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PARAMETRIC AND DYNAMIC PROGRAMMING
IN FOREST FIRE CONTROL MODELS

by

Richard J. Swersey

OPERATIONS RESEARCH CENTER
INSTITUTE OF ENGINEERING RESEARCH
UNIVERSITY OF CALIFORNIA-BERKELEY
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University of California, Berkeley

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ABSTRACT

The forest fire control models developed by Parks and Jewell [1] [3] assumed one type of suppression force with a particular effectiveness and cost of operations. These models are extended here to the multiple suppression force case.

If $N$ types of forces are dispatched to a fire, they will either arrive simultaneously or in a lagged sequence. Choosing the optimal (lowest cost of burn plus suppression) mix of forces for simultaneous arrival requires the use of a linear programming model that is simultaneously parametric in the right-hand sides and in the objective function. For non-simultaneous arrival, a dynamic programming algorithm, that compared pairs of forces, was developed and solved on an IBM 1620 computer. The lowest envelope of cost curves for all such pairs can be used as a planning tool in initial attack.
I. Introduction

The fire growth model of Parks and Jewell [1] was constructed under the assumption that the suppression force has a singular effectiveness denoted by $E$. In actual practice, however, suppression forces form a complex organization with different values of $E$ for each component. The extension of Parks' model to the multiple suppression crew case is necessary for its actual employment in initial attack planning.

This paper describes the construction of and application of some decision rules concerning the choice of forces sent to a fire given the essential fire characteristics, the values of $E$, and the attack time for each group of suppression forces. The decision criterion used here is minimization of total costs incurred after discovery of a fire. An alternative criterion of decision, commonly used by the U.S. Forest Service, is acreage control. Therefore, wherever possible in the mathematical solutions, the optimal solutions for all values of acres burned after detection will be included with the overall optimal solution.

The assumptions used here are basically those that appear in Reference [1]. The most important of these is,

$$\frac{dy}{dt} = G_d + Ht$$

where $dy/dt$ is the growth rate of the fire in acres per hour, $G_d$ is the observed growth rate at detection. $H$ is the acceleration in acres per hr$^2$, and $t$ is any time after detection. After initial attack, the growth rate is described by,

$$\frac{dy}{dt} = G_a - (EX - H)(t - T_a)$$
where \( X \) is the number of men sent, \( T_a \) is the time interval between detection and attack, \( E \) is the effectiveness of the men, and \( G_a \) is the growth rate at the time of attack. For multiple forces two problems arise: 1) measurement of the various \( E \) values for each type of force, and 2) description of the growth rate in terms of nonsingular attack times. Measurement of \( E \) values for hand crews shows a clear relationship of \( E \) with the basic line building rates. A more detailed analysis of this problem is treated by McMasters [4]. Let us concern ourselves with the second problem.

In general, any group of forces will not arrive at the fire simultaneously. The decision maker must decide how many of each type of force to send given their effectiveness and inter-arrival times. As a simplifying assumption, however, consider first the case of simultaneous arrival.

II. MODEL 1: Simultaneous Arrival

Simultaneous arrival is possible both in actual practice and as an assumption when the inter-arrival times are small compared to the detection-arrival interval. Suppose a forest or region has \( N \) types of suppression forces available. Let \( X_i \) be the amount of the \( i^{th} \) type of force sent to the fire

\[
E_i = \text{effectiveness of the } i^{th} \text{ force in acres per man hr}^2
\]

\( Y_c = \text{total acres burned at control} \)

\( Y_a = \text{total acres burned at initial attack} \)

\( G_a = \text{growth rate at initial attack} \)

The fire is controlled when

\[
\frac{dy}{dt} = 0;
\]

This can occur if

\[
\sum_{i} E_i X_i > H,
\]

-2-
and therefore by extension of Parks' equations,

\[(1) \quad Y_c - Y_a = \frac{G_a^2}{2(\sum E_i X_i - H)}\]

and \(T_c\), the control time, is given by,

\[(2) \quad T_c = \frac{G_a}{\sum E_i X_i - H}\]

The economics of fire control have been discussed in [1]. If fixed costs are neglected as decision variables, the only cost factors to consider are,

- \(L_i\) = the transportation cost per unit of the \(i^{th}\) force ($/man)
- \(W_i\) = the cost per unit hour of the \(i^{th}\) force ($/man hr)
- \(C\) = the cost of burn ($/acre).

An optimal decision requires the minimization of,

\[(3) \quad C(Y_c - Y_a) + \sum L_i X_i + \frac{\sum W_i X_i G_a}{\sum E_i X_i - H}\]

A simple adjustment transforms this nonlinear system into a parametric linear programming problem. If the cost of suppression is minimized for any given value of the burn cost, then the lowest value of burn cost plus suppression cost is the optimal solution.

If the total cost of burn is known, then \((Y_c - Y_a)\) is also known; hence,
\[ \sum_{i} E_i X_i - H = \frac{G_a^2}{2(Y_c - Y_a)} = A \]

(4) \[ \sum_{i} E_i X_i = A + H = B \]

(5) \[ T_c = \frac{G_a}{A} \]

(6) \[ \sum_{i} (L_i + W_i G_a / A) X_i + C(Y_c - Y_a) = \text{Minimum} \]

III. Generalization of the Simultaneous Arrival Problem

In order to generalize the above problem, let

\[ Y_c - Y_a = X_0 \]

and

\[ \sum_{i} E_i X_i - H = \alpha_0 + \sum_{i} \alpha_i X_i \], a linear combination of the \( X_i \)

for \( i = 1, \ldots, n \). Then,

(4) \[ X_0 \left[ \alpha_0 + \sum_{i} \alpha_i X_i \right] = \frac{G_a^2}{2} \]

(6) \[ C X_0 + \sum_{i} L_i X_i + \sum_{i} D_i X_i \left[ \alpha_0 + \sum_{i} \alpha_i X_i \right]^{-1} = Z \text{ (Min)} \]

where \( D_i = W_i G_a \)

(8) \[ X_i \geq 0 \]

We must minimize (6) subject to (4) and (8). If we assume \( X_0 > 0 \), then for
any particular value of $X_0$, say $X_0^*$,

\begin{align*}
(4a) & \quad \alpha_0 + \sum_{i} \alpha_i x_i = \frac{G_i^2}{2X_0} = R(X_0^*) \\
(6a) & \quad C^* x_0 + \sum_{i} D_i x_i = Z \text{ (Min.)}
\end{align*}

where $D_i = L_i + D_i / R(X_0^*)$,

\begin{equation}
(8) \quad x_i \geq 0.
\end{equation}

Equations (4a), (6a), and (8) describe an elementary linear programming problem that is parametric simultaneously in the right-hand side as well as in the objective function.

The addition of other constraints on the $x_i$ will not change the problem. Most generally we must solve,

\begin{align*}
(4a) & \quad \alpha_0 + \sum_{i} \alpha_i x_i = R(X_0^*) \\
(8a) & \quad 0 \leq x_i \leq K_i \quad i = 1, \ldots, n \\
(6a') & \quad \sum_{i} (L_i + D_i R(X_0^*)^P) x_i = Z \text{ (Min.)} \quad -\infty < P < \infty
\end{align*}
IV. Solution Algorithm for Simultaneous Arrival Problem

The following procedure yields the required optimal solution:

Set $Z' = \infty$  
Choose $X_0^*$  
Solve the linear program $(4a), (6a')$  
$(8a)$  
$Z' = \text{Min}(Z', Z^*)$ Store $X_0^*, X_1$ associated with $Z'$  
$x_0^* = x_0^* + \theta \ (j = 1, 2, \ldots, s)$

Does $j = s$?

No

Terminate with optimal solution $Z', X_0^*, X_1^*$

Yes

EXAMPLE: Simultaneous Arrival

In order to illustrate the above technique, consider the following example for two suppression crews that can arrive simultaneously at a fire.

\[
\begin{align*}
E_1 &= 4 \\
L_1 &= 25 \\
W_1 &= 10 \\
H &= 20 \\
E_2 &= 2 \\
L_2 &= 10 \\
W_2 &= 6 \\
T_a &= 2
\end{align*}
\]
For simplicity assume the fire is detected immediately after it starts. Then,

\[ G_a = HT_a = 40 \text{ acres per hr.} \]

\[ Y_a = \frac{1}{2} HT_a^2 = 40 \text{ acres}. \]

For \( C \) equal to $200 per acre, the following results were obtained:

<table>
<thead>
<tr>
<th>( X_0 )</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( C_{\text{supp}} )</th>
<th>( C_{\text{burn}} )</th>
<th>( C_{\text{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>acres</td>
<td>No. of forces</td>
<td>No. of forces</td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>60</td>
<td>( \frac{1}{3} )</td>
<td>0</td>
<td>458</td>
<td>12,000</td>
<td>12,458</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>20</td>
<td>440</td>
<td>8,000</td>
<td>8,440</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>30</td>
<td>480</td>
<td>4,000</td>
<td>4,480</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>90</td>
<td>1,035</td>
<td>1,000</td>
<td>2,035</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>120</td>
<td>2,226</td>
<td>400</td>
<td>2,626</td>
</tr>
</tbody>
</table>

Notice that the optimal solution occurs where the cost of burn after arrival of crews is about equal to the cost of suppression. This type of solution agrees with the intuitive idea that it is uneconomic to spend more money per acre on suppression than the value per acre of the burning land. The results are plotted below in Figure 1.
TOTAL COST

ACRES BURNED

Fig. 1
A further analysis of the optimal solution yields some useful results. The minimum cost occurs at $X_0 = 5$ acres. The equations of this system are,

$$4X_1 + 2X_2 = 180$$
$$27.5X_1 + 11.5X_2 = Z\text{(Min)}.$$

The optimal solution is $X_2 = 90$, and $Z = \$1,035$. Suppose we now consider the use of a bulldozer for which

$$E_{bu} = 50$$
$$W_{bu} = \$25.$$

What is the maximum transportation cost for which the bulldozer can be used? The transportation cost is important in the consideration of bulldozer usage because we must consider the cost of getting the bulldozer to the fire in the same time that the other forces require. This type of problem is easily analyzed by the dual linear programming problem.

V. Use of the Dual Linear Program

For every primal linear programming problem,

$$Ax = b$$
$$Cx = \text{Min}.$$

there is an associated dual,

$$yA \leq C$$
$$yb = \text{Max.}$$

such that at optimality

$$yb = Cx$$
Furthermore, by the theorem of complementary slackness,

\[ X_i = 0 \text{ whenever } y_{a_i} < c_i \]

and

\[ y_{a_i} = c_i \text{ whenever } X_i > 0. \]

For a full treatment of the subject, see Chapters 1 and 4 of Gale [2].

Let the number of bulldozers used by represented by \( X_{bu} \). From the above, if

\[ y_{abu} < c_{bu}, \text{ then } X_{bu} = 0 \text{ and hence the optimal solution does not change.} \]

For the optimal solution of Example 1, the dual is,

\[ 4Y_1 \leq 27.5 \]
\[ 2Y_1 \leq 11.5 \]

By the complementary slackness requirements,

\[ 2Y_1 = 11.5 \]
\[ Y_1 = 5.75 \]

The dual is optimal because,

\[ Y_{1} b = 5.75(180) = 1035 = Cx. \]

The bulldozer should not be used if,

\[ 50Y_1 < (I_{bu} + \frac{1}{4}W_{bu}) \]

or if,

\[ 287.5 < (I_{bu} + 6.25). \]
If \( L_{bu} > 281.25 \), the bulldozer should not be used. If the inequality is not satisfied, then another cycle of the simplex method will determine the correct number of bulldozers to use. Further, if they are used at the optimal acreage, the entire problem should be re-worked with the bulldozers considered in case there is a new optimal acreage allowance. Even for the case where bulldozers are not used at the optimum acreage, \( X^*_0 \), part of the problem must be re-solved. At some larger value of \( X_0 \), it is possible that bulldozers will be economical to use. Therefore, one should rework the problem for all \( X_0 > X^*_0 \). In the neighborhood of \( X^*_0 \), where the set of basic variables does not change, one can compute a region in which bulldozers are not used, thereby reducing the actual amount of re-computation of the linear program.

Note that the solution has not been restricted to integers. In practice a fraction of a man cannot be sent to a fire nor can you send a fraction of a bulldozer. Rather than complicate the linear programming problem with integer requirements one can arbitrarily send an additional man for each fraction greater than 0.3. Parks [1] has shown that it is cheaper to overkill a fire than to underman the suppression force. Therefore, little is to be gained by formulating complex integer programming problems. For problems with more constraints, one might also investigate fractional programming algorithms.

VI. Uncertain Growth Rates

The extension of Parks' model for uncertain growth rates follows directly from the argument in [1]. In actual practice \( G_a \) may only be measurable to a distribution function with mean \( \mu \) and variance \( \sigma^2 \).

Taking the expected value of \( (6), \)
\[
\mathcal{C} \left[ \sum \frac{L_1}{1} x^i + \sum \frac{W_i X_i}{A} \right] = \sum \mathcal{C}(L_i X_i) + \sum \mathcal{C} \frac{W_i X_i}{A} \\
= \sum (L_i X_i) + \sum \frac{W_i X_i}{A} \mathcal{C}(a_i)
\]

because
\[
\frac{G_a}{A} = \frac{2X_0}{G_a} \\
A = f(G_a).
\]

Therefore,
\[
\mathcal{C}(A) = \frac{\mathcal{C}(c_a)}{2X_0}
\]

and
\[
\mathcal{C}(c_a^2) = (K^2 + 1)(\mathcal{C}(c_a))^2
\]

where
\[
K^2 = \frac{c_a^2}{\mu^2}.
\]

Then as before,
\[
(4a) \quad \sum_{i} \alpha_i x_i = \frac{(K^2 + 1)(\mathcal{C}(c_a))^2}{2X_0} + \mathcal{H}
\]

\[
(6b) \quad \sum \left( L_i + \frac{2X_0 W_i}{\mathcal{C}(c_a)} \right) X_i = \text{Minimum}.
\]

The insertion of uncertainty into the problem will increase the number of men sent on the initial attack. The solution method here is the same as that outlined in Model 1.
VII. MODEL 2 Nonsimultaneous Arrival

The usual deployment situation is characterized by different arrival times for groups of forces dispatched at the same time. For example, a helitac crew should arrive early than a tanker truck crew under average conditions. Before developing a mathematical model of this situation note that values $E$ for each force are important in eliminating a class of problems. For those cases where the most effective force is closest to the fire one should usually send as many of these forces as possible. Only under an unusual cost spread between types of forces would there be a change from this strategy.

Under most circumstances there are usually only a few types of forces that are sent initially. Let us consider just two types of forces and develop the mathematical structure of the model.

Let

$G_1 = G_d + HT_{da}$ = growth rate at the time of attack of the first crew

$T_1$ = inter-arrival time between crews

$T_2$ = time from attack to control for crew 2

$T_1$ = time from attack to control for crew 1 (the first to arrive)

$E_i$ = effectiveness of the $i^{th}$ crew.

$T_{da}$ = time from detection to arrival of the first crew

Using the time of attack of the first crew as a base, the growth rate of the fire is described by,

$$\frac{dy}{dt} = G_1 - (E_1 X_1 - H) T_1 - (E_1 X_1 + E_2 X_2 - H) (t - T_1) .$$

Equation (9) holds for all non-negative values of $X_1$ and $X_2$. We must minimize,

-12-
Such a process is not trivial because $T_1$ and $T_2$ depend directly on $X_1$ and $X_2$. The variables, $X_0$, $X_1$, and $X_2$ are related by,

\[(11) \quad X_0 = \left[ \frac{g_1 - (E_1 X_1 - H) T_1}{2} \right] T_1 + \left[ \frac{g_1 - (E_1 X_1 - H) T_1}{2 (E_1 X_1 + E_2 X_2 - H)} \right]^2 \]

The control times are given by,

\[(12) \quad T_2 = \frac{g_1 - (E_1 X_1 - H) T_1}{(E_1 X_1 + E_2 X_2 - H)}; \quad T_1 = T_1 + T_2 \]

At the boundary, however, we get different expressions for control times.

For $X_2 = 0$, and $X_1 > 0$

\[(12a) \quad T_1 = \frac{g_1}{E_1 X_1 - H} \]

and for $X_1 = 0$, and $X_2 > 0$,

\[(12b) \quad T_2 = \frac{g_1 + H T_1}{E_2 X_2 - H} \]

The variable $X_1$ is also bounded above and below for $X_2$ positive; its minimum value must not allow more than $X_0$ acres to burn before the second crew arrives, and its maximum value must not allow control of the fire before the second crew arrives. Thus, for $X_2 > 0$

\[\frac{g_1^2}{2E_1 X_0} + \frac{H}{E_1} \leq X_1 \leq \frac{g_1}{E_1 T_1} + \frac{H}{E_1} \quad \text{or} \quad A(X_0) \leq X_1 \leq B\]
The following algorithm will solve Model 2:

1. Set $Z' = \infty$
2. Compute $A(X_0), B$
3. $X_0^* = X_0 + \Theta_j$
4. $j = 1, \ldots, s$
5. $X_1^* = X_1 + \alpha_k$
6. $K = 1, \ldots, r$
7. Compute $X_2$ from (11)
8. Compute $T_1, T_2$ and $Z^*$ from (10), $L_2 - L_2^a$
9. $Z' = \text{Min}(Z^*, Z')$
10. If $K = R$
11. Yes
12. No
13. If $J = s$
14. Yes
15. No
16. Terminate with optimal solution $Z', X_0', X_1', X_2'$
VIII. Generalization of Nonsimultaneous Arrival

This problem can be generalized to

\[(10a) \quad Z = \min f(X_0, X_1, X_2)\]

where

\[(11a) \quad X_2 = g(X_0, X_1)\]

and

\[(13) \quad h_2(X_0) \leq X_1 \leq h_1(a_1, \ldots, a_n) \quad (a_i \text{ are constants})\]

which is a dynamic programming problem.

The minimization process can be computed using the following scheme:

1. Set \(\bar{Z} = \infty\)
2. Compute \(h_1(X_0), h_2(a_1, \ldots, a_n)\)
3. \(X_0^* = X_0 + \Theta_j, \quad j = 1, \ldots, s\)
4. Compute \(h_1(X_0), h_2(a_1, \ldots, a_n)\)
5. \(X_1^* = X_1 + \alpha_k, \quad k = 1, \ldots, r\)
6. \(X_2^* = g(X_0^*, X_1^*)\)
7. \(Z^* = f(X_0^*, X_1^*, X_2^*)\)
8. \(\bar{Z} = \min(Z^*, \bar{Z})\)
9. \(X_0^*, X_1^*, X_2^*\) associated with \(\bar{Z}\)

Terminate with \(\bar{Z}\)
IX. Use of the Planning Model

An optimization process for a system such as (10)-(13) is tedious and is not intended for use on individual fires. The process lends itself to solution by an iterative computer program. Such a program was written for the IBM 1620 located at the Survey Research Center at the University of California, Berkeley. From the solution of many possible cases using the same set of forces, a family of cost curves can be drawn; then depending on the value of \( H, G \) and \( E \) for each force, a dispatcher can easily choose the best mix of suppression forces for most fires. If the estimates of \( H \) are too low then it will be necessary to employ reinforcements. As Parks has shown [1], the reinforcement problem is similar to the initial attack problem and hence presents no great difficulties.

Before using the model to solve a specific problem, consider intuitively the results that we should obtain. For all values of \( X_0 \) less than \( X_0^* \), the closer forces will begin to be used until at some point the control time is less than the inter-arrival time and only the earliest arriving forces can be used. For \( X_1 = 0 \), the equation for \( X_0 \) implies,

\[
X_0 \geq (G_1 + HT_1/2)T_1.
\]

The critical value of \( X_0 \) is reached where

\[
X_0^* = (G_1 + HT_1/2)T_1, \text{ the area burned until crew 2 arrives.}
\]

If the most effective force does not arrive first, one would expect to use it only when the values of \( X_0 \) are large. If the number of early arriving forces is limited, then so is the minimum acreage that can be controlled. Thus there will be three critical regions to define and they are all linearly dependent on \( H \). For large values of \( H \) the transitions should occur at higher values of \( X_0 \) than for smaller values of \( H \).
EXAMPLE: Nonsimultaneous Arrival

In order to illustrate the use of the planning model consider the following problem: Three crews are available for initial attack on fires in a particular region of a forest. They are, (1) a 2-man helitac crew, (2) a 4-man tanker crew and (3) a small bulldozer. For values of $H$ of 3, 10, and 50 acres per hr$^2$, find the optimal strategy.

<table>
<thead>
<tr>
<th>Crew</th>
<th>Inter-arrival Time*</th>
<th>$E$</th>
<th>$L_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>—</td>
<td>4</td>
<td>50</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2/3</td>
<td>8</td>
<td>25</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>2/3</td>
<td>25</td>
<td>100</td>
<td>28</td>
</tr>
</tbody>
</table>

*Inter-arrival time from Crew 1 to other crews.

Let $C = $200 per acre, $G_1 = 5$ acres per hour.

The two crew model can be solved for Crews 1 and 2 and for Crews 1 and 3; the lower envelope of cost curves will indicate the optimal policy among all three crews.

Some of the results of the computer solution are listed in Appendix A. There are 2 columns in the appendix that bear further explanation. MVS refers to the values saved in controlling the fire at the next lowest $X_0$ value. The value saved at $X_0 = 20$ acres compared to $X_0 = 30$ acres is $2,000 because MVS equals $C_{burn} \Delta X_0$. MC is the extra cost of suppression associated with controlling the fire at the lower $X_0$ value. These columns indicate the following requirement for an optimal solution:

$$X_0^* \text{ occurs where } MC = MVS. \quad (X_0^* \text{ is optimal } X_0)$$

Optimality then, is not defined as the $X_0$ value for which burn cost per
acre equals suppression cost per acre; such a condition may be true at lower values of \( H \), but it is not true in general. Notice that at high values of \( H \), the suppression effort is rather costly on a per unit acre basis; such a measure of spending efficiency is biased, however, because it does not reflect the potential acreage saved.

Returning to the simultaneous arrival case, notice that the marginal criterion of optimality holds there too. This concept is similar to the determination of optimal output in a production system; optimal output occurs where the marginal cost of the next unit produced equals the marginal revenue derived from the unit.

The results for the fixed inter-arrival problem are plotted in Figures 2, 3, and 4.

The optimal solutions of this model clearly agree with our intuitive assumptions. The cost curves have a typical convex shape and a minimum point. There is an error in the curves, however, caused by the lack of restrictions on using integral crews. For example, the cost of transporting a helitac crew is $25 per man, but the cost of sending only a one man crew is $50 per man. Therefore for each fractional crew there is an added fixed cost of transportation not considered here. The results will not be changed markedly because these fixed costs will only cause jumps in the cost curves at several points rather than causing changes in the cost curves.

For the lower values of \( H \), the cost of suppression decreases as \( X_0 \) decreases, and then increases again. This behavior is caused by the balancing of extra transportation costs with decreased hourly work costs.

With or without these refinements, the multiple crew model can be used to advantage in choosing among different types of forces.
X. Multiple Crews Fire Break Model

Jewell [3] has constructed the fire spread model illustrated in Figure 5.

\[ V_f \]

\[ L \]

\[ D \]

\[ W \]

\[ \alpha \]

\[ x \]

\[ W \]

\[ V_f \] is the velocity of the fire front. \( L \) is the length of the front, \( D \) is a distance away from the front at time zero that a crew must move in order to build a fire break of width \( W \) by the time the fire front reaches the crew.

\[ \alpha x/W = \text{feet of fire break constructed by } X \text{ men per hour.} \]

If the time to construct the break equals the travel time of the fire front,

\[ T = LW/\alpha x. \]

The area burned \( A = V_f L T = L^2 W (V_f/\alpha x) \). For the multiple crew case,

\[ A = \frac{L^2 W V_f}{\sum_i \alpha_i X_i} \]

Let \( \alpha_i/W = \beta_i = \text{the line building rate of the } i^{th} \text{ force at a standard width } W \).

\[ A = \frac{L^2 V_f}{\sum_i \beta_i X_i} \quad \text{and} \quad T = L/\sum_i X_i \]

If \( A \) is held constant, then
(13) \[ \sum_{i} \beta_i X_i = L^2 V_f / A \]

and

(14) \[ \sum_{i} C_i X_i A / L V_f = \text{Minimum} \text{ (where } C_i \text{ is the unit cost per man hour)} \]

forms a linear programming system whose solution yields the optimal mix of forces. An iterative procedure on \( A \) will define the overall optimal strategy given the unit cost of burn.

Now suppose that a fire spreads in \( N \) directions and \( N \) such fire breaks must be constructed. Because of the terrain and fuel type, let the area \( A_i \), be fixed. Then,

(15) \[ \sum_{i=1}^{N} \beta_{ij} X_{ij} = V_{f_j} L_j^2 / A_j \text{ for all } j = 1, 2, \ldots, M, \]

where \( \beta_{ij} = \text{rate for the } i^{th} \text{ force on the } j^{th} \text{ fire front and} \]

\( X_{ij} = \text{number of the } i^{th} \text{ type of force sent to the } j^{th} \text{ fire front,} \)

and

(16) \[ \sum_{j=1}^{M} X_{ij} \leq X_i \text{ for all } i \]

(17) \[ \sum_{i} C_i \sum_{j} X_{ij} A_j / L_j V_{f_j} = \text{Minimum} \]

describes a linear programming problem. We can solve these problems with the same parametric procedure outlined in Model 1. The optimal solution of this system is given the best deployment strategy for all forces. On a campaign fire, where there are a number of divisions and sectors, such a model would be a useful tool to a fire boss; the linear programming device
automatically investigates the trade offs among costs of crews, line building ability and values burned.

As models are constructed that analyze other types of fire line strategies similar extensions to multivariable case can be readily made. In conclusion, I would suggest that extensions of these models are easily attained through the use of parametric programs that are solvable on a computer.
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<th>C_{total}</th>
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**KEY**

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<td>X₀</td>
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<tr>
<td>MVS</td>
<td>Value of burned acres saved in reducing X₀ at each step</td>
</tr>
<tr>
<td>MC</td>
<td>Additional cost of suppression in reducing X₀ at each step</td>
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REFERENCES


