NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
AUTOMATIC DATA PROCESSING
TECHNIQUES FOR DETERMINING
ENERGY-MANAGEMENT RELIABILITY

6 NOVEMBER 1963
DDC AVAILABILITY NOTICE.

Qualified requesters may obtain copies of this report from the Defense Documentation Center for Scientific and Technical Information, Cameron Station, Alexandria, Virginia, 22314.

DESTRUCTION NOTICE

Destroy; do not return.
AUTOMATIC DATA PROCESSING TECHNIQUES FOR
DETERMINING ENERGY - MANAGEMENT RELIABILITY

by

William D. Guthrie and J. S. Sturdivant

Department of the Army Project No. 1B222801A205
Army Materiel Command Management Structure Code No. 5221.11.147

Advanced Propulsion Engineering Branch
Propulsion Laboratory
Directorate of Research and Development
U.S. Army Missile Command
Redstone Arsenal, Alabama
ABSTRACT

Two techniques for automated statistical evaluation of rocket motor performance envelopes have been developed: the absolute and the approximate techniques. This report describes the logic for the absolute technique, which is a mathematically accurate method applicable to a large, fast computer, such as the IBM 7090. The report also describes in detail an LGP-30 computer program for the analytical study of pressure time or thrust time data using the approximate technique. The approximate technique is designed for use with relatively slow, limited-storage computers, such as the LGP-30 and the RPC-4000. The approximate and absolute techniques make possible quality control and the evaluation of the effects of solid-propellant processing variations on performance reproducibility.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. FOURIER SERIES EQUATION</td>
<td>2</td>
</tr>
<tr>
<td>III. EVALUATION OF THE STATISTICAL PROFILE AS A FUNCTION OF TEMPERATURE</td>
<td>7</td>
</tr>
<tr>
<td>IV. REQUIREMENTS AND LIMITATIONS OF THE CURVE FIT METHOD</td>
<td>10</td>
</tr>
<tr>
<td>V. AN LGP-30 COMPUTER PROGRAM FOR THE STATISTICAL EVALUATION OF ENERGY MANAGEMENT PROFILES</td>
<td>11</td>
</tr>
<tr>
<td>A. Computer Memory Usage</td>
<td>12</td>
</tr>
<tr>
<td>B. Preparation of Data Tapes</td>
<td>13</td>
</tr>
<tr>
<td>C. Selectivity and Other Characteristics</td>
<td>14</td>
</tr>
<tr>
<td>VI. CONCLUSION</td>
<td>16</td>
</tr>
</tbody>
</table>

# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. An Attempt of Statistical Evaluation of an Energy Management Profile by the Use of the Linear Variance Method Normal to the X-Axis</td>
<td>3</td>
</tr>
<tr>
<td>2. An Attempt of Statistical Evaluation of an Energy Management Profile by the Use of the Linear Variance Method Normal to the Nominal Performance Curve</td>
<td>4</td>
</tr>
<tr>
<td>3. A Statistical Evaluation of the Energy Management Profile as a Function of Temperature Using the Approximate Technique</td>
<td>8</td>
</tr>
</tbody>
</table>
AUTOMATIC DATA PROCESSING TECHNIQUES FOR DETERMINING ENERGY-MANAGEMENT RELIABILITY

I. INTRODUCTION

The internal ballistics of rocket motors has in the past largely escaped quality control. In the rare cases where statistical evaluation of performance curves was attempted, laborious and erratic methods were utilized. The demand for a more dependable and accurate method of evaluation of more advanced solid-propellant propulsion systems has resulted in the development of two new techniques: the approximate and the absolute techniques. This report describes the approximate and absolute techniques for the automated statistical evaluation of rocket motor performance envelopes.

The approximate technique is applicable to relatively slow, limited-storage computers, such as the LGP-30 and the RPC-4000. An LGP-30 computer program for the analytical study of pressure-time or thrust-time data is described using the approximate technique.

The absolute technique is a mathematically accurate method of statistical evaluation applicable for large, fast computers, such as the IBM 7090. Logic for the absolute technique is examined, and the application of both techniques are compared.

The absolute and the approximate techniques utilize an empirical curve fit of the performance envelope to a multi-term Fourier series expansion. Each performance envelope is represented by a single equation, and the total number of equations (representing the total number of rounds) may be evaluated statistically to determine performance standard deviation. Thus, incorporation of variations in web time or grain temperature may be handled without the difficulties of ordinary statistical techniques. The absolute and approximate techniques have been designed for automatic data processing equipment and lend readily to quality control evaluation. The absolute and approximate techniques may be utilized as research tools for evaluating the effects of solid-propellant processing variations on performance reproducibility.

Information regarding the statistical variance of the rocket motor management profile (such as thrust-time and pressure-time histories), as a function of temperature, is often desirable. Analysis of the information using automatic data processing equipment without intermediate data handling is also very desirable.
Difficulties usually arise when attempts are made to statistically analyze an energy performance curve, because the y-parameter is usually inversely related to the x-parameter. The difficulties result in a non-valid comparison when using the linear variance technique for the evaluation of the y-axis variance normal to the x-axis. The shift on ignition and tailoff regions becomes quite severe, resulting in an unrealistically high variance and an extremely erratic analysis (see Figure 1).

A modified approach, which has been used with moderate success, uses the linear variance method but evaluates the y-axis normal to the average performance curve. The modified approach reduces the shift but requires certain approximations, such as "eye-balling" the tangent lines and interpreting graphic data, hence introducing possibility for human error. In addition, the modified approach results in an envelope of variances completely surrounding the average performance curve. Thus, no provision is made to retain the characteristic shape of the plus or minus, one, two, or three sigma curves. A result of the modified approach to statistical evaluation is illustrated in Figure 2.

Variance envelopes may be obtained for any set of conditions where a valid statistical population exists, but no method exists for analysis between points of evaluation. Therefore, random, out-of-specification performance curves (which often result when solid propellant rocket motors are tested at three environmental temperatures) cannot be included in the overall evaluation. Any single, off-temperature round is also lost to valid statistical examination.

Statistical evaluation is laborious and time consuming, and the linear variance technique and the modified approach are not easily compatible with automatic data processing equipment.

II. FOURIER SERIES EQUATION

The Propulsion Laboratory, desiring a technique for statistically evaluating performance envelopes which would be compatible with digital computers, investigated the concept of utilizing mathematical empirical relationships representing performance characteristics. A Fourier series equation was curve fit with relative ease to performance curve data by use of the digital computer. Since the use of electronic data processing equipment to collect and process data is now widespread, a resulting curve fit could be directly pre-programmed.
Figure 1. AN ATTEMPT OF STATISTICAL EVALUATION OF AN ENERGY MANAGEMENT PROFILE BY THE USE OF THE LINEAR VARIANCE METHOD NORMAL TO THE X-AXIS
Figure 2. AN ATTEMPT OF STATISTICAL EVALUATION OF AN ENERGY MANAGEMENT PROFILE BY THE USE OF THE LINEAR VARIANCE METHOD NORMAL TO THE NOMINAL PERFORMANCE CURVE
Investigation revealed that a 40-term Fourier series equation would contain only a 0.3% error (in ordinate for a given abscissa) for Army-developed solid-propellant rocket motors. However, the total amount of tolerable error may be controlled by the length of the equation used. In addition, the total integral of the Fourier equation between 0 and 2 agrees with 0.001% of pdt for the same performance curve. Thus, the equation format used to describe performance is

\[ y = A_0 + \sum_{i=1}^{40} \left( a_i \cos ix + b_i \sin ix \right) \quad (1) \]

where the y-axis represents pressure, thrust, or another desired parameter and the x-axis usually represents time. Time is nondimensional since \( X = 2\pi \) radians is the total time.

A close examination of the Fourier equation shows that an average \( y \) at any value of \( x \) may be obtained by averaging all coefficients of identical trigonometric functions, that is,

\[
\left( \frac{y_1 + y_2 + \ldots + y_n}{n} \right) = \left( \frac{A_{01} + A_{02} + \ldots + A_{0n}}{n} \right) + \sum_{i=1}^{40} \left( \frac{a_{i1} + a_{i2} + \ldots + a_{in}}{n} \right) \cos ix + \\
\left( \frac{b_{i1} + b_{i2} + \ldots + b_{in}}{n} \right) \sin ix \quad (2)
\]

or

\[ y_{ave} = A_{ave} + \sum \left( a_{ave} \cos ix + b_{ave} \sin ix \right). \quad (3) \]

Likewise, \( \Delta y \) for each value of \( x \) may be obtained by the difference in the coefficients of the curve and the average curve and may be expressed as

\[
\Delta y = y_{ave} - y = A_{ave} + \sum_{i=1}^{40} \left[ \left( a_{ave} - a_i \right) \cos ix + \left( b_{ave} - b_i \right) \sin ix \right] \\
\]

or

\[ \Delta y = \Delta A_o + \sum_{i=1}^{40} \left( \Delta a_i \cos ix + \Delta b_i \sin ix \right). \quad (5) \]
However, statistical evaluation of variance in the $y$-parameter (through statistical evaluation of coefficients for a 40-term Fourier series) would require the statistical evaluation of 6,561 columns of data followed by a matrix solution for the square root, since

$$
\sigma_y = \left\{ \sum_{o}^{n} (\Delta y)^2 \right\}^{\frac{1}{2}} = \left\{ \sum_{o}^{n} \Delta A_o + \sum_{i=1}^{40} (\Delta a_i \cos ix + \Delta b_i \sin ix) \right\}^{\frac{1}{2}}. \quad (6)
$$

The treatment shown in Equation (6) is obviously impractical, even for automatic data processing equipment. In the case of limited memory storage computers, such as those available within the propulsion laboratory, the above treatment is impossible. However, since

$$
\sigma_y = \left[ \sum_{o}^{n} (\Delta y)^2 \right]^{\frac{1}{2}} \quad (7)
$$

and since $\Delta y$ may be calculated from Equation (5), $\sigma_y$, as a function of $x$, may be evaluated by obtaining $\sigma_y$ at $(2i + 1)$ equal distant values of $x$. Then $\sigma_y$ may be curve fit to the Fourier series equation (Equation 8) of $\sigma_y$. Since

$$
\sigma_y = A_o + \sum_{i=1}^{40} \left( \frac{a_i}{\sigma_y} \cos ix + \frac{b_i}{\sigma_y} \sin ix \right), \quad (8)
$$

the complete variances of $y$ as a function of $x$ then can be expressed as

$$
y_{ave} + K\sigma y = A_{ave} + KA_o + \sum_{i=1}^{40} \left[ \left( \frac{a_i}{\sigma_y} \cos ix + \frac{b_i}{\sigma_y} \sin ix \right) \right], \quad (9)
$$

where $K$ represents the number of standard deviations, usually an integral value between -3 and +3.
The absolute technique is mathematically correct and very practical when used on large, fast, memory computers such as the IBM 7090. Nevertheless, the absolute technique is still not practical for the smaller and slower computers available in the propulsion laboratory.

Each term of the Fourier series equation is not abnormally weighted by certain values of $x$ as are the terms of a polynomial equation. Therefore, a simple statistical analysis of the coefficients of the equation is assumed to approximate the variance behavior of the $y$-parameter. Thus, the approximate technique is conceived. A comparison of the approximate technique and the absolute technique resulted in $\sigma y$ values that were within 3 percent for any value of $x$.

$$
\sigma y = \left[ \sum_{i=1}^{n} \frac{(\Delta A_{i})^2}{n-1} \right]^\frac{1}{2} + \sum_{i=1}^{40} \left[ \sum_{i=1}^{n} \frac{(\Delta a_{i})^2}{n-1} \right]^\frac{1}{2} \cos ix + \left[ \sum_{i=1}^{2} \frac{(\Delta b_{i})^2}{n-1} \right]^\frac{1}{2} \sin ix
$$

Like the absolute technique, the approximate technique sets all total times equivalent. The statistical variation of time is then independently evaluated. An LGP-30 program for approximated deviations is described in Section V.

The 70° F curve in Figure 3 presents a statistical evaluation of seven static test pressure-time histories of a typical solid-propellant motor using the approximate technique. Section V describes the computer program used to evaluate the performance profile.

III. EVALUATION OF THE STATISTICAL PROFILE AS A FUNCTION OF TEMPERATURE

The correlation of results obtained by one evaluation technique with the results from another variable, such as saturation grain temperature, is possible. Correlation of results may be accomplished by statistically evaluating performance at specific grain temperatures (usually a low, a nominal, and a high temperature) to obtain a statistically accepted population.

The average performance curve as a function of temperature may be found by a least mean square curve fit (of all coefficients of each $i$) to a binomial equation. Therefore,
Figure 3. A STATISTICAL EVALUATION OF THE ENERGY MANAGEMENT PROFILE AS A FUNCTION OF TEMPERATURE USING THE APPROXIMATE TECHNIQUE
\[ Y_{\text{ave}} = A_o \left( a_{A_o} + b_{A_o} T + c_{A_o} T^2 \right) + \sum_{i=1}^{i=40} \left[ a_i \left( a_{a_i} + b_{a_i} T + c_{a_i} T^2 \right) \right. \\
\left. \cos ix + b_i \left( a_{b_i} + b_{b_i} T + c_{b_i} T^2 \right) \sin ix \right] \]

where \( T \) equals grain environment temperature.

Thus, an average performance curve may be generated as a means of evaluating \( \Delta y \) for a random nonreferenced sample.

If the necessary computer storage is available, \( \Delta y \) for each individual round is determined at 81 equivalent intervals of \( x \) and then squared.

For each value of \( x \), \( (\Delta y)^2 \) is curve fit to a binomial expression of \( T \). After the evaluation of \( (\Delta y)^2 \) as a function of \( T \) for the 81 values of \( x \), the population moment as a function of \( T \) must be obtained. Sigma \( Y \) may now be determined at any value of \( x \) or \( T \) by

\[ \sigma_y = \left( \frac{\langle (\Delta y)^2 \rangle_{\text{ave}}}{N-1} \right)^{\frac{1}{2}}. \]

Now \( \sigma_y \) may be calculated and curve fit to obtain \( \sigma_y \) as a function of \( x \). The process is then repeated for critical temperatures. Then the coefficients of the Fourier series equation representing \( \sigma_y \) are curve fit to a binomial curve as a function of temperature, resulting in an equation identical in structure to Equation (9).

Wherever computer facility is limited in memory or speed, Equation (9) may be approximated by squaring and curve-fitting the coefficient and least mean square (as a function of temperature) to a binomial equation. Therefore,

\[ c_{i}^2 = \langle a + bT + cT^2 \rangle_{\text{ave}}. \]  

Then \( \sigma c_i \) can be found by applying the moment of the population, where
\[
\sigma c_i = \left( \frac{\left[ \sigma_i(a + bT + cT^2) \right]^2}{N} \right)^{\frac{1}{2}}
\]

(14)

Now \( \sigma c_i \) can be curve fit to the binomial expression in order to obtain an expression similar to Equation (8).

Using either the approximate or the absolute technique, a mathematical expression can be formulated for complete statistical description of the energy profile of a rocket motor. Although ballistics are evaluated at only three grain temperatures, a continuous basis for evaluation is possible which greatly reduces the number of tests lost from evaluation.

Figure 3 illustrates a complete statistical evaluation of the energy management profile of a typical solid-propellant rocket motor. For clearness the nominal performance curve is shown for lower and nominal environmental temperatures with +1σ variance envelope in the nominal temperature case and -1σ variance envelope in the lower temperature case. All characteristic changes that occur in the profile in Figure 5 are faithfully maintained. Although the data in Figure 3 was obtained through the approximate technique, the same results (although greater in accuracy) are obtained with the absolute technique.

IV. REQUIREMENTS AND LIMITATIONS OF THE CURVE FIT METHOD

The following requirements were found necessary to statistically evaluate performance curves by use of the curve fit method:

A. All data to be evaluated must be fit to equations of identical length.

B. Total time is set equivalent by removing time and arbitrarily evaluating \( y \) as units of \( x \) where total time equals \( 2\pi \) radians.

C. Time is then independently evaluated and incorporated into the overall variance pattern by coupling positive \( \sigma \) variations in \( y \) with equivalent negative \( \sigma \) variations in time. The sign of the variation for individual coefficients (when using the approximate technique) may vary independent of \( \sigma y \). Therefore, the direction of the variations must be considered when the approximate technique is used.
V. AN LGP-30 COMPUTER PROGRAM FOR THE STATISTICAL EVALUATION OF ENERGY MANAGEMENT PROFILES

The LGP-30 computer program is designed to determine the closest fit Fourier series to describe a given set of data points which may be graphed in two dimensions (that is, with an ordinate and an abscissa). The primary purpose of the program is the analytical study of pressure-time or thrust-time data. Since information is programmed in the LGP-30 fixed point routine, data must be storable at the given q's of the program.

In Part I of the program data is reduced to a Fourier series, the equation is stored in the computer memory for further operation, and the data is printed in decimal form. In Part I of the program several Fourier series can be stored in memory. The number of equations storable is dependent upon the number of terms of the series.

Part II is entered after the completion of Part I and replaces the logic of Part I without disturbing the stored Fourier coefficients. The logic of Part II is designed to average the Fourier coefficients of Part I and determine the sigma variation of each average coefficient. The average curve and the sigma values are stored in memory and printed in decimal form.

Part III is entered into the computer after the completion of Part II and replaces the logic of Part II without disruption of the storage of Parts I and II. Part III of the program takes stored data of Part II and calculates sigma maximum and sigma minimum deviations of each coefficient from the average coefficients. Thus, Part III supplies sigma maximum and sigma minimum Fourier series for the set of data traces under analysis. Part III calculates, prints in decimal form, and stores in memory the 1 sigma, 2 sigma, and 3 sigma maximum and minimum curves.

Part IV is entered into the computer in the same section as the logic of Parts I, II, and III. Thus, the logic of Parts I, II, and III is displaced without losing any previously stored data. The logic of Part IV evaluates the average and sigma series to produce the average and sigma traces associated with the original group of traces entered as data in Part I.

Part V is entered into the computer after Part IV and alters the logic of Part IV so that the sigma variations of the coefficients (as if constituting a Fourier series) are evaluated. (The sigma variations of the coefficients should actually represent the sigma variation of the data.)
Part VI is entered into the computer after Part IV and alters the logic of Part IV so that the original Fourier series representing the raw data is evaluated point by point. Part VI is intended primarily as a check of the accuracy of the Fourier series in reproducing the original curves.

Due to some confusion in Fourier series notation the reader should note that the program under consideration is a program of Fourier series in the following form:

\[ y = A_0 + \sum (A_i \cos \theta_i x + B_i \sin \theta_i x) \]

where \( \theta = \frac{360}{T} \) and where \( T \) is the terminal value of \( x \).

A. Computer Memory Usage

The logic portions of all parts of the LGP-30 computer program are stored in memory locations 2000 to 2500. Not all parts of the program completely fill the space from 2000 to 2500, but one or more parts eventually utilize the bulk of the space.

The program constants and counters are stored in memory locations 2500 to 2600. Since not all spaces from 2500 to 2600 are occupied, some room is left for expansion.

Values of \( T \) for each data trace are stored in memory locations 2600 to 2700. As previously noted, \( T \) is the terminal value of \( x \) in the Fourier series equation, that is, the value of the abscissa at the end of one cycle.

Memory locations 2700 to 2800 are retained for miscellaneous data, that is, data which has to be in a specific location but not in tubular storage. Data stored in memory locations 2700 to 2800 include both input data and calculated data.

Memory storage locations 2800 to 4300 are reserved in Part I of the program for the input of the raw data points to be converted to a Fourier series.

In all parts of the program succeeding Part I, memory locations 2800 to 4100 are reserved for storage of average Fourier coefficients and sigma Fourier coefficients.
Memory locations 4100 to 4300 are reserved for sigma variations of Fourier coefficients for all parts of the program following Part I.

Memory locations 4300 to 6300 are reserved for storage of Fourier series coefficients associated with individual raw data traces.

Memory locations 2732 to 2800 are reserved in Parts II, III, IV, and V to store $T_{ave}$. $T_{ave}$ is the sigma variation of the $T$'s of the set of traces and the sigma $T$'s associated with the individual sigma Fourier series.

B. Preparation of Data Tapes

As the LGP-30 computer program is written, only Part I has a data call in the logic. All other parts of the program depend upon data stored by previous parts of the total program. Thus, if a Fourier series is to be evaluated point by point, the coefficients must be entered by programming a data call in an unoccupied set of locations and then using Part IV of the program.

When utilization of the program, part by part beginning with Part I is desired, all necessary data may be entered on one data tape in Part I as follows:

1. The $T$'s are loaded sequentially from memory location 2600 at a q of 10. The numbers are loaded without a decimal point. For example, if the cycle ends at $x = 1.53$ seconds, 1.53 is entered as 153 and the units of $x$ or $T$ are entered as separate data. All $T$'s of a given set of traces must be in the same units.

2. The units of $T \circ x$ are entered in memory location 2700 at a q of 1. As stated in step 1, the units would be 0.01 or hundredths of a second.

3. The number of terms desired in the Fourier series is entered in memory location 2701 at a q of 29. The number of terms may be any whole number within certain limits defined by the available storage space.

4. The number of traces to be reduced to a Fourier series is entered in memory location 2702 at a q of 29. The number of traces may be any whole number within certain limits set by the number of terms desired in the resulting Fourier series.
5. The number of raw data points to be reduced to a Fourier series is stored in memory location 2704 at a q of 29. The number of raw points may be any whole number up to 960, which is the limit of the number of data points for any one trace for which storage space has been reserved. The number of raw points must be entered with each succeeding trace, since the computer has memory storage for only one trace at a time and cannot store the number of points associated with each trace in a tabular form.

6. The interval between data points $dx$ or $\Delta x$ is entered in location 2703 at a q of 10. The interval must be the same between all data points of a given trace. For example, if the interval between any two data points is 0.1 second, all data points must be at intervals of 0.1 second. Data point $dx$ may be either a fraction or a whole number.

7. The data points of each trace are loaded sequentially from memory location 2800 to memory location 4300 at a q of 20. As many as 960 data points may exist, all of which must be equidistant from each other in terms of the abscissa.

8. In order to avoid printing an excessive number of points in Parts IV, V, and VI, the program is designed to evaluate a trace from the Fourier series in three regions of different intervals along the abscissa. The magnitude of the intervals ($\Delta x$) are stored sequentially in memory locations 2725, 2726, and 2727 at a q of 10. Thus, a close trace at a region of rapid change of the ordinate may be obtained without spending time for computation in regions of slight change in the ordinate's magnitude.

9. The number of points desired in each region is stored in memory locations 2728, 2729, and 2730 at a q of 29. When the printout of greatest interest is the family of average and sigma curves, each region must be arranged to cover the regions of maximum slope of the entire family of curves. The last region must contain enough points to reach the end of the trace. The last region is programmed to go to another trace when the pressure drops below zero.

C. Selectivity and Other Characteristics

The number of terms may be chosen for the Fourier series which is most compatible with the traces to be analyzed. By the number of terms is meant the value of the letter $i$ in the equation,

$$ y = A_0 + \sum (A_i \cos \theta ix + B_i \sin \theta ix). $$
Therefore, $2i + 1$ coefficients actually exist in the Fourier series.

Since memory locations 4300 to 6300 are reserved for the storage of coefficients and 64 locations exist per track, 1,280 memory locations are available for storage of coefficients. Thus, if a 40-term Fourier series for all traces in a set were called for, 15 traces could be reduced to a Fourier series and the coefficients could be stored with 65 locations left over. In a 20-term Fourier series 31 traces could be reduced and 9 locations would be left over. Similarly, in a 50-term Fourier series 12 traces could be reduced and 68 locations would be left over.

The amount of available storage for average and sigma curves limits the number of terms for the Fourier series. Seven Fourier series (832 locations) exist which define average and sigma curves for which tracks 2800 to 4100 are available. Therefore, the maximum number of terms allowable is 58.

The fact that the maximum number of data points for a given trace is 960 and that the points must be equidistant from each other in terms of $dx$ is due to the method of evaluation of the coefficients. The method of evaluation, though leading to a slow program, is unavoidable. A greater value of $dx$ and a fewer number of points result in a faster program. A smaller value of $dx$ and a larger number of points result in greater accuracy of the resulting Fourier series.

The LGP-30 computer program, though designed for analysis of pressure-time traces, may be used with some versatility for other purposes. For example, any set of data points set at equal intervals according to the abscissa can be reduced to a Fourier series during Part I. Whether the points are positive or negative with regard to the ordinate is not significant in Part I of the program providing the abscissa begins at zero and increases along the positive axis. Such a Fourier series could be evaluated point by point by utilizing only two of the $ox$ regions of Parts IV or VI. (The program switches to the stop on minus in the third $ox$ region.)

A Fourier series in equation form may be evaluated point by point by entering a data input subroutine in unreserved locations, programming the data input subroutine with the Fourier coefficients in locations beginning at 2800, and then jumping into Part IV to evaluate the input data.

The LGP-30 computer program is written for the input of raw data points at a $q$ of 20. Data which cannot be held at a $q$ of 20 can be processed by shifting the decimal point to produce a number which
the computer can handle at a q of 20. Care must be taken that all data points are shifted the same number of decimal points and that the decimal point shift is reversed in the final point-by-point printout. (The Fourier coefficients are not subject to the same shift in decimal point as the data points.)

VI. CONCLUSION

Because of the demands placed upon solid-propellant propulsion systems, knowledge of the reproducibility of performance envelopes and the controlling parameters is essential. The absolute and approximate techniques are effective methods of automated statistical evaluation of rocket motor performance envelopes.

Currently, the absolute technique is being programmed for the IBM 7090 computer in Fortran (formula translator) language. The IBM 7090 program inputs original source test data recorded on frequency modulated magnetic tape and has the option of an output using an automatic plotting board. (At the completion of the IBM 7090 program, a copy of a working deck will be available.) The IBM 7090 program could feasibly complete statistical evaluation of a family of test data curves at an estimated cost of 45 seconds machine time per motor.
6 November 1963

APPROVED:

E. W. JAMES
Director, Propulsion Laboratory

DISTRIBUTION

This report is distributed in accordance with the Chemical Propulsion Mailing List dated June 1963 excluding addressee numbers: 45, 51, 52, 53, 54, 56, 57, 84, 87, 91, 93, 103, 120, 123, 125, 126, 128, 132, 159, 164, 165, 197, and 201. 1-153

AMSMI -FRS  154
AMSMI -R, Mr. McDaniel  155
   -RL  156
   -RS  157
   -RT  158
   -RR  159
   -RK  160
   -RE  161
   -RG  162
   -RKA, Mr. Guthrie  163-167
   -RBL  168-172
   -RAP  173