SPECIAL REPORT

INVESTIGATION OF ELECTRIFICATION OF POWDERS IN FLOW THROUGH TUBES AND NOZZLES. II. CHARGE ANALYSES OF DEAGGLOMERATED POWDERS

by

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CORPORATION
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Investigations Under U.S. Army Chemical Center


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ABSTRACT

Charge measurements were conducted with airborne powders flowing out of a capillary tube at flow rates between 100 and 500 cc/min., corresponding to 3 and 15 m/s linear flow rate. The relative humidity of the air was 10%. The powders were saccharin, Carbowax 6000, Cab-O-Sil, and the two former materials with 1% of Cab-O-Sil as a deagglomerant. The MMD particle sizes were 7, 14, and 0.02 μ, respectively.

The dependence of the positive and negative charges per gram of material and of the abundances of positively and negatively charged material upon the flow rate were determined. The bearing of these quantities upon agglomeration is discussed with respect to flowing powders and aerosols. The effects of the deagglomerant are different in the two cases and depend upon the nature of the powder and upon the flow rate.

It is concluded that a deagglomerant may be effective as a free-flowing agent by causing large agglomerates, although smaller than those in the pure material, and by causing nearly equal abundances of positive and negative particles. Both effects are ruinous to presized aerosol material.
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1. INTRODUCTION

When a powder flows through a tube, the tube wall acquires a coat of the powder, and the flowing powder rubs against this coat. As a result of the friction, the flowing powder becomes electrically charged. When a powder is milled or generally handled, there is also such friction and electrification. In all these cases, the charges affect the properties of the powder, particularly by promoting agglomeration. This is of considerable importance in the production of aerosols because agglomeration increases the particle size. Furthermore, the charges acquired in the spraying of an aerosol affect the stability of the aerosol and also the effectiveness of the aerosol, e.g., an insecticide.\(^1\)

These effects have been studied extensively in a current research program. They are of particular importance at low humidities because few materials in particulate form can be handled at all at high humidities, and because electrification in friction is greatest at low humidities. The effects of the relative humidity of the air upon electrification and adhesion between powder particles are the subjects of preceding reports\(^2-4\).

According to Nash et al\(^5\), the addition of a deagglomerant reduces the charge and the agglomeration. Thus, the addition of 1% Cab-O-Sil to saccharin reduced the charge on the powder as it flowed out of a hole in the bottom of the container and also reduced the shear strength of the compacted powder.

It appears, therefore, that a deagglomerant could have drastic effects upon the properties of an aerosol by suppressing the charge. In order to study this and other electrification phenomena, an investigation has been conducted, in which airborne particles with and without an added deagglomerant have been analysed with respect to charge and charge distribution as they emerge from a capillary tube. The technique used in this investigation was described in a previous report\(^6\). The results are presented and discussed in the following sections.

In order to keep this investigation within the frame of the current program, agglomeration per se was not included as an object of study. It turns out, however, that information on agglomeration may be derived from the results. It is therefore in order to discuss the results with respect to their bearing upon agglomeration in preparation for future investigations that include the study of agglomeration. This will be done in Sections 5 and 6.
The results show that a deagglomerant may affect the charge and the charge distribution more or less, depending upon the nature of the powder, to which it is added, and upon the flow rate. The deagglomerant may have different effects on different powders, and on the same powder at different flow rates. Thus, to take an extreme case, the deagglomerant could conceivably suppress agglomeration at low flow rates as in ordinary handling but promote agglomeration in the sprayed aerosol. As a consequence, a deagglomerant should be evaluated with respect to its effects upon the properties of the aerosol as well as those of the powder.

2. EXPERIMENTAL PROCEDURE

The experimental technique and the equipment used are described in a preceding report\(^6\).

The materials were given to us by Mr. Nash of General Mills, Inc. They were:

- Saccharin, MMD 7μ, size distribution ref.\(^5\), p. 2-6
- Carbowax 6000, MMD 14μ, size distribution ref.\(^5\), p. 2-7
- Cab-O-Sil, MMD 0.02, ref.\(^5\), p. 4-54
- Saccharin + 1% Cab-O-Sil
- Carbowax + 1% Cab-O-Sil

The powder was aerosolized in the apparatus shown in Figure 1. The powder is held in a container, which is vibrated, and the powder is stirred up by a stream of air of about 10% relative humidity. The aerosol is sucked into the charge analyzer by a separate pump. The flow rates in the two systems are thus controlled independently. The flow rate of the stirring air was kept constant at approximately 2000 cc/min, corresponding to a linear velocity of 3 cm/s. The flow rate in the charge analyzer was varied between 100 and 500 cc/min, corresponding to a linear velocity of 3 to 15 m/s.

There was noticeable agglomeration of the powder in the aerosol generator, and the particle size was certainly larger than the nominal size. The size of the agglomerates was not assessed.

No effort was made to maintain a constant aerosol density. Previous experience\(^6\) has shown that the charge per gram of material is independent of the density over a wide range. This holds also in this investigation.
Figure 1. Aerosol Generator (Dimensions in mm).
It was pointed out in the preceding report\(^6\) that the charge analyzer does not work well at high humidities of the air because the humidity reduces the insulation resistance. The most obnoxious result of this effect is a shift of the zero level of the charge record. This reduces the accuracy of the charge measurement much more than does the leakage current. In order to avoid this error, it is necessary to operate the charge analyzer at a low enough humidity. This is not considered to be a restriction to the usefulness of the charge analyzer because fine powders cannot be handled at high humidities anyway. It was found that a relative humidity of the air of 10\% is adequate. This happens to be the relative humidity of the air from our compressor, and the use of air of this humidity was therefore adopted in these experiments.

In another investigation, following upon the one treated in this report, it appeared that further drying of the air caused the data to become erratic, and occasionally no charge at all was recorded. This may be a result of poor adherence of the aerosol material to the tube wall so that the tube wall is not coated by the aerosol material but left bare. This issue has not been studied as yet.

3. EXPERIMENTAL RESULTS

The primary measured quantities are total positive charge \(Q_+\), total negative charge \(Q_-\), weight of positive deposit \(W_+\) and weight of negative deposit \(W_-\), all in a period of 1 min. There was no neutral deposit in these experiments. These primary quantities are plotted against the flow rate in Figures 2 to 8.

At a constant density of material, expressed as weight of powder per unit volume of air, the total deposit,

\[
W = W_+ + W_-
\]  

(1)

is proportional to the flow rate. This quantity is plotted against the flow rate in Figure 9.

Derived quantities of particular interest are the average positive and negative charges per gram,

\[
q_+ = \frac{Q_+}{W_+}
\]  

(2)

\[
q_- = \frac{Q_-}{W_-}
\]  

(3)
Figure 2. Total Positive Charge Deposited in $I \cdots \Omega_+$ as Function of Flow Rate $v$.

Ø Saccharin, × Saccharin + 1% Cab-O-Sil
Figure 3. Total positive charge deposited in 1 min $Q_+$ as function of low rate square $V^2$. ▲ Carbowax 6000, ◇ Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 4. Total negative charge deposited in 1 min $Q_-$ as a function of flow rate square $v^2$. • Saccharin. X Saccharin + 1% Cab-O-Sil.
Figure 5. Total negative charge deposited in 1 min $Q_-$ as a function of flow rate $v$. ▼ Carbowax 6000, ▼ Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 6. Weight of positive deposit in 1 min $W_+$ as a function of flow rate $v$. Saccharin, X Saccharin + 1% Cab-O-Sil.
Figure 7. Weight of positive deposit in 1 min $W_+$ as a function of flow rate $v$. Carbowax 6000, Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 8. Weight of negative deposit in 1 min \( W_\) as a function of flow rate \( v \). Saccharin, "Saccharin + 1% Cab-O-Sil,
Carbowax 6000, △ Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 9. Total weight of deposits in 1 min. $W = W_+ + W_-$ as a function of flow rate $v$. Saccharin, $\times$ Saccharin + 1% Cab-O-Sil, Carbowax 6000, Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
the average absolute charge per gram

\[ q = \frac{Q_+ + Q_-}{W} \]  

(4)

and the ratio \( \frac{W_+}{W_-} \). These quantities are plotted against the flow rate in Figures 10 to 16.

The scatter of the points in some of these plots is so great as to make it almost impossible to draw meaningful curves. This scatter is not a result of inaccuracy of the measurements but is a feature of the phenomena under study. Thus, the scatter in the values of \( q \), \( q_+ \), and \( q_- \) is little as compared to that in the values of the primary quantities, and the scatter in the data for the materials of a single component is little as compared to that in the same data for the mixtures.

In the case of Carbowax 6000 + 1% Cab-O-Sil, the large Carbowax 6000 particles are coated with the small Cab-O-Sil particles. This coating may be expected to be more or less complete, and one may therefore expect the data to scatter as much as the coverage varies. The data may then vary between low values and those for Carbowax 6000 or Cab-O-Sil alone.

Another reason for scatter is variation in the coat of the material on the tube wall. In order to eliminate such scatter, the powder was blown for some time through the tube before the data were taken.

At the side of these two factors, the variations in the humidity of the air are probably of lesser effect.

It follows from this discussion already that the data are meaningful in spite of the scatter, and that the scatter as such is informative of the phenomenon. This point will be further discussed in Sections 4 and 5. The curves drawn on the data are based upon the analysis of the data presented in Section 4.

4. ANALYSIS OF EXPERIMENTAL DATA

A casual glance at some of the data gives an impression of a wild scatter that does not permit an analytical representation. On the other hand, some of the data fall on a straight line with remarkable precision. An instance of the wild scatter are the \( Q_+ \), data
Figure 10. Average charge per gram of positive deposit $q_+$ as a function of flow rate $v$. ◦ Saccharin, × Saccharin + 1% Cab-O-Sil, Carbowax 6000, ▽ Carbowax 6000 + 1% Cab-O Sil.
Figure 11. Average charge per gram of positive deposit $q_+$ function of flow rate square $v^2$ for Cab-O-Sil.
Figure 12. Average charge per gram of negative deposit $q_-$ as a function of flow rate $v$. Saccharin, $\times$Saccharin + 1% Cab-O-Sil, Carbowax 6000, Cab-O-Sil, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 13. Total average absolute charge per gram \( q = \frac{Q_+ + Q_-}{W} \) as a function of flow rate \( v \). - Saccharin, \( \times \) Saccharin + 1\% Cab-O-Sil, . Cab-O-Sil.
Figure 1. Total average absolute charge per gram \( q = \frac{(Q_+ - Q_-)}{W} \) as a function of flow rate \( V \). Carbowax 6000, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 15. Ratio of weights of positive and negative deposits $W_+/W_-$ as a function of flow rate $v$. • Carbowax 6000, + Carbowax 6000 + 1% Cab-O-Sil.
Figure 16. Ratio of weights of negative and positive deposits $W_\text{-}/W_\text{+}$ as a function of flow rate $V$. : Saccharin, $\times$ Saccharin + 1% Cab-O-Sil, : Cab-O-Sil.
for Carbowax 6000 + 1% Cab-O-Sil in Figure 5. An instance of the high precision are the q data for Carbowax 6000 in Figure 14. It is clear that the accuracy of the measurements is very high and that the scatter is a feature of the phenomenon under study.

As already pointed out, the density of the airborne powder was not controlled, and there is no way to control it. This point will be discussed further in Section 6. Nevertheless, the plot in Figure 9 of W against v for Carbowax 6000 + 1% Cab-O-Sil is a remarkably good straight line, the slope of which is the density. Hence, there is nothing wrong with the aerosol generator. The corresponding plot for saccharin + 1% Cab-O-Sil is also fairly good. But the plots for the pure powders saccharin, Carbowax 6000, and Cab-O-Sil are far from linear. Since the deagglomerated powders have a constant density and the pure powders do not, the variations in the density must be a result of agglomeration. Accordingly, the deviation from proportionality between W and v give information on agglomeration in the powder.

The analysis of the data in this section is a purely mathematical operation. It is based upon the interrelation of the quantities under study, which makes it possible to calculate one from the others, and upon the observation referred to that the error of measurement is negligible. The interrelation between the quantities has the effect of increasing the number of observations. Actually, there are 5 values of v plus the origin, since $Q_+$, $Q_-$, $W_+$, $W_-$, and W must necessarily be zero at $v = 0$. This makes effectively 6 values of v. The interrelation between quantities effectively multiplies this number of observations.

4.1 SACCHARIN

The data for the positive deposits are fairly good. $Q_+$ and $q_+$ are proportional to $v$, and $W_+$ is constant at high flow rates. These data are thus consistent. The best fit of the data gives

Figure 6. $W_+ = 1.140 \frac{mg}{min} \cdot v > 200 \frac{cc}{min}$  \hspace{1cm} (5)

Figure 10. $q_+ = 7.22 \times 10^{-3} \frac{\mu C}{g} \cdot v > 200 \frac{cc}{min}$ \hspace{1cm} (6)

Figure 2. $Q_+ = 823 \times 10^{-8} \frac{\mu C}{min}$ \hspace{1cm} (7)
For the negative deposit we find

Figure 8. \[ W^- = 5.84 v \times 10^{-3} \text{ mg.min} \]  
(8)

Figure 12. \[ q^- = 8.91 v \times 10^{-3} \frac{\mu C}{g} \]  
(9)

Figure 4. \[ Q^- = 5.20 v^2 \times 10^{-8} \frac{\mu C}{\text{min}} \]  
(10)

The formulae (7) to (10) hold for all values of v. The ratio \( \frac{W^-}{W^+} \) calculated from formulae (8) and (5) is

Figure 16. \[ \frac{W^-}{W^+} = 5.10 v \times 10^{-3}, \quad v > 200 \frac{\text{cc}}{\text{min}} \]  
(11)

The product \( q_+ q^- \) calculated from formulae (6) and (9) is

Figure 17. \[ q_+ q^- = 64.4 v^2 \times 10^{-6} \frac{\mu C^2}{g}, \quad v > 200 \frac{\text{cc}}{\text{min}} \]  
(12)

There is only one measurement below \( v = 200 \text{ cc/min} \). However, the functions in the region \( v < 200 \text{ cc/min} \) can be constructed from this one measurement with good accuracy taking

Figure 6. \[ W^+ = 30 v^2 \times 10^{-6} \frac{\text{mg}}{\text{min}}, \quad v < 200 \frac{\text{cc}}{\text{min}} \]  
(13)

This together with the formula (7) for \( Q_+ \) gives

Figure 10. \[ q_+ = \frac{274}{v} \frac{\mu C}{g}, \quad v < 200 \frac{\text{cc}}{\text{min}} \]  
(14)

The formula for \( q_- \) can also be derived from the product \( q_+ q^- \) shown in Figure 17.
Figure 17. The product $q_+ \times q_-$ as a function of flow rate square $v^2$. : Saccharin, Saccharin + 1% Cab-O-Sil, Carbowax 6000, Carbowax 6000 + 1% Cab-O-Sil.
Below \( v = 200 \text{ cc/min} \) this product is a constant.

\[
\text{Figure 17. } q_+ q_- = 2.4 \frac{\mu C^2}{g}, \ v < 200 \frac{\text{cc}}{\text{min}} \quad (15)
\]

This divided by \( q_- \) from formula (9) gives

\[
\text{Figure 10. } q_+ = \frac{270}{v} \frac{\mu C}{g}, \ v < 200 \frac{\text{cc}}{\text{min}} \quad (16)
\]

in agreement with formula (14).

The ratio \( W_+ / W_- \) from formulae (13) and (8) is

\[
\text{Figure 16. } \frac{W_+}{W_-} = 5.14 \times 10^{-3}, \ v < 200 \frac{\text{cc}}{\text{min}} \quad (17)
\]

which is almost exactly the inverted value of that found at \( v \geq 200 \text{ cc/min} \) according to formula (11). Thus, \( W_+/W_- \) increases proportionally to \( v \) up to \( W_+ = W_- \), whereupon \( W_-/W_+ \) increases with \( v \) at the same rate. The measured value of \( W_-/W_+ \) given in Figure 16 for \( v = 100 \text{ cc/min} \) is considerably lower than that given by formula (17), but the measured value for saccharin + 1% Cab-O-Sil coincides with the calculated values. This point will be further discussed below.

The product \( P = Q_+ Q_-/W^2 \) is plotted in Figure 18. The plot gives

\[
\text{Figure 18. } \frac{Q_+ Q_-}{W^2} = 0.140 v^2 \times 10^{-4} \frac{\mu C^2}{g}, \ v > 200 \quad (18)
\]

\[
\frac{Q_+ Q_-}{W^2} = 0.56, \ v < 200 \quad (19)
\]
Figure 18. \( P = \frac{Q_+ x Q_-}{W^2} \) as a function of flow rate square \( v^2 \).

O Saccharin, x Saccharin + 1\% Cab-O-Sil.
These two formulae together with the formulae (7) and (10) give

\[
W = \left[ \frac{Q_+ Q_-}{Q_+ Q_-} \right]^{1/2}
\]

Figure 9.

\[
= 0.175 \sqrt{\frac{1}{2}} \frac{mg}{\min}, \quad v > 200 \frac{cc}{\min}
\]  

(20)

\[
W = \left[ \frac{Q_+ Q_-}{Q_+ Q_-} \right]^{1/2}
\]

Figure 9.

\[
= 0.874 \sqrt{\frac{3}{2}} \times 10^{-3} \frac{mg}{\min}, \quad v < 200 \frac{cc}{\min}
\]

(21)

These functions are plotted in Figure 9. They fit the data fairly well. Of course, by definition, \(W\) is the sum of \(W_+\) and \(W_-\), and the formulae (20) and (21) are therefore approximate.

Finally, \(q = Q_+ Q_-/W\) is shown in Figure 13. This plot is also linear. Using the formulae for \(Q_+\), \(Q_-\), \(W_+\), and \(W_-\) we obtain

\[
q = \frac{823 \nu \times 10^{-8} + 5.20 \nu^2 \times 10^{-8}}{1.14 \times 10^{-3} + 5.84 \nu \times 10^{-6}}
\]

\[
= 7.22 \nu \times 10^{-3} \left(1 + 1.20 \nu \times 10^{-3}\right) \frac{\mu C}{g}, \quad \nu > 200 \frac{cc}{\min}
\]

(22)

\[
q = \frac{823 \nu \times 10^{-8} + 5.20 \nu^2 \times 10^{-8}}{5.84 \nu \times 10^{-6} + 30 \nu^2 \times 10^{-9}}
\]

\[
= 1.41 \left(1 + 1.18 \nu \times 10^{-3}\right) \frac{\mu C}{g}, \quad \nu > 200 \frac{cc}{\min}
\]

(23)
These formulae agree with the data in Figure 13 by being linear and constant, respectively, but they differ in magnitudes and do not give a good fit. It appears that $q$ is not a simple function of $v$ for saccharin.

4.2 SACCHARIN + 1% CAB-O-SIL

The data for saccharin + 1% Cab-O-Sil are very similar in some respects to those for pure saccharin. The values of $W_+/W_-$ (Figure 16) differ less from the plot for saccharin than do the values for saccharin themselves. The values of $P = Q_+ Q_- / W_+^2$ (Figure 18) below $v = 200$ cc/min fall right on the curve for saccharin for $v \geq 200$ cc/min. The same applies to $q_+ q_-$ (Figure 17) and to $q_+$ (Figure 10). We may thus calculate $q_-$ from $q_+ q_-$ and $q_+$ as given in formulae (12) and (6), respectively. This gives

$$ Figure 12. \quad q_- = 8.91 v \times 10^{-3} \frac{\mu C}{g}, \quad v < 200 \frac{cc}{min} \quad (24) $$

Thus, $q_-$ for $v < 200$ cc/min follows the formula (9), which holds for saccharin at all values of $v$.

$W_-$ is a constant between 200 and 300 cc/min

$$ Figure 8. \quad W_+ = 3.32 \frac{mg}{min}, \quad 100 \leq v \leq 200 \frac{cc}{min} \quad (25) $$

We may thus calculate $W_+$ from $W_+/W_-$ and $W_-$ as given by formulae (17) and (25), respectively.

$$ Figure 6. \quad W_+ = 17.1 v \times 10^{-3} \frac{mg}{min}, \quad v < 200 \frac{cc}{min} \quad (26) $$

Combining the formulae (6) for $q_+$ and (26) for $W_+$ we obtain

$$ Figure 2. \quad Q_+ = q_+ W_+ = 12.3 v^2 \times 10^{-8} \frac{\mu C}{min}, \quad v \leq 200 \frac{cc}{min} \quad (27) $$
Combining the formulae (24) for $q_-$ and (25) for $W_-$ we obtain

$$Q_- = q_ - W_- = 0.296 v \times 10^{-4} \frac{\mu C}{\text{min}}, \quad v \leq 200 \frac{cc}{\text{min}}$$  \hspace{1cm} (28)

As may be expected, the data for saccharin + 1% Cab-O-Sil scatter much more at high flow rates than do those for pure saccharin. Among the primary variables, $Q_-$ shows comparatively little scatter.

$$Q_- = 17.8 v^2 \times 10^{-8} \frac{\mu C}{\text{min}}, \quad v \geq 200 \frac{cc}{\text{min}}$$  \hspace{1cm} (29)

The data for $W$ (Figure 9) in the range $v \geq 200 \text{ cc/min}$ follow a straight line except for the point at $v = 300 \text{ cc/min}$ that is too high. This plot gives

$$W = 35.0 v \times 10^{-3} \frac{mg}{\text{min}}, \quad v \geq 200 \frac{cc}{\text{min}}$$  \hspace{1cm} (30)

$W_-$ and $W_+$ are of the same order of magnitude in this range. Accordingly, they should both be proportional to $v$. The best straight lines on the data for $W_-$ and $W_+$ are

$$W_- = 25.2 v \times 10^{-3} \frac{mg}{\text{min}}, \quad v \geq 400 \frac{cc}{\text{min}}$$  \hspace{1cm} (31)

$$W_+ = 10.0 v \times 10^{-3} \frac{mg}{\text{min}}, \quad v \geq 400 \frac{cc}{\text{min}}$$  \hspace{1cm} (32)

These add up to the formula (30) for $W$.

The data for $P = Q_+ Q_- / W^2$ in Figure 18 fall on a straight line

$$\frac{Q_+ Q_-}{W^2} = 656 v^2 \times 10^{-8} \left(\frac{\mu C}{g}\right)^2, \quad v \geq 300 \frac{cc}{\text{min}}$$  \hspace{1cm} (33)

This gives us a formula for $Q_+$, inserting $Q_-$ and $Q_+$ from formulae (29) and (30), respectively,
Dividing by \( W \) from formula (32) we obtain

\[
Q_+ = 4.52 \, v^2 \times 10^{-8} \frac{\mu C}{\text{min}}, \quad v \geq 300 \frac{\text{cc}}{\text{min}}
\]  

(34)

The fit is much better than that for \( Q_+ \) since the density variations are eliminated.

From the formulae (29) and (31) we obtain

\[
q_+ = 4.52 \, v \times 10^{-3} \frac{\mu C}{\text{g}}, \quad v \geq 300 \frac{\text{cc}}{\text{min}}
\]

(35)

From formulae (35) and (36) we obtain

\[
q_+ q_- = 31.9 \, v^2 \times 10^{-6} \left( \frac{\mu C}{\text{g}} \right)^2, \quad v \geq 300 \frac{\text{cc}}{\text{min}}
\]

(37)

Among the plots of the charge data it is really only \( q_- \) that shows a poor fit with the data. This is associated with the poor fit for the data for \( W_- \).

It now remains to join the curves for low and for high values of \( v \). We see then from Figures 10 and 17 that \( q_+ \) and \( q_+ q_- \) are constant. Furthermore, \( Q_+ Q_- / W^2 \) is a constant but \( Q_- \) is proportional to \( v^2 \) and \( W \) to \( v \). Hence, \( Q_+ \) is a constant. Consequently \( W_+ \) is a constant. Since \( Q_- \) is proportional to \( v^2 \) and \( q_- \) is constant, \( W_- \) is proportional \( v^2 \). The function

\[
W_- = 0.41 \, v^2 \times 10^{-4} \, \frac{\text{mg}}{\text{min}}, \quad 200 \leq v \leq 300 \frac{\text{cc}}{\text{min}}
\]

(38)

connects the two branches of the curve at low and at high flow rates.

The data discussed above and presented in Section 3 are those from a second run. The data for the first run sometimes come out bad because the coat on the tube wall may not be well built up. However, the data from the first run shed some light on the scatter of the data and thereby upon the effect of the deagglomerant. They are therefore given in this section.
The data for the weights (Figure 19) all show breaks at $v = 200$ cc/min. Figure 20 shows $W_-/W_+$. The curves drawn are according to formulae (11) and (17).

In this case $Q_+$ and $W_+$ are

Figure 21.  \[ Q_+ = 2.16 \, v \times 10^{-5} \, \frac{\mu C}{\min} \]  \hspace{1cm} (39)

Figure 22.  \[ W_+ = 4 \, \frac{\text{mg}}{\text{min}}, \quad v \geq 200 \, \frac{\text{cc}}{\text{min}} \]  \hspace{1cm} (40)

This gives with formula (38)

Figure 22.  \[ q_+ = \frac{Q_+}{W_+} = 5.04 \, v \times 10^{-3} \, \frac{\mu C}{g}, \quad v \geq 200 \, \frac{\text{cc}}{\text{min}} \]  \hspace{1cm} (41)

which is about 10% higher than that given in formula (35).

At low flow rates

Figure 19.  \[ W_+ = v^2 \times 10^{-4} \, \frac{\text{mg}}{\text{min}}, \quad v \leq 200 \, \frac{\text{cc}}{\text{min}} \]  \hspace{1cm} (42)

This gives with formula (39)

Figure 22.  \[ q_+ = \frac{Q_+}{W_+} = \frac{216}{v}, \quad v \leq 200 \, \frac{\text{cc}}{\text{min}} \]  \hspace{1cm} (43)

This is similar to the formula (16) for saccharin. The observed value at $v = 100$ cc/min falls far below this curve.

4.3 CAB-O-SIL

The data for Cab-O-Sil that scatter least are those for $q_-$ and $Q_-$

Figure 12.  \[ q_- = 49.9 \, v \times 10^{-3} \, \frac{\mu C}{g} \]  \hspace{1cm} (44)

Figure 4.  \[ Q_- = 2.53 \, (v - 35) \times 10^{-4} \, \frac{\mu C}{\min} \]  \hspace{1cm} (45)
Figure 1. Weights of positive deposit in 1 min $W_+$, negative deposit $W_-$, and sum of positive and negative deposits $W = W_+ + W_-$ as functions of flow rate $v$ for the first run of Saccharin + 1% Cab-O-Sil.}
Figure 20. Ratio of weights of negative and positive deposits in 1 min $W_-/W_+$ as a function of flow rate $v$ for the first run of Saccharin + 1% Cab-O-Sil.
Figure 21. Total positive charge $Q_+$ and total negative charge $Q_-$ deposited in 1 min as functions of flow rate $v$ and square of flow rate $v^2$, respectively, for the first run of Saccharin + 1% Cab-O-Sil.
Figure 22. Average charge per gram of positive deposit $q_+$, negative deposit $q_-$, and absolute charge $q$ as functions of flow rate $v$ for the first run of Saccharin + 1% CaO-Sil. $q_+$, $q_-$, $q$. 
These formulae give

\[ W_+ = \frac{Q_+}{q_-} = 5.07 \left(1 - \frac{35}{v}\right) \times 10^3 \text{ mg} \text{ min}^{-1} \]  \hspace{1cm} (46)

At high flow rates, \( W_+ \) is a constant

\[ W_+ = 0.183 \text{ mg} \text{ min}^{-1}, v \geq 300 \text{ cc} \text{ min}^{-1} \]  \hspace{1cm} (47)

Assuming that \( Q_+ \) is proportional to \( v^2 \)

\[ Q_+ = 18.8 v^2 \times 10^{-8} \text{ } \mu\text{C} \text{ min}^{-1}, v \geq 200 \text{ cc} \text{ min}^{-1} \]  \hspace{1cm} (48)

we have

\[ q_+ = \frac{Q_+}{W_+} = 1.05 v^2 \times 10^{-3} \text{ } \mu\text{C} \text{ g}^{-1}, \]  \hspace{1cm} \( v \geq 300 \text{ cc} \text{ min}^{-1} \)  \hspace{1cm} (49)

Formulae (49) and (44) give

\[ q_+ q_- = 52 v^3 \times 10^{-6} \text{ } \mu\text{C} \text{ g}^{-1}, v \geq 200 \text{ cc} \text{ min}^{-1} \]  \hspace{1cm} (50)

Formulae (48), (45), (47), and (46) give

\[ \frac{Q_+ Q_-}{W^2} = 1.90 v^4 \times 10^{-6} \text{ } \mu\text{C}^2 \text{ g}^{-1} \text{ } v^2, v \geq 200 \text{ cc} \text{ min}^{-1} \]  \hspace{1cm} (51)

At small values of \( v \),

\[ W_+ = 2.35 v \times 10^{-3} \text{ mg} \text{ min}^{-1}, v \geq 200 \text{ cc} \text{ min}^{-1} \]  \hspace{1cm} (52)
Figure 23. Logarithm of the product $q_+ \times q_-$ as a function of flow rate $v$ for Cab-O-Sil.
Figure 24. \( P_4 = Q_+ \times Q_- / W^2 \) as a function of \( v^4 / v - 35 \) for Cab-O-Sil.
This gives with formula (46)

\[
\frac{W}{w} = \frac{2120}{v} \left( 1 - \frac{35}{v} \right), \quad v \leq 200 \frac{\text{cc}}{\text{min}}
\]  

(53)

The data at small values of \(v\) are difficult to interpret, but below \(v = 35 \ \text{cc/min}\), \(Q_- = 0\) and \(W_- = 0\) so that \(q_+ q_-\) and \(Q_+ Q_-/W^2\) are both zero.

At high values of \(v\), formulae (46) and (47) give

\[
\frac{W}{w} = 27.7 \left( 1 - \frac{35}{v} \right), \quad v > 300 \frac{\text{cc}}{\text{min}}
\]  

(54)

At small values of \(v\),

\[
Q_+ = 0.33 v \times 10^{-4} \mu \text{C/ min}, \quad v < 200 \frac{\text{cc}}{\text{min}}
\]  

(55)

The formulae (55) and (52) give

\[
q_+ = \frac{Q_+}{W_+} = 14 \frac{\mu \text{C}}{\text{g}}, \quad v < 200 \frac{\text{cc}}{\text{min}}
\]  

(56)

in fair agreement with the data.

Formulae (56) and (44) give

\[
q_+ q_- = 0.7 v \left( \frac{\mu \text{C}}{\text{g}} \right)^2, \quad v < 200 \frac{\text{cc}}{\text{min}}
\]  

(57)

Above \(v = 50 \ \text{cc/min}\), \(W_-\) is much larger than \(W_+\). We may then take \(W = W_-\) and obtain from formulae (55), (45), and (46)

\[
\frac{Q_+ Q_-}{W^2} = \frac{3.25 v^3}{v - 35} \times 10^{-4}, \quad 50 < v < 200 \frac{\text{cc}}{\text{min}}
\]  

(58)
4.4 CARBOWAX 6000

The data for $q$ give a very good straight line, which indicates a high degree of accuracy in the measurements.

$$q = 4.30 \times v \times 10^{-3} \frac{\mu C}{g}$$  (59)

$Q_+$ and $W_+$ are both very large as compared to $Q_-$ and $W_-$, and $q$ is therefore essentially equal to $q_+$ at high flow rates.

$$q_+ = 4.18 \times v \times 10^{-3} \frac{\mu C}{g}$$  (60)

In order that $q$ and $q_+$ be proportional to $v$, $Q$ and $Q_+$ should be proportional to $v^2$, and $W$ and $W_+$ to $v$. The scatter in these data is very bad, but $Q_+$ gives a reasonably straight line against $v^2$.

$$Q_+ = 3.05 v^2 \times 10^{-8} \frac{\mu C}{min}$$  (61)

Hence, from formulae (61) and (60)

$$W_+ = \frac{Q_+}{q_+} = 7.30 \times 10^{-3} \frac{mg}{min}$$  (62)

The data for $q_-$ give

$$q_- = 10.25 (v - 90) \times 10^{-3} \frac{\mu C}{min}$$  (64)

A similar plot fits the data for $Q_-$

$$Q_- = 123 (v - 90) \times 10^{-8} \frac{\mu C}{min}$$  (64)

Hence, from formulae (64 and 63)

$$W_- = \frac{Q_-}{q_-} = 0.120 \frac{mg}{min}$$  (65)
The data indicate the \( W_+ \) is constant and give roughly
\[
W_+ = 0.12 \quad (66)
\]

From formulae (62) and (65) we obtain
\[
\frac{W_+}{W_-} = 0.0609 \nu \quad (67)
\]

The data may indicate
\[
\text{Figure 15.} \quad \frac{W_+}{W_-} = 0.024 \nu \quad (68)
\]

There is a difference by a factor of 2.5 in the two formulae, but formula (67) is probably more accurate.

4.5 CARBOWAX 6000 + 1% CAB-O-SIL

The data for the weights scatter remarkably little

\[
\text{Figure 9.} \quad W = 50 \nu \times 10^{-3} \frac{mg}{min}, \nu \geq 200 \frac{cc}{min} \quad (69)
\]

\[
\text{Figure 7.} \quad W_+ = 31 \nu \times 10^{-3} \frac{mg}{min}, \nu \geq 200 \frac{cc}{min} \quad (70)
\]

The difference is

\[
\text{Figure 8.} \quad W_+ = W - W_- = 19 \nu \times 10^{-3} \frac{mg}{min}, \
\nu \geq 200 \frac{cc}{min} \quad (71)
\]

A straight line for \( q_+ \) gives:

\[
\text{Figure 10.} \quad q_+ = 2.22 \nu \times 10^{-3} \frac{mC}{g}, \nu > 200 \frac{cc}{min} \quad (72)
\]
Formulae (72) and (70) give

\[ Q_+ = q_+ W_+ = 6.88v^2 \times 10^{-8} \frac{\mu C}{\text{min}}, \]
\[ v > 200 \frac{\text{cc}}{\text{min}} \]  

(73)

Figure 3.

The data for \( q_- \) (Figure 12) suggest that \( q_- \) is a constant and that therefore, since \( W_- \) is proportional to \( v \), \( Q_- \) is proportional to \( v \).

On the other hand, the data for \( q_+ \) (Figure 17) indicate that \( q_+ \) is a constant and therefore, since \( q_+ \) is proportional to \( v \), that \( q_- \) is proportional to \( v^{-1} \) and \( Q_- \) a constant. Adopting the latter alternative we find

\[ Q_- = 7.0 \times 10^{-3} \frac{\mu C}{\text{min}}, \quad v > 300 \frac{\text{cc}}{\text{min}} \]  

(74)

and from formulae (74) and (71)

\[ q_- = \frac{Q_-}{W_-} = 368 \frac{\mu C}{g}, \quad v > 200 \frac{\text{cc}}{\text{min}} \]  

(75)

Figure 12.

It appears that the value of \( Q_- \) measured at \( v = 300 \text{ cc/min} \) is off by a factor of approximately 3 and that \( q_- \) therefore is off by the same amount.

Formulae (72) and (75) give

\[ q_+ q_- = 0.82 \frac{\mu C}{g}, \quad v > 200 \frac{\text{cc}}{\text{min}} \]  

(76)

Figure 17.

The average of the measured values is 0.7 \( \mu C/g \), but this average includes the low value at \( v = 300 \text{ cc/min} \). If this value is disregarded, the average is 0.8 \( \mu C/g \), in agreement with the calculated value.

Formulae (73), (74), and (69) give

\[ \frac{Q_+ Q_-}{W^2} = 0.19 \frac{\mu C}{g}, \quad v > 200 \frac{\text{cc}}{\text{min}} \]  

(77)

Figure 25.

This is in agreement with the data.
Figure 25. $P = Q_+ \times Q_- / W^2$ as a function of flow rate $v$.
- Carbowax 6000,
- Carbowax 6000 + 1% Cab-O-Sil.
The formulae (70) and (71) give

\[
\frac{W^+}{W^-} = 1.63, \ v > 200 \ \text{cc/min} \quad (78)
\]

At low flow rates, \( q^- \) appears to be proportional to \( v^2 \),

\[
q^- = 0.45 \ v^2 \times 10^{-4} \ \frac{\mu C}{g}, \ v \leq 200 \ \text{cc/min} \quad (79)
\]

This gives \( q^- = 1.80 \ \mu C/g \) at \( v = 200 \ \text{cc/min} \). Formulae (75) gives \( q^- = 1.84 \ \mu C/g \) at \( v = 200 \ \text{cc/min} \). Thus, the functions (75) and (79) intersect at \( v = 200 \ \text{cc/min} \). \( q^+ \) is proportional to \( v \),

\[
q^+ = 8.0 \ v \times 10^{-3} \ \frac{\mu C}{g}, \ v < 200 \ \text{cc/min} \quad (80)
\]

Formulae (80) and (79) give

\[
q^+ q^- = 0.36 \ v^3 \times 10^{-6} \ \left(\frac{\mu C}{g}\right)^2, \ v < 200 \ \text{cc/min} \quad (81)
\]

The measured values at \( v = 100 \ \text{cc/min} \) are \( q^+ = 0.8 \ \mu C/g \) and \( q^- = 0.45 \ \mu C/g \) and, thus, \( q^+ q^- = 0.36 \ (\mu C/g)^2 \). The corresponding values at \( v = 200 \ \text{cc/min} \) are \( q^+ = 1.53 \ \mu C/g \) and \( q^- = 2.35 \ \mu C/g \). If the latter value is replaced by that given by formulae (75) and (79) i.e., \( q^- = 1.8 \ \mu C/g \), the product is \( q^+ q^- = 2.8 \ (\mu C/g)^2 \) which is not far off the value \( q^+ q^- = 2.9 \ (\mu C/g)^2 \) calculated from formula (81).

The values of \( W \) at \( v = 100 \) and \( v = 200 \ \text{cc/min} \) are 1.25 and 10 mg/min, respectively. Hence,

\[
W = 1.25 \ v^3 \times 10^{-6} \ \frac{\text{mg}}{\text{min}}, \ v < 200 \ \text{cc/min} \quad (82)
\]

A similar formula holds for \( W^+ \)

\[
W^+ = 0.75 \ v^3 \times 10^{-6} \ \frac{\text{mg}}{\text{min}}, \ v < 200 \ \text{cc/min} \quad (83)
\]
Hence,

Figure 8. \[ W_+ = W - W_+ = 0.50 v^3 \times 10^{-6} \text{ mg/ min}, \]

\[ v \geq 200 \frac{\text{cc}}{\text{min}} \] (84)

Formulae (83) and (84) give

\[ \frac{W_+}{W_-} = 1.50, \quad v \leq 200 \frac{\text{cc}}{\text{min}} \] (85)

Formulae (80) and (83) give

Figure 3. \[ Q_+ = q_+ W_+ = 6.0 v^4 \times 10^{-12} \frac{\mu \text{C}}{\text{min}}, \]

\[ v \leq 200 \frac{\text{cc}}{\text{min}} \] (86)

in agreement with the measured value, \( Q_+ = 96 \times 10^{-4} \mu \text{C/ min} \)
at \( v = 200 \text{ cc/min}. \)

Formulae (79) and (84) give

Figure 5. \[ Q_- = q_- W_- = 2.25 v^5 \times 10^{-14} \frac{\mu \text{C}}{\text{min}}, \]

\[ v \leq 200 \frac{\text{cc}}{\text{min}} \] (87)

This gives \( Q_- = 2.25 \times 10^{-4} \mu \text{C/ min} \) at \( v = 100 \text{ cc/min} \) in fair agreement with the observed value \( 2.67 \times 10^{-4} \). The formula (87) gives \( Q_- = 71 \times 10^{-4} \mu \text{C/ min} \) at \( v = 200 \text{ cc/min} \) which is about 20% lower than the observed value \( 86.2 \times 10^{-4} \). It is obvious that a small error in \( v \) with such a steep curve causes a great error in \( Q_- \). The observed value \( Q_- = 86.5 \) corresponds according to formula (87) to \( v = 208 \text{ cc/min}. \)
Formulae (86), (87), and (82) give

\[
\frac{Q_+ Q^-}{W^2} = 6.65 \times 10^{-8} \left( \frac{\mu C}{g} \right)^2, \quad v \leq 200 \text{ cc/min} \quad (88)
\]

The function \(Q_\cdot\) can be constructed from formulae (75) and (71) in the interval 200-300 cc/min, because both formulae are valid in this entire range. Thus, the formula (74) holds for \(v \geq 200\) cc/min. The true value of \(Q_\cdot\) at \(v = 200\) cc/min is thus somewhere between 70 \(\times 10^{-3}\) and 72 \(\times 10^{-4}\) \(\mu\)C/min as calculated from formulae (74) and (87), respectively. The measured value, \(Q_\cdot = 86.5 \times 10^{-4}\) \(\mu\)C/min is probably too high.

Neither \(Q_+\) nor \(q_+\) permit extension in the range 200-300 cc/min. However, the ratio between the values of \(q_+\) at \(v = 200\) and \(v = 300\) cc/min is \(15.3/6.7 = 2.28\), which is close to \((3/2)^2 = 2.25\). This suggests that

\[
q_+ = \frac{6.1}{v^2} \times 10^{4} \frac{\mu C}{g}, \quad 200 \leq v \leq 300 \text{ cc/min} \quad (89)
\]

The formulae (89) and (70) give

\[
Q_+ = q_+ W_+ = \frac{1.89}{v} \frac{C}{\text{min}}, \quad 200 \leq v \leq 300 \text{ cc/min} \quad (90)
\]

5. AGGLOMERATION IN POWDERS AND AEROSOLS

It was shown in preceding investigations \(3, 4, 7\) that adhesion between powder particles is caused by electrostatic forces at low humidities and by hydrogen bonds with water at higher humidities. The limit is
at 4% RH for silica sand and at 7% RH for plexiglass. It is difficult to handle fine powders at high humidities, and, in practice, dry powders for aerosol production will be held at some technically achievable low humidity. In the work of Nash et al. in this field, the relative humidity was held at 2%. It appears that a still lower humidity, say, 0.5% RH, would be preferable and also attainable.

It was shown in other preceding investigations that collisions between aerosol particles are determined by electrostatic forces. Oppositely charged particles collide as a result of electrostatic attraction, particles of equal polarity are prevented from colliding by electrostatic repulsion. This holds at all humidities of the air.

The data presented in Section 2 were obtained at a comparatively high humidity, 10%. They apply directly to collisions in aerosols sprayed at 10% RH. They also apply directly to agglomeration in powders, for which electrostatic adhesion predominates at 10% RH. They apply indirectly to agglomeration in powders at lower humidities on the assumption that the charge distribution is essentially the same at the lower humidities, whereas the charge level may be higher at the lower humidities. The data will be discussed on this basis in this section and in Section 6.

Agglomeration may be discussed in terms of forces of adhesion within the agglomerates and the number of particles engaged in agglomerates. At low humidities, the former quantity is determined essentially by the magnitude of the charge; the latter quantity is determined essentially by the charge distribution. In the cases under consideration there were no neutral particles. Hence, agglomeration occurs with oppositely charged particles only.

The number of particles engaged in agglomerates, ν, may be expressed in terms of $W_+/W_-$. When this quantity is zero or infinite, i.e., for unipolar charge, there are no agglomerates, and $\nu$ is therefore zero for these two values of $W_+/W_-$. When $W_+/W_- = 1$, all the particles may be engaged in agglomerates, and $\nu$ has therefore a maximum at $W_+/W_- = 1$.

As the size of an agglomerate increases, the external forces acting upon it also increase. It is conceivable that the agglomerate grows by addition of particles of successively smaller charges. The size of the agglomerates should thus be determined by the force of adhesion.
and, therefore, by the magnitude of the charge or, specifically, by the product of the magnitudes of the positive and negative charges per particle. A rough estimate of the force of adhesion may be given by the product of \( q_+ \) and \( q_- \).

One may thus consider a "degree of agglomeration" determined by the two quantities \( W_+/W_- \) and \( q_+ q_- \). Figure 26 shows a schematic representation of the "degree of agglomeration" as a function of \( W_+/W_- \) for various values of \( q_+ q_- \) in arbitrary units.

These considerations hold equally for the two cases, agglomeration in milling, handling, and slow flow, and agglomeration in aerosols. But in the latter case, we may also consider the rate of agglomeration. This quantity is determined by the long-range electrostatic forces between the particles and by the density of the aerosol. If the numbers of positive and negative particles per unit volume are \( n_+ \) and \( n_- \), respectively, and the total number of particles per unit volume is \( n = n_+ + n_- \), the average distance between particles is \( 1/n \). At a random distribution \( q_+ q_-/n^2 \) thus gives a measure of the force of attraction between positive and negative particles. The frequency of collisions and thereby the rate of agglomeration may thus be expressed in terms of

\[
P = \frac{n_+ n_-}{n^2}
\]

But \( q_+ n_+ = Q_+^{-1} \) and \( q_- n_- = Q_-^{-1} \) are the total positive and negative charges per unit volume, and \( Q_+^{-1} / n \) and \( Q_-^{-1} / n \) are proportional to the positive and negative charges per gram of aerosol material. Hence,

\[
P = \frac{Q_+ Q_-}{W^2}
\]

disregarding a conversion factor and using the notations previously introduced.

The quantities \( q_+ q_- \) and \( P = Q_+ Q_-/W^2 \) are plotted in Figures 18, 24 and 25.
Figure 26. Schematic representation of "degree of agglomeration" as a function of the ratios $W_+/W_-$ and $W_-/W_+$ at different values of the product $q_+ \times q_-$. 
An attempt at a concise presentation of the data is offered in Tables 1 and 2, showing the data at low and at high flow rates, respectively. Unfortunately, not all the plots permit simple analytical expressions. Nevertheless, the data given in Tables 1 and 2 assist in the comparison between the powders and in the evaluation of the deagglomerant, but the actual plots should be consulted for additional information.

6. DISCUSSION

Since agglomeration was not studied directly the conclusions drawn from the data are necessarily tentative and subject to checking by direct observation. Furthermore, in spite of the inherent accuracy of the measurements, there are some large deviations, occasionally by a factor of 3, in the data that are difficult to account for and that also need checking.

However, the information that this investigation was planned to provide is sufficiently accurate for safe conclusions. The agglomeration factors discussed in Section 5 are all ratios, e.g., $W_0/W_+$. These ratios do not depend upon the density of the aerosol in the aerosol generator. They have been calculated from the data and from the formula derived in Section 4 at $v = 50$ and $v = 500$ cc/min. They are listed in Table 3.

The aerosol in the aerosol generator contains very little of small particles. Most of the aerosol particles are agglomerates of the powder particles. The data in Table 3 pertain to these agglomerates and to the agglomeration of these agglomerates.

Comparing the data for saccharin and saccharin + 1% Cab-O-Sil it is apparent that the deagglomerant has no effect upon $W_0/W_+$ but that it reduces $q_0q_-$ and $P = Q_0Q_+/W^2$, at low flow rates considerably, at high flow rates slightly. Thus, the deagglomerant is effective at low flow rates but ineffective at high flow rates. It reduces the strength of the agglomerates by a factor of 15 at $v = 50$ cc/min but only by a factor of 2 at $v = 500$ cc/min. It reduces the probability of collisions in the aerosol by a factor of 15 at $v = 50$ cc/min but only by a factor of 2 at $v = 500$ cc/min. Since the spraying of aerosols occurs at high flow rates, it follows that the deagglomerant has no effect upon agglomeration in sprayed aerosols, although it is effective in promoting free flow at low flow rates.

Comparing the data for Carbowax 6000 and Carbowax 6000 + 1% Cab-O-Sil we find a drastic change to the worse in the factor $W_0/W_+$. In the pure material there are only positive particles (or agglomerates)
Table 1: Data at Low Flow Rates

<table>
<thead>
<tr>
<th></th>
<th>Carbowax 6000</th>
<th>1% Carbowax-SII</th>
<th>Saccharin + Carbowax 6000</th>
<th>1% Carbowax-SII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{N - A}{2} \times 10^{-3}$</td>
<td>1.20</td>
<td>0.63</td>
<td>8.0</td>
<td>1.20</td>
</tr>
<tr>
<td>1.0 x 10$^6$</td>
<td>0.0</td>
<td>0.36</td>
<td>0.0</td>
<td>0.36</td>
</tr>
<tr>
<td>$\frac{\Lambda}{1200}$</td>
<td>0.48</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>1.4 x 10$^3$</td>
<td>123</td>
<td>63</td>
<td>11.1</td>
<td>63</td>
</tr>
<tr>
<td>2.35 x 10$^{-3}$</td>
<td>2960</td>
<td>255</td>
<td>2.5</td>
<td>255</td>
</tr>
<tr>
<td>2.93 x 10$^{-3}$</td>
<td>3.02</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>3.00 x 10$^{-3}$</td>
<td>123</td>
<td>10.4</td>
<td>10.4</td>
<td>10.4</td>
</tr>
<tr>
<td>3.05 x 10$^{-3}$</td>
<td>85</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3.06 x 10$^{-3}$</td>
<td>85</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3.18 x 10$^{-3}$</td>
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</tr>
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<td>3.72 x 10$^{-3}$</td>
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<td>5.07 x 10$^{-3}$</td>
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<td>6.23 x 10$^{-3}$</td>
<td>85</td>
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Note: The table entries are not clearly visible due to the image quality. The values in the table are not legible and may require manual transcription for accurate input.
Table 2: Data at High Flow Rates.

<table>
<thead>
<tr>
<th></th>
<th>% Carb-O-SII</th>
<th>% Carb-O-SII</th>
<th>% Carb-O-SII</th>
<th>% Carb-O-SII</th>
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<tbody>
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<td>100</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>8.7 (V - 90)</td>
<td>0.82 x 10^6</td>
<td>4.27 (V - 90)</td>
<td>3.19 x 10^6</td>
<td></td>
</tr>
<tr>
<td>656.2 x 10^-4</td>
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<td>0.140 x 10^-3</td>
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<td>5.10 x 10^-3</td>
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<td>7.06 x 10^-3</td>
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</tr>
<tr>
<td>1.025 (V - 90)</td>
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<tr>
<td>1.025 (V - 90)</td>
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### Table 3: Agglomeration Factors at V=50 cc/min and 500 cc/min.

<table>
<thead>
<tr>
<th></th>
<th>6000</th>
<th>28</th>
<th>1.8</th>
<th>1.6</th>
<th>1.5</th>
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<tr>
<td>250</td>
<td>0.19</td>
<td>0.35</td>
<td>1.6</td>
<td>3.5</td>
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<td>9</td>
<td>1.6</td>
<td>2.6</td>
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<td></td>
</tr>
<tr>
<td>6000</td>
<td>0.8</td>
<td>6</td>
<td>8</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>1.6</td>
<td></td>
<td>2.6</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{b}{\frac{C}{\pi^{2}}} - \frac{b}{\frac{C}{\pi^{2}}} = p
\]

ACCOLIMATION FACTORS AT V = 500 cc/min

<table>
<thead>
<tr>
<th></th>
<th>6000</th>
<th>28</th>
<th>1.8</th>
<th>1.6</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>1.6 × 10⁻⁹</td>
<td>0</td>
<td>0.035</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.45</td>
<td>0</td>
<td>1.6</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>1.5</td>
<td>0</td>
<td>0.0</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{b}{\frac{C}{\pi^{2}}} - \frac{b}{\frac{C}{\pi^{2}}} = p
\]

ACCOLIMATION FACTORS AT V = 50 cc/min

<table>
<thead>
<tr>
<th>% Carb-O-S III</th>
<th>Carbosil 6000</th>
<th>1% Carb-O-S III</th>
<th>% Sarcacrin</th>
<th>% Sarcacrin + 6000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbosil 6000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% Carb-O-S III</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Sarcacrin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Sarcacrin + 6000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
below \( v = 90 \text{ cc/min} \) and 12 times as many positive as negative particles at \( v = 500 \text{ cc/min} \). In both cases, the deagglomerant makes \( W_+ \) and \( W_- \) more equal, \( W_+/W_- = 1.5 \). As a result, the deagglomerant causes bonds between particles below \( v = 90 \text{ cc/min} \), but these bonds are comparatively weak, e.g., as compared to those in the case of saccharin + deagglomerant, and may not affect the flowability of the powder. At \( v = 500 \text{ cc/min} \), the bonds are stronger, but \( q_+ q_- \) is still lower by a factor of 10 as compared to Carbowax 6000. The probability of collisions is reduced by a factor of 2 only. Thus, the deleterious effects of the deagglomerant are not important at low flow rates but are significant in the case of aerosols sprayed at high flow rates.

The agglomerations in the powder and in the aerosol in the aerosol generator can be studied by means of the aerosol density as given by \( W_+ \) and \( W_- \). The conclusions drawn from these data are, of course, no more accurate than the data themselves. They will therefore be presented as tentative. They indicate a potential usefulness of the experimental technique for this type of investigation. It should be recalled that this use of the data was not anticipated during the actual experiments and that therefore no effort was made to control the aerosol density.

The weight and charge data show peculiarities at \( v = 200 \text{ cc/min} \) for all materials except Carbowax 6000, and at \( v = 90 \text{ cc/min} \) for Carbowax 6000 and at \( v = 35 \text{ cc/min} \) for Cab-O-Sil. At \( v = 90 \text{ cc/min} \), the linear flow rate is 300 cm/s and equal to that of the air stream at its entrance in the aerosol generator. At \( v = 35 \text{ cc/min} \), the linear flow rate is 120 cm/s and equal to that of the air stream at its exit from the aerosol generator. The particle sizes with these terminal velocities are approximately 600 and 240 \( \mu \text{m} \) diameter, respectively. Although particles (or agglomerates) this large and much larger were present in the powders, it is not possible to relate these flow velocities with particle sizes without further experiments, particularly at different velocities of the air stream in the aerosol generator.

The linear velocity of the air stream through the aerosol generator was maintained at 2 cm/s. The maximum particle size that can be suspended in the aerosol at this velocity is approximately 25\( \mu \text{m} \). Particles of this size have a considerable inertia. At low values of \( v \) they are not collected and the extent to which they are collected increases with \( v \). Since the drag force is proportional to \( v^2 \) one may expect \( W \) to be proportional to \( v^2 \) at low flow rates. At high flow rates, \( W \) should be proportional to \( v \) at a constant aerosol density.
These considerations are reflected by the curves for $W$, $W_+$, and $W_-$. For saccharin (Figure 6), and saccharin + 1% Cab-O-Sil (Figure 6), $W_+$ is proportional to $v^2$ at low flow rates. Comparing the curves in Figure 19, we see that

$$W_+ = k_+ v^2$$  \hspace{1cm} (93)

$$W_- = k_- v$$  \hspace{1cm} (94)

$$W = k_+ v^2 + k_- v$$  \hspace{1cm} (95)

This indicates that the negative charge is carried by small particles without inertia, and that the positive charge is carried by large particles with considerable inertia. However, the curve of $W_+$ for saccharin + 1% Cab-O-Sil in Figure 6 shows no inertia effect at low flow rates.

The data for Carbowax 6000 + 1% Cab-O-Sil indicate that $W$, $W_+$, and $W_-$ are all proportional to $v^3$ at low flow rates. In the case of Cab-O-Sil, $W_+$ is proportional to $v$ at low flow rates, whereas $W_-$ is zero below $v = 35$ cc/min.

Above $v = 200$ cc/min, $W_+$ for saccharin + 1% Cab-O-Sil appears to be independent of $v$. This means that the density $d_+$ is

$$d_+ = \frac{k}{v}, \hspace{1cm} v > 200 \text{ cc/min}$$  \hspace{1cm} (96)

Since $1/v$ is proportional to the time of residence $t$ of the particles in the aerosol

$$d_+ = kt, \hspace{1cm} v > 200 \text{ cc/min}$$  \hspace{1cm} (97)

Accordingly, $d_+$ should be the density of agglomerates that fall out below $v = 200$ cc/min and that are all collected above $v = 200$ cc/min. Thus,

$$d_+ = kt_0, \hspace{1cm} v \leq 200 \text{ cc/min}$$  \hspace{1cm} (98)

when $t_0$ is the residence time determined by fall-out.
Consequently, the positive charge should be carried by agglomerates formed in the aerosol. According to formulae (97) and (98), the rate of formation of these agglomerates is independent of $d_+$. The formula (93) together with the linear plot of $Q_+$ (Figure 2) gives

$$d_+ = \frac{274}{v}, \quad v \leq 200 \text{ cc/min}$$

(99)

Thus, $q_+$ decays with increasing friction in the capillary tube. This is not improbable. A similar initial decay of the charge with increasing friction has been observed with glass beads. Carbowax 6000 + 1% Cab-O-Sil shows a similar decay of $q_+$ at high flow rates. Since $W_-$ is proportional to $v$, it is composed of particles with little inertia.

The same reasoning applied to Carbowax 6000 + 1% Cab-O-Sil suggests that all particles are large in the aerosol and that they agglomerate very rapidly in the aerosol. Indeed, the absence of a linear term in $W$, indicates that all particles are large enough to show appreciable inertia. If we are to rely on the data, the particles are all of the maximum size, $25 \mu$, and agglomerate at a rate such that

$$d = kv = \frac{k}{t}, \quad v \leq 200 \text{ cc/min}$$

(100)

at low flow rates. This relation would apply to the case of rapid agglomeration according to the formula

$$- \frac{dd}{dt} = kd^2$$

(101)

which gives

$$\frac{1}{d} - \frac{1}{d_0} = kt$$

(102)
and, when $k_t$ and $d_o$ are large

$$d = \frac{1}{k_t}$$  \hspace{1cm} (103)

This seems to lead to a difficulty at high flow rates because the constant density at high flow rates requires that all particles and agglomerates are collected without inertia above $v = 200$ cc/min, whereas all agglomerates fall out below $v = 200$ cc/min. It appears, however, that if two $25\mu$ particles collide while each is being sucked toward the capillary tube, they will continue to move toward the capillary tube.

The data for Carbowax 6000 are rather complex. However, it is clear that the inertia effect persists beyond $v = 200$ cc/min. The straight lines drawn for $W$ and $W_+$ in Figures 9 and 7 are no more than very rough approximations. Above $v = 300$ cc/min, $W$ and $W_-$ decrease, probably as a result of extensive agglomeration in the powder after prolonged stirring. It is noteworthy that the aerosol density is small as compared to that for Carbowax 6000 + 1% Cab-O-Sil.

Comparing the plots of $W$ for saccharin + 1% Cab-O-Sil and Carbowax 6000 + 1% Cab-O-Sil at high flow rates, we see that both aerosols have small enough particles not to show an inertia effect, and that the density of the latter aerosol is larger than that of the former aerosol. But the data at low flow rates indicate larger particles in the latter than in the former aerosol. It should then follow that there are smaller particles in the former powder but less of them, i.e., the saccharin + 1% Cab-O-Sil powder is more agglomerated than is the Carbowax 6000 + 1% Cab-O-Sil powder. As a consequence, the former powder should have a range of particle size from small unagglomerated particles to very large agglomerates, whereas the latter powder should have a more uniform size of agglomerates of about $25\mu$ with none of the small unagglomerated particles and little of the very large agglomerates. This seems to be in accord with observations on the effects of the deagglomerant upon the flowabilities of the two materials.

An important conclusion, as yet tentative, is that the effect of the deagglomerant is to form agglomerates of the order of $25\mu$ and of
a fairly uniform size. These agglomerates flow more readily than the powder particles because the forces between the agglomerates are smaller than those between the original particles, which prevents them from agglomerating, and because the flow at a uniform particle size should be easier than at a large variation in particle size. However, the forces that break up agglomerates in flow are not present in the aerosol, and a reduction of the agglomerate size in a flowing powder by no means indicates a limitation to the agglomerate size in the aerosol. The decay of $q_-$ for Carbowax 6000 + 1% Cab-O-Sil for $v \geq 200$ cc/min and that of $q_+$ in the range 200 to 300 cc/min indicate a transfer of charge from the negative particles to the positive particles. Carbowax 6000 alone being predominantly positive and Cab-O-Sil alone being predominantly negative, the two combine such that the negative Cab-O-Sil particles form a coat on the positive Carbowax 6000 particles. 5) The Cab-O-Sil particles would thus tend to bind Carbowax 6000 particles together. If now two such agglomerates, one covered with Cab-O-Sil, the other bare, collide and stick together, their subsequent separation would break the bonds between the Cab-O-Sil and either the former or the latter agglomerate. In the former case, both agglomerates lose charge while remaining negative and positive, respectively. Thus $q_+$ and $q_-$, and $Q_+$ and $Q_-$ may change while $W_+$ and $W_-$ are unchanged. Thus, positive and negative particles may collide and agglomerate in the aerosol and then be broken up by friction and impact in the capillary tube on their way to the charge analyzer.

The charge distributions found with these materials differ very drastically from some of those found previously with other materials 6). Thus, for MgO and NH$_4$Cl, $W_+ = W_-$ at all flow rates. For polyvinyl chloride, both $W_+$ and $W_-$ were proportional to $v$ and $W_+/W_- = 2.5$ at all flow rates. For Buerker's powder $W_+/W_-$ was 4 at low flow rates and increased with the flow rate at high flow rates.

It appears that this difference may be a result of the technique for producing the aerosol. The aerosol generator shown in Figure 1 was used for polyvinyl chloride and Buerker's powder. MgO was produced by burning a strip of Mg metal and NH$_4$Cl was produced by mixing NH$_3$ and HCl vapors. In both cases the aerosol was produced in a comparatively large glass cylinder. The MgO and NH$_4$Cl aerosols were close to monodisperse, whereas the aerosols produced in the aerosol generator cannot be monodisperse because of
agglomeration into particles of widely varying sizes. $W_+ / W_-$ would be expected to vary with the particle size distribution.

The effect of the particle size distribution upon the particle charge distribution was not studied in this investigation. It appears to be an important factor in the control of particle charge.

It should be pointed out that the charge per gram is greater for small particles than for large particles. This was found in the preceding investigation\(^6\), and it is also shown clearly in the large charge values for Cab-O-Sil in Tables 1 and 2.

Reference was made in Section 2 to an observation, as yet tentative, that the flowing material may not coat the tube wall at very low humidities. This tentative observation should be investigated. If confirmed, it means that the charge distribution may be controlled by a suitable choice of nozzle material, e.g., by inserting a sleeve of a suitable material in the nozzle. It would also mean, of course, that the data obtained at 10% RH do not apply at lower humidities because if there is no coat there is no charge. However, these data should still apply to electrification under conditions where a coated tube wall is not involved, e.g., in milling and slow flow.

7. CONCLUSION

This investigation was undertaken in order to learn more about the electrification of airborne particles in the flow through a tube, and in order to determine the charge and the charge distribution on the particular materials used. The results show that these measurements are useful in the evaluation of deagglomerants. They also indicate means for the control of the charge and charge distribution of sprayed aerosols.

The data obtained yield information on the tendency to agglomeration in powders and aerosols. They show that the charge distribution may depend very much upon the flow rate, and that it may be possible to produce unipolar aerosols by using a high enough flow rate in spraying. The effect of a deagglomerant may vary very much with the flow rate and may be very different with different materials. Thus, Cab-O-Sil is effective with saccharin at low flow rates but has no effect at high flow rates.
In order to take full advantage of the information yielded by these measurements, one should correlate it with direct observations of agglomerates and agglomeration and with the particle size distribution. It is hoped that future investigations may be extended in these directions.

Meanwhile, the data obtained yield information on the electrification of the materials investigated. Of particular interest is that the charge distributions differ so much with the materials studied, namely, from almost equal amounts of positively and negatively charged particles to almost complete predominance of one polarity or the other.

To the problem of aerosol stability it is important that the charge increases with the flow rate, and that it becomes increasingly unipolar as the flow rate increases. This has a profound bearing upon the control of aerosol charge and aerosol stability.

In the application of measurements of this type to the spraying of aerosols, the coating of the tube wall by the aerosol material is a decisive factor. This factor should be investigated, particularly at low humidities. There is evidence to the effect that the composition of the gas used in spraying affects the charge and charge distribution of the aerosol. 6) This may be an important factor in the control of aerosol charges, and it should be investigated from this point of view.

Important conclusions, although as yet tentative, can be drawn on the effects of deagglomerants. A deagglomerant may cause free flow by forming large agglomerates, which are limited in size and smaller than those formed in the pure material. This is ruinous to presized aerosol particles. A deagglomerant may cause a change in charge distribution so that there are equal abundances of positive and negative particles, whereas the pure material may be close to unipolar. This promotes agglomeration in the aerosol.

It should be noted that there are no or very weak forces only on aerosol particles, whereas the forces on powder particles in flow are quite large. The fact that a powder flows freely is no indication that agglomeration in its aerosol would not occur.

It appears that the effects mentioned are inherent with deagglomerants. These effects should be carefully studied before a deagglomerant is added to a presized aerosol material.
4% RH for silica sand\(^3\) and at 7% RH for plexiglass\(^4\). It is difficult to handle fine powders at high humidities, and, in practice, dry powders for aerosol production will be held at some technically achievable low humidity. In the work of Nash et al\(^5\) in this field, the relative humidity was held at 2%. It appears that a still lower humidity, say, 0.5% RH, would be preferable and also attainable.

It was shown in other preceding investigations\(^8,9\) that collisions between aerosol particles are determined by electrostatic forces. Oppositely charged particles collide as a result of electrostatic attraction, particles of equal polarity are prevented from colliding by electrostatic repulsion. This holds at all humidities of the air.

The data presented in Section 2 were obtained at a comparatively high humidity, 10%. They apply directly to collisions in aerosols sprayed at 10% RH. They also apply directly to agglomeration in powders, for which electrostatic adhesion predominates at 10% RH. They apply indirectly to agglomeration in powders at lower humidities on the assumption that the charge distribution is essentially the same at the lower humidities, whereas the charge level may be higher at the lower humidities. The data will be discussed on this basis in this section and in Section 6.

Agglomeration may be discussed in terms of forces of adhesion within the agglomerates and the number of particles engaged in agglomerates. At low humidities, the former quantity is determined essentially by the magnitude of the charge; the latter quantity is determined essentially by the charge distribution. In the cases under consideration there were no neutral particles. Hence, agglomeration occurs with oppositely charged particles only.

The number of particles engaged in agglomerates, \(n\), may be expressed in terms of \(W_+/W_-\). When this quantity is zero or infinite, i.e., for unipolar charge, there are no agglomerates, and \(n\) is therefore zero for these two values of \(W_+/W_-\). When \(W_+/W_- = 1\), all the particles may be engaged in agglomerates, and \(n\) has therefore a maximum at \(W_+/W_- = 1\).

As the size of an agglomerate increases, the external forces acting upon it also increase. It is conceivable that the agglomerate grows by addition of particles of successively smaller charges. The size of the agglomerates should thus be determined by the force of adhesion.
REFERENCES


