Technical Note N-691

FEASIBILITY OF MODELING RUN-UP EFFECTS
OF DISPERSIVE WATER WAVES

BY

J. M. Jordaan, Jr.

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U. S. NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California
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Type C

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ABSTRACT

Studies were made in a laboratory test basin to determine the feasibility of modeling run-up effects of explosively generated water waves on beach and waterfront structures. Results were compared with (i) analytically derived predictions, and (ii) wave measurements (but not run-up) made in the ocean with high-energy (HE) explosives as the generating source.

The test basin is 92 feet by 94 feet in size with 1:5, 1:13.6 and 1:5 sand beaches on three sides and on the fourth a 14-foot-diameter semi-paraboloidal plunger which by sudden plunge or retraction generates dispersive waves with dominant period of 2 seconds, height of 0.2 feet, and length of 20 feet in water 2-1/2 feet deep.

It was found that wave motions are (i) predicted well by Green's Law, as modified for dispersive waves; i.e., wave amplitude proportional to $b^{-\frac{1}{2}}y^{-\frac{1}{2}}r^{-\frac{1}{2}}$ where $b$, $y$, $r$ are orthogonal spacing, water depth and radial distance from plunger center, respectively; and (ii) related to waves generated in the ocean by HE by the Froude scaling law. For example, the Hydra-II-A series with 10,000-pound charges of HE in 300 feet of water at various submergences conducted off San Clemente Island, California, yielded wave motions predictable from scale tests conducted in the Basin.

It is found that run-up measurements in the Basin are predicted reasonably well by an existing technique; i.e., that of Kaplan in BEB TM No. 60, 1955.
On the basis of the above findings and theoretical considerations, it is concluded that a scaled-down model presents a feasible means of studying the kinematics of wave propagation and run-up. Generation of the waves by a plunger rapidly immersed and/or retracted agrees with analytically predicted wave forms.

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OBJECTIVES

The ultimate objective is to predict the effects of large-amplitude wave run-up resulting from underwater nuclear explosions on water front installations and beaches.

The immediate objective (reported here) was to determine whether scale models of run-up due to explosively generated waves are feasible.
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INTRODUCTION

The desired end product of this DASA Subtask 14.083 is to obtain design criteria for waterfront structures related to the Naval Shore Establishments and National Defense to contend with the effects of large-amplitude water wave run-up caused by underwater nuclear explosions. The damage potential of water waves caused by offshore explosions compare to that of seismic sea waves.

Records of the latter over the past century show that predominantly large losses due to seismic sea waves (tsunamis) occur mostly due to run-up between 20 and 50 feet in height. The run-up is defined as the maximum vertical rise of the water's edge on the shore above the still water level. Run-up less than 20 feet high in general causes relatively minor damage, and run-up greater than 50 feet in populated areas is rare and is associated with major destruction and loss of life. Possibly run-up between 20 and 50 feet high warrants the major consideration. The coastline of the United States of America is relatively unprotected against such run-up since tsunamis are uncommon enough that hardly any coastal defense measures exist, and the possible damage by excessive run-up due to underwater nuclear explosions is a serious threat to the shore facilities of the Fleet.

Potentially high run-up may already be caused, for example, by detonation of an underwater nuclear weapon, of, say, 200 kilotons at about 200 miles from the continental shelf. It could threaten or disrupt defenses of the entire Gulf coast or could be effective over half the Atlantic coast and have perhaps only a localized effect if detonated off the Pacific coast. On the other hand, a nuclear weapon of, say, 20 megatons, if detonated some 2000 miles off either coast, would have the widespread destructive or annoyance effects commonly associated with seismic sea waves along most of the exposed coastline.

Figure 1 is a chart indicating the estimated wave effects emanating from various strength impulsive sources at various "ranges" or radial distances away from the source. The wave height is defined as the vertical dimension between crest and preceding trough. If the first wave is a rise above still-water level, the wave height is simply still-water level to crest for the first wave. The wave period and length are measured from crest to crest or twice the "point of initial rising to first crest" dimension.
SCOPE OF NCEL EFFORT

The modest aim of the current effort at NCEL on this task is to study the run-up of impulsively generated waves under controlled conditions so as to predict damage possibilities. Data will lead hopefully to improved planning, design and protection of vital waterfront facilities. At present there are international and practical limitations on conducting large-scale tests with high-yield underwater explosions. Efforts are mostly limited to laboratory tests and reduced-scale field tests.

Although field testing cannot be completely foregone to verify theory, it can be supplemented by small-scale experiments such as in the wave test basin facility at NCEL. The underwater explosion itself is here simulated by the action of a paraboloidal plunger, which can be programmed to excite a wave system with selected properties. The response of shore facilities such as docks, seawalls, buildings, moored platforms, cranes, etc., to these waves and the run-up on smooth or rough terrain can be experimentally determined to a certain degree of precision.

The present technical note reports on the study made to establish the predictability of run-up by underwater blast-generated water waves.

The wave basin facility provides the unique opportunity for obtaining reliable correlation between excitation and response energies, applicable to improvement of design and countermeasures, against run-up hazards from explosively generated waves.

CRITERIA FOR MODELING RUN-UP BY DISPERSIVE WATER WAVES

The question to be answered is: "Can run-up be predicted on the basis of model experiments, and if so, what determines the choice of type of model and scale?"

Similitude Considerations

The generation, propagation and terminal effects, including kinematics, dynamics and run-up of gravity water waves are governed by the Froude similitude relationship:

\[ \frac{V^2}{r} = \frac{y}{y_r} \]
where $V_r$ is the velocity ratio and $y_r$ the vertical-scale ratio between two geometrically similar events. This law relates inertia and gravity forces, and if both these forces are simulated in the same proportion in model and prototype, that is, if the Froude number $F^2 = V^2/yg$ is the same, then all dependent effects will also be in the correct scale relationship.

A Froude scale reduction results, however, in proportionally higher viscous damping forces in the model than in the prototype since the Reynolds number $R = Vy/\nu$, that is, the ratio between inertia and viscous forces, is different. The scale reduction permissible is limited by the Reynolds criterion that turbulent flow in nature should be modeled by turbulent flow in the model or that the model Reynolds number should be above a certain minimum value associated with incipient turbulent flow. Because of this limitation in the depth scale, models are quite frequently made to a larger vertical scale than the horizontal scale; i.e., distorted so as to conserve the model area required without making $R_m$ too small. Some adjustment in the model roughness is generally necessary.

As essential scaling condition is that the model should be a geometrically correct representation of its prototype; e.g., in the case of a breaking wave on a sloping sea defense works, a distorted model would violate the similitude criterion with respect to steep slopes, wave steepnesses and wave length to depth ratios. There are ways to partially compensate for this in a distorted model by making the slopes rougher and adopting wave length and height scale ratios other than the model length and depth scales. In this way, in certain cases, Froude's law can still be satisfied in a distorted model, while other effects such as viscosity, roughness, etc., can be compensated for sufficiently.

Decisions to be made in a particular case to meet the feasibility criterion are shown in Table I. The breakdown in Table I indicates that certain types; e.g., tidal models can be built to scales as small as $1:10^4$ or $10^2$ horizontal, $1:10^2$ to $10^4$ vertical. For accurate dispersive wave height simulation, the smallest permissible scales are $1:100$ vertical and $1:1000$ horizontally, using a maximum distortion of 10. At these scales a model wave basin of; e.g., $90 \times 80$ feet water area by 2.5 feet deep, will model an ocean area of only about 20 by 18 miles and 250 feet deep.
Table I. **Decisions Influencing Feasibility Criterion**

1. **May one use Froude Law** \( \frac{V^2}{y} = y \)?
   - **YES**, if viscous effects are unimportant; otherwise, allow for friction loss.

2. **May one use Distorted Scale** \( L_r \neq y_r \)?
   - **YES, if** refraction and shoaling are the only important effects; otherwise, use 3.

3. **Must one use Undistorted Scale** \( L_r = y_r \)?
   - **YES, if** diffraction, impact, and breaking effects are important.

4. **If 2, how much distortion is permissible**, \( \frac{y}{L} \)?
<table>
<thead>
<tr>
<th>Distortion</th>
<th>3X?</th>
<th>10X?</th>
<th>100X?</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO DISTORTION</td>
<td>YES</td>
<td>YES(?)</td>
<td>NO</td>
</tr>
<tr>
<td>Generation, terminal effects:</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Dispersive wave propagation:</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Non-dispersive (tidal) propagation:</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Sediment transport:</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

5. **What is smallest vertical scale permissible?**
<table>
<thead>
<tr>
<th>Vertical Scale</th>
<th>1/50 or greater</th>
<th>1/50 to 1/100</th>
<th>1/100 or less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation; terminal effects</td>
<td>YES</td>
<td>YES(?)</td>
<td>NO</td>
</tr>
<tr>
<td>Dispersive wave propagation</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Non-dispersive (tidal) propagation</td>
<td>YES</td>
<td>YES</td>
<td>YES to ( 10^{-4} )</td>
</tr>
<tr>
<td>Sediment transport</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Sacrificing, however, about 50 percent in the correct scaling of wave height will allow reduction of the scales to 1:300 vertical and 1:10,000 horizontal (using a distortion of 30). The wave travel time and general properties will still be fairly well represented, but the run-up will not be accurately scaled because of viscous attenuation of the waves in the model and surface tension effects preventing breaker and shock front formation. (The latter phases can, however, be separately modeled to a suitable scale using the information obtained from the small-scale model as input data.) Thus, a wave basin of above dimensions will then be sufficient for modeling an ocean area of about 200 by 180 miles and 750 feet deep. This would be sufficient for the purpose of scaling the wave-making effects of the detonation of a multikiloton nuclear weapon at the edge of the continental shelf. An accurate wave model of an entire major ocean appears unfeasible because of the earth's curvature and rotation, though there is no doubt that these effects could be either simulated or theoretically allowed for.

To model a multimegaton underwater explosion in the full ocean depth would require an area at least 1000 by 1000 feet by 3 feet deep permitting an undistorted scale of 1:5000 or 1 foot to a mile. Wave heights will necessarily have to be exaggerated to measurable proportions, and such a representation is feasible in a small lake using explosives and sensitive instrumentation under dead calm wind conditions.

Wave Generation

If the generation process is to be simulated by a device other than an actual explosive, a plunger may be used to simulate the action of the bubble, dome and crater following an underwater explosion. This has to conform with the model scaling, and if distortion is necessary, the plunger itself has to be geometrically distorted to conform. The detail representation of the near-source waves will suffer, and only the distant effects, which are relative insensitive to exact source geometry, will be modeled to a fair degree. Similarly, the shore effects of the waves will only be approximately modeled on a small-scale model encompassing the entire wave history from generation to run-up. E.g. refraction, see Fig 2.

For studying run-up and impact effects on waterfront structures of dispersive waves such as generated by megaton-range explosions several hundreds of miles offshore, only the terminal regions, say, the last
three or four wave lengths, need be modeled. A model representing an area of about 10 by 10 miles of ocean fronting a particular or hypothetical shore facility would be adequate. To get sensible wave heights, the scale should be undistorted and not less than 1:100. A basin of 500 by 500 feet is then needed to be able to say that all dispersive characteristics of the waves have been included. Practically, however, the waves in their final stage are individually indistinguishable from waves of permanent form (non-dispersive) such as a solitary wave or a cnoidal wave. It is, therefore, feasible to generate an approximate wave form of suitable proportions representing a typical wave in a dispersive train in a conventional wave flume or basin and experimentally determining its run-up, impact on a structure, or its flow field and calculating the force indirectly.

The input wave forms to such a simplified model can be mechanically programmed to conform to a calculated or otherwise determined prototype wave at a distance of, say, a few miles offshore in, say, about 20 fathoms of water. Such waves will be in the nature of long surges of wave length comparable to the flume or basin dimensions so that effects of boundary reflection will have to be carefully controlled.

Above considerations lead to the conclusion that the NCEL wave test basin with paraboloidal plunger is capable of simulating impulsive water wave systems. As an undistorted model, it can be used to simulate waves up to the maximum properties given in Tables II, III, and IV. The waves and effects will be adequately modeled for these scale ratios.

EXPERIMENTAL VERIFICATION THAT SHOALING AND RUN-UP OF DISPERSIVE WAVES CAN BE MODELED

Dispersion

Water waves in nature generally are dispersive because wave celerity is a function of wave length, and each fraction of a wave spectrum travels at its own speed so that the water surface profile is not conserved. Only uniform and continuous, or solitary waves, are non-dispersive, i.e., the period of any wave is constant with time. An impulsively produced parcel of waves disperses its energy through an ever-increasing frequency band so that wave periods, lengths, duration time, and disturbed area are not constant but grow larger as the wave system radiates outward. Ever decreasing heights,
Table II. Dispersive Impulsively Generated Waves

<table>
<thead>
<tr>
<th>Existing Wave Basin MODEL</th>
<th>PROTOTYPE, Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Ratios:</td>
<td>For Generation and Propagation only For Scale Ratios:</td>
</tr>
<tr>
<td>$x_r$ = horiz. dist. scale</td>
<td>$x_r = y_r = t_r^2 = 1:400$</td>
</tr>
<tr>
<td>$y_r$ = vert. dist. scale</td>
<td>$x_r = y_r = t_r^2 = 1:100$</td>
</tr>
<tr>
<td>$t_r$ = time scale</td>
<td></td>
</tr>
</tbody>
</table>

| Range $B$ = 80 ft.        | 32,000 ft.              | 8,000 ft.              |
| Width $B$ = 90 ft.        | 36,000 ft.              | 9,000 ft.              |
| Depth $y$ = 2.5 ft.       | 1,000 ft.               | 250 ft.                |
| Period $\tau$ = 3 sec.    | 60 sec.                 | 30 sec.                |
| Height $H$ = .1 ft.       | 40 ft.                  | 10 ft.                 |
| w.l. $\lambda$ = 18 ft.   | 7,200 ft.               | 1,800 ft.              |

Equivalent Yield:

1 lb. TNT
at surface at 500 ft. depth at 1-1/2 mi. range
at 80 ft. range at 6 mi. range

200 KT 2 KT
at mid-depth
Table III. Non-Dispersive Impulsively Generated Waves

<table>
<thead>
<tr>
<th>Existing Wave Basin Model</th>
<th>PROTOTYPE, Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(shallow depth, small plunger)</td>
<td>Propagation Only $x_r = 1:100,000, \gamma_r = 1:10,000, \nu_r = 1:100$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range E</td>
<td>60 ft.</td>
<td>1,200 mi.</td>
</tr>
<tr>
<td>Width B</td>
<td>45 ft.</td>
<td>900 mi.</td>
</tr>
<tr>
<td>Depth y</td>
<td>1 ft.</td>
<td>10,000 ft.</td>
</tr>
<tr>
<td>Period T</td>
<td>1 sec.</td>
<td>15 min.</td>
</tr>
<tr>
<td>Equivalent Yield</td>
<td>1 oz. TNT</td>
<td>20 MT</td>
</tr>
</tbody>
</table>

at 3000 ft. depth

at 1000 mi. range
Table IV. Non-Dispersive Non-Impulsively Generated (in Model) Waves

<table>
<thead>
<tr>
<th>Existing Wave Basin MODEL</th>
<th>PART OF PROTOTYPE, Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Plunger, Full Depth and PROGRAMMED STROKING</td>
<td>Terminal Effects (Run-Up, Wave Impact Forces, etc.) only</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale Ratios:</th>
<th>$x_r = y_r = 100$, $t_r = 1:10$, $v_r = 1:10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 3$ to $6$ sec., uniform repetitive</td>
<td>$\tau = 30$ to $60$ sec.</td>
</tr>
<tr>
<td>$H = 0.1$ to $0.5$ ft</td>
<td>$H = 10$ to $50$ ft.</td>
</tr>
<tr>
<td>$\lambda = 20$ to $40$ ft.</td>
<td>$\lambda = 2000$ to $4000$ ft.</td>
</tr>
</tbody>
</table>

Equivalent Yield:
- Up to 10 lbs. TNT simulated by five plunger strokes.

Shore Effects of a 20-megaton explosion at 100J-mile range simulated.
periods and wave lengths again are found in the trailing part of the system all the way back to the origin. The maximum wave height in the group consequently attenuates inversely proportional to distance of travel in conservance of energy. The energy density per unit area decreases with dispersion time. If the total nodal length of the main part of the disturbed water zone is \( L \), radially, and the peripheral length is \( S \), than \( L \) and \( S \) are both proportional to \( R \), range from source; and since energy is proportional to \( \int \eta^2 dA = \text{constant} \), hence \( H^2 R^2 = \text{constant} \). Figures 3 and 4 show that the waves disperse according to theory.

Shoaling Effects (Figure 7)

Green's Law states that for shallow-water (non-dispersive) waves shoaling over a beach, the water surface oscillation \( \eta \) is \( \eta = B^{-\frac{1}{4}} y^\frac{1}{4} \), where \( y \) = depth and \( B \) = width between orthogonals. Impulsively generated waves begin to be less dispersive when and where the limited water depth prevents the celerity of the leading waves from increasing without limit. Due to shoaling, an originally dispersive wave train eventually becomes non-dispersive in its leading portion, at which time dispersive small-period waves are still being generated in the trailing portion due to the continuing transfer of energy to the rear of the group. The shoaling of impulsively generated waves gives rise to a hybrid state of affairs which neither Green's Law \( \eta = B^{-\frac{1}{2}} y^{-\frac{1}{4}} \) nor the dispersive decay law \( H \propto R^{-1} \) strictly represents. The conservation of energy principle yields no ready solution because no fixed frame of reference exists, as the entire wave system occupies the entire region of constant and variable depth. It was found, however, in this work at NCEL that a compromise relationship:

\[
\eta \propto B^{-\frac{1}{2}} y^{\frac{1}{4}} R^{-\frac{1}{2}}
\]

closely agrees with the data from the wave basin as shown in Figure 5a,b. \( B \) is the distance between the rays (orthogonals to crests), and \( R \) is the range from the source. The use of \( R^{-\frac{1}{2}} \) instead of \( R^{-1} \) is because the term \( B^{-\frac{1}{2}} \) accounts for the dispersion in the peripheral
direction and replaces an $R^{-\frac{1}{2}}$. The remaining $R^{-\frac{1}{2}}$ accounts as before for the lingering dispersion in the radial direction, i.e., wave period or length. Because of the refractional bending of the rays, $S$ and $B$ are no longer proportional to $R$, hence the form of the equation above, giving a slower decay rate.

Figure 5a shows that the leading wave closely follows this compromised Green's Law curve in the shoaling region, while the shorter and more dispersive following waves tend to bring the average over the first few waves closer to the $\frac{\sqrt{H}}{R}$ curve. The leading waves are therefore proven to be less dispersive than the trailing waves and can be treated to a first approximation as long shallow water waves following Green's law, provided the bottom slope is gentle, i.e., of smaller order of magnitude than the depth to wave length ratio: slopes $< \frac{d}{\lambda}$. Green's Law is not valid beyond the point where wave height $H = \text{local depth } d$; see Figure 6.

Figure 5b shows that the agreement between experiment and the compromised Green's Law expression holds for a strongly refracted ray (30° incidence with beach normal). The agreement is in fact better for the diffracted ray because of less interference from reflection off the beach of the initial wave.

Figure 9 shows that the run-up height is not simply related to the wave height, but depends on the initial as well as following wave modes. The highest run-up appears to be three times the height of the wave height at breaking, or less, depending on the sense of the leading wave motion. Compare with Figure 7, theoretical.

Figure 10 gives a theoretical estimate of run-up heights and horizontal inundation reach for various deep-water wave heights. It has been found that the deep ocean height of a tsunami is of the order of 1 foot. A 1-foot-high tsunami from the chart is seen to generate a 6-foot bore at the shoreline producing about 20 feet run-up independent of the foreshore slope. On the other hand, a 10-foot-high dispersive wave, generated by an explosion in deep ocean, could produce a breaking wave of 22 to 40 feet high, by the same theory, at the shoreline. Even allowing a possible 50-percent reduction due to dispersive effects, the threat to coastal regions closest to the source could be considerably greater than from distant tsunamis. The United States coastlines are left relatively unprotected against tsunamis as their effects have been relatively small.
From model studies such as these predictions may be made of the unprecedented run-up and the devastating effects possible on unprotected terrain if waves of the heights given are explosively generated near the edge of the continental shelf. No field data of this kind is likely to be obtainable.

EXAMPLES

1. Figure 11 provides an example from an actual test series where the run-up effects of a simulated underwater nuclear explosion on the continental shelf (100 fathoms) at a distance of 4 miles off the shoreline are represented. (The instantaneous water surface profiles at successive time instants were drawn from time histories of the run-up on the submerged and dry beach, recorded at various distance intervals.) The geometrical scale ratio is taken as 1:240 at which scale the wave generator plunger, of wetted diameter 12.8 feet, would correspond to the crater of a 200-kiloton nuclear device detonated at mid-water depth.

The wave elevation at the start of the slope of the foreshore will be 40 feet high and will produce a run-up height of about 70 feet, reducing to about half this value some 3 miles up or down the coast. This explosion was simulated by a sudden retraction of the plunger because the water depth is too small to contain the entire explosion bubble and an initial upheaval of water into the atmosphere follows the detonation, simulated by the lost volume due to the plunger displacement. In the actual case (Ref. Glasstone) this displaced water does not contribute materially to wave motion but forms a condensation cloud and a frothy base surge; the waves are formed by the crater motions only.

2. For deep-water generation and the explosion depth greater than the maximum "bubble" radius, hardly any water is lost to the atmosphere. The plunger motion required to simulate this is a withdrawal to simulate the crater formed when the bubble breaks the surface, followed by a drop of the plunger to full depth to reoccupy its initial position, so as to maintain volume constancy. A known quantity of potential energy or impulsive kinetic energy may thus be imparted to the water and related to the wave-making effects. Figures 12 and 13 show the simulation of the run-up effects of a deep water underwater explosion. For this case the plunger position is, perhaps, unrealistically close to the beach; however, it is best to neglect the early history of these waves and concentrate on the ultimate effects as if
the waves had come from a distant source of formidable magnitude. Figure 13 is therefore considered to be a scale representation of the run-up effects produced by an underwater blast in the megaton range, detonated off the continental shelf. At a scale ratio of 1:240, it is seen that the run-up surge will have a maximum velocity of 85 miles per hour and the maximum run-up height will be 120 feet for a shoreline wave elevation (above mean sea level) of 50 feet.

Figures A-3a and b show a comparison between a moderate yield (5 tons HBX explosive) field experiment and its simulation to a 1:55 linear scale in the NCEL wave basin, using a plunger one sixth the scale of the large plunger of the former case. The field experiment is Shot No. 11 of the Hydra IIA test series, reported by Van Dorn, and is essentially a deep-water-generated impulsive wave study. The model results are seen to provide a close check with theory and are essentially free of background effects, compared to the field test which was conducted in the Pacific Ocean off San Clemente Island of California. The agreement between nature and model is considered good, and except for the slight attenuation due to scale effects, the model's results are easier to analyze by direct methods. Details of the two tests are given in the next section.

FEASIBILITY OF SIMULATION OF A FIELD EXPERIMENT IN WAVE BASIN

An actual field test, 10,000-pound HBX underwater explosion at 15-foot depth in 300 feet deep water (Van Dorn, Hydra IIA, Shot No. 11) was simulated in the wave basin. A 28-inch-diameter plunger, weighing 60 pounds, was dropped from various heights into the quiet water of the wave basin. It was found that the wave height was proportional to the square root of the drop distance and that the wave traces were inverted from those obtained with a sudden pull-out of the plunger. The wave arrival times at various points and periods, wave lengths and number were the same for all cases. A test run with a 4-foot drop distance was used for comparison with the field test and inverted because the downward impulse is used to simulate the upward escaping explosion bubble. The linear scale ratio was 1:55 velocity and time scale was 1:7.4. The model represented the effects of a charge weight of 1/16 pound, and the charge weight scale ratio was 1:55$^3$ or 1:167,000. The prototype crater was calculated from field data to be 99 feet in radius at the rim, 30 feet deep in the center with a mean effective radius of 70.5 feet. The latter
is determined from the propagation velocity of the group $V_g = 19$ fps, from the relationship $a = V_g^2 \cdot 2\pi/g$. The model crater produced in the wave basin was approximately 2 feet radius at the rim and of an effective mean radius of 1.28 feet; the plunger itself was 1.16 feet in radius.

Figure A-1 shows the cross-sectional topography of the model and of the prototype reduced by the crater scale ratio of 1:55. Although the water depth was not modeled fully to scale, the model depth, though less than ideal for this particular case, was sufficient for essentially correct reproduction of the wave motion, except for a minor retarding effect on the initial crest only.

Figure A-2 shows the comparative wave trains and envelopes (reduced to the same scale by the scale ratio of 1:55) for the Hydra test and the model at corresponding distances from the source. The agreement is good as far as the general behavior and the time effects are concerned. The influence of scale is seen in the about 40 percent damping of the model waves in the final stages because of influence of viscosity and surface tension on the rather small waves in the model. On the other hand the Hydra wave height data at Station A is considerably smaller in amplitude than indicated by a model extrapolation. Even the reconstructed spectrum gives a height of only 3.1 feet compared to a model scaled-up value of 6.8 feet. This is ascribed to the combined effects of the pressure attenuation with depth, on the pressure transducer method of wave recording used at Station A in the field test as well as imperfections in the field data processing and reconstruction. A direct method of wave recording such as at Stations B and C is preferred.

Figure A-3b and A-3c compare in more detail the model (inverted trace) and the Hydra IIA Shot No. 11 data for similar locations, model data from a similar run as in Figure A-2. The uncorrected wave pressure-head record is fairly similar to the theoretical for the highest wave crests but deteriorates as the wave periodicity decreases in the trailing waves. The leading crests are masked by background activity. The model results Figure A-3b are free of background, or attenuation effects due to recording limitations, and duplicates the theory very closely. The two theoretical curves A-3a and the envelope of the reconstructed spectrum of the Hydra IIA test are in mutual agreement.

The effect of finite depth on the leading crest is (by the linear theory of Kranzer and Keller) seen to be a small-time lag and an increased amplitude over the deep water theoretical case.
given by Penney. The properties such as periodicity, group celerity, propagation of peaks, and nodes of waves and envelopes are found to agree very closely with the theory presented in Technical Report R-330 (taking "a" as given in that report as the effective mean crater radius, i.e., \(a = 0.71 \times \) the radius at the rim of a paraboloidal crater or approximately "a" is equal to the explosion bubble radius at maximum, \(A_{\text{max}}\)).

The annexed table gives some of these properties for comparison.

Conclusion

It is concluded from this comparison between a field test and its scale-model counterpart that, without necessarily using explosives, simulation of the time-related effects (wave number, period, length, celerity, arrival time) of a prototype field experiment is feasible in a scaled-down form in a wave basin.

In addition, if suitable allowance is made for viscous and surface tension damping effects in the scale model or if model wave heights are not less than about 2 cm, a reasonably good simulation of generation and run-up effects of a prototype field experiment is possible in a scale model.

COMPARISON OF NCEL DATA ON RUN-UP WITH OTHER MATERIAL AVAILABLE

The following comparative data for a continuous uniform slope, starting from a constant depth ocean, \(y_0\), demonstrates feasibility of laboratory measurements, such as at NCEL (1964), to predict prototype run-up. The waves are long, that is, they travel with speed
\[c = \sqrt{g y_0}\]
in the constant depth portion. R here denotes vertical run-up.

Figure B-1 shows Kaplan's (1) results for relative run-up, \(R/H\), versus wave steepness, \(H/L\), for two slopes, 1:30 and 1:60. Kaplan found an exponential relationship to hold
\[\frac{R}{H} = A_8 \left(\frac{H}{L_0}\right)^{-p},\]
with \(p\) near 0.3. Figure B-2 shows Kaplan's results for the relative shoreline height, \(H_s/H\), versus wave steepness, \(H/L\). Again, an exponential relationship:
\[\frac{H_s}{H} = B_8 \left(\frac{H}{L}\right)^{-q},\]
was found with \(q\) near 0.4.
ANNEXURE: Table of Comparative Data, Field and Basin (foot and second units)

<table>
<thead>
<tr>
<th>Station</th>
<th>Depth y</th>
<th>Range R₀</th>
<th>Maximum Wave</th>
<th>Envelope Max</th>
<th>Eff. cr. Radius (deriv.)</th>
<th>R₀/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>558</td>
<td>932</td>
<td>3.1</td>
<td>19.3</td>
<td>48.4</td>
<td>72.5</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
<td>0</td>
<td>(At source, 10,000 lbs HBX fired at -15.6 ft.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>205</td>
<td>1568</td>
<td>2.1</td>
<td>-18.8</td>
<td>83.8</td>
<td>69.0</td>
</tr>
<tr>
<td>C</td>
<td>28</td>
<td>2254</td>
<td>1.7</td>
<td>18.9</td>
<td>119.5</td>
<td>70.0</td>
</tr>
<tr>
<td>Shore</td>
<td>0</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Eff. crater size a = 70.5 ft.; period max. wave Tₚ = 7.43 sec.

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5</td>
<td>1.4</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>33</td>
<td>42</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>0.0375</td>
<td>0.025</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2.57</td>
<td>2.55</td>
<td>2.62</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>13</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>1.28</td>
<td>1.26</td>
<td>1.33</td>
<td>(1.56)</td>
</tr>
</tbody>
</table>

Eff. crater size a = 1.28; period max. wave Tₚ = 1.0 sec.

(Values of R₀/a for Station A nearly same as for Station a, B with b, etc., hence data can be directly related A to a, B to b, etc.)
Energy conservation concepts by Van Dorn (2) led to the interpretation that \( p \) and \( q \) should be equal and \( R/H_s \), a constant dependent only on the slope. \( R \) here denotes vertical run-up, \( L \) here denotes wave length.

Computer results by Amein (3) using the method of characteristics also finds \( R \) to be practically independent of the slope for slopes 1:10 to 1:20 and the same wave steepness, \( H/L \), at the toe of the slope. (This is in conformance with data on Figure 1.) Furthermore, by Green's Law \( H^2 \propto \frac{1}{L} \), so that writing, according to Kaplan

\[
\frac{R}{H} = A_s \left( \frac{H}{L} \right)^{-p}, \quad \text{or} \quad R = A_s \frac{H(1 - p)}{L^{-p}}\text{ constant;}
\]

for a particular wave, it is found

\[
H(1 - p)L^p = \text{constant, and } H^2/L = \text{constant;}
\]

yielding

\[
\frac{1 - p}{p} = 2, \quad \text{and } p = \frac{1}{3}.
\]

Therefore, the slopes of the straight lines through the plots should theoretically be \(-1:3\), as is confirmed by the lines drawn through the plotted points at this slope in Figure B-1. The conservation of energy and validity of Green's Law for dispersive wave run-up is thus proven. The equations of best fit are, therefore

\[
\frac{R}{H} = A_s \left( \frac{H}{L} \right)^{-0.333} \quad \text{and} \quad \frac{H_s}{H} = B_s \left( \frac{H}{L} \right)^{-0.333}.
\]

RELATIONSHIP BETWEEN SHORELINE WAVE HEIGHT AND RUN-UP

The above result also shows that for a particular slope a constant ratio between run-up and shoreline height \( R/H_s \) exists of between 2 and 3, as suggested earlier by Munk (Kaplan, p. 19) et al. For the 1:60 slope,

\[
\frac{R}{H_s} = \frac{A_s}{B_s} = \frac{0.19}{0.08} = 2.4.\]
The points for the NCEL data in Figure B-2 display a trend in good agreement with Kaplan's data, with the NCEL H/L values computed in the same way as Kaplan's data (see definition sketch, Figure B-2).

The experimental runs and computed data (by method of characteristics) are presented in Table V supporting the conclusion that the run-up R* bears indeed a constant ratio to the shoreline wave height H. See Figures 8 and 9 for typical Profiles.

Non-Uniform Slope

Where the slope is non-uniform or interrupted in other ways such as by berms and dikes at the top of the slope, the previous section's results cannot directly be applied.

Results for the run-up of dispersive waves on a slope topped by a dike-type wall and reflecting (vertical) wall were also obtained by Kaplan for the case of a 1:60 beach slope. The run-up on the face of such a wall, compared to the run-up on a uniform (1:60) beach, is from 65 to 100 percent larger, hence about 3.3 to 4 times the shoreline wave height H. (This is in general agreement with preliminary qualitative sea-wall observations in the NCEL wave basin.)

*The values of R by Kaplan and NCEL were mostly derived based on the run-up from the leading initial water surface rise where this produced the maximum run-up. Results for dispersive waves preceded by an initial depression also generally agree with the above conclusions if H is taken as trough to crest height at the toe of the slope, and R is taken as run-up above the initial drawdown line, whereas in the case of an initial rise, H is taken as SWL to crest height at toe and R as run-up above SWL.
Table V. Run-up Versus Shoreline Wave Height

<table>
<thead>
<tr>
<th>Run</th>
<th>$H_s$</th>
<th>$R$</th>
<th>$R/H_s$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Shoreline wave height</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Run-up above still water elevation or initial</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>drawdown elevation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ratio $R$ to $H_s$</td>
</tr>
<tr>
<td>NCEL &quot;A&quot;</td>
<td>1.3 in.</td>
<td>3.5 in.</td>
<td>2.7</td>
<td>Plunger drop of 2.5 feet deep.</td>
</tr>
<tr>
<td>&quot;B&quot;</td>
<td>1.22 in.</td>
<td>3.1 in.</td>
<td>2.85</td>
<td>Plunger drop and raise 2.5 feet.</td>
</tr>
<tr>
<td>&quot;C&quot;</td>
<td>1.9 in.</td>
<td>5.22 in.</td>
<td>2.73</td>
<td>Plunger pull-out from 2.5 feet.</td>
</tr>
<tr>
<td>&quot;D&quot;</td>
<td>3.5 in.</td>
<td>10.00 in.</td>
<td>2.85</td>
<td>Plunger pull-out and drop 2.5 feet.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Waves 3 second period 1 to 3 inches high)</td>
</tr>
<tr>
<td>Amein &quot;1&quot;</td>
<td>33 ft.</td>
<td>74 ft.</td>
<td>2.25</td>
<td>Slope 1:20</td>
</tr>
<tr>
<td>&quot;2&quot;</td>
<td>41 ft.</td>
<td>94 ft.</td>
<td>2.25</td>
<td>Slope 1:13.4</td>
</tr>
<tr>
<td>&quot;3&quot;</td>
<td>40 ft</td>
<td>95 ft.</td>
<td>2.38</td>
<td>Slope 1:10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Waves 2 minute period, 20 feet high at toe of slope)</td>
</tr>
</tbody>
</table>
OSCILLATORY WAVES (AND UNIFORM CONTINUOUS SLOPE)

For comparison, data for oscillatory (non-dispersive) waves from Savage* (4) is presented in Figure B-3 for slopes of from 1:30 up to 1:2-1/2. These results indicate that the run-up of oscillatory waves is different from that of dispersive waves. There is for one a much stronger dependence on the slope and for a 1:30 slope, and flatter slopes in particular, the waves run-up very little.

COMPARATIVE RESULTS

Finally, in Figure B-4 the data for both non-dispersive and dispersive waves are presented together as a function of slope for various wave steepnesses. Some additional data by Granthem (5) for slopes steeper than 1:5 is included.

FINDINGS

1. Water level variations, by measurements in the basin, are well predicted by theory, such as Kranzer and Keller and Penney (NCEL Technical Report R-330) in the dispersion process away from the impulsive disturbance. The waves produced by the simulation facility are thus truly dispersive.

2. The shoaling of dispersive waves over a sloping beach, by measurements in the wave basin, are found to be predictable by Green's Law for waves with dominant height of 0.2 feet, length of 20 feet, period of 3 seconds in water 2.5 feet deep.

3. Measurements in basin when extrapolated to ocean by Froude criteria check well those measured in ocean in the Hydra IIA experiments near San Clemente.

*The term $H_0/L_0$, relating to wave height and length in infinite depth water, has no meaning for long waves as the average ocean depth of 14,400 feet (Pacific) limits the likely long period waves (tsunamis) to "shallow-water or $C = \sqrt{g} \gamma$ waves", therefore, the data of Savage had to be reconverted from $H_0/L_0$ to $H/L$, and of Savage's data only that pertaining to "shallow-water" waves was used.
4. Run-up data from several sources compare well with NCEL wave basin results. The following observations were made with respect to the behavior and predictability of run-up due to dispersive waves such as generated by explosions (for slopes flatter than 1:10):

a. The leading wave of a dispersive wave system will run-up considerably higher than an oscillatory wave of equal height and steepness, namely, up to two or four times as high.

b. It is followed by pseudo-oscillatory type-waves (yet still dispersive), possibly higher than the leading wave, but producing generally a much lower run-up, a result which was reported by Kaplan (1955, p. 30) and rightly ascribed as due to interaction with the backwash of the preceding wave. Since no backwash precedes an initial elevation wave, it runs up without loss of energy, hence the higher run-up value.

c. The curves on Figure B4 therefore permit run-up predictions known *g* slope, *H* and *L*, at the toe of the slope. Where *H*/*L* is less than 0.04, Figure B1 is to be used which is confirmed by run-up data in Hilo Harbor, Hawaii, for 1946 tsunami. Furthermore, prototype data of wave height at the shoreline may be used to predict run-up equal to the ratio two to three times the shoreline wave height (also Kaplan, p. 28, 29).

<table>
<thead>
<tr>
<th>Slope</th>
<th>R/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:60</td>
<td>2.4</td>
</tr>
<tr>
<td>1:13.6</td>
<td>2.75</td>
</tr>
</tbody>
</table>

5. General Comments. The above considerations lead to the conclusion that the NCEL wave basin constitutes a facility whereby the properties and effects of dispersive waves generated by large underwater nuclear explosions at or near the edge of the continental shelf, upon waterfront facilities in the vicinity may be modeled to scale. To this end it is highly suitable.*

*The existing wave basin facility is best for modeling dispersive wave processes in depths up to 600 feet; that is, *H*/d > 0.02; *λ*/d < 10 and *H*/λ > 0.0001. Because transoceanic dispersive wave systems have *H*/d < 0.0001, *λ*/d > 10 and *H*/λ < 10^-5 and still produce destructive effects on shorelines; their true scaling on a hydraulic model is not feasible with this or even a much larger facility. The waves may be studied by analogy with the relatively steep waves practical in a scale model, but the shore effects such as run-up and impact forces will not be quantitatively correctly modeled for transoceanic dispersive waves because of the limiting effects of viscosity and surface tension in the model scale. Depth denoted by *d* here, and wavelength by *λ*. 

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Furthermore, and to a lesser extent, it is suitable for studying the properties but not effects of tsunamis and transoceanic impulsively generated dispersive water waves. To study the effects of these, a separate model of the end-effects to a suitable scale is necessary and could be carried out in the same or a similar test facility concurrently. Alternatively, a small lake or basin of about 1000 feet in diameter could be employed for studying such deep-ocean dispersive wave systems.

CONCLUSIONS

Experimental verification of theoretical and available prototype informations shows that:

1. A scaled-down model study of pertinent aspects of impulsive wave generation, dispersion and run-up is feasible.

2. The kinematics of wave motion and run-up: celerity, travel time, period, wave length are represented truly according to the Froude scaling law.

3. The dynamics of wave motion are scaled to a lesser accuracy because of "scale effects" on energy transfer and dissipation. Wave height and breaking behavior are less accurately scaled.

The run-up depends upon the energy dissipated and is influenced by scaling. Nevertheless, the model may be verified and adjusted to reproduce observed or derived prototype events. It is a valid means for analyzing a given situation.

4. The early history of the wave motion has much less effect on the run-up than the travel path profile. Run-up of impulsively generated waves (dispersive) is higher relative to shoreline wave height than run-up of oscillatory waves. Best results can be expected from simulating a known wave on a larger-scale model of the near shore topography.

5. In summary, feasibility has been demonstrated that data can be obtained by means of scale models of the run-up produced by explosively generated dispersive water waves, sufficiently accurate to predict the extent of the run-up on beaches of gentle slope.
REFERENCES


3. Saville, Thorndyke, Jr., "Laboratory data on wave run-up and overtopping on shore structures," Beach Erosion Board, Technical Memorandum No. 64, October 1955.


13. Amein, M., "Long waves on a sloping beach and wave forces on a pier deck," prepared at the University of North Carolina under contract NBy-32236 for the U. S. Naval Civil Engineering Laboratory, September 1, 1964, 26 pp.


Figure 1. Wave making effects of underwater explosions (Ref 8330)
90° & 50° (42" sphere, 600 lbs, dropped 8.4 ft.)

Uniform.
2.5 ft. deep

LEGEND:
- First crest position
- Envelope maximum position
- Ray

Note: (1) Stronger refraction of first wave crest.
(2) Envelope maximum arrives earlier at shore than in deep water.

Figure 2. Selective refraction of dispersive waves generated in deep water.
Figure 3. Verification of dispersion (height) of waves according to theory. Leading wave $H \propto R_w^{-2.5}$
Max of envelope $H \propto R_w^{-1}$
Figure 4. Experimental verification of dispersion period according to expression giving wave period as function of distance and rank of wave $\tau_n \rightarrow \frac{2\pi n}{\sqrt{ng}}$.  

**LEGEND:**
- $\circ$ Successive wave periods
- $\times$ Maximum wave
- $\_\_\_$ Theoretical $\tau_n = \sqrt{\frac{8\pi n}{g}} \cdot \{f_n\}$ or, approx. $\tau_n = \sqrt{\frac{2\pi n}{ng}}$
Figure 5b. Same as preceding but for refracted ray 30° to normal angle of incidence.
Experimental Data for Impulsive Generation (dispersive waves)
either 2.5' immersion (single)
or 2.5' withdrawal (single)
in 1.0 seconds

Figure 6. Typical data for wave height versus distance for impulsive generation. (Note: Multiple stroking non-dispersive wave can have η > 4" everywhere.)
Figure 8. Wave profiles for 2.5 ft stroke retract & plunge test (Ω).
A. PLUNGE

B. PLUNGE & RETRACT

C. RETRACT

D. RETRACT AND PLUNGE

E. PLUNGE & RETRACT, 3X

FIGURE 9  Experimental Profiles of Extreme Wetted Region of Wave Motion Following an Impulsive Disturbance.  Compare Effect on Run-up
Figure 10-a. Critical runup zone affecting shore-based facilities

Figure 10-b. Deep water wave height $H_o$ and corresponding wave heights at shoreline $H_B$. 

RU = $3H_B$
Runup in ft.

$D = \text{horizontal run-up distance (inland from -10' contour)}$

$H_B / H_0$

$W = \text{runup height}$

Rule for runup height

$W = 30 / D_{ave}$

upper limit of likely runup

lower limit

1:10

1:100

1:1000

1:10,000

1:10,000

Hawaii tsunami

breakwater

$D = \text{horizontal run-up distance (inland from -10' contour)}$

$H_B / H_0$

$W = \text{runup height}$

Rule for runup height

$W = 30 / D_{ave}$

upper limit of likely runup

lower limit

1:10

1:100

1:1000

1:10,000

1:10,000

Hawaii tsunami

breakwater

$D = \text{horizontal run-up distance (inland from -10' contour)}$

$H_B / H_0$

$W = \text{runup height}$

Rule for runup height

$W = 30 / D_{ave}$

upper limit of likely runup

lower limit

1:10

1:100

1:1000

1:10,000

1:10,000

Hawaii tsunami

breakwater

$D = \text{horizontal run-up distance (inland from -10' contour)}$
Figure II. RUN-UP PROFILES FOR 75 FT. RETRACTION TEST (C)
FIGURE 12 Diagram to Scale Showing Maximum Run-up Produced by Plunger Withdrawal (1 SEC) Followed by Immersion (1 SEC). Summary of Properties.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>20°</th>
<th>2.4°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>+22°</td>
<td>3.0°</td>
</tr>
<tr>
<td>$\theta$</td>
<td>45°</td>
<td>(7° More)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>25°</td>
<td>(-3° More)</td>
</tr>
</tbody>
</table>

- $\theta$: Angle of Plunger To Beach
- Run-up:
  - Ratio of Run-up
  - Random
  - Wave at Breaking
Figure 13
Run-up Profiles for
2.5 ft RETRACT AND PLUNGE
Note: Limited bottom depth, effects only sensible on the leading wave.

Note: Horizontal and Vertical Scales are the same in both figures

Figure A-1
Comparison of a field test (below) and its simulation in the wave basin (above)
Comparison of water level oscillation (wave time trace) of a field test (HYDRA) with its simulation in wave basin (NCEL).

Increased by 60% to allow for pressure attenuation of wave gage; this trace only.
**Figure B1.** Relative run-up ($A_s$) versus wave steepness ($H/L$) at the toe of various values of uniform slopes, for dispersive leading waves. Comparison of data from Kaplan, REB TM 60, pp. 8 and 12 with NCEL data, 1964, theoretical results by Amin. Extrapolation verified by 1946 tsunami, also plotted.
Figure 82: $H_s$, Shoreline height over wave height at toe of slope, for various values of $H$. Data from Kaplan, BEB TM 60, p. 25.
LEGEND

NON-DISPERSIVE WAVE RUN-UP
Data presented are taken from SAVAGE, BEB TM 109 pp. 8, 9, 19, 21, 27, and was converted from \( \frac{R}{H_0} \) VS \( \frac{H_0}{T^2} \) to \( R/H \) VS \( H/L^0 \).

DATA FOR 1:2 1/2 to 1:30 SLOPES

\[
\text{Figure 23. Relative run-up } \frac{R}{H} \text{ versus wave steepness } \frac{H}{L} \text{ at the toe of various values of uniform slopes, for non-dispersive (osc.) waves. Comparison of data from Savage, BEB TM 109, pp. 8-27 with data for dispersive waves on preceding graph.}
\]
Vert.

1:1 1:2 1:5 1:10 1:15 1:30 1:60

LEGEND

- - - Non-Dispersive

Dispersive • •

X Granthem WRL 348
○ BEB TM 109 Savage
• BEB TM 60 Kaplan
○ plus NCEL data

Figure B4. Wave run-up on smooth slopes, relative to wave height at TOE of slope, R/H, vs. Slope.

Notes:

1. For Non-dispersive (oscillatory) waves, wave height H is taken as trough to crest elevation.

2. For Dispersive waves, wave height H is taken as still water level to first crest elevation in the case of an initial rise, or first trough to first crest elevation in the case of an initial depression.

3. R, Run-up is the vertical extreme reach above the still water level of the wave uprush.

4. L, Wave length is taken as crest to crest distance, at toe of slope. For dispersive it is twice distance init. rise to first crest.