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TRANSLATION

SUBLIMATION OF A BLUNT BODY IN THE VICINITY OF THE STAGNATION POINT IN PLANE AND AXISYMMETRICAL FLOW OF A GAS MIXTURE

By

G. A. Tirskiy

FOREIGN TECHNOLOGY DIVISION

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SUBLIMATION OF A BLUNT BODY IN THE VICINITY OF THE STAGNATION POINT IN PLANE AND AXISYMMETRICAL FLOW OF A GAS MIXTURE

G. A. Tirskiy
(Moscow)

The problem of the sublimation of a body in the vicinity of the stagnation point was examined in an earlier work [1], where a numerical solution was carried out for the case of quasistationary "adiabatic" (disregarding heat transfer into the body) boiling of solid carbon dioxide in an axisymmetrical case where the Mach number of the incident flow was 6.2. However, the conditions for attaining such a maximum regime of sublimation were not obtained in the work and no solution was given which permitted the calculation of the sublimation of bodies with other physical properties under arbitrary flow conditions.

A simplified analysis of the sublimation of a solid at a given sublimation temperature T_w has been presented [2] which is based on the method of integral relationships. This case also corresponds to the "boiling" of a solid since during sublimation, the temperature prior to boiling temperature on the vaporizing surface is unknown and must be determined while solving the problem.

In the present article we obtained an accurate solution (solution of the type of a uniformly propagating wave) of the problem of equilibrium and nonequilibrium sublimation of a blunt body in the stagnation region with an arbitrary dependence of the physical properties of the body on temperature.

For a Prandtl number $\sigma = 0.7; 1$ and a Schmidt number $Sc = 1$ the solution was obtained numerically. By means of asymptotic integration a solution was also obtained for arbitrary σ and $Sc > 0.5$. It was demonstrated that if the coefficient of accomodation $f > 0.1$ then, with sufficient accuracy for application, vaporization of the body will proceed according to diffusion kinetics (equilibrium vaporization) for $u_\infty \sim 10^3$ m/sec, when $f < 0.1$ nonequilibrium vaporization must be taken into account.

The necessary and sufficient condition under which boiling at the sublimation front takes place was obtained (7). When boiling is attained the sublimation rate and the mass velocity of sublimation are in a final form (Eqs. 7.4 and 7.5).

The temperature profile in the body for an arbitrary dependence of thermophysical properties of the body on temperature are found in quadratures. The calculation of any specific problem in the general case reduces to a solution of a system of three finite equations for the determination of concentration, temperature at the vaporization front, and rate of sublimation.

1. Statement of the Problem

If a mixture of gas flows past a solid body with a vapor partial pressure of the body in the incident flow lower than the pressure of the saturated vapors at the body surface temperature and if the surface temperature is lower than the temperature of the triple-point

of the phase diagram, then the body will sublimate. For this purpose we will work out for the sublimation front, the appropriate temperature and mixture composition which, like the position of the sublimation front, are not known beforehand and must be determined while solving the problem.

The process of sublimation of a multicomponent body must be considered as a simple heterogeneous chemical reaction with the stoichiometric equation

$$\sum_{k=1}^{N_0} \nu_k' A_k' \rightarrow \sum_{k=1}^{N'} \nu_k A_k, \tag{1.1}$$

where A_i', A_i are the chemical symbols corresponding to the solid and gaseous components; ν_i', ν_i are the stoichiometric coefficients of the solid and gaseous components respectively; N_0 is the number of components in the solid phase; and N' is the number of components produced during sublimation of the body (for example, during the vaporization of SiO_2 a four-component ion vapor of SiO_2^+, SiO^+, O_2^+ and O^+ is formed [3]).

The mass concentrations c_{is} $(i = 1, \ldots, N')$ of the gaseous products approaching the sublimation surface from the side of the solid phase are associated with N' by the equation

$$\frac{c_{1s}}{\nu_1 M_1} = \cdots = \frac{c_{N's}}{\nu_{N'} M_{N'}}, \quad \sum_{k=1}^{N'} c_{ks} = 1,$$

where M_i is the molar mass of the i-th component. Hence the composition of the gaseous products produced by vaporization of the body will be

$$c_{is} = \frac{\nu_i M_i}{\sum_{k=1}^{N'} \nu_k M_k}, \qquad i = 1, \ldots, N'. \tag{1.2}$$

Then the problem of a stationary regime of sublimation in the stagnation line (plane case) or in the stagnation region (axisymmetrical case)

of a blunt body placed in a stationary gas flow consisting of N = N' + + N" components leads to the solution of a system of equations of a nonstationary boundary layer for a multicomponent gas mixture:

$$\frac{\partial \rho}{\partial t} + \frac{1}{x^{n-1}}\frac{\partial}{\partial x}(\rho u x^{n-1}) + \frac{\partial}{\partial y}(\rho v) = 0, \qquad n = 1, 2; \tag{1.3}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho_\infty \beta^2 x + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \qquad \beta = \left(\frac{du_e}{dx}\right)_{x=0}; \tag{1.4}$$

$$\rho\left(\frac{\partial c_i}{\partial t} + u\frac{\partial c_i}{\partial x} + v\frac{\partial c_i}{\partial y}\right) = \frac{\partial}{\partial y}\left(\rho D_{12}\frac{\partial c_i}{\partial y}\right), \qquad i = 1, \ldots, N; \tag{1.5}$$

$$\rho\left(\frac{\partial h^*}{\partial t} + u\frac{\partial h^*}{\partial x} + v\frac{\partial h^*}{\partial y}\right) = \frac{\partial}{\partial y}\left\{\frac{\mu}{\sigma}\left[\frac{\partial h^*}{\partial y} + (\mathrm{Lc}^{-1} - 1)\sum_{k=1}^{N} h_k^*\frac{\partial c_k}{\partial y}\right]\right\}; \tag{1.6}$$

$$p = \rho RT\sum_{k=1}^{N}\frac{c_k}{M_k}, \quad h = \sum_{k=1}^{N} c_k h_k = \sum_{k=1}^{N} c_k(h_k^* + h_k^0) = h^* + h^0; \tag{1.7}$$

$$h_k^* = c_{p_k}T, \quad h^* = \sum_{k=1}^{N} c_k h_k^* = c_p T, \quad c_p = \sum_{k=1}^{N} c_k c_{p_k}, \quad h^0 = \sum_{k=1}^{N} c_k h_k^0;$$

$$\sigma = \frac{\mu c_p}{\lambda}, \quad \mathrm{Lc} = \frac{\lambda}{\rho c_p D_{12}},$$

together with the equation of thermal conductivity in the solid body

$$\rho_1 c_* \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial y}\left(\lambda_1\frac{\partial T_1}{\partial y}\right), \quad \rho_1 c_* = \bar{\rho}_1\bar{c}_* N(T_1), \quad \lambda_1 = \bar{\lambda}_1 L(T_1). \tag{1.8}$$

Here x, y are the coordinates associated with the body along and normal to the initial surface of the body respectively; u and v are the components of the velocity vector with respect to x and y respectively; ρ is the density; p - pressure; T and T_1 — temperature of the gas and body; c_i — mass concentration of the i-th component of the mixture; $h_i = h_i^* + h_i^0 = c_{p_i} T + h_i^0$; and c_{p_i} — specific partial enthropy and specific heat for constant pressure of the i-th component respectively; h^0 — "zero enthalpy" of the i-th component [4]; μ, λ, D_{12} — coefficients of viscosity, thermal conductivity and diffusion of the mixture; R — universal gas constant; ρ_1, c_*, λ_1 — density, heat capacity, and coefficient of thermal conductivity of the solid body; $L(T_1)$ and $N(T_1)$ — given functions of temperature.

It is easy to show that in an approximation of the boundary-layer theory a term with baro diffusion drops out of the equations of diffusion (1.5) and the equation of energy (1.6).

When writing equations (1.5)-(1.7) we must also assume, in addition to the usual assumptions of boundary-layer theory in the stagnation region that all of the Maxwellian diffusion coefficients are equal (for a binary mixture this supposition is satisfied exactly), that the effect of thermal diffusion is small, and that the mixture is ideal. The equation of thermal conductivity (1.8) is written on the assumption that the thickness of the thermal boundary layer in the body is much smaller than the radius of curvature of the surface of the body in the vicinity of the stagnation point (line).

System of Equations (1.3)-(1.8) is solved for the following boundary-value conditions:

1) on the outer edge of the boundary layer

$$u = \beta x, \quad c_i = c_{i\infty} \quad (i = 1, \cdots, N), \quad h = h_\infty; \tag{1.9}$$

2. from the law of conservation of mass and energy on the surface of sublimation which is unknown before the solution of the problem we obtain [15] $\rho\ (D - v) = \rho_1 D$;

$$\rho c_i (D - v) + \rho D_{12} \frac{\partial c_i}{\partial y} = \rho_1 c_{i_0} D \quad (i = 1, \cdots, N'), \tag{1.10}$$

$$\rho c_j (D - v) + \rho D_{12} \frac{\partial c_j}{\partial y} = 0 \quad (j = 1, \cdots, N^*);$$

$$\rho (D - v) \left[l (T_0) + \mathrm{Le} \left(h - \sum_{i=1}^{N'} c_{i_0} h_i \right) \right] = \lambda_1 \frac{\partial T_1}{\partial y} - \rho D_{12} \mathrm{Le} \frac{\partial h}{\partial y}$$

or

$$\rho (D - v) \left[l (T_0) + \mathrm{Le} \left(h^* - \sum_{i=1}^{N'} c_{i_0} h_i^* \right) \right] = \lambda_1 \frac{\partial T_1}{\partial y} - \rho D_{12} \mathrm{Le} \frac{\partial h^*}{\partial y};$$

3) at infinity, in the solid body

$$T_1 = T_{-\infty}, \tag{1.11}$$

where D is the displacement velocity, normal to the surface, of the sublimation front relative to the body and $l(T_0)$ is the latent heat of sublimation which is dependent, generally speaking, on the surface of sublimation T_0.

Since hereafter we will trace the stationary regime of sublimation, we will not put forth the initial conditions. The formulated problem is a parabolic system of nonlinear equations (1.3)-(1.6), (1.8) of the 2N + 7th order with 2N + 5 conditions (1.9)-(1.11) which contain, in addition, the two unknown quantities T_0 and D. In order to complete the problem it is necessary to introduce four additional relationships which characterize the kinetics of sublimation.

For a sufficiently dense viscous mixture we can, of course, assume an equality of tangential velocities at the sublimation front, i.e.

$$u = 0. \tag{1.12}$$

It is known [6] that during weak vaporization, i.e., when convection heat transfer from the vaporizing surface is significantly lower than molecular heat transfer, the temperature jump is of the order of λ/a (the ratio of the length of free path to the characteristic dimension of the problem), i.e., $T = T_1 = T_0$. When convection heat transfer is significantly greater than molecular heat transfer, the temperature jump and deviation of vapor partial pressure p_0 form equilibrium $p(0)$ at the sublimation front will be of the order of v/c (the ratio of the vapor flow velocity to the average velocity of the thermal motion of the molecules). If it is assumed that during vaporization the thickness of the boundary layer remains small with respect to the characteristic linear dimension, then $v \sim \sqrt{\beta\nu}$ ($\beta \sim u_\infty/d$), where u_∞ is the velocity of an undisturbed flow, d is the radius of

curvature of blunting, and $\underline{v}$ is the kinematic coefficient of viscosity. Then $v/c \ll 1$, ($c \sim 10^2$ to 10^3 m/sec) and in this case we can assume

$$T = T_1 = T_0 \tag{1.13}$$

and

$$p_0 = p(0) = p_1 + \ldots + p_{N'} = p_* \Psi\left(\frac{T_0}{T_*}\right),$$

where the last relationship is the equation of the vapor tension curve; p_* and T_* are some values of pressure and temperature on this curve; $p_i (i = 1, \ldots, N')$ is the partial pressure of the i-th vapor component. If mass concentrations are introduced then the equation of the vapor tension curve may be written in the form

$$\frac{p_*}{p} \Psi\left(\frac{T_0}{T_*}\right) \sum_{k=1}^{N} \frac{c_k}{M_k} = \sum_{k=1}^{N'} \frac{c_k}{M_k}. \tag{1.14}$$

For a binary system $\underline{i}$, $\underline{j}$, with use of the van der Waals formula [4] for the vapor tension curve, Eq. (1.14) is rewritten as

$$c_{i0} = \left\{1 + \frac{M_j}{M_i}\left[\frac{p}{p_*} \exp \frac{l}{R}\left(\frac{1}{T_0} - \frac{1}{T_*}\right) - 1\right]\right\}^{-1}. \tag{1.15}$$

In order to clarify the influence of nonequilibrium vaporization, i.e., when $p_0 \neq p(0)$, in place of (1.15) we will use the fundamental formula of Knudsen-Langmuir for the mass velocity of vaporization:

$$\rho_1 D = f \sqrt{\frac{M_i}{2\pi R T_0}} [p_0 - p(0)], \quad p(0) = p_* \Psi\left(\frac{T_0}{T_*}\right), \tag{1.16}$$

where $\underline{f}$ is the coefficient of accomodation expressing probability that a molecule of vapor striking the vaporization surface will adhere to it. Formula (1.16) was derived on the assumption that the flow of molecules striking vaporizing surface has a maximum distribution for a quiescent gas with a temperature T_0.

As has been demonstrated [6], for a mass vaporization rate (if

the temperature jump is neglected) when using the distribution function in an approximation of "13 moments" for the calculation of the molecular flux impenging on the surface, a formula analogous to (1.16) is obtained but in place of f, we must use 2f. Since the law according to which dispersion of the molecules of vapor takes place is unknown, we will use the relationship in (1.16). We note that certain authors [7] maintain that f ~ 1. Introducing the molar vapor concentration c* into (1.16) we obtain for a binary gas-vapor mixture

$$\rho_1 D = f p(0)(R-1)\sqrt{\frac{M_i}{2\pi R T_0}}, \quad R = \frac{p_0}{p(0)} = \frac{c_0^*}{c^*(0)} \tag{1.17}$$

Henceforth we will consider nonequilibrium vaporization for the case of a binary gas-vapor mixture and use Eq. (1.17). Thus, additional conditions (1.12), (1.13), (1.14) or, for the case of nonequilibrium vaporization, (1.17) close the problem. The effect of dissociation and radiation are not taken into account when formulating the problem. It is known [8] that for flow stagnation in the vicinity of the stagnation point, the influence of equilibrium dissociation and radiation on the magnitude of the thermal flux is small if the temperature of the wall is lower than the temperature of dissociation (1600-2200°K). But at such wall temperatures sublimation for most materials may attain a significant value.

2. Coefficient of Molecular Transfer and Prandt and Lewis Numbers

Generally speaking, accurate expressions for the coefficients of transfer (coefficients of viscosity, thermal conductivity, diffusion, and thermal diffusion) for gases and gas mixtures may be obtained by calculations according to the law of statistical mechanics and the kinetic theory of gases when the necessary data concerning inter-

molecular forces is available. However, these formulas are too complex for practical calculations.

Recently approximation formulas have been derived from exact formulas for the coefficients of viscosity [9], and thermal conductivity [9,10] of multicomponent gas mixtures, which over a wide range of temperatures [11] give values differing little from the exact formulas of the kinetic theory of gases. Comparison of these formulas with experiments [9,10] also gives satisfactory agreement. These formulas, in essence, are corrected first approximation obtained from a rigorous kinetic theory of gases and are very convenient for numerical calculations. In order to calculate the coefficient of viscosity of a mixture it is necessary to know only the molecular weight and coefficient of viscosity of the individual components of the mixture at the same temperature at which the gas mixture is found. In order to calculate the coefficient of thermal conductivity, we need to know the molecular weight, the coefficients of thermal conductivity of the pure components, and also either the viscosity of the individual components or the specific heat of the components at the same temperature at which the gas mixture is found.

1. Mixture viscosity. The approximation formula for the coefficient of viscosity of a mixture obtained from the elementary kinetic theory of gases [12] and verified from considerations of the strict kinetic theory of gases [19] has the form

$$\mu = \sum_{i=1}^{N} \mu_i \left(1 + \sum_{j \neq i}^{N} G_{ij} \frac{c_j}{c_i}\right)^{-1}, \quad G_{ij} = \frac{1}{2\sqrt{2}} \left(1 + \frac{M_i}{M_j}\right)^{-1/2} (1 + \varkappa_{ji})^2 \frac{M_i}{M_j}; \tag{2.1}$$

$$\frac{[\mu_i]_3}{[\mu_j]_3} = \left(\frac{M_i}{M_j}\right)^{1/2} \varkappa_{ji}^2, \quad \varkappa_{ji}^2 = \frac{(r_0)_j^2 Y(2,2;\tau_j) I(\tau_i)}{(r_0)_i^2 Y(2,2;\tau_i) I(\tau_j)}, \quad \tau_i = \frac{kT}{\varepsilon_i}, \tag{2.2}$$

where μ_i — coefficient of viscosity of the individual components; $(r_0)_i = (r_0)_{ii}$ — distance between molecules at which the energy of interaction is zero; $\varepsilon_i = \varepsilon_{ii}$ — absolute value of the maximum energy of attraction; k — Boltzmann constant; $Y(2;\ 2;\ \tau_i)$ — average reduced collision cross-section which depends on the temperature and potential energy of molecular interaction $I(t_i)$ — correcting multiplier, dependent on τ_i, which, for a wide temperature range, differs from unity by less than + 0.08 [13].

When $\tau_i > 3$ the function $Y\ (2;\ 2;\ \tau_i)$ is slightly dependent on temperature. Therefore the magnitude of $\varkappa^2_{ji}$, within the limits of accuracy of formula (2.1) may be considered constant for the given vapor. Actually, for example, for vapors of carbon dioxide CO_2 (i) and air (j) we have [13] [$(r_0)_i = 3.996$A, $(r_0)_j = 3.617$, $\varepsilon_i/k = 190^\circ$K and $\varepsilon_j/k = 97^\circ$K]:

$$\frac{Y(2;2;\tau_j)\,I(\tau_i)}{Y(2;2;\tau_i)\,I(\tau_j)} = 0{,}855\ (500^\circ\text{K}),\ = 0{,}892\ (1000^\circ\text{K}),\ = 0{,}904\ (2000^\circ\text{K}),$$
$$= 0{,}907\ (4000^\circ\text{K}).$$

Whence $\varkappa^2_{ji} \approx 0.729$ for a mixture of CO_2 and air in this temperature range (disregarding dissociation).

Formula (2.1) may then be written as the product of the function which depends only on temperature times the function which depends only on composition:

$$\mu = \mu_j\,(T)\,\varphi_\mu\,(c_i), \tag{2.3}$$

where μ_j (T) is the coefficient of viscosity of some component of the mixture,

$$\varphi_\mu(c) = \sum_{i=1}^{N} \frac{\mu_i}{\mu_j}\left(1 + \sum_{j\neq i}^{N} G_{ij}\frac{c_j}{c_i}\right)^{-1} = \sum_{i=1}^{N}\left(\frac{M_i}{M_j}\right)^{1/2}\varkappa^2_{ji}\left(1 + \sum_{j\neq i}^{N} G_{ji}\frac{c_j}{c_i}\right)^{-1}. \tag{2.4}$$

2. For the coefficient of thermal conductivity of the mixture we will use an approximation formula derived from the expressions of the rigorous kinetic theory of gases [10]. For monoatomic gases or for gas mixtures with frozen internal degrees of freedom this formula has the form

$$\lambda^0 = \sum_{i=1}^{N} \lambda_i^0 \Big(1 + \sum_{j \neq i}^{N} G'_{ij} \frac{c_j}{c_i}\Big)^{-1},$$

$$G'_{ij} = \frac{1.065}{2\sqrt{2}} \Big(1 + \frac{M_i}{M_j}\Big)^{-1/2} \Big[1 + \Big(\frac{\lambda_i^0}{\lambda_j^0}\Big)^{1/2} \Big(\frac{M_i}{M_j}\Big)^{1/4}\Big] \frac{M_i}{M_j}. \tag{2.5}$$

A comparison of this formula with the exact formulas of the kinetic theory of gases and with experiment yields satisfactory agreement [10].

For a mixture of polyatomic gases the formula takes the form

$$\lambda = \sum_{i=1}^{N} \lambda_i \Big(1 + \sum_{j \neq i}^{N} G'_{ij} \frac{c_j}{c_i}\Big)^{-1}. \tag{2.6}$$

When $\lambda_1 = \lambda_1^0$ Formula (2.6) changes into (2.5). Since

$$\frac{\lambda_i^0}{\lambda_j^0} = \frac{M_j}{M_i} \frac{\mu_i}{\mu_j} = \Big(\frac{M_j}{M_i}\Big)^{1/2} \varkappa_{ji}^2,$$

then from (2.5) and (2.6) it follows that

$$G'_{ij} = 1{,}065 G_{ij}. \tag{2.7}$$

The ratio λ_i/λ_j for polyatomic molecules with consideration of the internal degrees of freedom is given by the expression

$$\frac{\lambda_i}{\lambda_j} = \frac{M_j}{M_i} \frac{\mu_i}{\mu_j} E_{ij} = \Big(\frac{M_j}{M_i}\Big)^{1/2} \varkappa_{ji}^2 E_{ij}, \quad E_{ij} = \frac{E_i}{E_j}, \quad E_i = 0{,}115 + 0{,}354 \frac{c_{p_i}}{R},$$

where E_i is a correction multiplier recently refined by Hirschfelder [14] for gases with polyatomic chemically reactive molecules and molecules with excited electron levels.

Formula (2.6) may also be written in the form

$$\lambda = \lambda_j (T) \varphi_\lambda (c_i), \tag{2.8}$$

where $\lambda_i(T)$ is the coefficient of thermal conductivity of some component of the mixture

$$q_\lambda(c) = \sum_{i=1}^{N} \frac{\lambda_i}{\lambda_j}\left(1 + \sum_{j \neq i}^{N} G'_{ij}\frac{c_j}{c_i}\right)^{-1} = \sum_{i=1}^{N}\left(\frac{M_j}{M_i}\right)^{1/2} \varkappa^2_{ji} E_{ij}\left(1 + \sum_{j \neq i}^{N} G_{ij}\frac{c_j}{c_i}\right)^{-1}. \qquad (2.9)$$

3. The <u>diffusion coefficients</u> in a multicomponent mixture are expressed by the diffusion coefficients of a binary mixture of all possible pairs of components of the mixture, by the molar concentrations, and by molecular weights of the components proportional to the corresponding binary diffusion coefficients [13].

4. Schmidt number $Sc = \mu/\rho D_{12}$. By virtue of the structure of Formula (2.3) it suffices to calculate the complex $\mu_j/\rho D_{1j}$; then it is possible to calculate $\mu/\rho D_{1j}$ also. We have

$$Sc_j = \frac{[\mu_j]_2}{\rho\,[D_{ij}]_2} = \frac{5}{3\sqrt{2}} M_j\left(\sum_{i=1}^{N}\frac{c_i}{M_i}\right)\left(1 + \frac{M_j}{M_i}\right)^{-1/2} \varkappa^2_{ij,j}, \qquad (2.10)$$

where

$$\varkappa^2_{ij,j} = \frac{(r_0)^2_{ij}}{(r_0)^2_j}\frac{Y(1,1;\tau_{ij})}{Y(2,2;\tau_j)} J(\tau_j)(1 - \Delta_{ij}),$$

$$(r_0)_{ij} = \frac{1}{2}[(r_0)_i + (r_0)_j], \quad \tau_{ij} = \frac{kT}{\varepsilon_{ij}}, \quad \varepsilon_{ij} = \sqrt{\varepsilon_i \varepsilon_j}.$$

The product $J(\tau_j)$ $(1 - \Delta_{ij})$ is weakly dependent on temperature and composition of the mixture [13] and its value varies between 0.97 and 1 for a wide range of temperature and composition changes [13]. The ratio $Y(1.1;\ \tau_{ij})/Y\ (2.2;\tau_j)$ is also weakly dependent on temperature for sufficiently large values of τ_i and τ_j $(r > 3)$. For example, for a mixture of molecular hydrogen and air this ratio is equal to 0.830 for T = 600°K and 0.819 for T = 2400°K etc. Therefore, the value of $\varkappa^2_{ij,j}$ within the limits of accuracy of Formula (2.3) may be considered constant and characteristic for the given pair ij. For example, the Schmidt number for a binary mixture will be

$$\mathrm{Sc} = \frac{5}{3\sqrt{2}}\left[1 + c_i\left(\frac{M_j}{M_i} - 1\right)\right]\left(1 + \frac{M_j}{M_i}\right)^{-1/2} \varkappa^*_{ij,j}\left[\left(\frac{M_i}{M_j}\right)^{1/2} \varkappa^*_{ji}\left(1 + G_{ij}\frac{1-c_i}{c_i}\right)^{-1} + \right.$$
$$\left. + \left(1 + G_{ji}\frac{c_i}{1-c_i}\right)^{-1}\right], \tag{2.11}$$

i.e., it will not depend on temperature buy only on composition. The maximum variation of Sc with respect to $\underline{c}$ is equal to

$$\frac{\mathrm{Sc}\,(c_i = 0)}{\mathrm{Sc}\,(c_i = 1)} = \left(\frac{M_i}{M_j}\right)^{1/2} \varkappa^*_{ij},$$

for example, for vapors of hydrogen and air ~ 0.9.

5. Lewis number, $\mathrm{Le} = \lambda/\rho c_p D_{ij}$. By virtue of the structure of Formula (2.8) it suffices to calculate the complex $\lambda_j/\rho c_p D_{ij}$; then it is possible to calculate Le also. We have

$$\mathrm{Le}_j = \frac{\lambda_j}{\rho c_p D_{ij}} = \frac{15}{4}\frac{R}{M_j}\frac{E_j}{c_p}\mathrm{Sc}_j. \tag{2.12}$$

The maximum variation of Le for a binary mixture is equal to

$$\frac{\mathrm{Le}\,(c_i = 0)}{\mathrm{Le}\,(c_i = 1)} = \frac{c_{p_i}}{c_{p_j}}\frac{E_j}{E_i}\left(\frac{M_i}{M_j}\right)^{1/2} \varkappa^*_{ij} = \frac{c^m_{p_i}}{c^m_{p_j}}\frac{E_j}{E_i}\left(\frac{M_i}{M_j}\right)^{1/2} \varkappa^*_{ij}, \tag{2.13}$$

where $c^m_{pi} = c_{pi} M_i$ is the molar heat capacity of the i-th component. Since the first two factors in (2.13) are close to 1, the variation of Le with a change of composition depends mainly on the ratio of the molecular weights of the components.

6. Prandtl number, $\sigma = \mu c_p/\lambda = \mathrm{Sc}/\mathrm{Le}$.

3. Formulation of the Boundary Value Problem for a System of Ordinary Differential Equations

Proceeding from the form of the boundary-value conditions (1.9) and (1.11) and taking into account that the solution yielding a stationary regime of sublimation must be a type of uniformly propagation wave, we will search for a solution of Eqs. (1.3)-(1.8) with conditions (1.9)-(1.17) in the form:

for a gas-vapor mixture

$$
\begin{gathered}
u = \beta x \varphi'(\eta), \quad v = -\frac{\rho_\infty}{\rho}\sqrt{\beta \nu_\infty}\left[\sqrt{\frac{\rho_0\mu_0}{\rho_\infty\mu_\infty K}}\, n\varphi(\eta) - D^* \frac{\rho}{\rho_\infty}\right], \quad n = 1, 2, \\
h^* = h_0^* + (h_\infty^* - h_0^*)\, z(\eta), \quad c_i = c_{i0} + (c_{i\infty} - c_{i0})\, g_i(\eta), \quad i = 1, \ldots, N, \\
\int_0^\eta \frac{\rho_\infty}{\rho} d\eta = \left(\frac{\beta \rho_\infty \mu_\infty K}{\nu_\infty \rho_0 \mu_0}\right)^{1/2} (y - Dt), \quad D^* = \frac{D}{\sqrt{\beta \nu_\infty}}, \quad \nu_\infty = \frac{\mu_\infty}{\rho_\infty};
\end{gathered}
\tag{3.1}
$$

for a solid body

$$
T_1 = T_{-\infty}\, \theta_1(\eta_1), \; \eta_1 = \left(\frac{\beta}{\nu_\infty}\right)^{1/2} (y - Dt), \tag{3.2}
$$

where K is an arbitrary constant, the subscript 0 refers to unknown values of the parameters at the sublimation front and the subscript ∞ refers to conditions outside the boundary layer.

The law of motion of the sublimation front for a stationary regime of vaporization will be

$$
y_0 = Dt, \tag{3.3}
$$

where D is not known before solution of the problem of the displacement velocity of the sublimation front.

Substituting Eqs. (3.1)-(3.3) into Eqs. (1.3)-(1.8) and conditions (1.9)-(1.17) we obtain for determination of the four functions φ, z, $g_1 = g(i = 1, \ldots, N)$, θ_1 and the parameters D^*, $\theta_0 = T_0/T_\infty$, $c_{i0}(i = 1, \ldots, N)$ the following nonlinear boundary-value problem:

$$
\left.
\begin{aligned}
& K(\overline{\rho\mu}\varphi'')' + n\varphi\varphi'' = \varphi'^2 - \rho_\infty \rho^{-1}, \\
& \frac{\rho_\infty}{\rho} = \frac{M_\infty}{M}\frac{c_{p\infty}}{c_p}\left[\frac{h_0^*}{h_\infty^*} + \left(1 - \frac{h_0^*}{h_\infty^*}\right) z\right], \quad M^{-1} = \sum_{i=1}^{N} \frac{c_i}{M_i}, \\
& K(\overline{\rho\mu}\, \mathrm{Sc}^{-1} g')' + n\varphi g' = 0, \\
& K\left\{\overline{\rho\mu}\sigma^{-1}\left[z' + (\mathrm{Le}^{-1} - 1)\frac{\bar{c}_{p\infty} - 1}{\bar{c}_{p\infty} - \theta_0}\theta g'\right]\right\}' + n\varphi z' = 0,
\end{aligned}
\right\}
\tag{3.4}
$$

$$\left.\begin{aligned}&[L(\theta_1)\,\theta_1']' + D^*\sigma_1 N(\theta_1)\,\theta_1' = 0, \quad \frac{d\theta_1}{d\eta_1} = D^*\sigma_1\frac{Q(\theta_1)-Q(1)}{L(\theta_1)},\\ &Q(\theta_1) = \int N(\theta_1)\,d\theta_1,\end{aligned}\right\} \tag{3.5}$$

$$\varphi'(\infty) = 1, \quad g(\infty) = z(\infty) = 1, \tag{3.6}$$

$$\left.\begin{aligned}&\varphi'(0)=0, \quad g(0)=z(0)=0, \quad r_1\sqrt{\frac{\rho_1\mu_0}{\rho_\infty\mu_\infty K}}\,n\varphi(0) = D^*, \quad \theta_1(0) = \theta_{10},\\ &\frac{c_{i0}-c_{i\infty}}{c_{i\infty}-c_{i0}}\,n\varphi(0) + \frac{K}{Sc_0}g'(0) = 0 \qquad (i = 1,\dots,N'),\\ &\frac{c_{j0}}{c_{j\infty}-c_{j0}}\,n\varphi(0) + \frac{K}{Sc_0}g'(0) = 0 \qquad (j = N'+1,\dots,N),\\ &n\varphi(0)\Big[\mathcal{L}(T_0) + \mathrm{Le}_0\Big(h_0^* - \sum_{i=1}^{N'} c_{i0}h_{i0}^*\Big)\Big] + c_0^{-1}K(h_\infty^* - h_0^*)\,z'(0) = 0,\end{aligned}\right\} \tag{3.7}$$

$$\left.\begin{aligned}&\frac{p_*}{p_\infty}\psi\Big(\frac{T_0}{T_*}\Big)\sum_{k=1}^{N}\frac{c_{k0}}{M_k} = \sum_{k=1}^{N'}\frac{c_{k0}}{M_k} \quad \text{or}\\ &c_{i0} = \Big\{1 + \frac{M_j}{M_i}\Big[\frac{p_\infty}{p_*}\exp\frac{l}{R}\Big(\frac{1}{T_0}-\frac{1}{T_*}\Big) - 1\Big]\Big\}^{-1}, \quad p = p_\infty - \frac{\rho_\infty\beta^2x^2}{2}, \quad p \approx p_\infty.\end{aligned}\right\} \tag{3.8}$$

$$\theta_1(-\infty) = 1. \tag{3.9}$$

For nonequilibrium vaporization in the case of a binary gas-vapor mixture with the use of (1.17) in place of (3.8) we obtain

$$\frac{D^*}{r_1} = \sqrt{\frac{\rho_1\mu_0}{\rho_\infty\mu_\infty K}}\,n\varphi(0) = \frac{f\,p(0)}{\rho_\infty\sqrt{\beta v_\infty}}\sqrt{\frac{M_i}{2\pi RT_0}}\,(R-1); \tag{3.10}$$

here the following designations are introduced:

$$\begin{gathered}\mathcal{L}(T_0) = l(T_0) + \bar{\rho}_1\rho_{10}^{-1}\bar{c}_*T_{-\infty}[Q(\theta_{10}) - Q(1)],\\ \overline{\rho\mu} = \frac{\rho\mu}{\rho_0\mu_0}, \quad \theta = \frac{T}{T_\infty}, \quad \theta_0 = \frac{T_0}{T_\infty}, \quad \theta_{10} = \frac{T_0}{T_{-\infty}}, \quad \bar{c}_{p\infty} = \frac{c_{p\infty}}{c_{p0}},\\ \sigma_1 = \frac{v_\infty}{\bar{\lambda}_1}\bar{\rho}_1\bar{c}_*, \quad r_1 = \frac{\rho_\infty}{\rho_{10}}, \quad \rho_{10} = \rho_1(T_0).\end{gathered} \tag{3.11}$$

The system (3.4)-(3.5) is a system of nonlinear ordinary differential equations of the 9th order with N + 2 unknown parameters c_{i0} ($i = 1, \dots, N$), D^* and T_0, which enter into conditions (3.7), (3.8) or (3.10).

Relationships (3.6)-(3.10) give exactly N + 11 conditions. Consequently it may be expected that the solution of problem (3.4)-(3.10) is determined uniquely.

After solution of the problem, the mass velocity of sublimation is determined by the formula

$$\rho_{10}D = \rho_{10}D^{*}\sqrt{\beta\nu_{\infty}} = n\varphi(0)\rho_{\infty}\sqrt{\frac{\rho_0\mu_0\beta\nu_{\infty}}{\mu_{\infty}\mu_{\infty}K}} = \\ = -\frac{\rho_{\infty}}{Sc_0}g'(0)\frac{c_{i\infty}-c_{i0}}{c_{i0}-c_{is}}\sqrt{\frac{\rho_0\mu_0K}{\rho_{\infty}\mu_{\infty}}\beta\nu_{\infty}}. \tag{3.12}$$

The temperature profile in the body is found after this in quadratures [15]:

$$D^{*}\sigma_1\eta_1 = -\int_{\theta_{w0}}^{\theta_1}\frac{L(t)\,dt}{Q(t)-Q(1)}. \tag{3.13}$$

From (3.10) it is possible, after solution of the problem, to calculate the deviation of the vapor partial pressure p_0 over the sublimation surface form the saturation pressure $p(0)$ at a temperature T_0:

$$R = \frac{p_0}{p(0)} = 1 + n\varphi(0)\,\varepsilon, \quad \varepsilon = \frac{\rho_{\infty}\sqrt{\beta\nu_{\infty}}}{fp(0)}\sqrt{\frac{2\pi RT_0}{M_i}\cdot\frac{\rho_0\mu_0}{\rho_{\infty}\mu_{\infty}K}}, \tag{3.14}$$

We will evaluate the order of the parameter ε. For $T_{\infty} \sim 1000^{\circ}K$, $\nu_{\infty} \sim 1$ cm^2/sec, $\rho_{\infty} \sim 10^{-4}$ gr/cm^3, $\beta \sim 10^3$sec^{-1} (with $u_{\infty} \sim 10^3$m/sec, $d \sim 1$ m), $T_0 \sim 1000^{\circ}K$ and $p(0) \sim 100$ mm Hg we obtain $\varepsilon \sim 10^{-5/2} f^{-1}$.

Since $|\varphi(0)| < 1$ for moderate vaporization, then $R = 1 + 10^{-3}f^{-1}$. Consequently for $f > 0.1$ it is possible to obtain with a high accuracy the equilibrium vaporization and to use Eq. (3.8). For $f < 0.01$ it is necessary to calculate nonequilibrium vaporization and to use Eq. (3.10). In this case the coefficient of accomodation must be known exactly.

4. The Case of Le = 1 (σ = Sc)

This case is interesting in that a number of conclusions may be reached before complete solution of problem (3.4)-(3.10). Since for many mixtures the Lewis number is close to unity (2), these conclusions will be of practical interest.

Actually, if sublimation proceeds according to diffusion kinetics (condition 3.8) and Le $\approx$ 1, then $z(\eta) \equiv g(\eta)$ and determination of the condition and temperature at the sublimation surface leads to a solution of the final system of N equations

$$\frac{\mathcal{L}(T_0) + h_0^* - \sum_{i=1}^{N'} c_{is} h_{i0}^*}{h_\infty^* - h_0^*} = \frac{c_{i0} - c_{is}}{c_{i\infty} - c_{i0}} \quad (i = 1, \ldots, N-1),$$
$$c_{is} = 0 \; (i > N' + 1), \quad c_{N0} = 1 - \sum_{k=1}^{N-1} c_{0k} \tag{4.1}$$

together with the equation of the vapor tension curve (3.8) for N + 1 unknown c_{i0} (i = 1, ..., N) and T_0.

For a binary mixture ($c_i = c$, $c_j = 1 - c$, $c_s = 1$) the concentrations of vapor and temperature at the sublimation surface are determined from a system of two equations

$$\frac{(h_\infty^* - h_{j0}^* + c_\infty (h_{j0}^* - h_{i0}^*)}{\mathcal{L}(T_0)} = \frac{c_0 - c_\infty}{1 - c_0}, \quad c_0 = \left\{1 + \frac{M_j}{M_i}\left[\frac{p_{i0}}{p_{*0}} \exp \frac{l}{R}\left(\frac{1}{T_0} - \frac{1}{T_*}\right) - 1\right]\right\}^{-1} \tag{4.2}$$

It is interesting to note that these results do not depend on the character of the dependence of transfer coefficients on temperature and composition and are associated only with the assumption that Le = = 1.

It follows from (4.1) and (4.2) that "boiling" ($\Sigma_{i=1}^{N'} C_{i0} = 1$) on the surface of the body in the case of equilibrium vaporization is attained only in the limiting case of an infinitely large thermal flux

from the gas. From (3.7) and (3.10) it follows that this fact does not depend on the assumption that Le = 1 and is associated only with the assumption that sublimation proceeds according to diffusion kinetics.

If sublimation is nonequilibrium ($f \ll 1$) and the coefficient of accomodation is known, then when "boiling" is attained, a solution to the problem (3.4)-(3.10) may be obtained in final form.

Actually for "boiling" $p(0) = p_{00}$ is the stagnation pressure, from which, if the pressure at the stagnation point p_{00} is taken as the characteristic pressure p_*, we will obtain $T_0 = T_* = T_{00}$, where T_{00} is the temperature on the vapor tension curve which corresponds to the pressure of the vapors p_{00}. From the first equation in (4.2), substituting in the left hand part $T_0 = T_{00}$, we find the concentration of vapors c_0 on the sublimation surface. From (3.10) and the first equation in (4.2), assuming for simplicity that $c_\infty = 0$, we obtain the rate of sublimation in the final form:

$$D = -\frac{f r_1 p_{00}}{\rho_\infty}\sqrt{\frac{M_i}{2\pi R T_\infty}}\left[1 + \frac{c_{pj}(T_\infty - T_{00})}{l(T_\infty) + \bar{\rho}_1 \rho_{10}^{-1} \bar{c}_* T_{-\infty}[Q(0_{10}) - Q(1)]}\frac{M_j}{M_i}\right]^{-1}, \qquad (4.3)$$

where c_{pj} is the specific heat for a constant pressure of the incident flow.

The mass velocity of sublimation will be

$$\rho_{10} D = -f p_{00}\sqrt{\frac{M_i}{2\pi R T_\infty}}\left[1 + \frac{c_{pj}(T_\infty - T_{00})}{l(T_\infty) + \bar{\rho}_1 \rho_{10}^{-1} \bar{c}_* T_{-\infty}[Q(0_{10}) - Q(1)]}\frac{M_j}{M_i}\right]^{-1}. \qquad (4.4)$$

The temperature profile in the body is found from Formula (3.13).

5. Numerical Solution of the Problem for $\sigma = 0.7$; 1 and Sc = 1

Solution of Problem (3.4)-(3.10) in the general form is associated with numerous calculations owing to numerous defining parameters;

$$n,\ \sigma,\ \mathrm{Sc},\ \frac{c_{p_i}}{c_{pN}},\ \frac{M_i}{M_N}\ (i=1,\ldots,N-1),\quad c_{is}\ (i=1,\ldots,N),\ r_1,$$
$$\frac{\bar{\rho}_1}{\rho_{10}},\ \frac{\bar{c}_* T_{-\infty}}{h^*_\infty},\quad \frac{n\rho_\infty\sqrt{\beta v_\infty}}{p(0)}\sqrt{\frac{2\pi RT_\infty}{M_1}} \tag{5.1}$$

and dimensionless functions $\overline{\rho\mu} = \overline{\rho\mu}(g,\ z)$, $Q(0;)$, $\psi(T_0/T)$, $l(T_0)/h^*_\infty$. We therefore made certain simplifications which reduce the number of these parameters.

First we will assume that

$$\left|\frac{\bar{c}_{p\infty}-1}{\bar{c}_{p\infty}-\theta_0}\left(\frac{1}{\mathrm{Le}}-1\right)\right|\theta = \left|\frac{c_{p\infty}-c_{p0}}{h^*_\infty-h^*_0}\left(\frac{1}{\mathrm{Le}}-1\right)\right|T \ll 1. \tag{5.2}$$

This corresponds to the case where Le $\approx$ 1 or where the components of the mixture have similar specific heats at a constant pressure ($c_{p\infty} \approx c_{r_0}$) or when sublimation is weak ($c_1 \ll 1$). Since for a large number of gaseous mixtures these assumptions are always fulfilled to a certain degree simultaneously, then for these mixtures assumption (5.2) will be valid.

Using Formula (2.3) for the viscosity of the mixture and Sutherland's formula for the viscosity of the j-th component, we obtain

$$\overline{\mu\rho} = \frac{\mu\rho}{\mu_0\rho_0} = \frac{\varphi_\mu(c_i)}{\varphi_\mu(c_{i0})}\frac{M}{M_0}\left(\frac{c_{p_0}}{c_p}\right)^{1/2}\frac{(z_0+s_{j0})\left[1+(z_0^{-1}-1)z(\eta)\right]^{1/2}}{s_{j0}+(c_{p_0}/c_p)\left[z_0+(1-z_0)z(\eta)\right]},$$
$$s_{j0} = \frac{S_j c_{p_0}}{T_\infty c_{p\infty}},\quad z_0 = \frac{h^*_0}{h^*_\infty}, \tag{5.3}$$

where Sj is the Sutherland constant and is equal to 107 for T = 97°K, 180 for T = 970°K and 823 for T = 4850°K [13]. During vaporization of most condensed media in air we have $c_{p_0}/c_p > 1$, $M/M_0 < 1$. The ratio $\varphi_\mu(c_i)/\varphi_\mu(c_{i0})$ is close to unity. Therefore, the product of the first three factors in (5.3) is close to unity and for certain mixtures is a slightly varying function of the composition (this function may be calculated on the basis of the formulas of (2). Therefore, we set this product equal to some average constant value and designate it by

$$K^{-1} = \frac{\varphi_\mu(\bar{c}_i)}{\varphi_\mu(c_{i0})} \frac{\overline{M}}{M_0} \left(\frac{c_{p_0}}{\bar{c}_p}\right)^{1/2}. \tag{5.4}$$

where the upper bar indicates the average value of the parameter between its value at the outer edge of the boundary layer and at the sublimation surface. Obviously the parameter K is unknown until solution of the problem. Taking into account also that $c_{po} \approx c_p$ we obtain

$$\overline{\mu\rho} = K^{-1}\Phi(z), \quad \Phi(z) = \frac{(z_{j0} + z_0)[1 + (z_0^{-1} - 1)z]^{1/2}}{z_{j0} + z_0 + (1 - z_0)z}. \tag{5.5}$$

and finally

$$\rho_\infty \rho^{-1} = z_0 + (1 - z_0)z. \tag{5.6}$$

We note that this approximation will effect mainly the value of $\varphi''(0) = a$. But since $g'(0)$ and $z'(0) \sim a^{1/3}$ it follows from an asymptotic solution (6) that the simplification (5.6) has little effect on the product of $g'(0)$ and $z'(0)$, through which the rate of sublimation and temperature at the sublimation front are determined from (3.7).

After these assumptions, System (3.4) is rewritten in the form

$$(\Phi\varphi'')' + n\varphi\varphi'' = \varphi'^2 - z_0 - (1 - z_0)z, \tag{5.7}$$

$$(\Phi\sigma^{-1}z')' + n\varphi z' = 0, \quad (\Phi\,\mathrm{Sc}^{-1}g')' + n\varphi g' = 0. \tag{5.8}$$

A formal solution of the last two equations in terms of the constants σ and Sc with regard to (3.6) and (3.7) will be

$$z(\eta) = \frac{\omega(\eta; \sigma, z_0, \alpha)}{\omega(\infty; \sigma, z_0, \alpha)}, \quad g(\eta) = \frac{\omega(\eta; \mathrm{Sc}, z_0, \alpha)}{\omega(\infty; \mathrm{Sc}, z_0, \alpha)}, \tag{5.9}$$

where

$$\omega(\eta; \tau, z_0, \alpha) = \int_0^\infty \Phi^{-1} \exp\left(-n\tau \int_0^\eta \varphi(\eta)\Phi^{-1} d\eta\right) d\eta, \qquad \alpha = \varphi(0).$$

Determining $z'(0)$ and $g'(0)$ from (5.9) we obtain from (3.7) N transcendental equations

$$-K^{-1}\sigma n a \omega(\infty; \sigma, z_0, \alpha) = \frac{h_\infty^* - h_0^*}{\mathscr{L}(T_0)}, \tag{5.10}$$

$$\frac{h_\infty^* - h_0^*}{\mathscr{L}(T_0)} = \frac{\sigma}{Sc}\frac{\omega(\infty; \sigma, z_0, \alpha)}{\omega(\infty; Sc, z_0, \alpha)}\frac{c_{i0} - c_{i\infty}}{c_{i0} - c_{i0}}, \qquad i = 1, \ldots, N-1. \tag{5.11}$$

If sublimation proceeds according to diffusion kinetics ($f > 0.1$), then, adding the first Eq. (3.88) to (5.10) and (5.11) we obtain a system of N + 1 equations for the determining of N + 1 unknowns: α, z_0, c_{i_0} ($i = 1, \ldots, N - 1$).

If sublimation is nonequilibrium, then, adding Eq. (3.10) to the system (5.10), (5.11) for N = 2 we obtain a system of three equations for the determination of three unknowns: α, z_0 and c_0. For an actual solution of systems (5.10), (5.11) and (3.8) it is necessary to compute the function $\omega(\infty; \tau; z_0, \alpha)$. For Sc = 1 and n = 1 the system (5.8) with the boundary conditions

$$\varphi(0) = \alpha < 0, \quad \varphi'(0) = z(0) = g(0) = 0, \quad \varphi'(\infty) = z(\infty) = g(\infty) = 1 \tag{5.12}$$

coincides with a boundary-value problem flow past an infinite cylinder in the vicinity of the stagnation line in the presence of slip and injection and was solved numerically by Beckwith [16] for $\alpha = 0$, -0.5, -1, for $z_0 = 0$, 0.5, 1 for two values of Prandtl numbers $\sigma = 0.7$ and 1. Numerical integration was performed for $\Phi(z) = 1$, $s_{j0} = 0.2$ and 0.02. In addition, for $\alpha = 0$ the results of numerical integration for the axisymmetrical case (n = 2) with $\sigma = 0.7$ and 1 are presented in this work for several values of the parameters z_0.

The results of Bechwith's calculations [16] for the case of a plane, supplemented by numerical integration of the system (5.8) for the axisymmetrical case where $\alpha = -0.5$, -1, $\sigma = 0.7$, 1, and Sc = 1 are

presented in Tables 1-4. Thus, for Sc = 1 and $\sigma = 0.7$, 1 System of Eqs. (5.10), (5.11), (3.8) or (5.10), (5.11), (3.10) together with Tables 1-4 solve the given problem in the plane and axisymmetrical case.

TABLE 1

n = 1 (plane case)

$\varphi(0) = \alpha$	z_0	$\varphi''(0) = \alpha$			
		$\sigma = 1$, $\Phi = 1$	$\sigma = 0.7$		
			$\Phi = 1$	$s_j = 0.2$	$s_j = 0.02$
0	0	0.6489	0.6071	—	—
	0.05	—	—	0.6891	0.4576
	0.5	0.9548	0.9362	0.9109	0.8894
	1	1.2326	1.2326	1.2326	1.2326
−0.5	0	0.3067	0.2968	—	—
	0.05	—	—	0.3858	0.1949
	0.5	0.6669	0.6610	0.6430	0.6276
	1	0.9692	0.9692	0.9692	0.9692
−1	0	0.0474	0.0705	—	—
	0.05	—	—	0.1572	—
	0.5	0.4519	0.4551	0.4458	0.4380
	1	0.7566	0.7566	0.7566	0.7566

TABLE 2

n = 1 (plane case)

$\varphi(0) = \alpha$	z_0	$z'(0)$, $\Phi = 1$		$\frac{z''(0)}{z'(0)}$, $\Phi = 1$	$z'(0)$	$\frac{z''(0)}{z'(0)}$	$z'(0)$	$\frac{z''(0)}{z'(0)}$	$\frac{z''(0)}{z'(0)}$, $\Phi = 1$
		$\sigma = 1$	$\sigma = 0.7$	$\sigma = 0.7$	$\sigma = 0.7$, $s_j = 0.2$		$\sigma = 0.7$, $s_j = 0.02$		$\sigma = 1$ (0.5)
0	0	0.5067	0.4362	1.140	—	—	—	—	1.127
	0.05	—	—	—	0.4661	1.147	0.2969	1.160	—
	0.5	0.5421	0.4696	1.146	0.4484	1.150	0.4317	1.153	1.139
	1	0.5705	0.4958	1.151	0.4958	1.151	0.4958	1.151	1.143
−0.5	0	0.2031	0.2103	0.9439	—	—	—	—	0.9818
	0.05	—	—	—	0.2452	0.9751	0.1025	0.8663	—
	0.5	0.2580	0.2600	0.9846	0.2430	0.9819	0.2295	0.9791	1.015
	1	0.2950	0.2934	1.005	0.2934	1.005	0.2934	1.005	1.028
−1	0	0.0211	0.0496	0.5565	—	—	—	—	0.841
	0.05	—	—	—	0.0819	0.6752	0.0432	—	—
	0.5	0.0823	0.1112	0.7419	0.1010	0.7326	0.0932	0.7221	0.892
	1	0.1168	0.1457	0.8016	0.1457	0.8016	0.1457	0.8016	0.918

TABLE 3

n = 2 (axisymmetrical case)

$\varphi(0) = a$	z_0	$\varphi''(0) = a$, $\Phi = 1$		z_0	$\varphi''(0) = a$, $\sigma = 0.7$	
		$\sigma = 1$	$\sigma = 0.7$		$s_j = 0.2$	$s_j = 0.02$
0	0	0.8217	0.7820	0.05	0.8073	—
	0.5	1.076	1.055	0.06	—	0.5820
	0.6	—	1.109	0.2	—	0.7483
	0.8	—	1.208	0.5	1.024	0.9956
	1	1.312	1.312	1	1.312	1.312
−0.5	0	0.248	0.240	—	—	—
	0.5	0.551	0.446	—	—	—
	0.8	0.704	0.702	—	—	—
	1	0.805	0.805	—	—	—
−1	0	—	—	—	—	—
	0.5	0.281	0.285	—	—	—
	1	0.491	0.491	—	—	—

TABLE 4

n = 2 (axisymmetrical case)

$\varphi(0) = a$	z_0	$z'(0)$, $\Phi = 1$		$\frac{z''(0)}{z'(0)}$, $\Phi = 1$	z_0	$z'(0)$, $\sigma = 0.7$	$\frac{z''(0)}{z'(0)}$, $\sigma = 0.7$	
		$\sigma = 1$	$\sigma = 0.7$	$\sigma = 0.7$		$s_j = 0.2$	$s_j = 0.2$	$s_j = 0.02$
0	0	0.6988	0.5995	1.137	0.05	0.6445	—	—
	0.5	0.7332	0.6360	1.145	0.06	—	—	—
	0.6	—	0.6485	—	0.2	—	—	—
	0.8	0.7450	0.6550	—	0.5	0.6987	1.146	1.149
	1	0.7621	0.6620	1.153	1	0.7621	—	—
−0.5	0	0.195	0.229	—	—	—	—	—
	0.5	0.226	0.252	—	—	—	—	—
	0.8	—	—	—	—	—	—	—
	1	0.252	0.276	—	—	—	—	—
−1	0	0.023	—	—	—	—	—	—
	0.5	0.053	—	—	—	—	—	—
	1	0.098	—	—	—	—	—	—

6. Solution of the Problem for Any Prandtl and Schmidt Numbers

In order to obtain a solution to the problem for any values of σ and Sc we will evaluate the integrals which enter into system (5.10), (5.11) by the asymptotic method. Assuming that the parameter $\tau = \sigma$ (or Sc) is sufficiently large and that parameter α is sufficiently small ($a \sim 1/\tau$) we will have

$$\omega(\infty;\tau,z_0,a)=\int_0^\infty \exp\left(-n\tau\int_0^\eta \varphi(\eta)\,d\eta\right)d\eta=\int_0^\infty \exp\left(-n\tau\alpha\eta-n\tau\xi(\eta)\right)d\eta, \tag{6.1}$$

where

$$\xi(\eta)=\int_0^\eta \varphi(\eta)\,d\eta-\alpha\eta=\eta^3\sum_{n=0}^{\infty}c_n\eta^n,$$

$$c_0=\frac{a}{6},\quad c_1=-\frac{z_0+n\alpha a}{4!},\quad c_2=\frac{n_\alpha(z_0+n\alpha a)-(1-z_0)z_0'}{5!},$$

$$c_3=\frac{(2-n)a^2-n^2\alpha^2(z_0+n\alpha a)+n\alpha(1+\sigma)(1-z_0)z_0'}{6!}, \tag{6.2}$$

$$c_4=\frac{n\alpha(5n-8)a^2+n[n^2\alpha^4+z_0(4n-6)]+z_0n^2\alpha^3-n^2\alpha^2(1+\sigma+\sigma^2)(1-z_0)z_0'}{7!},\ \ldots,$$

$$a=\varphi''(0),\ z_0'=z'(0).$$

Changing under the integral in (6.1) to the variable of integration $\bar{z}$ we obtain [17]

$$\omega(\infty;\tau,z_0,a)=\int_0^\infty e^{-n\tau\alpha\eta}\frac{d\eta}{d\bar{z}}e^{-n\tau\bar{z}}d\bar{z}\sim\sum_{m=0}^{\infty}d_m\,\Gamma\left(\frac{m+1}{3}\right)(n\tau)^{-1/3(m+1)}=$$

$$=\sum_{m=0}^{\infty}\frac{1}{3}\bar{d}_m\left(\frac{6}{an\tau}\right)^{1/3(m+1)}\Gamma\left(\frac{m+1}{3}\right), \tag{6.3}$$

where

$$d_m=\frac{1}{6\pi i}\oint \exp(-n\tau\alpha\eta)\,\bar{z}^{-1/3(m+1)}d\eta=\frac{1}{3}\left(\frac{6}{a}\right)^{1/3(m+1)}\bar{d}_m.$$

From which, using (6.2), we obtain

$$\bar{d}_0=1,\quad \bar{d}_1=\frac{z_0+n\alpha a}{6a}-n\tau\alpha,$$

$$\bar{d}_2=\frac{n^2\tau^2\alpha^2}{2}-\frac{n\tau\alpha(z_0+n\alpha a)}{4a}+\frac{(z_0+n\alpha a)^2}{16a^2}-\frac{n\alpha(z_0+n\alpha a)-(1-z_0)z_0'}{20a},$$

$$\bar{d}_3=-\frac{n^3\tau^3\alpha^3}{6}+\frac{n^2\tau^2\alpha^2(z_0+n\alpha a)}{6a}-\frac{7n\tau\alpha(z_0+n\alpha a)^2}{72a^2}+\frac{n\tau\alpha[n\alpha(z_0+n\alpha a)-(1-z_0)z_0']}{15a}+$$

$$+\frac{35(z_0+n\alpha a)^3}{16\cdot 81a^3}-\frac{7(z_0+n\alpha a)}{180a^2}[n\alpha(z_0+n\alpha a)-(1-z_0)z_0']-$$

$$-\frac{(2-n)a^2-n^2\alpha^2(z_0+n\alpha a)+n\alpha(1+\sigma)(1-z_0)z_0'}{90a},$$

$$\bar{d}_4=\frac{n^4\tau^4\alpha^4}{24}-\frac{5n^3\tau^3\alpha^3(z_0+n\alpha a)}{72a}+\frac{5n^2\tau^2\alpha^2(z_0+n\alpha a)^2}{72a^2}-$$

$$-\frac{n^2\tau^2\alpha^2[n\alpha(z_0+n\alpha a)-(1-z_0)z_0']}{24a}-$$

$$-\frac{35n\tau\alpha(z_0+n\alpha a)^3}{16\cdot 81a^2}+\frac{n\tau\alpha(z_0+n\alpha a)[n\alpha(z_0+n\alpha a)-(1-z_0)z_0']}{18a^2}+$$

$$+\frac{n\tau\alpha[(2-n)a^2-n^2\alpha^2(z_0+n\alpha a)+n\alpha(1+\sigma)(1-z_0)z_0']}{72a}+\frac{385}{128\cdot 243}\frac{(z_0+n\alpha a)^4}{a^4}-$$

$$
\begin{aligned}
&- \frac{11(z_0+nxa)^2}{10\cdot 27a^2}[nx(z_0+nxa)-(1-z_0)z_0'] + \frac{[nx(z_0+nxa)-(1-z_0)z_0']^2}{180a^2} - \\
&- \frac{(2-n)a^2 - n^2x^2(z_0-nxa) + nx(1+\sigma)(1-z_0)z_0'}{108a^2}(z_0+nxa) - \\
&- \frac{nx(5n-8)a^2 + a[n^2x^2 + z_0(4n-6)] + z_0n^2x^2 - n^2x^2(1+\sigma+\sigma^2)(1-z_0)z_0'}{504a}.
\end{aligned}
\tag{6.4}
$$

Since z_0' enters into the coefficients $\bar{d}_2$, $\bar{d}_3$, ... then expansion of (6.3), generally speaking, yields a transcendental equation for the determination of ω in terms of τ, z_0 and α. But since the main contribution to the value of the function $\omega(\infty;\ \tau,\ z_0,\ \alpha)$ is introduced by the first terms of the series, then substituting $\bar{d}_2$, ..., $z_0' = 3\,(an\sigma/6)^{1/3}\Gamma^{-1}(\frac{1}{3})$, in the coefficinets, sufficient accuracy can be obtained. Due to the weak dependence of the magnituded of α on σ (see Tables 1 and 3) in (6.3) the value of α may be substituted when $\sigma = 0.7$. Calculations according to Formula (6.3) indicate that for $\alpha = 0$ and $\tau > 0.5$ the first three terms yield a value of $\omega(\infty;\ \tau,\ z_0,\ 0)$ with an error less than 1% (see Ref. 18). When $\alpha = -0.5$ it is necessary to calculate five terms of the series (6.3) in order to obtain an error not exceeding 1%.

By expanding (6.3) and limiting ourselves to the first term, we obtain

$$
\begin{aligned}
\frac{g'(0)}{z'(0)} = \frac{\omega(\infty;\ \sigma,\ z_0,\ \alpha)}{\omega(\infty,\ \mathrm{Sc},\ z_0,\ \alpha)} = \left(\frac{\mathrm{Sc}}{\sigma}\right)^{1/3} \Bigg\{ 1 + \frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)}\left(\frac{6}{an}\right)^{1/3} \Bigg[\frac{z_0}{6a}\left(\frac{1}{\sigma^{1/3}} - \frac{1}{\mathrm{Sc}^{1/3}}\right) + \\
+ \frac{an}{6}\left(\frac{1}{\sigma^{1/3}} - \frac{1}{\mathrm{Sc}^{1/3}}\right) - an\left(\sigma^{1/3} - \mathrm{Sc}^{1/3}\right)\Bigg] + \cdots \Bigg\}.
\end{aligned}
\tag{6.5}
$$

Table 2 gives for comparison the values of $g'(0)/z'(0)$ as calculated according to Formula (6.5). When $|\alpha| \leq 0.5$, Formula (6.5) gives an error not exceeding 4.5% for all z_0. When $\alpha = -1$ the error reaches a significant value and in Formula (6.5) it is necessary to take into account subsequent terms. From the system (5.10), (5.11) it follows that the case of $\alpha = -1$ corresponds to "low-enthalpy"

materials, i.e., materials having $(h_\infty^* - h_0^*)/\mathcal{L} \gg 1$ and a high vaporization rate. For example, when $\sigma = 1$, $n = 1$ $(h_\infty^* - h_0^*)/\mathcal{L} = \omega(\infty, \sigma, z_0, -1)$ when $z_0 = 0.5$ and 47.7 when $z_0 = 0$. Thus, this case is of little practical interest and we will correct formula (6.5) for $\alpha < -0.5$.

Using asymptotic representation of (6.5), Eq. (5.11) may be written in the form

$$\frac{h_\infty^* - h_0^*}{\mathcal{L}(T_0)} = \left(\frac{\sigma}{\mathrm{Sc}}\right)^{1/6} I(\alpha, z_0, \sigma, \mathrm{Sc}) \frac{c_{in} - c_{i\infty}}{c_{i0} - c_{i0}}, \qquad i = 1, \cdots, N-1. \tag{6.6}$$

where

$$I(\alpha, z_0, \sigma, \mathrm{Sc}) = 1 + 0{,}506\left(\frac{\sigma}{\alpha n}\right)^{1/6} \left[\frac{z_0}{6\alpha}\left(\frac{1}{\sigma^{1/6}} - \frac{1}{\mathrm{Sc}^{1/6}}\right) + \frac{\alpha n}{6}\left(\frac{1}{\sigma^{1/6}} - \frac{1}{\mathrm{Sc}^{1/6}}\right) - \right.$$
$$\left. - \alpha n(\sigma^{1/6} - \mathrm{Sc}^{1/6})\right].$$

The System of Eqs. (5.10), (6.6) and (3.8) or (5.10), (6.6) and (3.10) determines a solution to the problem for any σ and Sc.

7. Necessary and Sufficient Conditions for Boiling at the Sublimation Surface

If vaporization is equilibrium, then, as was pointed out in 4, boiling on the sublimation front is attained when there is infinitely of large thermal fluxes from the gas.

We will now examine the conditions of reaching the boiling point during nonequilibrium sublimation (condition 3.10). During boiling, the equilibrium vapor pressure $p(0)$ becomes equal to the external pressure p_{00} and the temperature at the sublimation front reaches a maximum value $T_0 = T_{00}$ for a given external pressure p_{00} at the stagnation point. Assuming then that $p_0 = p_{00}$ and $T_0 = T_{00}$ in (3.10) and eliminating the parameters α and c_0 from (5.10), (5.11) and (3.10), we obtain the necessary and sufficient conditions of boiling or attainment of maximum temperature at the sublimation front for a binary mixture ($c_\infty = 0$, $K = 1$, $\mathrm{Le} = 1$) in the form

$$\varepsilon \frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\left(1 + \frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\frac{M_j}{M_i}\right) = \sigma\omega\left[\infty;\sigma,z_0,-\frac{1}{n\varepsilon\left(1+\frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\frac{M_j}{M_i}\right)}\right], \tag{7.1}$$

the value of c_0 which enters into the parameter

$$z_0 = \frac{h^*_0}{h^*_\infty} = \frac{c_{p_0}T_0}{c_{p_\infty}T_\infty} = \frac{c_{p_i}c_0 + c_{p_j}(1-c_0)}{c_{p_\infty}}\theta_0,$$

being determined from the ratio

$$\frac{c_0}{1-c_0} = \frac{c_{p_j}(T_\infty - T_0)}{\mathscr{L}(T_\infty)}, \tag{7.2}$$

from which it follows that there are always finite values of determining parameters, satisfying condition (7.1) at which boiling ensures. In fact, when the parameter $(h^*_\infty - h^*_{j0})/\mathscr{L}(T_{00})$ is increased from 0 to ∞, the left-hand part of Equality (7.1) increases monotonically from 0 to ∞ while at the same time the right-hand part decreases monotonically from $\sigma\omega(\infty;\sigma, z_0, -(n\varepsilon)^{-1}) > 0$ to $\sigma\omega(\infty;\sigma, z_0, 0) < \infty$. Consequently for given values of the parameters ε, M_j/M_i, c_{p_i}/c_{p_j} and σ there is an unique value of the parameter $(h^*_\infty - h_{j00})/\mathscr{L}$ at which boiling sets in.

If Le $\neq$ 1 then the necessary and sufficient condition of boiling is obtained by eliminating parameter α from the system of two transcendental equations ($c_\infty = 0$, $K = 1$):

$$\begin{gathered}\varepsilon \frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\left[1 + \frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\left(\frac{\mathrm{Sc}}{\sigma}\right)^{1/3} I^{-1}(\alpha, z_0, \sigma, \mathrm{Sc})\right] = \sigma\omega(\infty;\sigma, z_0, \alpha) \\ \alpha n\varepsilon\left[1 + \frac{h^*_\infty - h^*_{j\infty}}{\mathscr{L}(T_\infty)}\left(\frac{\mathrm{Sc}}{\sigma}\right)^{1/3} I^{-1}(\alpha, z_0, \sigma, \mathrm{Sc})\right] + 1 = 0,\end{gathered} \tag{7.3}$$

which has a unique solution.

When the condition (7.1) is satisfied, the velocity and mass velocity of sublimation are found from (4.3) and (4.4) respectively. When Le $\neq$ 1 the velocity and mass velocity of sublimation during boiling may be obtained according to the formulas (if for simplicity

we set I ≈ 1)

$$D = -r_1 \frac{fp_{\infty}}{\rho_{\infty}} \sqrt{\frac{M_i}{2\pi R T_{\infty}}} \left[1 + \frac{c_{p_j}(T_{\infty} - T_{0\infty})}{l(T_{\infty}) + \bar{\rho}_1 \rho_{10}^{-1} \bar{c}_{\bullet} T_{-\infty} [Q(0_{10}) - Q(1)]} \frac{M_j}{M_i} \mathrm{Le}^{2/3} \right],$$

$$\rho_{10} D = -fp_{\infty} \sqrt{\frac{M_i}{2\pi R T_{\infty}}} \left[1 + \frac{c_{p_j}(T_{\infty} - T_{0\infty})}{l(T_{\infty}) + \bar{\rho}_1 \rho_{10}^{-1} \bar{c}_{\bullet} T_{-\infty} [Q(0_{10}) - Q(1)]} \frac{M_j}{M_i} \mathrm{Le}^{2/3} \right]^{-1}.$$

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