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MEMORANDUM
RM-3687-PR
JUNE 1963

THE EFFECT OF VERTICAL AIR MOTION ON ATMOSPHERIC DENSITY DETERMINATION FROM "ROBIN" FLIGHTS

L. C. Kern and R. R. Rapp

PREPARED FOR:

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The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum represents a minor effort under the Project RAND program for general geophysics. It has been noted that data from the ROBINs (rocket balloon instruments) appear inconsistent and that these falling spheres appear to collapse more often than would seem reasonable. This Memorandum may offer a significant insight into why some of the data from the ROBIN experiments are lost and how some may be misconstrued. It should be of general interest to groups studying the characteristics of the mesosphere or operating rockets in and through that region.

ABSTRACT

In order to study possible perturbations of the ROBIN density calculations, a consistent set of velocities was computed between 70 km and 38 km for a standard atmosphere using simplified fall-velocity equations. Consistency was achieved by iterating the calculations until variations in the drag coefficient produced insignificant changes in the fall velocities. Fall velocities were then computed using an equation that included an arbitrary distribution of vertical atmospheric motions. The resulting fall velocities were used to compute air density, assuming, as is done in the ROBIN calculations, that there is no vertical air motion. These computations are used to demonstrate the error in density resulting from the neglect of the vertical-motion term. The effect of the vertical motion on the iterative process used to obtain the drag coefficients and on the process used to check the balloon's sphericity is also discussed.

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I. INTRODUCTION

The ROBIN (Rocket Balloon Instrument) is a spheroid of 1/4-mil Mylar, 1 meter in diameter, having a metalized Mylar corner plate used as a reflector, which when ejected from a rocket at some altitude, inflates and is tracked by radar as it falls. With the information recorded by the radar, the ROBIN becomes a tool for determining atmospheric density. Because of seemingly inconsistent density data from ROBIN experiments (Ref. 1, Fig. 5), this study was undertaken to examine vertical winds as a possible cause of these inconsistencies in the computed densities.

One of the first problems to be solved is that of determining proper drag coefficients. Since drag coefficients are not known as a function of altitude and, in reality, cannot be determined without knowledge of the Mach and Reynolds numbers, it is necessary to assume their values and observe what errors may ensue by back calculation. As a result of his work with the ROBIN, Engler (Ref. 2, p. 93) presents a table of drag coefficients as a function of Mach and Reynolds numbers, which provides the values necessary for the initial assumptions. If the calculations are iterated, variations in drag coefficients will be reduced to the point of producing insignificant changes in the velocity.

The introduction of vertical winds into the problem produces interesting changes in velocity measurements and density calculations. It is not possible, from ROBIN measurements, to distinguish between changes in density and vertical wind perturbations; we will discuss this dependency of calculated-density variations upon vertical-wind magnitudes. Also,

II. DEVELOPMENT OF A CONSISTENT SET OF ρ VS. w VALUES

Using the coordinate system in which altitude, z, is positive downward, the equations of motion of Lally and Leviton[3] become

$$\frac{dw}{dt} = g - a_D \quad ,$$

in which

$$a_D = \frac{C_D \pi r^2 \rho w^2}{2m} \quad ,$$

where g is gravity, C_D is the drag coefficient, r is the radius, m is the mass of the balloon, ρ is the density, and w denotes the balloon's vertical velocity.* Thus

$$\frac{dw}{dt} = g - \frac{KC_D \rho w^2}{2} , \qquad (1)$$

where $K = \pi r^2/m$ is a constant. Since

$$\frac{dw}{dt} = w \frac{dw}{dz} = \frac{d(w^2/2)}{dz},$$

vertical acceleration can be expressed as a function of altitude as

$$\frac{d(w^2/2)}{dz} = g - \frac{KC_D \rho w^2}{2} \quad . \qquad (2)$$

*In this case, the air is assumed to be still, and the balloon's velocity relative to the earth is the same as the balloon's velocity relative to the air.

any variation in drag coefficient will result in a variation in computed density; therefore, as will be shown, vertical winds, drag coefficients, and densities are quite closely related.

Using the simple equation defining the fall rate of a ROBIN as determined by Lally and Leviton[3] and the equations defining a standard atmosphere (Ref. 4, pp. 2,6), we have incorporated vertical winds, as proposed by Woodbridge[5] in the calculation of the balloon's vertical velocity. The hypothesis of the presence of vertical winds will be further illustrated by the use of data from ROBIN flights (Ref. 2, pp. 3-4) to calculate these vertical perturbations.

A discussion of the λ-check as proposed by Engler (Ref. 2, pp. 11-14) is included to point out the difficulty of differentiating between fall rates resulting from a vertical wind and those resulting from malfunction of the balloon.

The IBM-7090 was used to perform the numerical calculations to determine the balloon's velocity. An appendix is included to illustrate the mechanisms of the FORTRAN program.

By defining $q = w^2/2$, Eq. (2) becomes

$$\frac{dq}{dz} = g - KC_D\rho q \quad , \qquad (3)$$

which, by numerical integration (see Appendix), yields the vertical velocity of the balloon.

Standard density and temperature are calculated by use of a consistent set of equations, i.e., those applying to the standard atmosphere,[4] which are (in a coordinate system with z positive downward) as follows:

$$\frac{dP}{dz} = \rho g$$

$$P = \rho RT \qquad (4)$$

$$T = T_b - \gamma(z - z_b)$$

with mks units where

g = gravity (m sec^{-2}),

ρ = density (kg m^{-3}),

P = atmospheric pressure (kg $m^{-1} sec^{-2}$),

R = universal gas constant, 2.87×10^2 joules $(^oK)^{-1} kg^{-1}$,

T = temperature (^oK),

T_b = value of T at altitude z_b,

γ = temperature gradient (^oK m^{-1}).

Using Eqs. (4) to solve for density, ρ, we find

$$\frac{dP}{dz} = \rho g \quad ,$$

$$\frac{dP}{dz} = RT\frac{d\rho}{dz} + R\rho\frac{dT}{dz} \quad ,$$

$$\frac{dT}{dz} = -\gamma \ .$$

So,

$$RT\frac{d\rho}{dz} - R\rho\gamma - \rho g = 0,$$

and

$$\frac{1}{\rho}\frac{d\rho}{dz} = \frac{g + R\gamma}{RT} \quad ,$$

or

$$\frac{d\ell n\rho}{dz} = \frac{g + R\gamma}{R[T_b - \gamma(z - z_b)]} \quad . \qquad (5)$$

Upon integration of Eq. (5) from z_b to z, where z_b is the altitude at the base of the interval,

$$\rho = \rho_b\left[\frac{T_b}{T_b - \gamma(z-z_b)}\right]^{(1 + g/R\gamma)} \qquad (6)$$

for $\gamma \neq 0$.

For $\gamma = 0$, Eq. (5) becomes

$$\frac{d\ell n\rho}{dz} = \frac{g}{RT_b} \quad ,$$

yielding

$$\rho = \rho_b \exp\left[\frac{g(z - z_b)}{RT_b}\right] \quad . \qquad (7)$$

The equation used to calculate temperature is simply

$$T = T_b - \gamma(z - z_b)$$

as noted above.

Drag coefficients present a problem in these calculations for velocity since they are not expressible as a function of altitude. Because the drag coefficients are actually unknown until a velocity is determined and Mach and Reynolds numbers can be found, it is necessary to assume a drag coefficient, C_D, to compute a velocity, then by computing the corresponding Mach and Reynolds numbers, determine a new drag coefficient, C_D', to be compared with the original assumption as a check.

We made the original, arbitrary assumption of a set of C_D values, using the drag coefficients of Engler (Ref. 2, p. 93) as a basis; these are shown in Fig. 1 as C_D. With these, we determined corresponding velocities and Mach and Reynolds numbers, as discussed above, to obtain a more refined drag coefficient for the problem; the results of this process to obtain a new drag coefficient, C_D', are also shown in Fig. 1. Then C_D' is the drag-coefficient value used in all subsequent calculations.

Upon comparison of the original and the refined drag coefficients, it can be seen that C_D' varies from C_D by 0% to 2.5%. Thus, by refining the drag coefficient values only once, a more accurate approximation of velocity can be achieved.

Let us consider the results of the refined drag coefficients on the subsequent velocity calculations. A range of deviation of drag-coefficient values of 0% to 2.5% results in a range of deviation for velocity

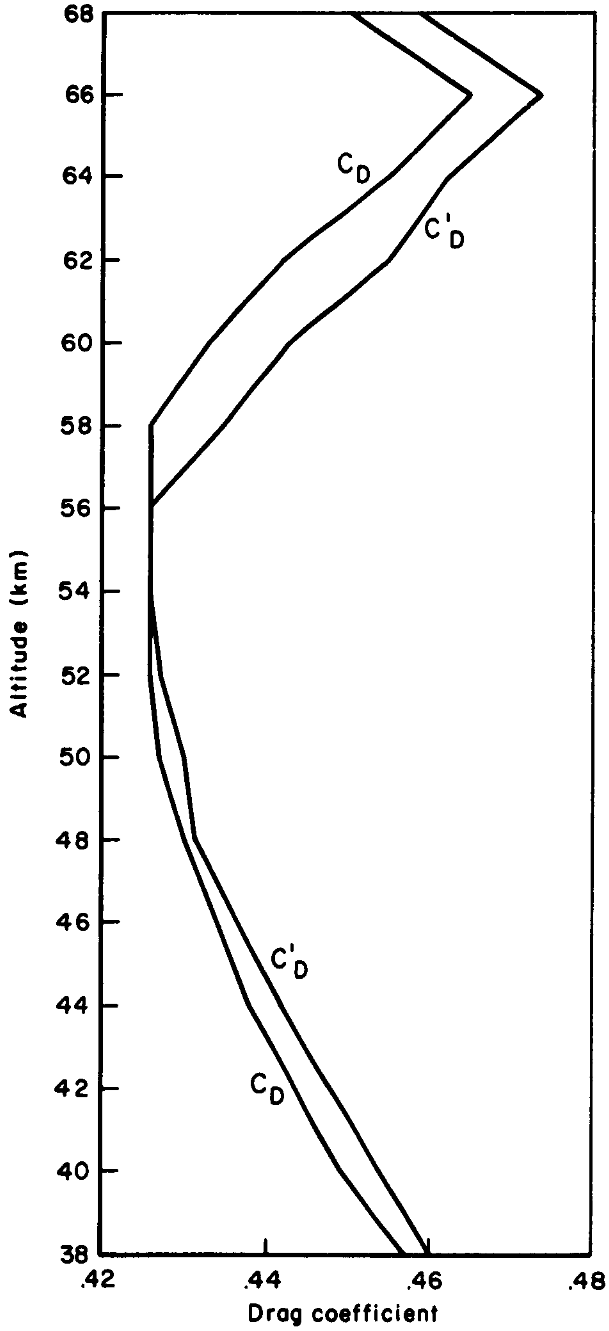

Fig. 1—Drag coefficient originally assumed and refined drag coefficient

calculations of 0% to 1.3%. So it can be seen that absolute accuracy in drag-coefficient values is not required for approximations of velocity within a minimal range of error in measurement.

III. INTRODUCTION OF VERTICAL WIND

Using the vertical-wind data plotted by Woodbridge[5] as a basis,* we constructed a smooth arbitrary curve over the 30-km interval between 40 km and 70 km in which the initial and terminal vertical wind velocity, defined as u, was zero. From this curve, values of u, selected at given intervals as a function of altitude, were entered into the calculations for the vertical velocity of the balloon, w.

Upon consideration of a vertical wind not equal to zero, the equation of motion (1) becomes, after replacing w with (w-u) in the drag term,

$$\frac{dw}{dt} = g - KC_D'\rho[(w-u)^2/2] . \qquad (8)$$

From Eq. (3), we find

$$\frac{dq}{dz} = g - KC_D'\rho\left(q - u\sqrt{2q} + \frac{u^2}{2}\right) , \qquad (9)$$

where z is positive downward; the drag force is now proportional to the relative vertical wind, and the balloon's vertical acceleration is relative to the earth. Since, over each subinterval of integration, g, K, C_D', ρ, and u are given as constant, this equation is easily solved for q, and thus, w is determined.

The vertical velocity of the balloon as determined by Eq. (9) is now a function of a vertical wind, u, as well as being a function

*Although the values from Woodbridge are used as a basis for defining a possible vertical wind velocity distribution, we do not mean to imply that his values are accepted as correct.

of altitude, z. We shall designate this velocity as w_u and show that when it is used to calculate density according to Eq. (12), below, in an atmosphere containing a vertical wind, an erroneous density, ρ_u, results.

By assuming that altitude is positive downward and taking the differential equation

$$\frac{d(w^2/2)}{dz} = g - KC_D'\rho(w^2/2), \tag{10}$$

by defining the balloon's motion with no vertical wind, by making the substitution for $w^2/2$, and by solving for ρ by taking finite differences, we find for Eq. (10)

$$\rho = \frac{1}{KC_D' q}\left[g + \frac{\Delta q}{\Delta z}\right] . \tag{11}$$

When $u \neq 0$, a similar form can be found:

$$\rho_{u_i} = \frac{1}{KC_{D_i}' q_i}\left[g_i + \frac{(q_{i-1} - q_{i+1})}{\Delta z}\right] , \tag{12}$$

where i denotes the subinterval of evaluation. Using this method of finite differences, the balloon's vertical velocity, including a vertical wind, determines a density, ρ_u. It should be mentioned in passing that, even though errors are small, there is some error in the results when the finite-difference method of back calculation is used, as compared with the results supplied by the IBM-7090 computer.

The results of Eqs. (9) and (12), i.e., the results of the calculations for velocity and density assuming nonzero vertical air motion, can be seen in Fig. 2. Figure 2a illustrates the arbitrarily

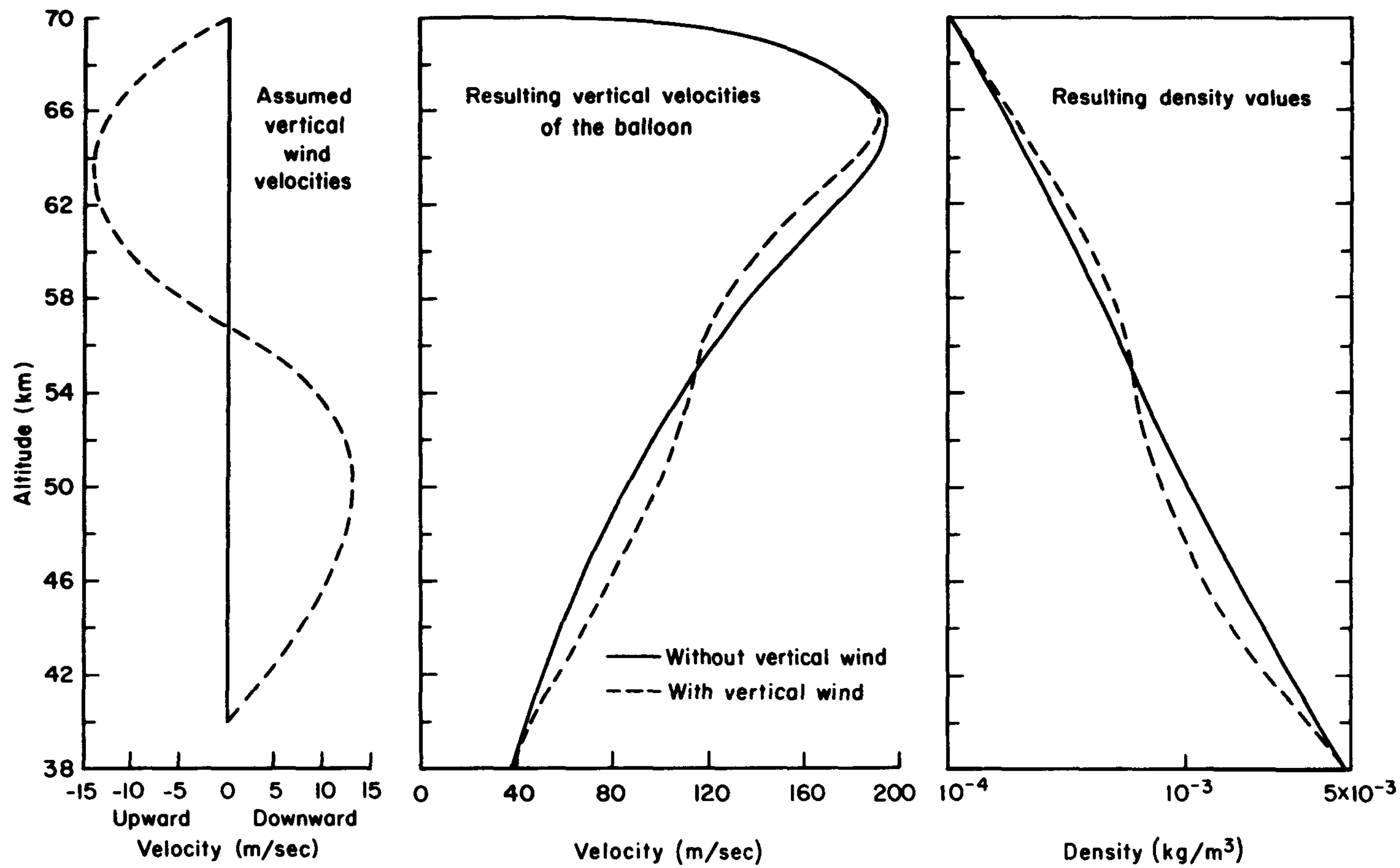

Fig. 2 — The effect of assumed vertical wind velocities on velocity of falling ROBIN, and the resulting miscalculation of density

chosen variation in the vertical wind; Fig. 2b shows the effect of this vertical air motion on the vertical motion of the balloon; Fig. 2c gives the erroneous density corresponding to the shown velocity variation which, in turn, is a result of the given vertical wind. As should be expected, an upward air thrust results in a lesser balloon velocity and a greater calculated density; downward air motion results in the opposite — greater balloon velocity and a smaller calculated density.

IV. CALCULATION OF PERCENTAGE ERROR DUE TO VERTICAL WINDS

Since, when a nonzero vertical wind exists, an erroneous calculated density directly results as discussed in Section III, it is necessary to be able to make an approximation as to what magnitude relative error should be expected. Here we will develop an expression for the approximation of relative errors in density calculations as a function of a vertical air motion, u, and thus, show the dependence of density calculations on this vertical motion. Equation (1), u = 0, becomes

$$\frac{d(w^2/2)}{dz} = g - \frac{KC_D' \rho_{std} w^2}{2} \quad ,$$

and Eq. (8), $u \neq 0$, becomes

$$\frac{d(w^2/2)}{dz} = g - \frac{KC_D' \rho_u (w-u)^2}{2} \quad .$$

Therefore,

$$\frac{2(\frac{dw^2/2}{dz} - g)/(-KC_D')}{2(\frac{dw^2/2}{dz} - g)/(-KC_D')} = \frac{\rho_{std} w^2}{\rho_u (w-u)^2} \quad ,$$

or

$$1 = \frac{\rho_{std} w^2}{\rho_u (w-u)^2} \quad ,$$

and

$$\frac{\rho_{std}}{\rho_u} = \frac{(w-u)^2}{w^2} \quad . \qquad (13)$$

By expanding and simplifying Eq. (13),

$$\frac{\rho_{std}}{\rho_u} = 1 - \frac{2u}{w} + \frac{u^2}{w^2} \quad ,$$

and

$$\frac{\rho_u - \rho_{std}}{\rho_u} = \frac{2u}{w} - (u/w)^2 \quad . \tag{14}$$

Therefore, the relative error in the determined density may be expressed as Eq. (14). Since $u \ll w$, the term (u^2/w^2) in Eq. (14) was omitted; its contribution to the determination of relative error in density calculation is negligible. The approximation of the relative error in the calculated density is made primarily by $(2u/w)$. Thus

$$\frac{\rho_u - \rho_{std}}{\rho_u} \approx \frac{2u}{w} \quad .$$

Let us now assume that we should expect a density-calculation error of 2% to 10%; we wish to know the vertical wind velocity that would cause a relative error in this range. Since the relative error in density calculation is approximated by $2u/w$, we can say, for 2% error in density, that $0.02 \approx 2u/w$, or $u \approx 0.01w$; for a 10% error in density, $u \approx 0.05w$. Figure 3 illustrates the balloon's vertical velocity with no vertical wind perturbation and the vertical winds that will cause 2% and 10% density-calculation errors. It can be seen here that, in the hypothesized model, density-determination accuracy and vertical-wind magnitude are inseparable.

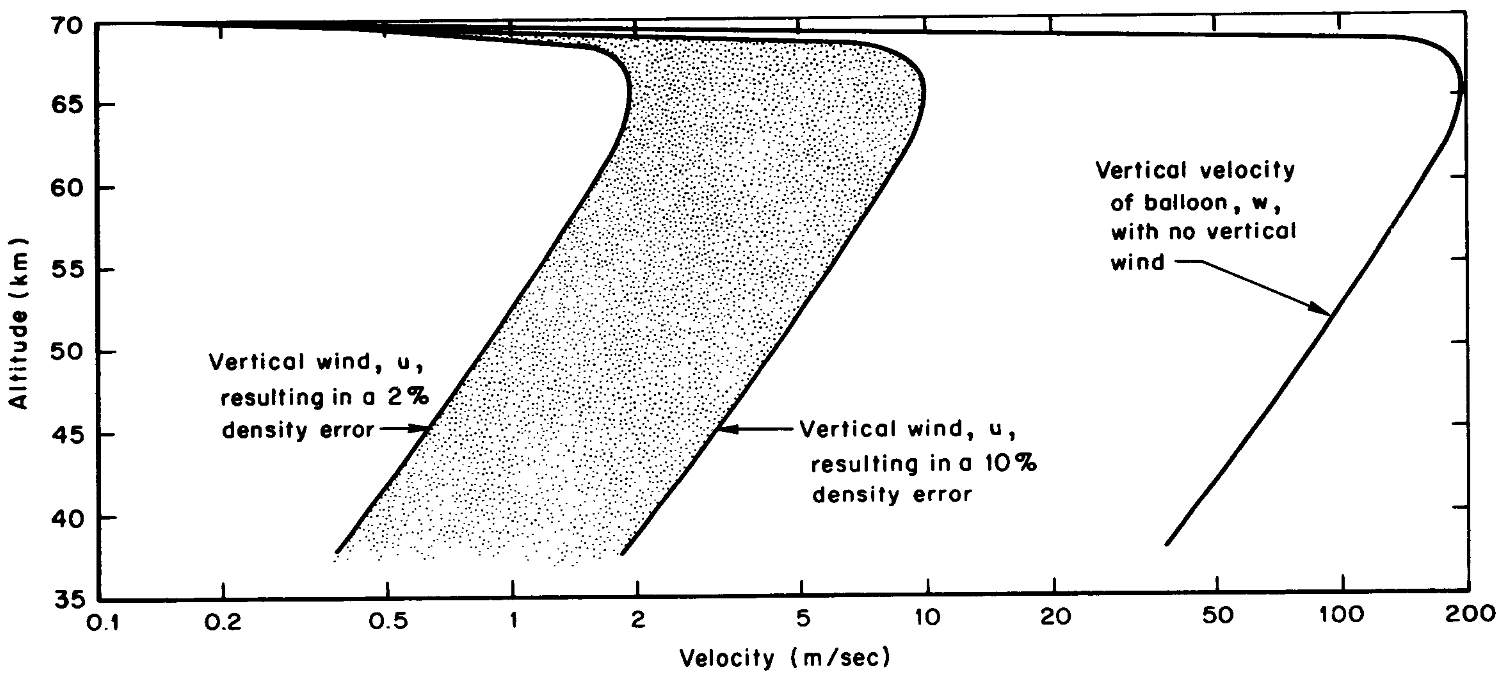

Fig. 3 — Vertical winds causing 2% to 10% relative error in density calculations and velocity of balloon with no vertical wind

V. THE EFFECT OF VERTICAL WINDS ON THE ITERATION FORMULA

Vertical winds will introduce errors into the determination of drag coefficients which will, in turn, cause an error in calculated density additional to that caused by vertical wind directly. The procedure below was followed to show these variations in drag-coefficient and density calculations. With the values for C_D' and ρ and the resulting computer calculations for velocity, w_u, Mach and Reynolds numbers were determined from

$$M = \frac{w_u}{KT^{\frac{1}{2}}} \quad , \qquad R_e = \frac{2\rho w_u r(T+S)}{\beta^{3/2}} \quad ,$$

where $K = 20.05$ m $sec^{-1}(^oK)^{-\frac{1}{2}}$, $\beta = 1.458 \times 10^{-6}$ kg m^{-1} $sec^{-1}(^oK)^{-\frac{1}{2}}$, $S = 110.4^oK$, T is the atmospheric temperature, and r is the radius of the balloon; M and R_e then determine a drag coefficient value, C_D'', by use of Engler's table (Ref. 2, p. 93). With C_D'', a new density value, ρ' is found from

$$\rho' = \frac{1}{KC_D'' \, q_u} \left[g + \Delta q_u/\Delta z\right] \quad ,$$

where

$$q_u = w_u^2/2 \quad ,$$

which is similar to Eq. (11). As can be seen, the variation in the density is inversely related to the variation in the drag coefficient. In Table 1, a few examples of this are shown. Whenever indeterminable vertical winds exist there will be errors in drag-coefficient determinations from the ROBIN data and, therefore, errors in the relative density. Thus it is evident that in the presence of vertical winds, density measurements will not necessarily converge to a correct value.

Table 1

VARIATION OF DRAG-COEFFICIENT AND DENSITY CALCULATIONS

C_D'	C_D''	ρ (kg m^{-3})	ρ' (kg m^{-3})	Altitude km
.473	.468	1.657×10^{-4}	1.824×10^{-4}	66
.455	.446	2.717	2.771	62
.435	.426	4.325	4.416	58
.426	.427	6.707×10^{-4}	6.694×10^{-4}	54
.430	.434	1.045×10^{-3}	1.036×10^{-3}	50
.437	.443	1.671×10^{-3}	1.648×10^{-3}	46

VI. THE EFFECT OF VERTICAL WINDS ON THE λ-CHECK

An approximation of the λ-check is used by Engler (Ref. 2, pp. 11-14) as a method for determining whether or not the balloon is spherical. With a properly inflated balloon and no vertical wind, the value of a calculated parameter, λ, can be shown to be limited to a specific range. The existence of a vertical wind acting on a ROBIN might produce the same effect during a λ-check as a balloon malfunction.

The value λ in Engler's method is approximated by λ_o, which is defined by

$$\lambda_o = \frac{2(dw/dt)}{(dz/dt)^2} = \frac{2\ddot{z}}{(\dot{z})^2}$$

At the maxima and minima of dw/dt, λ_o is computed by Engler, and if $\lambda_o > 2 \times 10^{-4}\ m^{-1}$, or $\lambda_o < 0.5 \times 10^{-4}\ m^{-1}$, it is assumed that the balloon is not rigid, and density calculations stop.

By definition,

$$\ddot{z} = \frac{dw}{dt} = w\frac{dw}{dz} = \frac{d(w^2/2)}{dz} \quad .$$

Therefore, since $\dot{z} = w$,

$$\ddot{z} = g - K\rho C_D \left[(\dot{z})^2/2\right] ,$$

and

$$(\dot{z})^2/2 = q \quad ;$$

so,

$$\lambda_o = \frac{\ddot{z}}{q} = \frac{g - K\rho C_D q}{q} \quad ,$$

or

$$\lambda_o = g/q - K\rho C_D$$

Table 2 below shows the results of the λ-check on the maxima and minima of the acceleration in our model ROBIN experiment as determined by the IBM-7090.

Table 2

RESULTS OF THE λ-CHECK ON THE ACCELERATION OF A ROBIN RESPONDING TO A NONZERO VERTICAL AIR MOTION

Acceleration	u (m/sec)	w_u (m/sec)	g (m/sec^2)	C'_D	ρ (kg/m^3)	q (m/sec)^2	λ_o (m^{-1})
Max	-5.	117.14	9.63	.426	5.402×10^{-4}	6.861×10^{3}	$.23 \times 10^{-4}$
Min	+4.	83.69	9.64	.426	5.705×10^{-4}	7.004×10^{3}	$-.82 \times 10^{-4}$

As can be seen, the λ-check at both the maximum and minimum accelerations yields a $\lambda_o < 0.5 \times 10^{-4} m^{-1}$, and density measurements should, according to the criteria, cease. Lambda measurements at the maximum and minimum accelerations would normally be considered to be valid since, at this point, the ROBIN has passed its peak velocity. Here we can see an example of a vertical wind being mistaken for a malfunction in the balloon.

VII. CONCLUSIONS

The process of deducing the atmospheric density from the actual tracking of ROBIN spheres is, of necessity, based on the assumption of zero vertical wind. By constructing a consistent set of hypothetical vertical sphere velocities based on a standard atmosphere, and a given set of vertical winds, we have been able to illustrate the effect of vertical winds on the computed densities. Our results indicate

(1) that vertical winds of reasonable velocity can directly increase the errors of ROBIN-derived densities beyond the published error;

(2) that these vertical winds affect the iteration technique for determining drag coefficients to a degree that casts doubt on the validity of the drag estimates;

(3) that they might also cause accelerations that could be mistaken for malfunctions of the balloons.

These indications suggest that the falling-sphere technique needs a critical re-examination. Perhaps a relaxation of the stated accuracy and stated resolution of the measurements of the spheres would produce more useful and more nearly consistent results.

Appendix

COMPUTER PROGRAM

The FORTRAN program discussed here was written for the IBM-7090 to provide a step-wise solution to the first-order, differential equation, Eq. (1), defining the fall rate of the ROBIN. Although a block diagram is included (Fig. 4), some discussion is necessary.

The solutions were obtained by means of a Runge-Kutta fourth-order method, modified by Blum.[6] Conte and Titus[7] discuss the integration technique that is incorporated in RAND routine (FORTRAN) RS-X013; it is also available in the SQUOZE deck as X004. The calling sequence for the set-up of the integration routine, which must be used before the integration is started and once per solution, is

CALL INT (T, N, K1, A2, A3, A4, A5, A6, A7)

where T, a region of at least 12N+3, is defined as follows:

T(1): set up by CALL INT

T(2): value of independent variable, z

T(3): value of step-size, Δz

T(4): value of dependent variable, q, supplied initially by user

T(5): value of derivative, dq/dz, stored by DAUX.

The calling sequence for integrating one step is CALL INTM and requires no arguments. Over each increment of integration, Δz, INTM calls up DAUX four times to evaluate the derivative dq/dz. The parameters of the integration routine are the following:

N : number of equations

K1 : option word for determination of integration mode (0 applies to the Adams-Moulton variable step-size mode, 1 applies to the

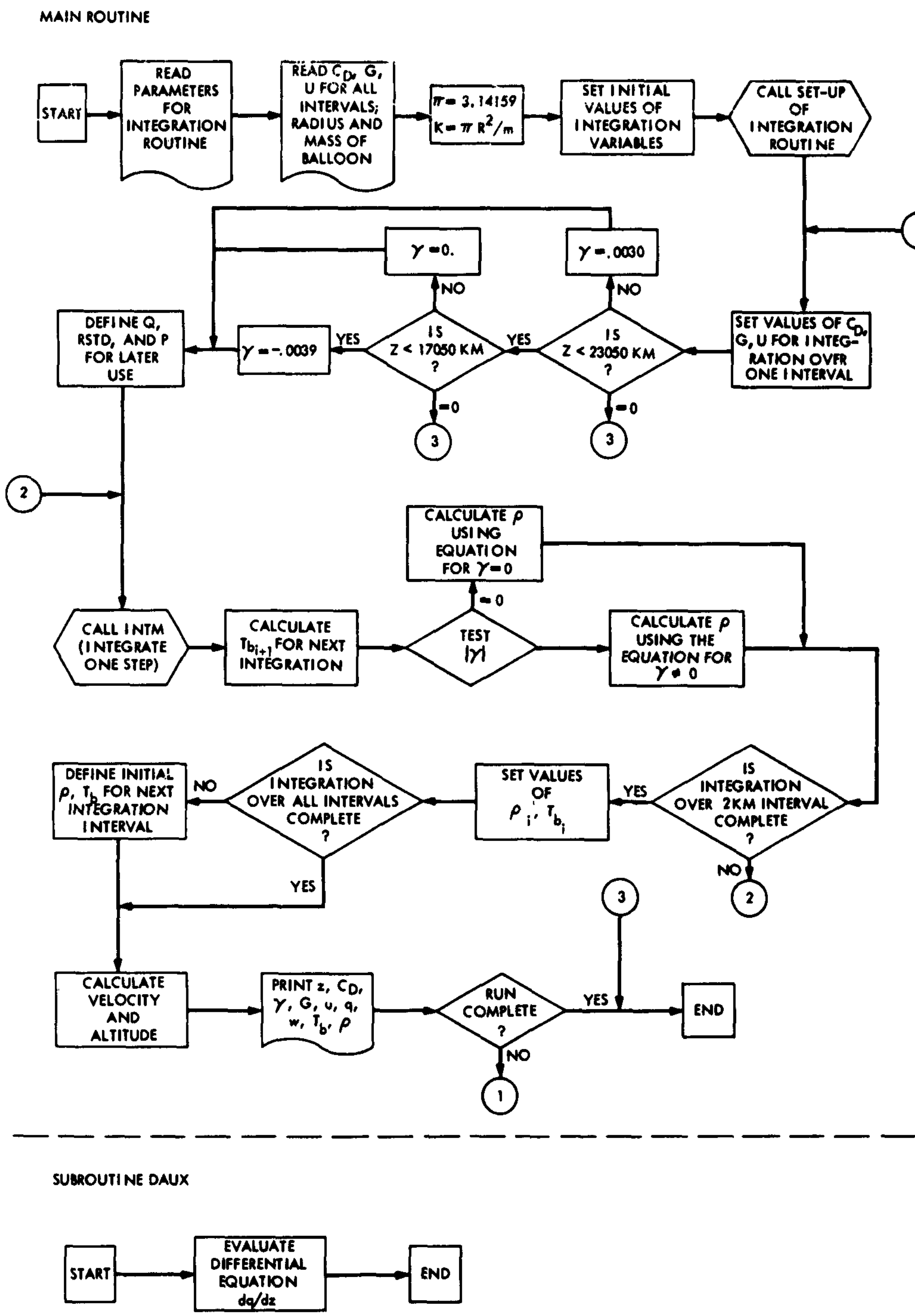

Fig. 4 — Block diagram of FORTRAN program to study ROBIN fall speeds

Runge-Kutta-Blum mode used for this problem, and 2 applies to the Adams-Moulton fixed step-size mode.)

A2, A3, A4, A5, A6, A7 :

apply only to the Adams-Moulton variable step-size mode.

A sample (Fig. 5) of the input formats, along with a table that relates FORTRAN nomenclature to the text, is included to illustrate the manner in which the data are read, as listed in the main routine and subroutine (Fig. 6). A sample of the output format is also included (Fig. 7).

A comment should be made regarding the technique used in the FORTRAN program to determine the fall speeds of the ROBIN. Each given 2-km interval is divided into twenty 100-meter subintervals. Values for C_D, drag coefficient, g, gravity, and u, vertical wind, are read as data and held constant over each given 2-km interval (controlled by I); ρ, density, and T_b, temperature at the base of an interval, having only initial values supplied, are calculated over every 100-meter subinterval (controlled by K). Values of ρ and T_b are assumed to change very little over 100 meters and therefore, are calculated outside of DAUX and held constant over each subinterval, or increment, of evaluation.

FN DATA FORM

± INTEGER		± INTEGER		± INTEGER		± INTEGER		± INTEGER		± INTEGER		I. D.
± FRACTION	± EXP.	± FRACTION	± EXP.	± FRACTION	± EXP.	± FRACTION	± EXP.	± FRACTION	± EXP.	± FRACTION	± EXP.	
01–09	10–12	13–21	22–24	25–33	34–36	37–45	46–48	49–57	58–60	61–69	70–72	73–80
	1		16									
0.		0.		0.		0.		0.		0.		
.450		9.59		0.								70-68
.465		9.60		-6.								68-66
.455		9.61		-12.								66-64
.444		9.61		-14.								64-62
.433		9.62		-13.								62-60
.426		9.62		-10.								60-58
.426		9.63		-5.								58-56
.426		9.64		4.								56-54
.426		9.64		9.								54-52
.427		9.65		12.								52-50
.430		9.65		13.								50-48
.434		9.65		11.								48-46
.438		9.67		10.								46-44
.444		9.67		6.								44-42
.449		9.68		2.								42-40
.457		9.68		0.								40-38
.5		.130										

Fig. 5 — Sample imput format

Table 3

DEFINITION OF FORTRAN NOMENCLATURE

FORTRAN Program	Text
BMASS	m, mass of the balloon
CD	C_D, drag coefficient
CONST	K, defined as $\pi r^2/m$
DZ	ΔZ, increment of altitude
G	g, acceleration due to gravity
GAMMAX	γ, temperature gradient
PI	π
R	r, radius of the balloon
RHO	ρ, density
RSTD	R, universal constant
TB	T_b, temperature at base of given interval
QZ	q, defined as $w^2/2$
VWIND	u, vertical wind
WZ	w, velocity of the balloon
Z	z, altitude

```
      COMMON T,N,I,K,P,CONST,RHO,Z,CDX,GX,GAMMAX,TBX,VWINDX,DZ,Q
      DIMENSION T(500),RHO(350),CD(16),G(16),VWIND(16),TB(16),GAMMA(16)
     1,GZ(350),WZ(16),Z(16)
C INPUT CODE NUMBER FOR FORTRAN INTEGRATION ROUTINE (K1), AND THE NUMBER
C OF INTERVALS OF INTEGRATION (IT).
      READ 1, K1, IT
    1 FORMAT(2I12)
C PRINT CODE NUMBER, NUMBER OF INTERVALS OF INTEGRATION, AND INTEGRATION
C MODE USED.
      PRINT 10, K1, IT
   10 FORMAT(2I10)
      PRINT 20
   20 FORMAT(24HORUNGE-KUTTA INTEGRATION)
    2 FORMAT(6E12.8)
    3 FORMAT(3E12.8)
C INPUT AND PRINT INTEGRATION ROUTINE FORMAT.
      READ 2, A2, A3, A4, A5, A6, A7
      PRINT 30, A2,A3,A4,A5,A6,A7
   30 FORMAT(100H        EBAR               M                  A
     1     DZMAX                DZMIN               BETA/1H 6E18.6)
      PRINT 40
   40 FORMAT (8HODZ=2 KM)
C INPUT DRAG COEFFICIENT, GRAVITY, AND VERTICAL WIND WHICH WILL BE CON-
C STANT OVER EACH INTERVAL BUT WILL CHANGE EVERY 2KM.
C ALTITUDE IS POSITIVE DOWNWARD.
      READ 3, (CD(J),G(J),VWIND(J), J=1,IT)
C INPUT THE MASS AND RADIUS OF THE ROBIN. CALCULATE CONSTANT, K. PRINT
C HEADING AND OUTPUT.
      READ 3, R, BMASS
        PI=3.14159
      CONST=PI*(R**2)/BMASS
      PRINT 50, R,BMASS,CONST
   50 FORMAT(65H0        R                     BMASS                CONST
     1              /3E20.8)
C SET UP INITIAL CONSTANTS AND CONDITIONS FOR INTEGRATION.
      QZ(1)=0.
      DZ =100.
      RHO(1)=9.906/(10.**5)
      TB(1)=216.36
      T(2)=0.
      T(3)=DZ
      T(4)=QZ(1)
      T(5)=9.59
C CALL SET-UP OF INTEGRATION ROUTINE
      CALL INT(T,1,K1,A2,A3,A4,A5,A6,A7)
C BEGIN DO LOOP CONTROLLING NUMBER OF INTERVALS OF INTEGRATION.
           DO 666  I=1,IT
           I=I
C SET VALUES OF DRAG, GRAVITY, AND VERTICAL WIND FOR 2KM INTERVAL.
           CDX=CD(I)
           GX=G(I)
           VWINDX=VWIND(I)
C TEST ALTITUDE TO DETERMINE VALUE OF TEMPERATURE GRADIENT, GAMMAX.
           IF(T(2)-23050.) 51,667,52
   51      IF(17050.-T(2)) 53,667,52
   52       GAMMAX=-.0039
             GO TO 55
   53       GAMMAX=0.
             GO TO 55
   54       GAMMAX=.0030
   55      CONTINUE
C DEFINE Q, RSTD(UNIVERSAL CONSTANT), P TO BE USED LATER.
            Q=ABSF(GAMMAX)
            RSTD=287.0
            P=1.+(GX/(RSTD*GAMMAX))
```

Fig. 6 — Program listing

```
PRINT HEADING.
         PRINT 60
 60      FORMAT(60H1           RHO                    TB                    GZ
   1          )
BEGIN DO LOOP WHICH CONTROLS CALCULATION OF DENSITY, TEMPERATURE, AND
((W**2)/2) OVER 2KM INTERVAL.
              DO 66  K=1,20
              K=K
INTEGRATE ONE ALTITUDE INCREMENT.
              CALL INTM
              PRINT 70, RHO(K),TB(K),T(4)
 70           FORMAT(1H03E20.8)
CALCULATE TB FOR NEXT INTERVAL.
              TB(K+1)=TB(K)-(GAMMAX*DZ)
TEST GAMMAX AND CALCULATE DENSITY ACCORDINGLY.
              IF(Q) 667,61,62
 61           RHO(K+1)=RHO(K)*(EXPF((GX*DZ)/(RSTD*TB(K))))
               GO TO 65
 62           RHO(K+1)=RHO(K)*(TB(K)/(TB(K+1)))**P
 65           CONTINUE
 66           CONTINUE
SET CALCULATED VALUES OF DENSITY AND TEMPERATURE.
          RHOX=RHO(20)
          TBX=TB(20)
TEST NUMBER OF INTEGRATED INTERVALS. IF INCOMPLETE, CONTINUE. IF COM-
PLETE, PRINT FINAL ANSWERS AND STOP.
         IF(I-IT) 67,75,75
 67       RHO(1)=RHO(21)
          TB(1)=TB(21)
 75      CONTINUE
         WZ(I)=SQRTF(2.*T(4))
         Z(I)=7.*(10.**4)-T(2)
         PRINT 80
 80      FORMAT(/////)
         PRINT 90
 90      FORMAT(1H0,10X,1HZ,18X,2HCD,16X,6HGAMMA,17X,1HG,17X,5HVWIND/
   1            1H ,9X,2HQZ,18X,2HWZ,18X,2HTB,18X,3HRHO)
         PRINT 100,Z(I),CDX,GAMMAX,GX,VWINDX,T(4),WZ(I),TBX,RHOX
100      FORMAT(1H05E20.8/(1H 4E20.8))
666      CONTINUE
    GO TO 1000
667 PRINT 668
668 FORMAT(19H1SOMETHING IS WRONG)
1000 CONTINUE
    CALL EXIT
    END
```

```
                        SUBROUTINE DAUX

    COMMON T,N,I,K,P,CONST,RHO,Z,CDX,GX,GAMMAX,TBX,VWINDX,DZ,Q
    DIMENSION T(500),RHO(350),CD(16),G(16),VWIND(16),TB(16),GAMMA(16)
   1,QZ(350),WZ(16),Z(16)
ROBIN'S EQUATION OF MOTION INCLUDING VERTICAL WIND TERM.
      T(5)=GX-(CONST*RHO(K)*CDX)*(T(4)-VWINDX*(SQRTF(2.*T(4)))
   1        +((VWINDX)**2)/2.)
    RETURN
    END
```

Fig. 6 (cont) — Program listing

```
      EXECUTION
      1         16

RUNGE-KUTTA INTEGRATION
      EBAR      M      A       DZMAX      DZMIN      BET
      0.        0.     0.      0.         0.         0.

DZ=2 KM

        R                     BMASS                  CONST
     0.50000000E 00        0.13000000E-00         0.60415192E 01
```

RHO	TB	QZ
0.99059998E-04	0.21636000E 03	0.78807286E 03
0.10041935E-03	0.21675000E 03	0.17028362E 04
0.10179487E-03	0.21714000E 03	0.25847488E 04
0.10318671E-03	0.21752999E 03	0.34341113E 04
0.10459504E-03	0.21791999E 03	0.42512459E 04
0.10602002E-03	0.21830999E 03	0.50364258E 04
0.10746182E-03	0.21869999E 03	0.57902249E 04
0.10892062E-03	0.21908999E 03	0.65128166E 04
0.11039656E-03	0.21947999E 03	0.72046741E 04
0.11188982E-03	0.21986999E 03	0.78662192E 04
0.11340059E-03	0.22025999E 03	0.84978916E 04
0.11492903E-03	0.22064999E 03	0.91001490E 04
0.11647531E-03	0.22103999E 03	0.96734650E 04
0.11803962E-03	0.22142999E 03	0.10218330E 05
0.11962213E-03	0.22181999E 03	0.10735248E 05
0.12122301E-03	0.22220998E 03	0.11224740E 05
0.12284246E-03	0.22259998E 03	0.11687339E 05
0.12448065E-03	0.22298998E 03	0.12123590E 05
0.12613776E-03	0.22337998E 03	0.12534051E 05
0.12781399E-03	0.22376999E 03	0.12919292E 05

Z QZ	CD WZ	GAMMA, TB	G RHO	VWIND
0.68000000E 05 0.12919292E 05	0.58499999E 00 0.16074385E 03	-0.38599999E-02 0.22376999E 03	0.95899999E 01 0.12781399E-03	0.

Fig. 7 — Sample output format

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