_p, \phi))d\phi = A(k_p) \quad . \tag{2.21}$$

The response spectrum can now be given explicitly in terms of known features of the pressure field that acts on a rigid surface. The spectrum has the value derived for the flat plate by Ffowcs Williams and Lyon (1963) when z_w and z_p are appropriately chosen.

The radiation from the simply supported surface can also be estimated by inserting the response spectrum in Eq. 2.10.

$$V_o^*(\omega) = \frac{p_h^2 \; P_\omega^*(\omega) \; A(k_p)}{(z_R+z_w) \; z_p} \tag{2.22}$$

$$P^*(\underset{\sim}{x},\omega) = T^* + |1-R|^2 \frac{\omega^2 \sin^2\theta}{16\pi^2 \; a_o^2 \; r^2} \frac{z_p}{(z_R+z_w)} \left|\frac{z_q}{z_q+z_p}\right|^2 p_h^2 \; P_\omega^*(\omega) \; A(k_p) \; . \tag{2.23}$$

Although this result could be used to evaluate the sound radiated from various flows and support structures, the general form is not very revealing of the important role played by surface inhomogeneities. It is clear that if the surface is supported on soft, or resonant undamped mounts, so that z_q approaches zero, there will be no additional radiation from the support. It is also clear that the support plays a minor role in high impedance structures where the reflection coefficient approaches unity. However, many practical instances occur where the reflection coefficient is close to zero, as in the case in sonar dome construction where optimum sound transmission is essential. That situation is illustrated below by an example in which the support impedance z_q is infinite, the surface is assumed to be loss-free, and the flow is a fully develcped turbulent boundary layer. The intensity of the radiation from the turbulence is identical to that from the image system which can be computed from a knowledge of the pressure field on the boundary surface. This has been done in an approximate form by Ffowcs Williams and Lyon 1963 with the result:

$$T^* = \frac{p_h^2}{16\pi^2} \frac{\cos^2\theta \; \sin^2\theta}{} \; 880\pi \; \frac{\delta_1^4}{\lambda^4} \; P_\omega^*(\omega) \; \frac{A}{r^2} \quad . \tag{2.24}$$

The notation is that already defined with the addition that δ_1 is the boundary layer displacement thickness, λ is the inverse acoustic wavenumber a_o/ω, and A is the area of the radiating surface. This estimate, when inserted in Eq. 2.23 yields an approximate expression for the total radiation from a turbulent boundary layer formed on a surface resting on a single rigid support.

$$P^*(\underset{\sim}{x},\omega) = \frac{p_h^2}{16\pi^2} \frac{\delta_1^4}{r^2\lambda^2} \sin^2\theta \; P^*_\omega(\omega)\left\{880\pi \frac{A}{\lambda^2} \cos^2\theta + \frac{z_p}{\delta_1^2 \; z_w} \frac{A(k_p)}{\delta_1^2}\right\} \quad . \tag{2.25}$$

The leading term in the brackets is that due to the free surface and the other represents the support. When this expression is evaluated for a typical underwater situation one finds that the support induces an intensity at $\theta = 45^\circ$ equivalent to that radiated by approximately 2 square metres of unsupported structure. This figure is worked out near the maximum value of $A(k_p)$ which occurs at a frequency of 3.5 kc for 0.25 inch steel plate and the boundary layer displacement thickness is taken as 0.1 inch.

A more general result showing the area of free surface to which the radiation from the support is equivalent at a particular radiation angle is obtained by equating the two terms within the brackets of Eq. 2.25. The value of $A(k_p)/\delta_1^2$ has been taken as 10, that being an estimate of its upper limiting value as shown in Fig. 1. Then it is seen that at an angle θ the radiation from a rigid support on a surface with high transmission factor cannot exceed that from an unsupported area equal to,

$$\frac{1}{88\pi \cos^2\theta} \frac{z_p}{z_w} \frac{\lambda^2}{\delta_1^2} \quad . \tag{2.26}$$

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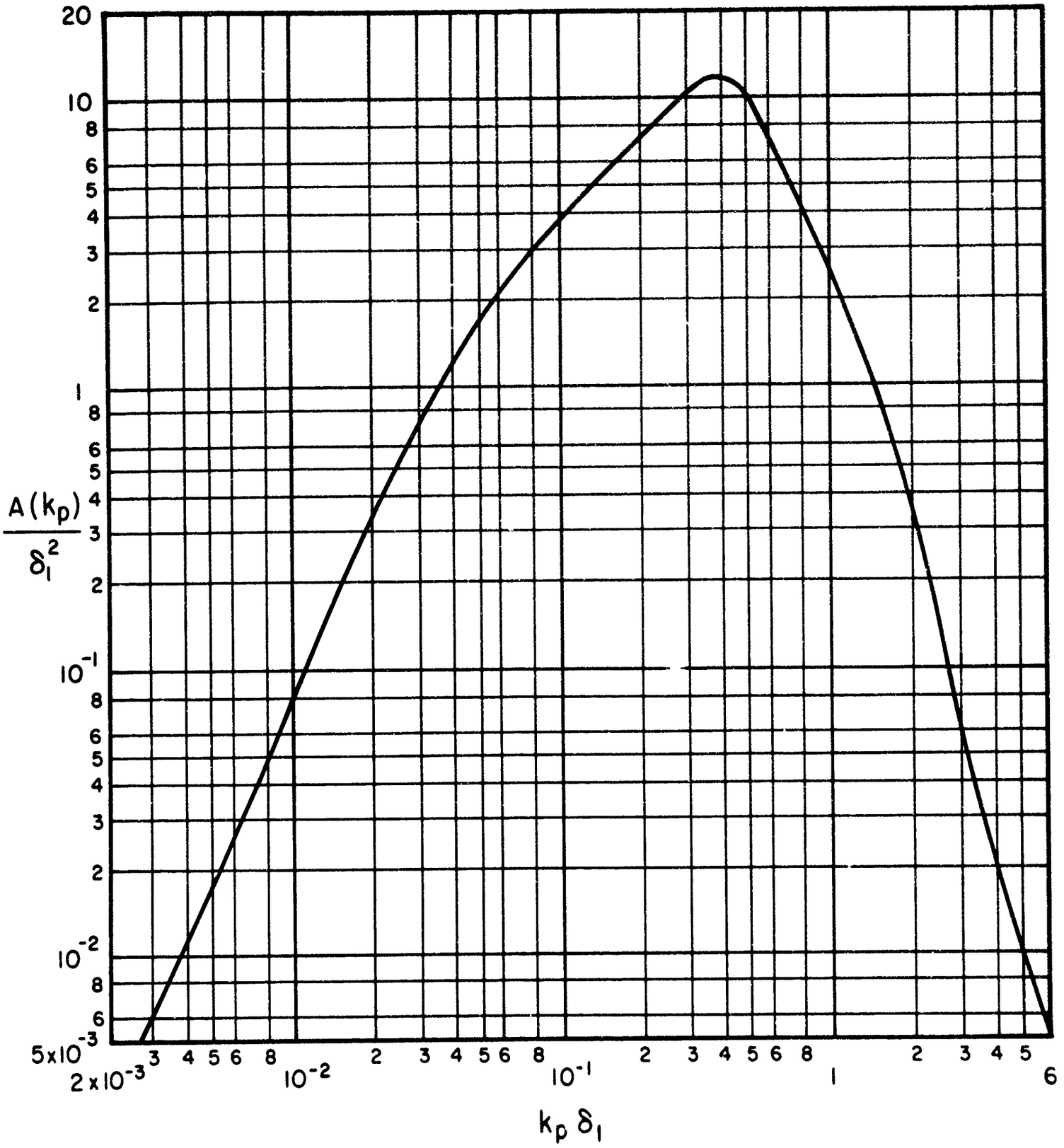

FIG. 1 FFOWCS WILLIAMS AND LYON'S (1963) ESTIMATE OF THE EQUIVALENT CORRELATION AREA $A(k_p)$ (DEFINED BY EQ. 2.21 OF TEXT).

VII. CONCLUSION AND DISCUSSION

In the general area of sound radiation by turbulent flow interacting with flexible surfaces it would seem helpful to distinguish between two essentially different situations. The discussion arises in establishing whether or not the sources induced by surface terms arise from hydrodynamic or compressible features of the turbulent field. It is confusion on this issue that has been primarily responsible for the erroneous estimates of radiation strength that have appeared in the literature from time to time.

There seems to be no doubt at all that once the vibration field of a structure is established, the radiation from the induced vibration is accessible by what are now standard techniques. The influence of compressibility is simply and properly accounted for in those procedures. It is in the computation of the surface stresses and response that the confusion arises. Two examples of this situation are to be seen in recent survey articles on the subject. The first is due to M. J. Lighthill in the Bakerian Lecture to the Royal Society, where the radiation from a rigid surface was estimated to be dipole in character and the dipole strength computed from boundary layer experiments that were not intended to be accurate in regimes where any effects of compressibility were significant. It is now known that the situation treated by Lighthill is entirely dependent in those usually small effects arising from fluid compressibility so that his computation becomes meaningless. The second example is due to Ffowcs Williams and Lyon in an AGARDograph on aerodynamic noise edited by G. M. Lilley. There the radiation from an infinite plane homogeneous structure was computed by equating input power to the vibration field to the energy loss by radiation. The input power was computed on the basis of the

equations of incompressible fluid motion and the result is quite an erroneous overestimate by several orders of magnitude (at least the square of Mach number) of the true radiation field. Again, it is now known that the properties of the vibration field responsible for radiation are intimately connected with the sound field of the turbulence and are rather unrelated to the hydrodynamic near field. Both these points are evident from the papers whose abstracts form sections IV and V of this report.

On the other hand where surface inhomogeneities occur there is very little that is new to be gained from a more precise treatment that consider effects of compressibility. This is evident from section VI of this report where both the vibration level and the radiation from the point support that are calculated are quite uninfluenced by fluid compressibility. In fact the vibration level is that previously calculated by Ffowcs Williams and Lyon in the above mentioned AGARDograph, and the radiation from the support is the familiar radiation from a point driven structure.

The essential feature determining whether or not compressibility terms should be considered in computations of the surface stress field is whether or not there is a unique wave matching between the excitation and radiated field. If such a matching exists, as is the case in the homogeneous surface, compressibility effects must play a major role in establishing the significant components of surface stress and response. On the other hand where there is not a unique matching, and energy is scattered from one spectral range to another, the relatively low intensity region of the spectrum where compressibility effects are important can play no major role in the radiation problem. The radiation energy is simply acquired from the more intense spectral regions by a scattering process.

Intermediate regimes are more difficult to deal with. The line supported structure acts as a wave scatterer to waves travelling in a direction normal to the support but cannot scatter waves travelling parallel to the support. In a similar way the edge of an otherwise uniform surface behaves as a wave scatterer to a degree depending on direction. The calculation of the radiation field by turbulent flow acting on such structures would consequently require that the theory take into account the sound field of the turbulence in addition to its hydrodynamic field. Of course it could be argued that the radiation from structures that do not possess significant scattering devices is essentially quadrupole and must be insignificant in any practical underwater application. There would seem no good reason to doubt this point of view so that the emphasis should be placed on the reduction of inhomogeneities that could cause efficient radiation through a scattering mechanism.

However, regarding the question of sound generation by turbulence near surfaces, with supports or edges that extend in a single direction for distances in excess of an acoustic wavelength, very little work has yet been attempted. The initial work on these lines carried out under the present effort has not led to any significant new contribution. The problem is difficult on two counts. First the pressure induced by the turbulent flow must be assessed. In doing this care must be taken that compressibility effects are accounted for in spectral regions where they are important. Secondly the radiation from sources in the vicinity of the edge must be computed taking proper account of the edge diffraction field. Both these questions seem difficult ones that require a substantial fundamental study before they are adequately understood. The pertinence of the problem to the sound generation properties of turbulent flow near propellor blades or hydrofoil sections makes it an obvious target for future concentrated effort.

VIII.

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13. ABSTRACT

This is the final report describing research work on the Theory of Boundary Layer Noise. The main effort has been concentrated on the effects of surface motion. Four published reports describe the details of the effort.

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